The ATLAS SemiConductor Tracker Endcap
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Chapter 6
Detector noise

"The mathematics is not there till we put it there."

Sir Arthur Eddington

This chapter discusses the consequences of noise using binary read-out. The noise can be reduced by increasing the threshold, however this influences the efficiency. A discussion on the effect of noise on efficiency is followed by the development of a model for the module noise. Finally this model is used to predict the module noise. Chapter 7 compares measurements of module noise to these predictions.

6.1 The consequences of noise with binary readout

In binary read-out, the threshold that is set determines the tracking performance. If the threshold is lowered, the noise occupancy grows. More hits that are not associated to any track will be found, deteriorating the tracking performance. Also more data needs to be read out which may result in inefficiencies, due to either buffer overflows or limited bandwidth in the read-out chain. The tracking is not significantly affected by the expected noise-levels: the read-out speed is the limiting parameter. On the other hand, raising the threshold setting decreases the detector efficiency. However, the efficiency will be mainly limited by the number of dead channels, chips and modules: the sensors need to have an intrinsic detection efficiency that does not increase the inefficiency already caused by these errors. The ATLAS SCT TDR (ATLAS Inner Detector Collaboration 1997b) gives the requirement for both parameters. It states a minimum efficiency of 99% and a maximum noise-occupancy of $5 \times 10^{-4}$.

Noise influences both requirements. At a set threshold, if the noise increases, the noise occupancy increases. In the same situation, the efficiency also drops, because the signal distribution is broadened by the noise. This section discusses the influence of the threshold setting on the detection efficiency and the noise occupancy.
6.1.1 Detection efficiency

The probability-density function of losing an energy $\Delta$ in traversing a thickness $t$ of material for thin silicon detectors is quite well approximated by the original Landau (Landau 1944) distribution, $f_L$:

$$f_L(\Delta) = \frac{1}{\xi} \phi(\lambda) \quad (6.1)$$

$$\phi(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{\lambda u + u \ln u} du \quad (6.2)$$

$$\lambda = \frac{\Delta}{\xi} - 1 + \Gamma + \beta^2 - \ln \frac{\epsilon m e}{\beta^2} \quad (6.3)$$

The dependence on the material and thickness traversed is contained in the parameters $\xi$, the maximum energy-deposit in a single collision $\epsilon_m$, and the average (logarithmic) ionisation energy $I$ of the material. $\beta$ is the particle velocity and $\Gamma$ is the Euler constant. The parameter $\xi$ for the Landau distribution is:

$$\xi = \frac{2\pi e^4 N_A z t}{m_e \beta^2} \quad (6.4)$$

where $e$ and $m_e$ are the electron charge and mass, $N_A$ is Avagadro’s number and $z$ is the atomic number of the material.

Noting that $\phi$ has a maximum at $\lambda \simeq -0.22278$, which corresponds to the most probable energy deposit $\Delta_{mp}$, Equation 6.3 becomes:

$$\lambda = \frac{\Delta - \Delta_{mp}}{\xi} + \lambda_0 \quad (6.5)$$

where $\lambda_0 \simeq -0.22278$ and:

$$\Delta_{mp} = \xi \left[ \ln \frac{\epsilon m e}{I^2} + 0.2000 - \beta^2 - \delta \right] \quad (6.6)$$

where $\delta$ describes the density effect\(^1\). This distribution is, expressed in units of electrons, shown by the solid line in Figure 6.1.

The original Landau distribution $f_L(\Delta; \Delta_{mp}, \xi)$ assumes scattering off free electrons. However, electrons in silicon are bound, and it is found that a convolution of the Landau distribution with a Gaussian with a width $\sqrt{t\delta^2}$ is a better approximation (Hancock et al. 1983, Bichsel 1988):

$$f_{LC}(\Delta; \Delta_{mp}, \xi, \sqrt{t\delta^2}) = \frac{1}{\sqrt{2\pi t\delta^2}} \int_0^\infty \exp \left( \frac{-\left(\Delta' - \Delta\right)^2}{2t\delta^2} \right) \frac{\phi(\lambda(\Delta'; \Delta_{mp}; \xi))}{\xi} d\Delta' \quad (6.7)$$

The energy-loss of the particle generates charge in the silicon. We use a naive way of translating the energy-deposit into a detected number of electrons $q_c$:

$$q_c = \frac{\epsilon_e \Delta}{\Delta_i} \quad (6.8)$$

\(^1\)The density effect was not accounted for in the original Landau function.
where $\Delta_i$ is the average number of electrons detected per unit of energy and $\epsilon_c$ is the charge-collection efficiency.

In general, if the probability density-function $P(x; \bar{a})$ depending on variable $x$ and the variables $\bar{a}$, the probability-density function $Q(y; \bar{a})$ where $y$ is a function of $x$, can be calculated using:

$$Q(y; \bar{a}) = \frac{P(x; \bar{a})}{dy/dx} \quad \text{with} \quad y = y(x) \quad (6.9)$$

It can be shown that for a scaling of $x$ with a constant factor $c$, $y = cx$ as in Equation 6.8, has the following relationship for a Gaussian, Landau distribution and their convolution:

$$Q(y; \bar{a}) = P(y; ca) \quad \text{if} \quad y = cx \quad (6.10)$$

with $\bar{a} = (\Delta_{mp}, \xi, \sqrt{t\delta^2})$. Using this result, we can rewrite Equation 6.7 as:

$$f_{LG}(q_c; \Delta'_{mp}, \xi', \sqrt{t\delta^2}) = \frac{1}{\sqrt{2\pi t\delta^2}} \int_0^\infty \exp \left( \frac{-(q - q_c)^2}{2t\delta^2} \right) \phi \left( \frac{\lambda(q; \Delta'_{mp}, \xi')}{\xi'} \right) dq \quad (6.11)$$

where:

$$\Delta'_{mp} = \frac{\epsilon_c\Delta_{mp}}{\Delta_i} , \xi' = \frac{\epsilon_c\xi}{\Delta_i} , \text{and} \quad \sqrt{t\delta^2} = \frac{\epsilon_c\sqrt{t\delta^2}}{\Delta_i} \quad (6.12)$$

The dashed line in Figure 6.1 shows the corrected Landau-distribution with 100% charge-collection efficiency. In reality, the distribution of collected electrons is further broadened by many sources. We make the assumption that these effects are dominated by the electronic noise in SCT modules. The distribution of the detected number of electrons can be calculated by convoluting Equation 6.7 with a Gaussian with a width $\sigma_N$, the equivalent noise-charge of the detector module. One can show that first convoluting a function with a Gaussian with a width $\sigma_1$ and then with a Gaussian with a width $\sigma_2$ is the same as convoluting the function with a Gaussian with a width $\sqrt{\sigma_1^2 + \sigma_2^2}$: the probability density-function of measuring an input charge $q_i$ can thus finally be written as:

$$f_{LG}(q_i; \Delta'_{mp}, \xi', \sqrt{t\delta^2}, \sigma_N) = \frac{1}{\sqrt{2\pi (t\delta^2 + \sigma_N^2)}} \int_0^\infty \exp \left( \frac{-(q - q_i)^2}{2(t\delta^2 + \sigma_N^2)} \right) \phi \left( \frac{\lambda(q; \Delta'_{mp}, \xi')}{\xi'} \right) dq \quad (6.13)$$

This leaves the calculation of the different parameters: $\Delta_{mp}, \xi, t\delta^2, \text{and } \Delta_i$. The (effective) charge-collection efficiency $\epsilon_c$ is not well known and depends on the position of the charged track traversing the sensor relative to the strips. The noise $\sigma_N$ is left to be determined in the remainder of this chapter.

H. Bichsel describes in detail the energy-loss processes in silicon detectors. I follow his approach in Appendix E of Bichsel (1988) for the Landau approximation in order to calculate the parameters of the original Landau-distribution $\Delta_{mp}$ and $\xi$.

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2This is the same as a change of variables, since this is in fact a scaling.
Detector noise

Figure 6.1: Four cases of the Landau distribution, as described in the text, assuming a charge-collection efficiency of 100%. All distributions have the values for $\xi$ and $\Delta_{mp}$ as mentioned in the text. The straight line is the original Landau function. The dashed line is the corrected Landau, with no electronic noise. The dotted line shows the corrected Landau, convoluted with the noise distribution of an outer module expected after ten years running at ATLAS.

The maximum energy-transfer is:

$$\epsilon_m = 2m_e c^2 \beta^2 \gamma^2$$  \hspace{1cm} (6.14)

For high-energy particles ($\gamma \equiv (1 - \beta^2)^{-\frac{1}{2}} > 100$), Bichsel takes $\beta \simeq 1$ and $\delta \simeq \ln \gamma^2 - 4.447$ so that Equation 6.6 can be written as:

$$\Delta_{mp} \simeq \xi \left[ \ln \frac{2m_e c^2 \xi}{I^2} + 3.647 \right]$$  \hspace{1cm} (6.15)

Putting in the constants for Equation 6.4 and $t$ in micrometer gives:

$$\xi (\text{keV}) \simeq 0.017825 t$$  \hspace{1cm} (6.16)

so that, with $I = 174$ eV and $t$ in micrometer:

$$\Delta_{mp} (\text{keV}) \simeq (0.1791 + 0.01783 \ln t) t$$  \hspace{1cm} (6.17)

For our sensor thickness of 285 $\mu$m we have:

$$\xi \simeq 5.08 \text{ keV}$$

$$\Delta_{mp} \simeq 79.8 \text{ keV}$$

The Gaussian width $\sqrt{\delta\sigma}$ has been measured for 300 $\mu$m detectors (Hancock et al. 1983): I take 5 keV as a reasonable value for high momentum particles for the SCT sensors.

When all charge is collected in one strip, the charge-collection efficiency $\epsilon_c$ is a little less than 100%, but the charge can be distributed between two strips. We assume a maximum charge-collection efficiency (CCE) of 90%. The worst case is for tracks exactly between two strips,
6.1. The consequences of noise with binary readout

The bandgap energy of silicon is 1.12 eV. However, energy is also transferred to the lattice via Raman scattering and the electron can have kinetic energy left after the ionisation process (Shockley 1961). We use the effective value of $\Delta_i = 3.6 \text{ eV}$.

Equation 6.13 has been evaluated numerically, where $\phi$ was calculated using the CERNLIB implementation (Kölbig & Schorr 1984). Figure 6.1 shows the convoluted Landau-distribution $f_{LC}$ for different values of the noise. Notice that the convolution with a Gaussian moves the effective most-probable value of the detected charge to higher values, but broadens it leading to lower efficiencies. The most-probable value before irradiation is $22.9 \times 10^3$ electrons and after irradiation is $23.1 \times 10^3$ electrons.

Figure 6.2 shows the efficiency as function of threshold for different values of the noise, for the estimated maximum charge-collection efficiency of 90% and the most pessimistic charge-collection efficiency of 45%. For the latter case the efficiency at 1 fC threshold is too low, independent of the noise. However, charge division is only significant when the tracks pass through the sensor in a very small region around the midpoint between two strips. The average charge-collection is therefore expected to be close to 90% giving an efficiency much higher than 99%, which is confirmed by testbeam (Lacasta et al. 2002). Testbeam results also show that the dependence of charge-collection and angle of incidence is small (Barr et al. 2001).

The detection inefficiencies of the ATLAS SCT will be dominated by dead or defective strips, chips, and modules. By ensuring the intrinsic detection efficiency is close to 100%, the system has maximum tolerance of production errors and other failures that lead to inefficiencies.
6.1.2 Noise Occupancy

![Graph showing noise occupancy](image)

\[ \text{threshold (fC)} \]

\[ \sigma_N = 1300 \text{ electrons} \]
\[ \sigma_N = 1800 \text{ electrons} \]
\[ \sigma_N = 2000 \text{ electrons} \]
\[ \sigma_N = 2200 \text{ electrons} \]

Figure 6.3: The noise occupancy of an SCT-sensor, for different values of the electronics noise \( \sigma_N \). The grey area is outside the specification for the ATLAS SCT: a maximum noise occupancy of \( 5 \times 10^{-4} \). The grey line indicates the anticipated threshold setting of 1 fC.

Assuming the bandwidth of the electronics is small compared to the range of the 'white' noise sources (see Section 6.2), the amplitude distribution of the noise can be assumed to be Gaussian. The noise occupancy as function of the threshold is described by the complementary error-function. Figure 6.3 shows the dependence for several noise levels.

6.1.3 Performance after 10 years

An estimate for the noise level after 10 years of operation of the ATLAS detector is 1800 electrons (ATLAS Inner Detector Collaboration 1997b). Recent measurements have shown that a noise level of 2000-2200 electrons is actually to be expected for the modules that receive the most irradiation (Lacasta et al. 2002).

Figure 6.3 shows that the noise-occupancy after 10 years requires the threshold to be raised to at most 1.16 fC, compared to the anticipated threshold of 1 fC, which has been confirmed by testbeam.

Figure 6.2 demonstrates that such a small change in threshold does not affect the detection efficiency significantly. This also has been confirmed by testbeam. However, to accomplish a high detection-efficiency after full irradiation, the sensor-bias needs to be raised to 500 V, which is above the maximum specification of 450 V. It has to be investigated whether this specification can be altered.
6.2 Noise sources

6.2.1 Noise mechanisms

In ideal electronics, there are two sources of noise that cannot be reduced: shot noise and thermal noise. Electric currents have shot noise due to the quantisation of charge. Shot noise has a white spectrum. For a current source $I$ the mean square of the shot-noise current per unit of frequency is:

$$\langle i_n^2 \rangle = 2q_e I$$

(6.18)

where $q_e$ is the electron charge.

Resistors give thermal noise due to the thermal movement of the electrons; it also has a white noise-spectrum. For a resistance $R$ the mean square of the thermal noise voltage per unit of frequency is:

$$\langle e_n^2 \rangle = 4kTR$$

(6.19)

where $k$ is the Boltzmann constant and $T$ is the temperature of the resistor. Using Norton's equivalence, thermal noise can also be expressed as a current noise-source in parallel with the resistance:

$$\langle i_n^2 \rangle = \frac{4kT}{R}$$

(6.20)

Real devices, however, have various sources of extra noise. This extra noise mostly depends linearly on the inverse of the frequency and is therefore called $1/f$-noise. In the case of the ABCD these noise sources can be neglected (Posh 1999).

6.2.2 Noise sources in the amplifier

The amplifier of the ABCD is based on a so-called 'Bipolar Junction Transistor' (BJT) (see Section 3.3.3). The operation of the BJT as well as the implementation in CMOS technology is shown in Figure 6.4. Both elements are needed to understand the total noise contribution of the amplifier. Four noise sources are considered.

The operation point of the amplifier is determined by the collector current $I_c$ that is set. The shot noise caused by this current directly influences the output voltage $U_{out}$. However, for convenience, all noise sources will be considered as input noise. To convert the output noise to an input noise, the hybrid-$\pi$ model is used (see for instance (Gray & Meyer 1993)). Using this model, the effective current-noise caused by $I_c$, $\sqrt{\langle i_c^2 \rangle}$, can be described as an effective voltage noise-source $\sqrt{\langle e_c^2 \rangle}$ at the input:

$$\langle e_c^2 \rangle = \frac{\langle i_c^2 \rangle}{g_m^2} \quad \text{and} \quad g_m = \frac{q_e I_c}{kT}$$

(6.21)

Using Equation 6.18 to calculate $\langle i_c^2 \rangle$ gives:

$$\langle e_c^2 \rangle = 4kT \frac{kT}{2q_e I_c}$$

(6.22)
Detectornoisecollectcontactbasestersemitterbasecollector

\[ R_c = \frac{kT}{2q_e I_c} \]  \hspace{1cm} (6.23)

The second noise source is the input or base current, \( I_b \), which is a shot noise source directly connected at the input. Using the current amplification factor \( \beta \) of the transistor, the base current can be expressed in terms of the collector current \( I_c \):

\[ I_b = \frac{I_c}{\beta} \]  \hspace{1cm} (6.24)

In this model, all noise sources are independent. In reality, the base current and the collector current are correlated and thus the noise is not simply the quadratic sum of both contributions. However, a more advanced model will involve more internal parameters of the BJT, while the above approximation only depends on the internal BJT parameter \( \beta \). The BJT parameters are different from batch to batch and are difficult to measure and therefore inaccurately known. Applying a model that is more dependent on these parameters will not have much predictive power. Also, at low sensor capacitance (approximately 10 pF) and high collector-current of the BJT (approximately 220 \( \mu \)A), which are approximately the parameters used in the SCT, the differences

Figure 6.4: Cross section and the principle of operation of a bipolar transistor in CMOS technology. The operation point of the BJT is determined by the collector current \( I_c \) that is set. The output voltage is \( U_{out} \), the input is \( U_{in} \). The feedback circuit consists of a resistance \( R_f \) and a capacitance \( C_f \).
6.2 Noise sources

Three noise sources are considered for the sensor itself; two current noise sources and one voltage noise source.

The first current noise-source is the leakage current $I_l$ of the silicon strip sensor. This is a small contribution for a newly built sensor, but since the leakage current increases a lot due to radiation damage, the shot noise it causes also increases.

The second current noise-source is the sensor bias-resistance $R_d$. Since this resistor is connected to ground, it is treated using the Norton equivalent of the resistor noise.

The voltage source is due to the aluminium traces that are used to read out the silicon. These have a resistance $R_s$.

6.2.4 Summary of the noise sources

Figure 6.5 shows all the noise sources (sensor and amplifier) discussed above, represented as input-noise sources to the amplifier.
Table 6.1: An overview giving the names of the noise sources in a SCT detector module considered in this chapter. The parameters correspond to Figure 6.5.

<table>
<thead>
<tr>
<th>type</th>
<th>name</th>
<th>symbol</th>
<th>related parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>voltage noise</td>
<td>BJT base-spread resistance</td>
<td>$e_{bb}$</td>
<td>$R_{bb}$</td>
</tr>
<tr>
<td></td>
<td>BJT collector current</td>
<td>$e_{c}$</td>
<td>$I_{c}$</td>
</tr>
<tr>
<td></td>
<td>sensor strip resistance</td>
<td>$e_{s}$</td>
<td>$R_{s}$</td>
</tr>
<tr>
<td>current noise</td>
<td>BJT base current</td>
<td>$i_{b}$</td>
<td>$I_{c}$</td>
</tr>
<tr>
<td></td>
<td>feedback resistance</td>
<td>$i_{f}$</td>
<td>$R_{f}$</td>
</tr>
<tr>
<td></td>
<td>strip leakage current</td>
<td>$i_{l}$</td>
<td>$I_{l}$</td>
</tr>
<tr>
<td></td>
<td>strip bias resistance</td>
<td>$i_{d}$</td>
<td>$R_{d}$</td>
</tr>
</tbody>
</table>

To evaluate the total noise at the amplifier they are added in quadrature, i.e. assuming all sources are uncorrelated:

$$\langle e_n^2 \rangle = \sum_k \langle e_k^2 \rangle$$  \hspace{1cm} (6.25)

$$\langle i_n^2 \rangle = \sum_m \langle i_m^2 \rangle$$  \hspace{1cm} (6.26)

where $e_k (k = bb, c, s)$ and $i_m (m = b, f, l, d)$ are the individual voltage and current noise sources.

This simplification is possible because the impedances of $C_d$, $R_d$ and $R_f$ are very much larger than $R_s$, $R_{bb}$, and $R_c$, so that almost all noise passes through the amplifier. From Equation 6.20 one can see that $R_d$ and $R_f$ have to be chosen as large as possible to get as low noise as possible. The sensor capacitance is a high impedance at the frequencies that the amplifier is sensitive to ($\sim$10 MHz).

Table 6.1 gives the names of the individual noise sources that will be used throughout this chapter. Any noise due to the comparator or binary circuit is neglected. To calculate the output noise the response of the system to voltage and current sources needs to be calculated; this is covered in the next section and the output noise is evaluated in Section 6.4.

### 6.3 Signal model

To calculate the output of the analogue part of the ABCD, the model in Figure 6.6 is used. The charge $Q$ is the total charge that is produced by a charged particle in the silicon and reaches the electronics. It will be assumed to be collected in a negligible amount of time (Riedler 1998). Hence the current generated can be represent by a Dirac pulse $\delta(t)$. For signal evaluation, the noise sources $i_n$ and $e_n$ are neglected. The dominant impedance for evaluation of the signal is the total capacitance:

$$C_{tot} = C_d + C_{str} + C_s + C_f$$  \hspace{1cm} (6.27)

where $C_d$ is the total strip capacitance, $C_s$ is the amplifier input capacitance and $C_{str}$ represents stray capacitance from the bond and fan-in. The output of the amplifier can be considered as a virtual ground, and the feedback capacitance $C_f$ is therefore from the signal point of view in
parallel to $C_d$ and $C_a$. The amplifier is capacitively coupled to the strips, however this coupling capacitance is much larger than $C_{\text{tot}}$ and is neglected. The amplifier will be assumed to be ideal with amplification factor $A$. The shaping circuit $H$, as used in the ABCD, can be approximated by a ‘$\text{CR}-(\text{RC})^3$’ filter (Posh 1999).

To calculate the output of this circuit, it is convenient to use the Laplace transformation of a function $x(t)$:

$$\mathcal{L} \left[ x(t) \right] \equiv X(s) \equiv \int_0^\infty e^{-st}x(t) \, dt$$

where, if $t$ is a time variable, $s$ can be interpreted as the complex frequency $i\omega$. The Laplace transformation has two convenient properties that will be used:

$$\mathcal{L} \left[ \delta(t) \right] = 1$$

$$\mathcal{L} \left[ \int_0^t x(u) \, du \right] = \frac{X(s)}{s}$$

The decay time of the input voltage is $R_f C_{\text{tot}}$ which is in the order of 1 $\mu$s, and much larger than the amplifier shaping time (23 ns). So the input voltage can be considered to be a step function and can be written as:

$$U_{A,\text{in}}(s) = \mathcal{L} \left[ \int_0^t \frac{Q\delta(u)}{C_{\text{tot}}} \, du \right] = \frac{Q}{sC_{\text{tot}}}$$

Since it has been assumed that the amplifier is ideal, the output is:

$$U_{A,\text{out}}(s) = \frac{AQ}{sC_{\text{tot}}}$$

The output of the analogue part of the ABCD is:

![Figure 6.6: A simplified diagram of the analogue front-end of the ABCD used to calculate the signal-to-noise ratio.](image)
Detector noise

\[ U_{\text{RC, in}}(s) \]

\[ U_{\text{RC, out}}(s) \]

Figure 6.7: RC diagram, used to calculate the response function.

\[ H(s) = \frac{1}{1+s\tau} \]

\[ |H(s)| \]

\[ 10 \quad 10^2 \quad 10^3 \]

\[ 10 \quad 1 \quad 10^{-1} \quad 10^{-2} \]

\[ 1 \quad 10 \quad 10^2 \quad 10^3 \]

\[ 10^4 \quad 10^5 \quad 10^6 \]

\[ 10^7 \quad 10^8 \]

\[ H(s) = \frac{s\tau}{1+s\tau} \]

\[ |H(s)| \]

\[ 10 \quad 10^2 \quad 10^3 \]

\[ 10 \quad 1 \quad 10^{-1} \quad 10^{-2} \]

\[ 1 \quad 10 \quad 10^2 \quad 10^3 \]

\[ 10^4 \quad 10^5 \quad 10^6 \]

\[ 10^7 \quad 10^8 \]

\[ U_{\text{out}}(s) = H(s) \cdot U_{\text{A, out}}(s) \] (6.32)

This leaves the calculation of the response function \( H(s) \) of the shaping circuit, which can be split into the calculation of the response function of an ‘RC’-circuit and a ‘CR’-circuit cubed:

\[ H(s) = \frac{U_{\text{H, out}}(s)}{U_{\text{H, in}}(s)} = \frac{U_{\text{CR, out}}(s)}{U_{\text{CR, in}}(s)} \cdot \left( \frac{U_{\text{RC, out}}(s)}{U_{\text{RC, in}}(s)} \right)^3 \] (6.33)

The response function of the ‘RC’-circuit can be calculated using Figure 6.7:

\[ \frac{U_{\text{RC, out}}(s)}{U_{\text{RC, in}}(s)} = \frac{i_{\text{RC}}/sC}{i_{\text{RC}}R + i_{\text{RC}}/sC} = \frac{1}{1 + s\tau} \] (6.34)

with \( \tau = RC \). The response function for the ‘CR’-circuit is derived in the same way:

\[ \frac{U_{\text{CR, out}}(s)}{U_{\text{CR, in}}(s)} = \frac{s\tau}{1 + s\tau} \] (6.35)

Figure 6.8 shows the amplitude response of the RC- and CR-circuits.

Combining Equations 6.31 - 6.35 gives the voltage output of the analogue part of the ABCD chip as:

\[ v_s(t) = \mathcal{L}^{-1} \left[ U_{\text{out}}(s) \right] = \frac{QA e^{-\frac{t}{\tau}} t^2}{C_{\text{tot}}} \] (6.36)
6.4. Noise model

Because it can be measured directly, it is more convenient to use the peaking time $T_p$ instead of $\tau$. The peaking time is the time it takes from the start of the signal until the signal maximum, which from Equation 6.36 is:

$$T_p = 3\tau$$  \hspace{1cm} (6.37)

with peak value:

$$v_s(T_p) = \frac{9}{2e^3} \frac{QA}{C_{tot}}$$  \hspace{1cm} (6.38)

In terms of $T_p$ the output voltage is:

$$v_s(t) = \frac{9}{2} \frac{QAe^{-\frac{3t^3}{T_p^3}}}{C_{tot} T_p^3}$$  \hspace{1cm} (6.39)

which can be seen in Figure 6.9.

6.4 Noise model

To calculate the noise output of the analogue part of the ABCD, the model in Figure 6.6 is used (with the input pulse height $Q = 0$). Obviously this is similar to the signal model. All the noise sources are represented in a single voltage noise-source $e_n$ and a single current noise-source $i_n$ per unit frequency, at the input. The total output-noise can be written as:

$$\sqrt{\langle v_n^2 \rangle} = \sqrt{\frac{A^2}{\langle e_n^2 \rangle} \Delta \nu + \frac{\langle i_n^2 \rangle}{\omega^2 C_{tot}^2} \Delta \nu}$$  \hspace{1cm} (6.40)

The effective noise-bandwidth (in Hz) is (Motchenbacher & Fitchen 1973):

$$\Delta \nu = \frac{1}{2\pi} \int_0^\infty |H(s)|^2 d\omega = \frac{1}{2\pi} \int_0^\infty \frac{\omega^2 r^2}{(1 + \omega^2 r^2)^4} d\omega$$  \hspace{1cm} (6.41)

Figure 6.9: Theoretical output signal of the ABCD analogue circuit for a delta pulse input (in units of $\frac{QA}{C_{tot}}$).
using Equations 6.33 - 6.35. Combining Equation 6.40 and 6.41 gives for the total input-noise:

\[
\sqrt{\langle v_n^2 \rangle} = \sqrt{\frac{A^2}{2\pi} \left[ \langle e_n^2 \rangle \int_0^\infty \frac{\omega^2}{(1 + \omega^2\tau^2)^4} d\omega + \frac{\langle i_n^2 \rangle}{C_{tot}^2} \int_0^\infty \frac{\tau^2}{(1 + \omega^2\tau^2)^4} d\omega \right]}
\]

(6.42)

The integrals are:

\[
\int_0^\infty \frac{\omega^2}{(1 + \omega^2\tau^2)^4} d\omega = \frac{\pi}{32\tau}, \quad \int_0^\infty \frac{1}{(1 + \omega^2\tau^2)^4} d\omega = \frac{5\pi}{32\tau}
\]

(6.43)

Combining these with the peaking time from Equation 6.37 finally gives:

\[
\sqrt{\langle v_n^2 \rangle} = \frac{A}{8} \sqrt{\frac{3\langle e_n^2 \rangle}{T_p} + \frac{5T_p \langle i_n^2 \rangle}{3C_{tot}^2}}
\]

(6.44)

If another shaper is chosen, the outcome of the integrals becomes different and the effective noise output changes. One can show that there is an optimal filter, the so-called ‘cusp’ or ‘matched filter’, that minimises the effective output noise (Radeka 1988). The shaper circuit used in the ABCD-chip is a practical approximation to this filter (Posh 1999).

### 6.5 Equivalent Noise Charge

The noise is only important with respect to the signal, so the Signal-to-Noise ratio (SNR) is the important quantity. From Equation 6.44 and Equation 6.38, assuming we probe the signal at its maximum, the ratio is:

\[
\text{SNR} = \frac{v_n(T_p)}{\sqrt{\langle v_n^2 \rangle}} = \frac{36Q}{e^3 \sqrt{\frac{3C_{tot}^2 \langle e_n^2 \rangle}{T_p} + \frac{5T_p \langle i_n^2 \rangle}{3}}}
\]

(6.45)

The value generally used to quantify the noise is derived from this: the Equivalent Noise Charge (ENC). The ENC is the value of the input charge which gives a signal as big as the effective output noise, or, in other words, the charge $Q$ giving SNR = 1.

From Equation 6.45 follows:

\[
\text{ENC} = \frac{e^3}{36q_e} \sqrt{\frac{3C_{tot}^2 \langle e_n^2 \rangle}{T_p} + \frac{5T_p \langle i_n^2 \rangle}{3}} \text{ (electrons)}
\]

(6.46)

where $q_e$ is the electron charge.

### 6.6 Measurement of the module properties

Calculating the noise of ATLAS SCT detector modules requires the knowledge of the amplifier and sensor properties. The properties of the chips are known via the group responsible for the chip design (Dabrowski 2001). Most sensor properties can be measured directly.

The following section discusses the necessary chip parameters. The next section describes the measurement of sensor parameters, including a discussion of the effects of external factors on them. In Section 6.7 the amplifier and sensor properties are used with Equation 6.46 to calculate the ENC for SCT modules. A comparison with measurements on prototype modules is made in Chapter 7.
6.6.1 Chip parameters

Resistors and capacitors in the chip are accurately known due to the use of test structures on the chip wafers. This enables the batch-dependent deviations of the process parameters with respect to the nominal values to be measured. Therefore the feedback impedances are accurately known:

\[ R_f = 77 \pm 1 \, \text{k}\Omega, \quad C_f = 170 \pm 2 \, \text{nF}. \]

The parameters of the BJT are less precisely known, simply because they cannot directly be measured. The current gain \( \beta \) is believed to be 200-250; we take the central value and assess the error by assuming that the real value of \( \beta \) is equally probable along this range. Therefore, the error is \( 1/\sqrt{12} \) times the range: \( \beta = 225 \pm 14 \).

The temperature dependence of the current gain is significant. According to the hybrid \( \pi \)-model the current gain is:

\[ \beta = \frac{qI_c}{kT g_e} \quad (6.47) \]

where \( g_e \) is the emitter resistance. The emitter is a highly doped \( n^+ \) region. The temperature dependence of highly doped material can be assumed constant in a large region around room temperature.

The base-spread resistance is 300 \( \Omega \) according to the process information. This parameter is determined by the effective thickness of the base, which is not very accurately known. Therefore we assume a large error of 10%.

The temperature dependence of the base-spread resistance is also significant. The base is lightly doped \( p \) silicon. The temperature dependence of this material in a large region around room temperature is described by lattice scattering and has a \( T^{-3/2} \) dependence.

The amplifier input-capacitance is 1 \( \pm 0.1 \) pF. The peaking time \( T_p \) has been measured and is found to be 23 \( \pm 0.5 \) ns (Gadomski & Reznicek 2001).

6.6.2 Sensor parameters

6.6.2.1 Strip and strip bias resistance

To avoid the influence of probe contact-resistance, the strip resistance was measured using two probes to send a current through the strip and two other probes to measure the voltage across the strip (a so-called four-point measurement). The voltage was measured at different currents; fitting a straight line through the data gave the resistance as 86 \( \pm 1 \) \( \Omega \) for a 7.2 cm strip (the wafer was for inner modules before they were shortened). Hence the resistance per unit length of the aluminium is 11.9 \( \pm 0.1 \) \( \Omega \)/cm, which is in agreement with other measurements (Richter 2002). This is well within the ATLAS specification, 15 \( \Omega \)/cm (ATLAS Semi-Conducter Tracker Collaboration 1999).

The strip bias resistance is specified to be 1.25 \( \pm 0.75 \) k\( \Omega \).

6.6.2.2 Strip leakage current

The main contribution of the leakage current in a read-out strip is caused by the generation current, which is proportional to the number of intrinsic charge carriers in the depleted region. The
dependence of the leakage current $I_l$ on the temperature $T$ can therefore be expressed as (Sze 1985):

$$I_l \sim T^{3/2} \cdot e^{-\frac{E_g}{2kT}}$$

(6.48)

where $E_g$ is the silicon band gap energy and $k$ is the Boltzmann constant. Since it is only an approximation that the generation current dominates, the SCT community uses the following empirical expression:

$$I_l \sim T^2 \cdot e^{-\frac{1}{2kT}}$$

(6.49)

which fits the leakage current of an SCT sensor well. The bandgap energy has been increased from 1.12 to 1.2 eV (see Table 3.3). A rule of thumb is that the leakage current doubles when the temperature changes approximately 8 K.

The leakage current is measured using the Detector Control-System of the module test set-up (see Section 5.2.2). The disadvantage of this method is that the leakage current measured is the sum of the current through the strips and the current through the guard ring structure. The latter is not well described by the generation current alone. Also an Ohmic resistance has to be taken into account, describing surface leakage currents. The total leakage current can be expressed as a function of the applied sensor bias (Koffeman 1996):

$$I_l = \begin{cases} A \cdot V_d + B \cdot V_d + C & V_d \leq V_{dep} \\ A \cdot V_{dep} + B \cdot V_d + C & V_d > V_{dep} \end{cases}$$

(6.50)

where $A$, $B$ and $C$ are constants. The first term describes the charge generation current contribution, the second term describes the surface current contribution and the last term accounts for a possible measurement offset. Figure 6.10 shows Equation 6.50 fitted to real data, and Table 6.2 gives the resulting constants\(^3\). Assuming the contribution for the generation current term in Equation 6.50 comes mainly from the strips gives a leakage current per strip of 0.2 nA or 0.04 nA/cm.

The leakage current after irradiation with the maximum expected dose after ten years of operation of ATLAS is predicted to be approximately 0.1 $\mu$A/cm (Riedler 1998).

### Capacitance measurements

The ABCD amplifier sees not only the capacitance of a strip to the sensor backplane, but also an extra capacitance due the neighbouring strips (see Figure 6.11). The total capacitance from

<table>
<thead>
<tr>
<th></th>
<th>A ($\mu$A/$\sqrt{V}$)</th>
<th>B ($\mu$A/V)</th>
<th>C ($\mu$A)</th>
<th>$V_{dep}$ (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>front side</td>
<td>$6.3 \times 10^{-2}$</td>
<td>$2.3 \times 10^{-3}$</td>
<td>$-4.2 \times 10^{-3}$</td>
<td>78</td>
</tr>
<tr>
<td>back side</td>
<td>$2.4 \times 10^{-1}$</td>
<td>$0.4 \times 10^{-3}$</td>
<td>$2.7 \times 10^{-2}$</td>
<td>58</td>
</tr>
</tbody>
</table>

Table 6.2: Results of the fit of the leakage current vs sensor bias voltage of the two sensors of NIKHEF module number 13.

\(^3\)The sensors used here are second-grade material, since these were the only ones available for the first prototypes. The sensors to be used at ATLAS have a typical total leakage current of 0.15 $\mu$A at 100 V sensor bias.
6.6. Measurement of the module properties

Figure 6.10: Fit of the leakage current vs sensor bias voltage of the two sensors of NIKHEF module number 13 with Equation 6.50

Figure 6.11: Simplified representation of several adjacent strips connected to the ABCD. $C_{ss}$ and $C_{sb}$ are the strip-to-strip and strip-to-backplane capacitances.

detector ($C_d$) that is seen at the input of the amplifier can be expressed as:

$$C_d = C_{sb} + C_{is}$$  \hspace{1cm} (6.51)

where $C_{sb}$ is the strip-to-backplane capacitance and $C_{is}$ is the effective capacitance due to the neighbouring strips.

Figure 6.12 shows the set-up that has been used to measure the capacitance of a strip of a
sensor from CiS for a forward Inner-Detector module. Since only the two direct neighbours are connected with ground, the measured capacitance is different from when all strips are connected, as is the case for a bonded module.

To disentangle the strip-to-backplane and interstrip capacitances the results of numerical calculations with ToSCA of the two capacitances have been used (Richter 2002). For a barrel sensor (strip pitch: 80 μm) the model predicts for the situation of Figure 6.12: $C_{sb} + 2 \cdot C_{ss} = 1.031 \text{ pF/cm}$, where $C_{ss} = 0.331 \text{ pF/cm}$ and $C_{sb} = 0.370 \text{ pF/cm}$. Also the situation where nine neighbours are connected to ground were simulated, as an approximation to a sensor with all strips bonded. For this situation the following values were found: $C_{ss} = 0.399 \text{ pF/cm}$ and $C_{sb} = 0.263 \text{ pF/cm}$. These predictions agree with measurements. The strip-to-backplane capacitance for a fully bonded sensor is close to what is expected for a plate capacitor with the dimensions of a barrel strip with pure silicon in between the plates, if one ignores edge effects. A sensor is a complicated structure of different types of silicon. The difference between the numerical model and the plate-capacitor model is used to calculate an effective dielectric constant $\epsilon_r$ for the plate-capacitor model of SCT detectors. With this $\epsilon_r$, the strip-to-backplane capacitance $C_{sb}$ for any SCT sensor-type can be estimated, using the average pitch. The interstrip capacitance can then be derived from the capacitance $C_{tot, meas}$, measured with the set-up shown in Figure 6.12:

$$C_{ss} = \frac{1}{2} \left( C_{tot, meas} \cdot 1.03 - C_{sb} \right)$$  \hspace{1cm} (6.52)

where the constant has been calculated comparing the ToSCA predictions for the interstrip capacitance when two neighbouring strips are grounded and for when all strips are grounded.
6.6. Measurement of the module properties

<table>
<thead>
<tr>
<th>type</th>
<th>( \langle p \rangle ) (( \mu )m)</th>
<th>( C_{\text{tot}} ) (pF/cm)</th>
<th>( C_{ss} ) (pF/cm)</th>
<th>( C_{sb} ) (pF/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12</td>
<td>63</td>
<td>0.79</td>
<td>0.30</td>
<td>0.21</td>
</tr>
<tr>
<td>W21</td>
<td>76</td>
<td>0.73</td>
<td>0.24</td>
<td>0.25</td>
</tr>
<tr>
<td>W22</td>
<td>89</td>
<td>0.67</td>
<td>0.19</td>
<td>0.29</td>
</tr>
<tr>
<td>W31</td>
<td>76</td>
<td>0.75</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>W32</td>
<td>86</td>
<td>0.69</td>
<td>0.21</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 6.3: The total strip capacitance, the interstrip, and the strip-to-backplane capacitances of all forward module type sensors. The average pitch \( \langle p \rangle \) is also indicated.

Figure 6.13: Comparison of results \( C_{ss} \) for the five different types of SCT end-cap sensors.

The results are summarised in Table 6.3. The measurements of the total capacitance of W12 were performed at NIKHEF, the other total capacitance measurements were at the Max Planck Institute in Münich (MPI) (Richter 2002). On all results, an error of 5% is assumed.

Figure 6.13 shows the interstrip capacitance of all sensor types as function of the average strip pitch for comparison with other sensors. The interstrip capacitance decreases linearly with increasing strip pitch: the capacitive coupling becomes smaller. This dependence has been carefully evaluated for many sensors in (Demaria et al. 2000), and is found to be correct. Figure 6.13 shows that the measurement result from NIKHEF (the point at average pitch of 63 \( \mu \)m) is consistent with the measurements made at MPI (the other four points).

The strip-to-backplane capacitance depends on the thickness of the depletion layer and thus the applied sensor voltage. This dependence can be expressed as (Sze 1985):

\[
\frac{1}{C_{sb}^2} = \begin{cases} 
\frac{V_d}{S} & \text{for } V \leq V_{\text{dep}} \\
\frac{V_{\text{dep}}}{S} & \text{for } V > V_{\text{dep}} 
\end{cases}
\]

(6.53)

where \( S \) is a constant, depending on the doping concentration. The built-in \( V_{bi} \) in junction potential, changing the effective sensor bias, can be neglected (Lutz 1999).

The depletion voltage was more accurately determined than in Section 6.6.2.2 by using this Equation 6.38, by measuring the signal height as function of sensor bias for an Atlas baby sensor. This sensor consisted of 60 strips of 1 cm length. The strips are bonded to form one single read-out strip. All the capacitance is assumed to be strip-to-backplane capacitance. The baby sensor
Detector noise

Figure 6.14: Fit of the pulse height vs bias. Figure (a) shows the fit of a signal spectrum at 100 V sensor bias. Figure (b) shows the square of the most-probable value of the Landau distribution from the fit as function of the bias voltage. The intersection of the two lines is quoted as $V_{dep}$.

was biased and read-out using set-up available at NIKHEF (Adam et al. 2002). The signal was generated with a $\beta$-source.

Figure 6.14 (a) shows the measured signal-height spectrum, fitted with Equation 6.13, assuming no offset and a linear gain. The fit parameters were the gain in (mV/\text{fC}), assuming a charge collection efficiency of 100%, and the electronic noise. In principle, the effective detector thickness $t$ should be fitted as well. This is not done in this case, since the fit here is only used to find the peak-position and the method as described is found to be effective. When the sensor is fully depleted, and the approximately correct thickness is used, the electronic noise found using this fit is $\sigma_N = 7.5 \pm 2.7$ mV. Measurements of the signal spectrum when the $\beta$-source is removed show a Gaussian distribution with a width of $6.27 \pm 0.05$ mV, which confirms the measurement of the noise from fitting the signal-height spectrum.

Figure 6.14 (b) shows the dependence of the signal on the sensor bias. It fits the description of Equation 6.53 well, except for the region close to full depletion. The depletion region grows from the $p^+$-side of the strip to the backplane (see Section 3.3.2.1). The observed deviation is suspected to be due to the $n^+$-region that ensures contact to the metalised backside of the sensor. The deviation is small and the model fits the other data well. Ignoring the data-points in the affected region, the depletion voltage is found to be $43$ V. This value is used in the remainder of this thesis. The values for the depletion voltage found in Table 6.2 do not agree with the value found here, however the sensors used for the leakage-current measurement are of second-grade quality.

The stray input-capacitances have to be estimated. These are expected to be high, due to
the large pitch adapters and long bonds. For the barrel the stray capacitance is known to be 0.5 pF (Dabrowski 2001). From geometrical arguments, it is estimated that the stray capacitance for a forward module is about 1.5 pF. Since this is only an educated guess, the error is taken to be approximately 10%: $C_{str} = 1.5 \pm 0.2$ pF.

### 6.7 Simulation

We now have all the ingredients needed to calculate the noise, including the temperature dependence. Combining Equation 6.46 with the specific noise equations from Section 6.2 and Equations 6.49-6.53 leads to the following expression for the total noise in an SCT silicon-strip detector:

$$ENC = \frac{e^3}{36q_c} \left\{ \frac{3}{T_p} (C_{is} + C_{sb} + C_{str} + C_a + C_f) \left[ 4kT_c R_{bb} + \frac{k^2 T_c^2}{2q_e I_c} + 4kT_s R_s \right] + \frac{5T_p}{3} \left( \frac{2q_e I_c}{\beta} + \frac{4kT_c}{R_f} + 2q_e I_t + \frac{4kT_s}{R_s} \right) \right\}^{\frac{1}{2}}$$

with the temperature and bias dependence of the parameters given by:

$$C_{sb} = \begin{cases} C_0 \cdot \sqrt{\frac{V_d}{V_{dep}}} & V_d \leq V_{dep} \\ C_0 & V_d > V_{dep} \end{cases}$$

$$I_t = \begin{cases} I_0 \cdot T_s^2 \cdot e^{\frac{1.24(T_s - T_{s,0})}{2kT_{s,0}}} \cdot \sqrt{\frac{V_d}{V_{dep}}} & V_d \leq V_{dep} \\ I_0 \cdot T_s^2 \cdot e^{\frac{1.24(T_s - T_{s,0})}{2kT_{s,0}}} & V_d > V_{dep} \end{cases}$$

$$R_{bb} = R_0 \frac{T_c^{1.5}}{T_{c,0}^{1.5}}$$

$$\beta = \beta_0 \frac{T_c}{T_e}$$

where $C_0$ is the fully depleted strip-to-backplane capacitance, $T_c$ and $T_s$ are the chip and sensor temperatures, $I_0$ is the leakage current at the sensor temperature $T_{s,0}$, $R_0$ is the bias-spread resistance at the chip temperature $T_{c,0}$, and $\beta_0$ is the current gain at the chip temperature $T_{c,0}$.

Table 6.4 summarises the values used for the noise calculation for SCT modules as discussed in Section 6.6.

Table 6.5 shows the individual contribution of the seven different noise sources and the total noise for different module configurations. To calculate the SNR, a signal of $22.9 \times 10^3$ electrons was assumed, as predicted in Section 6.1.1.

The extrapolation to the other module types from the long inner type has been made by scaling the values for $R_s$ and $I_0$ with the nominal strip length of the modules and using the capacitances in Table 6.3. The base-spread resistance gives the highest contribution for all modules.

Table 6.6 shows which parameter dominates the error on each noise source.
Detecto rr nois e
name | parameter | value
feedback resistance | $R_f$ | $77 \pm 1 \text{k}\Omega$
feedback capacitance | $C_f$ | $170 \pm 2 \text{fF}$
current gain | $\beta_0$ | $225 \pm 14$
base-spread resistance | $R_0$ | $300 \pm 30 \text{\Omega}$
amplifier input capacitance | $C_a$ | $1.00 \pm 0.1 \text{pF}$
peaking time | $T_p$ | $23 \pm 0.5 \text{ns}$
sensor strip resistance | $R_s$ | $12.0 \pm 0.1 \text{\Omega/cm}$
strip bias resistance | $R_b$ | $1.25 \pm 0.75 \text{k}\Omega$
interstrip capacitance | $C_{is}$ | $0.58 \pm 0.01 \text{pF/cm}$
strip-to-backplane cap. | $C_0$ | $0.21 \pm 0.01 \text{pF/cm}$
stray capacitance | $C_{str}$ | $1.5 \pm 0.2 \text{pF}$
collector current | $I_c$ | $220 \text{\mu A}$
chip temperature | $T_c$ | $8 \text{\degree C}$
strip leakage current | $I_0$ | $0.04 \text{nA/cm}$
sensor bias voltage | $V_d$ | $100 \text{V}$
depletion voltage | $V_{dep}$ | $43 \text{V}$
sensor temperature | $T_s$ | $-7 \text{\degree C}$
reference temperature sensor | $T_{s,0}$ | $300 \text{K}$
reference temperature chip | $T_{c,0}$ | $300 \text{K}$

Table 6.4: Summary of the parameters needed for the noise calculation. The values are for an ATLAS SCT inner detector module. The errors stated are mostly estimates.

<table>
<thead>
<tr>
<th>noise source</th>
<th>hybrid</th>
<th>inner</th>
<th>middle</th>
<th>outer</th>
</tr>
</thead>
<tbody>
<tr>
<td>base-spread resistance</td>
<td>$218 \pm 21$</td>
<td>$684 \pm 50$</td>
<td>$879 \pm 58$</td>
<td>$929 \pm 60$</td>
</tr>
<tr>
<td>sensor strip resistance</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
</tr>
<tr>
<td>collector current</td>
<td>$98 \pm 8$</td>
<td>$375 \pm 20$</td>
<td>$611 \pm 26$</td>
<td>$665 \pm 26$</td>
</tr>
<tr>
<td>base current</td>
<td>$369 \pm 12$</td>
<td>$369 \pm 12$</td>
<td>$369 \pm 12$</td>
<td>$369 \pm 12$</td>
</tr>
<tr>
<td>feedback resistance</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
<td>$306 \pm 3$</td>
</tr>
<tr>
<td>sensor leakage current</td>
<td>$1 \pm 0$</td>
<td>$1 \pm 0$</td>
<td>$1 \pm 0$</td>
<td>$1 \pm 0$</td>
</tr>
<tr>
<td>sensor bias resistance</td>
<td>$73 \pm 22$</td>
<td>$73 \pm 22$</td>
<td>$73 \pm 22$</td>
<td>$73 \pm 22$</td>
</tr>
<tr>
<td>total noise</td>
<td>$536 \pm 12$</td>
<td>$969 \pm 38$</td>
<td>$1241 \pm 39$</td>
<td>$1304 \pm 46$</td>
</tr>
<tr>
<td>SNR</td>
<td>43</td>
<td>24</td>
<td>18</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 6.5: The relative contributions of the seven different noise sources calculated for a hybrid and all different forward module configurations as calculated using Equation 6.54. The first three noise sources are the voltage noise-sources, the last four noise sources are the current noise-sources.
Table 6.6: The parameters giving the main error contributions.

<table>
<thead>
<tr>
<th>noise source</th>
<th>dominant parameter</th>
<th>index</th>
</tr>
</thead>
<tbody>
<tr>
<td>base-spread resistance</td>
<td>$R_{bb}$</td>
<td>1</td>
</tr>
<tr>
<td>collector current</td>
<td>$C_{tot}$</td>
<td>2</td>
</tr>
<tr>
<td>sensor strip resistance</td>
<td>$C_{tot}$</td>
<td>3</td>
</tr>
<tr>
<td>base current</td>
<td>$\beta$</td>
<td>4</td>
</tr>
<tr>
<td>feedback resistance</td>
<td>$R_f$</td>
<td>5</td>
</tr>
<tr>
<td>sensor leakage current</td>
<td>$T_p$</td>
<td>6</td>
</tr>
<tr>
<td>sensor bias resistance</td>
<td>$T_p$</td>
<td>7</td>
</tr>
</tbody>
</table>

Figure 6.15: Results of the calculated noise for a (long) inner module as function of the BJT collector current using Equation 6.54. The values of the parameters are taken from Table 6.4. The two noise sources that vary with collector current are also shown. The grey band indicates the one-sigma error.

From this table it follows that some of the errors of the individual noise sources are strongly correlated. To allow for this, the error on the total noise is calculated as:

\[
\begin{align*}
\sigma_{n_1}^2 &= \sqrt{\sigma_1^2 + (\sigma_2 + \sigma_3)^2} \\
\sigma_{n_2}^2 &= \sqrt{\sigma_3^2 + \sigma_5^2 + (\sigma_6 + \sigma_7)^2} \\
\sigma_{N^2}^2 &= \sqrt{\sigma_{n_1}^2 + \sigma_{n_2}^2} \\
\sigma_N &= \frac{\sigma_{N^2}}{2N}
\end{align*}
\]
where $\sigma_{V^2}$ is the error on the total noise squared caused by the voltage noise sources, $\sigma_{I^2}$ is the error on the total noise squared caused by the current noise sources, $\sigma_N$ is the error on the total noise squared and $\sigma_N$ is the error on the noise $N$. The error on the square of the individual contributions $\sigma_i$ are the contributions of the individual noise sources following the labelling from Table 6.6.

Figure 6.15 shows the dependence of the noise on the most important parameter of the amplifier operation: the collector current. The two noise sources influenced by the collector current are shown as well. Both noise sources counteract, causing the total noise to change not more than approximately 30 electrons over a region of 100 $\mu$A. The minimum noise is found at 200 $\mu$A. The module is normally operated using a collector current of 220 $\mu$A.

Table 6.5 enables an evaluation of an centre-tapped configuration with respect to a end-tapped configuration. The forward modules are end-tapped: the amplifier is connected at the end of one strip. The barrel modules are centre-tapped: the amplifier is connected in between two sensors. This has no consequences for the capacitance, but quarters the strip-resistance $R_s$ as seen from the amplifier, because the strips are in parallel for the centre-tapped case. This halves the noise contribution from $R_s$. For outer modules, this would reduce the noise from 1304 electrons end-tapped to 1253 electrons centre-tapped.

The model also predicts a temperature dependence of the noise. The sensor temperature was varied from 250 to 350 K with the chip temperature always 15 °C hotter. In this region, the model predicts a linear temperature dependence. Table 6.7 gives the slopes of this dependence. As seen in Chapter 7 this is close to what is observed in real modules.

<table>
<thead>
<tr>
<th>module type</th>
<th>temperature dependence ENC (electrons/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>hybrid</td>
<td>$1.22 \pm 0.08$</td>
</tr>
<tr>
<td>inner</td>
<td>$2.84 \pm 0.10$</td>
</tr>
<tr>
<td>middle</td>
<td>$4.18 \pm 0.12$</td>
</tr>
<tr>
<td>outer</td>
<td>$4.41 \pm 0.13$</td>
</tr>
</tbody>
</table>

*Table 6.7: The predicted dependence of noise on temperature.*

Finally notice that in Table 6.5 the strip leakage-current does not contribute to the noise at all. In the fast electronics used here, the noise is dominated by sources in the amplifier. Even after full irradiation, the leakage current is expected to be 0.1 $\mu$A per centimetre of strip, contribution about 70 electrons to the noise, still insignificant compared to the 2000 electrons noise in irradiated modules. Increase in the interstrip capacitance, reduction of the amplifier current gain $\beta$, and increase of the base-spread resistance through radiation damage are all far more important. The main problem of increased leakage current is that it can lead to thermal runaway (see Section 3.2.3).
6.8 Conclusion

The noise model in this chapter demonstrates a few crucial points. First, the temperature dependence is an important effect (see Table 6.7). The different module testing systems used in the SCT collaboration have different cooling systems giving widely varying chip temperatures; a difference of 20 °C changes the noise by 100 electrons. Therefore, in comparing results it is important to quote the chip (or hybrid) temperature during the measurement.

Second, most of the noise is generated by the amplifier (see Table 6.5). The biggest noise source of the ABCD is the base-spread resistance $R_{bb}$. Since this a voltage noise-source, the effect of this noise is proportional to the total capacitance seen by the amplifier. The influence of the sensor is mostly its capacitance.

Third, the end-tap configuration is not ideal; a centre-tap configuration would reduce the strip resistance, reducing the noise by about 50 electrons. An extra advantage of the centre-tap configuration is the possibility to use smaller fan-ins and bonds, reducing the stray input-capacitance.
Detector noise