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Publication date
2002

Published in
Physical Review B

Citation for published version (APA):
Separation of spin and charge in paired spin-singlet quantum Hall states

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Received 8 October 2001; published 2 January 2002

We propose a series of paired spin-singlet quantum Hall states, which exhibit a separation of spin and charge degrees of freedom. The fundamental excitations over these states, which have filling fraction \( \nu = 2/(2m + 1) \) with \( m \) an odd integer, are spinons (spin-\( \frac{1}{2} \) and charge zero) or fractional holons (charge \( \pm 1/(2m+1) \) and spin zero). The braid statistics of these excitations are non-Abelian. The mechanism for the separation of spin and charge in these states is topological: spin and charge excitations are liberated by binding to a vortex in a \( p \)-wave pairing condensate. We briefly discuss related, Abelian spin-singlet states and possible transitions.

DOI: 10.1103/PhysRevB.65.041305 PACS number(s): 73.43.-f, 71.10.Pm

Strongly correlated electrons in low dimensional systems are known to exhibit physical phenomena that are surprising and, at first sight, counterintuitive. Among these is the remarkable phenomenon of quantum number fractionalization: elementary excitations in strongly interacting many-electron systems can have quantum numbers (for spin and charge) that are fractions of those of the electron. This fractionalization can take the form of a separation of spin and charge, or of a fractionalization of the electric charge of the electron.

In one spatial dimension, the separation of spin and charge is well understood. It is seen in explicit solutions of specific integrable model systems (Hubbard and supersymmetric \( t-J \) models). The general framework of the Luttinger Liquid has made it clear that in \( 1+1 \) dimensions the separation of spin and charge is a generic feature, which does not require any fine tuning of the interactions among the electrons.

In spatial dimensions \( D = 2 \) or higher, spin and charge tend to confine and a separation of the two is only possible under very special conditions. It has been proposed that the key feature underlying the anomalous behavior of the cuprate high-\( T_c \) materials is precisely a separation of spin and charge, and concrete scenarios, based on \( \mathbb{Z}_2 \) or \( U(1) \) gauge theories, have been put forward.

In this paper, we focus on the quantum Hall (QH) regime, which is relevant for two-dimensional (2D) electrons in strong magnetic fields, and for rotating Bose-Einstein condensates. In particular, we discuss the separation of spin and charge in the QH regime. Specifically, we propose a series of paired spin-singlet QH states, of filling fraction \( \nu = 2/(2m + 1) \), which are generalizations of the Moore-Read or pfaffian states for spin polarized electrons. The fundamental excitations over these states are spinons (with spin-\( \frac{1}{2} \) and zero charge) and holons (with zero spin and fractional charge \( \pm 1/(2m+1) \), in units of the charge of the electron). The braid statistics of these excitations are non-Abelian, and thereby the paired spin-singlet states fall in the category of "non-Abelian QH states."

It is important to stress that the more conventional Abelian spin-singlet QH states [such as the Halperin states with label \( (m + 1,m + 1,m) \), see below] do not exhibit a separation of spin and charge. The excitations over such states are conveniently analyzed in terms of a "spin-charge decomposi-
below we display reduced QH wave functions \( \Psi(x) \), which are related to the actual wave functions \( \Psi(x) \) via \( \Psi(x) = \Psi(x) \exp(-\Sigma_i (x_i^2 l^2)) \) with \( x_i = z_i^\ast, z_i^\dagger \) and \( l = \sqrt{\hbar eB} \) the magnetic length. Hierarchies of more general (Abelian) spin-singlet states were studied in Refs. 6 and 10–12.

In Ref. 4, Moore and Read introduced the notion of a paired QH state and discussed the so-called \( q \)-pfaffian states at filling \( \nu = 1/q \) (with \( q \) even). It is believed that this state (with \( q = 2 \)) is at the origin of the observed QH plateau at filling fraction \( \nu = \frac{1}{2} \) (see Ref. 13 for a recent review). The wave function for the \( q \)-pfaffian is given by

\[
\Psi_{\text{pf}}^q(z_1, \ldots, z_N) = \prod_{i<j} (z_i - z_j)^q,
\]

where the pfaffian factor for an antisymmetric matrix \( M_{ij} \) is defined as \( \text{Pf}(M_{ij}) = \mathcal{A} \prod_i M_{i+1,i} \), with \( \mathcal{A} \) denoting antisymmetrization. In Ref. 14, the pfaffian states were generaliz-
That condition can be seen by noting that it is identical to

\[ \left( x_i - x_j \right)^{1/2} \prod_j \left( z_j^1 - w_j \right)^{1/2}, \]  

where

\[ \Phi(x_i, x_j; w_e, w_s) = \frac{x_i - x_j}{x_j - w_e} \left( x_j - w_s \right)^{1/2} + i \leftrightarrow j. \]  

That (7) in fact defines a well-behaved electronic wave function can be seen by noting that it is identical to

\[ \prod_j \left( z_j^1 - w_j \right)^{1/2} \]  

in the limit where \( w_s, w_c \rightarrow w, \) spin and charge recombine and the wave function reduces to Eq. (6). Note that the factor \( \Pi_j \left( z_j^1 - w_j \right)^{-1} \) should be regularized and projected onto the lowest Landau level in the same way as the wave functions for quasiparticles over the Laughlin states.\(^{16}\)

The charge of the holon excitation equals \( \frac{1}{2} \Phi_0 \sigma_H \) (with \( \Phi_0 = \hbar/e \) the flux quantum), showing that the creation of a single holon involves the insertion of a half-quantum of magnetic flux, which is the canonical flux quantum in the presence of a pairing condensate. This flux insertion is accompanied by a vortex in the pairing condensate, and this brings in the factor \( \sigma(z) \) in the expressions Eq. (5). The role of the vortices in this discussion is similar to the role of visons in the Senthil-Fisher theory.\(^{17}\)

An important feature that is implied by the presence of spin-fields \( \sigma(z) \) in the expressions Eq. (5) for the spinons and holons, is that the braid statistics of these excitations will be non-Abelian. This feature is analogous to the non-Abelian statistics of the charge \( 1/(2q) \) excitations over the (spin-polarized) \( q \)-pfaffian state, and we refer to the literature for a discussion.\(^{3,18,19}\)

It is well-known that the \( q \)-pfaffian spin-polarized state is closely related to two Abelian states at filling \( \nu = 1/q \): the two-layer \( (q+1, q+1, q-1) \) state and a strong pairing state which is a Laughlin state of strongly paired electrons. Possible transitions among these three states have been discussed in the literature (see, e.g., Refs. 19–21). In the spin-singlet situation, we may similarly identify two series of Abelian spin-singlet states at \( \nu = 2/(2m+1) \) that allow for a transition into the pfaffian spin-singlet state Eq. (3): a two-layer state associated to \( \text{SO}(6) \) and a strong pairing state. The wave function for the two-layer state reads as

\[
\Psi_{2\text{-layer}}^{(m)}(\{z_j^1, z_j^3, z_j^5, z_j^7\}) = \prod_{i<j} (z_i^1 - z_j^1)^{m} \prod_{i<j} (z_i^1 - z_j^3)^{m+2} \prod_{i<j} (z_i^1 - z_j^5)^{m+2} \prod_{i<j} (z_i^1 - z_j^7)^{m+2} \prod_{i<j} (z_i^3 - z_j^3)^{m+2} \prod_{i<j} (z_i^3 - z_j^5)^{m+2} \prod_{i<j} (z_i^3 - z_j^7)^{m+2} \prod_{i<j} (z_i^5 - z_j^5)^{m+2} \prod_{i<j} (z_i^5 - z_j^7)^{m+2} \prod_{i<j} (z_i^7 - z_j^7)^{m+2} \prod_{i<j} (z_i^7 - z_j^1)^{m+2} \prod_{i<j} (z_i^7 - z_j^3)^{m+2} \prod_{i<j} (z_i^7 - z_j^5)^{m+2} \prod_{i<j} (z_i^7 - z_j^7)^{m+2} 
\]

where the indices \( t, b \) refer to the top and bottom layers. This wave function arises as a correlator of two-layer spinful electron operators which, in the case \( m = 0 \), generate an \( \text{SO}(6) \) affine Kac-Moody algebra.

The strong pairing state is an Abelian state of strongly paired electrons in the two-layer state are subjected to increasing interlayer interactions. A useful framework is that of \( \text{SU}(2) \times \text{SU}(2) \) matrices describing the topological order of the various states.\(^{22}\) For this discussion, we refer to the states via their associated \( \text{SO}(6), \text{SO}(5) \), or \( \text{SO}(4) \) symmetries.] For the \( \text{SO}(6) \) states, the naive \( K \) matrix for the four electron operators \( (\downarrow, \uparrow), (\downarrow, \downarrow), (\uparrow, b), (\downarrow, b) \) is singular. After a reduction to three independent condensate operators we find the following \( \text{QH} \) data

\[
K_e = \begin{pmatrix}
    m + 2 & m & 2m + 1 \\
    m & m + 2 & 2m + 1 \\
    2m + 1 & 2m + 1 & 4m + 2
\end{pmatrix},
\]

\[
q_e = \begin{pmatrix}
    1, 1, 2 \\
    -1, 1, 0 \\
    1, b, c
\end{pmatrix}
\]

where \( q_e, s_e, \) and \( \mathbf{1}_e \) specify the charge, spin and layer index for an appropriate basis of electron operators, which build the \( \text{QH} \) condensate. By applying a duality transformation [\( K_{\phi} = K_e^{-1} \), \( q_{\phi} = -K_e q_e \), etc.] one obtains the topological data for a basis of quasi-hole excitations.\(^{22,23}\)

Starting from this characterization of the topological order in the \( \text{SO}(6) \) state, the topological order of the \( \text{SO}(5) \) and \( \text{SO}(4) \) states can be obtained in a systematic manner.\(^{21,23}\) For the \( \text{SO}(5) \) state, the resulting description employs a so-called pseudoparticle whose role it is to account for the degeneracies that are associated to the non-Abelian braid statistics. Choosing \( \Psi^{(s)} \), \( \Delta^{(s)\uparrow} \), and \( \Delta^{(s)\downarrow} \) as the fundamental condensate operators, we find

\[
K_s = \begin{pmatrix}
    m + 2 & 1 & 2m + 1 \\
    1 & 2 & 0 \\
    2m + 1 & 0 & 4m + 2
\end{pmatrix},
\]

\[
s_e = \begin{pmatrix}
    \uparrow, \uparrow, \downarrow \\
    \downarrow, \uparrow, \downarrow \\
    \uparrow, \downarrow, \downarrow
\end{pmatrix}
\]
To distinguish the different states, one may described by composite fermions with antiparallel flux filling fraction, there exists an Abelian spin-singlet state, \( K \).

The simplest filling fraction where the paired spin-singlet states that we propose are possible is \( \nu = \frac{5}{2} \). At that same filling fraction, there exists an Abelian spin-singlet state, described by composite fermions with antiparallel flux attachment. To distinguish the different states, one may consider the exponents for various tunneling processes. For the paired spin-singlet state the scaling dimensions for electrons, holons and spinons are \( s_{el} = \frac{m + 2}{2} \), \( s_{hol} = \frac{(2m + 5)}{(16m + 8)} \), and \( s_{sp} = \frac{5}{2} \), respectively. Thus, for tunneling through the bulk, the holon is the most relevant particle (for \( m = 1 \)), while the \( I-V \) for tunneling electrons from a Fermi-liquid into the edge is \( I \sim V_{el} = V^{m+2} \). According to Ref. 25, the scaling dimensions for the composite fermion spin-singlet state at \( \nu = \frac{5}{2} \) are \( s_{el} = 2 \), \( s_{sp} = \frac{5}{2} \). They give rise to a quadratic \( I-V \) for electron tunneling, in contrast to the cubic \( I-V \) for the paired state. Another way to distinguish the two states is via the spin-Hall conductance, which has opposite sign as compared to the ordinary Hall conductance for the Abelian state. For the paired spin-singlet state both conductances have the same sign.

There are two ways in which the paired state Eq. (3) can be relevant in a double-layer geometry. First, as already mentioned, there is the possibility of a transition from a double-layer state for spin-full electrons, Eq. (10), into a single-layer paired state. A second possibility is a realization of the paired state as a double-layer state for spin-polarized electrons, with the layer index playing the role of the spin index.

As is the case for the pfaffian and the NASS states, these states can be generalized to states which show clustering instead of pairing. Starting from an SO(5)\(_k\) symmetry structure, one derives states that allow clusters of up to \( 2m \) particles of equal spin, with filling fractions given by \( \nu = 2k/(2km + 1) \).

This research was supported in part by the Foundation FOM of the Netherlands and by the Netherlands Organization for Scientific Research (NWO). A.W.W.L. acknowledges the Institute for Theoretical Physics at the University of Amsterdam for hospitality. His research was supported by NSF under Grant No. DMR-00-75064.

\[ \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & -1 \\ -2 & 1 & 2 \\ -2 & 3 & 1 \\ -4 & 1 & 4 \\ 2 & 4 & 8m + 4 \end{bmatrix} \]

\( \mathbf{K}_\phi = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 2 & -1 \\ -2 & 1 & 2 \end{bmatrix} \)

\( \mathbf{q}_\phi = \begin{bmatrix} 0, 0, \frac{1}{2} (m + 1) \end{bmatrix} \)

\( \mathbf{s}_\phi = (0, 0, 0) \).

\( \mathbf{K}_c = \begin{bmatrix} 2 & 0 \\ 0 & 4m + 2 \end{bmatrix} \)

\( \mathbf{q}_c = - (0, 2) \)

\( \mathbf{s}_c = (1, 1, 0) \).

(12)

(13)


7 For a review, see J.P. Eisenstein, in Perspectives in Quantum Hall Effects, edited by S. Das Sarma and A. Pinczuk (Wiley, New York, 1997).


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