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Separation of spin and charge in paired spin-singlet quantum Hall states

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We propose a series of paired spin-singlet quantum Hall states, which exhibit a separation of spin and charge degrees of freedom. The fundamental excitations over these states, which have filling fraction \( \nu = 2/(2m + 1) \) with \( m \) an odd integer, are spinons (spin \(-\frac{1}{2}\) and charge zero) or fractional holons (charge \( \pm 1/(2m+1) \) and spin zero). The braid statistics of these excitations are non-Abelian. The mechanism for the separation of spin and charge in these states is topological: spin and charge excitations are liberated by binding to a vortex in a \( p \)-wave pairing condensate. We briefly discuss related, Abelian spin-singlet states and possible transitions.

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Strongly correlated electrons in low dimensional systems are known to exhibit physical phenomena that are surprising and, at first sight, counterintuitive. Among these is the remarkable phenomenon of quantum number fractionalization: elementary excitations in strongly interacting many-electron systems can have quantum numbers (for spin and charge) that are fractions of those of the electron. This fractionalization can take the form of a separation of spin and charge, or of a fractionalization of the electric charge of the electron.

In \( D = 1 \) spatial dimension, the separation of spin and charge is well understood. It is seen in explicit solutions of specific integrable model systems (Hubbard and supersymmetric \( t-J \) models). The general framework of the Luttinger Liquid has made it clear that in \( 1 + 1 \) dimensions the separation of spin and charge is a generic feature, which does not require any fine tuning of the interactions among the electrons.

In spatial dimensions \( D = 2 \) or higher, spin and charge tend to confine and a separation of the two is only possible under very special conditions. It has been proposed that the key feature underlying the anomalous behavior of the cuprate high-\( T_c \) materials is precisely a separation of spin and charge,1 and concrete scenarios, based on \( \mathbb{Z}_2 \) or \( U(1) \) gauge theories, have been put forward.2

In this paper, we focus on the quantum Hall (QH) regime, which is relevant for two-dimensional (2D) electrons in strong magnetic fields, and for rotating Bose-Einstein condensates.3 In particular, we discuss the separation of spin and charge in the QH regime. Specifically, we propose a series of paired spin-singlet QH states, of filling fraction \( \nu = 2/(2m+1) \), which are generalizations of the Moore-Read or pfaffian states for spin polarized electrons. The fundamental excitations over these states are spinons (spin \(-\frac{1}{2}\) and zero charge) and holons (with zero spin and fractional charge \( \pm 1/(2m+1) \), in units of the charge of the electron). The braid statistics of these excitations are non-Abelian, and thereby the paired spin-singlet states fall in the category of “non-Abelian QH states.”

It is important to stress that the more conventional Abelian spin-singlet QH states [such as the Halperin states with label \( (m+1,m+1,m) \), see below] do not exhibit a separation of spin and charge. The excitations over such states are conveniently analyzed in terms of a “spin-charge decomposition”4–6 but this is subject to certain gluing conditions (expressing locality of the excitation w.r.t. the electrons), which exclude single spinons or holons from the (bulk) physical spectrum. The essential feature that liberates spin and charge in the paired states proposed here is the presence of the pairing condensate: by binding to a vortex in the pairing condensate, the spin and charge excitations become local with respect to the electrons in the ground state, and they can propagate independently.

It is illustrative to compare the separation of spin and charge in the paired spin-singlet states with the fractionalization of charge in paired, so-called \( q \)-pfaffian, spin-polarized states. For the \( q \)-pfaffian states, Laughlin’s gauge argument gives that the adiabatic insertion of a single flux quantum will produce an excitation of charge \( 1/q \). However, as in the case of BCS superconductors, the presence of the pairing condensate leads to a reduction of the elementary flux quantum by a factor of \( 2 \), and thereby the unit-flux Laughlin quasiparticles are separated into two constituents each carrying a charge \( 1/(2q) \). In a similar way, conventional quasiparticles (carrying spin and charge) over a paired spin-singlet state are separated into spinons and holons.

Before we present the paired spin-singlet states, we briefly recall some facts about spin-singlet states and paired states in the QH regime. Despite the presence of strong magnetic fields in the QH regime, there is experimental motivation to study states that are not (fully) spin polarized (see, e.g., Ref. 7). In many QH systems, the energy scale for the Zeeman splitting is relatively low, and it can be further suppressed by the application of hydrostatic pressure. Using this technique, combined with a tilted field technique, spin transitions in the QH regime can be studied.8 The simplest QH states that are singlets w.r.t. the SU(2) spin symmetry are the Halperin states9

\[
\Psi_{SS}^{(m+1,m+1,m)}(z_1, \ldots, z_n; \bar{z}_1, \ldots, \bar{z}_n) = \Pi_{i<j}(z_i^* - z_j^*)^{m+1} \Pi_{i<j}(z_i - z_j)^m \Pi_{i,j}(z_i^* - \bar{z}_j)^m,
\]

(1)

where \( z_i^* \) and \( z_i^\dagger \) are the coordinates of the spin up and spin down electrons, respectively, and \( m \) is an even integer. The state Eq. (1) has filling fraction \( \nu = 2/(2m+1) \). Here and
ized to a series of non-Abelian spin-singlet symmetrization. In Ref. 14, the pfaffian states were generalized to a series of non-Abelian spin-singlet (NASS) states, at filling \( \nu = 1/q \) (with \( q \) even). It is believed that this state (with \( q = 2 \)) is at the origin of the observed QH plateau at filling fraction \( \nu = \frac{2}{5} \) (see Ref. 13 for a recent review). The wave function for the \( q \)-pfaffian is given by

\[
\Psi_{pf}^{(q)}(z_1, \ldots, z_N) = \frac{1}{\prod_{i<j} (z_i - z_j)} \prod_{i} (z_i - z_j)^q, \tag{2}
\]

where the pfaffian factor for an antisymmetric matrix \( M_{ij} \) is defined as \( \text{Pf}(M_{ij}) = \sum_{\sigma} \text{det}(A_{\sigma}) \), with \( A \) denoting antisymmetrization. In Ref. 14, the pfaffian states were generalized to a series of non-Abelian spin-singlet (NASS) states, at filling \( \nu = 4/(4M + 3) \) with \( M \) an odd integer. These states exhibit a pairing of like spins. The excitations over these NASS states have non-Abelian statistics, but there is no separation of spin and charge.

In the paired spin-singlet states that we propose here, the pairing takes place in the charge sector, irrespective of the spin of the electrons. This leads to a wave function

\[
\Psi_{\text{paired}}^{(m)}(z_1^r, \ldots, z_N^r; z_1^s, \ldots, z_N^s) = \frac{1}{\prod_{i<j} (x_i^r - x_j^r)} \frac{1}{\prod_{i<j} (x_i^s - x_j^s)} \Psi_{SS}^{(m+1, m+1, m)}(z_1^r; z_1^s), \tag{3}
\]

where \( x_i^r = z_i^r, x_i^s = z_i^s \), \( m \) is now an odd integer and the filling fraction is \( \nu = 2/(2m + 1) \). There exists a Hamiltonian for which the unique ground state. One way to study the excitations over this state is by using this Hamiltonian. Here we will proceed by analyzing the state Eq. (3) and its excitations using an associated conformal field theory (CFT).

Following the CFT-QH correspondence outlined in Ref. 4, one quickly finds that the CFT associated to the (bosonic) paired spin-singlet state at \( m = 0 \) is the (chiral) CFT based on the affine Kac-Moody algebra \( SO(5)_1 \). For this algebra, the eight currents associated to the roots of \( SO(5) \) can be written in terms of spin and charge bosons \( \varphi, \bar{\varphi} \) and a Majorana fermion \( \psi \). [The assignment of spin and charge quantum numbers to the weights and roots of \( SO(5) \) is indicated in Fig. 1.] For general \( m \), the “condensate” operators \( \Psi \) and \( \Delta \) are obtained from these currents by the substitution \( \varphi \rightarrow \sqrt{2m + 1} \varphi \),

\[
\Psi = e^{i(2m+1)\bar{\varphi}/2} \varphi, \quad \bar{\Psi} = e^{-i(2m+1)\varphi/2} \bar{\varphi}, \tag{4}
\]

where \( \alpha = \uparrow, \downarrow \) referring to the spin eigenvalue \( s_z = \pm \frac{1}{2} \) and \( \Delta = \uparrow \uparrow, \downarrow \downarrow \). The quantum numbers \( q \) (charge) and \( s_z \) are measured by the operators \( Q = -i\sqrt{2/(2m + 1)} \varphi (dz/2 \pi i) \partial \varphi \) and \( S_c = i\sqrt{2} \varphi (dz/2 \pi i) \partial \varphi \). The wave function Eq. (3) is obtained as a correlator of \( N \) spin-up electrons \( \Psi^1 \) and \( N \) spin-down electrons \( \Psi^{-1} \), together with a neutralizing background charge. The CFT description makes it easy to identify the fundamental (quasiparticle) excitations. For \( m = 0 \) they are the operators that generate the spinor (four-dimensional) representation of the \( SO(5)_1 \) current algebra. For general \( m \) these become

\[
\phi = e^{i(\sqrt{2}m+1)\varphi/2}, \quad \bar{\phi} = e^{-i(\sqrt{2}m+1)\varphi/2}, \tag{5}
\]

where \( \sigma(z) \) is the so-called spin field associated to the Majorana (Ising) fermion \( \psi(z) \). Higher excitations, such as those constituting the vector representation, can be generated by bringing together two or more of the fundamental excitations. The expressions Eq. (5) show that the fundamental excitations can be characterized as spinons \( \phi^a \) (spin-\( \frac{1}{2} \) but no charge) and holons \( \bar{\phi}, \phi \) (of charge \( \pm 1/(2m + 1) \) and zero spin).

To illustrate the separation of spin and charge, we present explicit wave functions for excited states. We first consider an Abelian excitation, with spin down \( (s_z = -\frac{1}{2}) \) and charge \( 1/(2m + 1) \), at location \( w \). Its wave function takes the familiar form

\[
\prod_i (z_i - w) \Psi_{\text{paired}}^{(m)} \tag{6}
\]

The important observation is now that, starting from this wave function, one can separate the locations of the spin and charge parts of this excitation, creating a spinon at position \( w \) and a holon at \( w \). In the corresponding wave function, the pfaffian factor in Eq. (3) is replaced by (compare with Ref. 4)
Spin and charge are decoupled from the start, and the wave function arises as a correlator of two-layer spinful electrons. A useful framework is that of bound pairs with quantum numbers \((m^s, m^s)\), which are the operators \(\tilde{c}^\dagger(\vec{r})\tilde{c}(\vec{r}+\vec{b})\) for an appropriate basis of electron operators, which build up the two-layer state associated to SO(6) and a strong pairing state. Possible transitions among these three states have been discussed in the literature (see, e.g., Refs. 19–21). In the spin-singlet situation, we may similarly identify two series of Abelian spin-singlet states at \(v = 2/(2m + 1)\) that allow for a transition into the Pfaffian spin-singlet state Eq. (3): a two-layer state associated to SO(6) and a strong pairing state. The wave function for the two-layer state reads as

\[
\Psi_{2\text{-layer}}^{(m)}(\{z_j^\dagger, z_i^\dagger, z_i^\dagger, z_j^\dagger\}) = \prod_{i<j}(z_i^\dagger - z_j^\dagger)^{m+2}\prod_{i<j}(z_i^\dagger - z_j^\dagger)^{m+2}\prod_{i<j}(z_i^\dagger - z_j^\dagger)^{m+2}\prod_{i<j}(z_i^\dagger - z_j^\dagger)^{m+2}
\]

\[
\times \prod_{i,j}(z_i^\dagger - z_j^\dagger)^{m+1}\prod_{i,j}(z_i^\dagger - z_j^\dagger)^{m+1}\prod_{i,j}(z_i^\dagger - z_j^\dagger)^{m+1}\prod_{i,j}(z_i^\dagger - z_j^\dagger)^{m+1},
\]

where the indices \(t, b\) refer to the top and bottom layers. This wave function arises as a correlator of two-layer spinful electronic operators which, in the case \(m=0\), generate an SO(6), affine Kac-Moody algebra. The strong pairing state is an Abelian state of strongly paired electrons in the two-layer state subjected to increasing interlayer interactions. A useful framework is that of the QH condensate. By applying a duality transformation (\(K_b = K_e^{-1}\)), one obtains the topological data for a basis of quasi-hole excitations.22,23

Starting from this characterization of the topological order in the SO(6) state, the topological order of the SO(5) and SO(4) states can be obtained in a systematic manner.21,23 For the SO(5) state, the resulting description employs a so-called pseudoparticle whose role it is to account for the degeneracies that are associated to the non-Abelian braid statistics. Choosing \(\Psi^L, \Delta^I, \text{ and } \Delta_e\) as the fundamental condensate operators, we find

\[
K_e = \begin{pmatrix}
  m + 2 & m & 2m + 1 \\
  m & m + 2 & 2m + 1 \\
  2m + 1 & 2m + 1 & 4m + 2
\end{pmatrix},
\]

\[
q_e = -(1,1,2), \quad s_e = (\uparrow,\uparrow,0), \quad l_e = (t,b,\cdot),
\]

where \(q_e, s_e, \text{ and } l_e\) specify the charge, spin and layer index for an appropriate basis of electron operators, which build the QH condensate. By applying a duality transformation (\(K_b = K_e^{-1}\)), \(q_b = K_b q_e, \text{ etc.}\) one obtains the topological data for a basis of quasi-hole excitations.22,23
attachment.11 To distinguish the different states, one may
filling fraction, there exists an Abelian spin-singlet state,
first proposed in Ref. 24; for the interpretation of
It is the first particle in the
This same set of QH data can be obtained by starting from
A further reduction leads to the following QH data for the
strong pairing SO(4) state (the data for the \( \phi \) sector is
obtained by the duality mentioned above)

\[
K_{\phi} = \begin{pmatrix}
1 & -\frac{1}{2} & -\frac{1}{2} \\
-\frac{1}{2} & 3 & \frac{1}{4} \\
1 & \frac{1}{2} & \frac{2m+3}{8m+4}
\end{pmatrix}, \quad q_{\phi} = \left(0, \frac{1}{2}, m+1\right), \quad s_{\phi} = (0, \frac{1}{2}, 0).
\]

(12)

It is the first particle in the \( \phi \) sector that is interpreted as a
pseudoparticle, the other two have quantum numbers corre-
sponding to \( \phi_{+} \) and \( \phi_{-} \). The matrix \( K_{\phi} \) is of a general form
first proposed in Ref. 24; for the interpretation of \( K \) matrices
for non-Abelian QH states we refer to Ref. 23. We remark
that the ground state degeneracy on the torus is not simply
given by \( |\det K_{\phi}| \), as is the case for Abelian QH states; the
actual value here is \( 3(2m+1) \).

A further reduction leads to the following QH data for the
strong pairing SO(4) state (the data for the \( \phi \) sector is
obtained by the duality mentioned above)

\[
K_{\phi} = \begin{pmatrix}
2 & 0 \\
0 & 4m+2
\end{pmatrix}, \quad q_{\phi} = -(0, 2), \quad s_{\phi} = (\uparrow \uparrow, 0).
\]

(13)

This same set of QH data can be obtained by starting from the
SO(6) data Eq. (11) and condensing quasiparticle-
quasihole pairs, following Ref. 21.

The simplest filling fraction where the paired spin-singlet
states that we propose are possible is \( \nu = \frac{2}{5} \). At that same
filling fraction, there exists an Abelian spin-singlet state,
described by composite fermions with antiparallel flux
attachment. To distinguish the different states, one may
consider the exponents for various tunneling processes. For
the paired spin-singlet state the scaling dimensions
for electrons, holons and spinons are \( g_{e} = m + 2 \),
\( g_{h} = (2m+5)/(16m+8) \), and \( g_{sp} = \frac{2}{5} \), respectively. Thus,
for tunneling through the bulk, the holon is the most relevant
particle (for \( m = 1 \)), while the \( I-V \) for tunneling electrons
from a Fermi-liquid into the edge is \( I \sim \nu_{c} \nu^{m+2} \). Accord-
ing to Ref. 25, the scaling dimensions for the composite
fermion spin-singlet state at \( \nu = \frac{2}{5} \) are \( g_{e} = 2 \), \( g_{sp} = \frac{2}{5} \). They
give rise to a quadratic \( I-V \) for electron tunneling, in contrast
to the cubic \( I-V \) for the paired state. Another way to distin-
guish the two states is via the spin-Hall conductance, which
has opposite sign as compared to the ordinary Hall conduc-
tance for the Abelian state. For the paired spin-singlet state
both conductances have the same sign.

There are two ways in which the paired state Eq. (3) can
be relevant in a double-layer geometry. First, as already men-
tioned, there is the possibility of a transition from a double-
layer state for spin-full electrons, Eq. (10), into a single-layer
paired state. A second possibility is a realization of the paired
state as a double-layer state for spin-polarized electrons, with
the layer index playing the role of the spin index.

As is the case for the pfaffian and the NASS states, these
states can be generalized to states which show clustering
instead of pairing. Starting from an SO(5) symmetry structure,
one derives states that allow clusters of up to
\( 2k \) particles of equal spin, with filling fractions given by
\( \nu = 2k/(2km+1) \).

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