Measurement of Z boson pair production and a search for the Higgs boson in e+e-collisions at LEP
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Chapter 6

NC02 (ZZ) cross section measurement

In 1997 LEP reached a centre-of-mass energy of 183 GeV, allowing for the first time on-shell Z bosons to be pair produced. In this chapter the measurement of the ZZ production cross section (through NC02 diagrams, see figure 2.8) is presented in case both Z bosons decay hadronically.

Measuring this small (~ 0.5 pb) and well predicted cross section at all centre-of-mass energies is interesting for various reasons. It allows to test the SM prediction, and, since both the cross section and the angular distribution of the produced Z bosons are sensitive to contributions from physics beyond the SM it also allows to set direct limits on parameters like for example anomalous neutral trilinear gauge couplings. Another important reason however to obtain a good estimation of the cross section is that ZZ events provide an environment that is both in terms of experimental signature\(^1\) and in cross section similar to a possible Higgs signal. All the ingredients in the Higgs search (b-tagging, heavy boson mass reconstruction and topological variables) also play an important role in this analysis. By performing the measurement of the ZZ cross section the existing Higgs analyses can be tested for sensitivity to such a signature and important systematic effects involved in examining final states with these specific characteristics will show up in this channel as well.

In section 6.1 the ZZ signal is defined on the level of matrix elements and the characteristic (experimental) properties of its 4-quark final state are discussed in section 6.2. In section 6.3 all the information from the measured event is combined and the distribution of the ZZ-probability is shown. The performance and separation power together with the procedure used to extract the ZZ cross section from this distribution are described in section 6.4. An evaluation of the main systematic effects contributing to the uncertainty on the extracted ZZ cross section (using the full LEP2 data set) is described in section 6.4.5. Finally the measured ZZ cross sections are translated into pure NC02 cross sections and compared to the SM expectation in section 6.5.

A dedicated ZZ → b\(\bar{b}\)q\(\bar{q}\) measurement is performed in section 6.6 to illustrate the flexibility of the event-by-event probability computations and because this particular final state is almost identical to a possible Higgs signal (a heavy object decaying into a b\(\bar{b}\) pair recoiling against a Z boson). The chapter ends with an indication of possible improvements in the analysis and a conclusion.

\(^1\) At LEP the Higgs boson is almost always pair-produced together with a Z boson.
6.1 ZZ signal definition

In the SM the possible 4-quark final states originating from pure NC02 diagrams also receive contributions from other diagrams (non-NC02 diagrams and interferences). To measure the cross section for on-shell Z boson pair production it is necessary to define an unambiguous experimentally accessible NC02-like (ZZ) signal for generated 4-fermion final states (EXCALIBUR). Once this ZZ cross section is experimentally extracted it can be extrapolated to a pure NC02 cross section (see the right plot of figure 6.1) using the relationship between the predicted ZZ and NC02 (YFSZZ calculations [40]) cross sections. In the analysis ZZ (W^+W^-) events are defined using for each generated event both the full matrix element (all diagrams) and that calculated using only the NC02 (CC03) diagrams:

\[
\mathcal{R}_{\text{NC02}} = \frac{|\mathcal{M}_{\text{NC02}}|^2}{|\mathcal{M}_{\text{full}}|^2} \quad \mathcal{R}_{\text{CC03}} = \frac{|\mathcal{M}_{\text{CC03}}|^2}{|\mathcal{M}_{\text{full}}|^2}
\]

\[
ZZ = \mathcal{R}_{\text{NC02}} \geq 0.5 \quad W^+W^- = \mathcal{R}_{\text{CC03}} \geq 0.5
\]

The distribution of \( \mathcal{R}_{\text{NC02}} \) for generated 4-fermion events with 4 quarks in the final state at a centre-of-mass energy of 200 GeV is shown in the left plot of figure 6.1.

![ZZ events](image)

Figure 6.1: The left plot shows the distribution of \( R_{\text{NC02}} \) for generated (EXCALIBUR) 4 quarks final states at a centre-of-mass energy of 200 GeV. The right plot shows the ratio of \( \sigma_{\text{NC02}} \) and \( \sigma_{\text{ZZ}} \).

Events are now classified as either ZZ, W^+W^- or, if none of the two conditions in equation 6.2 is satisfied, as \( Z/\gamma^* \). The pre-selection efficiencies for hadronic ZZ events are evaluated in section 4.5 for different centre-of-mass energies and are close to 90%.

\[2\] Due to the interference between (sets of) diagrams, a small fraction of the 4-fermion events (\( \sim 5 \cdot 10^{-5} \)) is simultaneously ZZ and W^+W^- according to the definition presented in equation 6.2. In this analysis these events are defined to be ZZ.
6.2 Characteristics of the ZZ final state

The ZZ fully hadronic final state has some characteristics that are quite distinct. To show their potential in separating the ZZ signal from the W⁺W⁻ and q̅q(γ) background the two most important ones are discussed: a pair of b-quarks present in the final state and one of the jet-pairings has two reconstructed di-jet masses close to M_Z.

B-tag configurations

The decay of the Z boson has been studied extensively using the millions of Z bosons collected at LEP1. This resulted not only in a precise measurement of the hadronic decay ratio, but also of the relative branching fraction of the Z boson into quark-pairs of various flavours. In short: about half (48.9%) of the ZZ decays is predicted to result in 4 quarks in the final state and in 38.7% of those fully hadronic events there are at least 2 b-quarks in the final state (see table 5.1).

In section 5.1 it was shown that the b-tag information is quite powerful in identifying a jet from b-quark fragmentation. As an illustration of the separation power of the b-tag information the distribution of the combined b-tag per event (defined as the sum of the combined b-tag per jet of the two most b-tagged jets in the event) is shown in figure 6.2 for the two main backgrounds and two ZZ final states: with and without b-quarks. There is clear separation between events with and without a pair of b-quarks and in the distribution for q̅q(γ) events the contribution from b-quark pair production (∼ 16%, see section 5.1.1) can be clearly identified. The agreement between the data and predictions in the full 183-207 GeV data set is shown in the right plot of figure 6.2.

Figure 6.2: The left plot shows the (normalised) distribution of the combined b-tag per event (sum of the combined b-tag per jet of the two most b-tagged jets in the event) for various event types. The right plot shows the comparison between data and Monte Carlo using the full 183-207 GeV data set.
**Mass distributions**

The distribution of the masses of the two Z bosons in a ZZ event is given by equation (5.4), with $M_Z = 91.2 \text{ GeV}/c^2$ and $\Gamma_Z = 2.49 \text{ GeV}/c^2$. The method to assign a compatibility with a ZZ hypothesis for each jet-pairing has been described in section 5.2 and the separation power in identifying ZZ events using only the mass information is shown in figure 5.7. As another illustration of the separation power from the mass information alone, the procedure used by most analyses at LEP to extract mass information from processes where two heavy bosons are pair-produced is discussed: a 5C(a)-fit to define the so-called *equal mass* in the event.

**5C(a)-fit:** Like a 4C-fit and in addition for a given pairing both di-jet masses are equal.

In addition to requiring energy and momentum conservation (4C-fit) on the reconstructed event, an additional constraint is imposed that for a given pairing both di-jet masses are equal:

$$\sum_{i=1}^{n_{jets}} (p_i^{(fitted)}) \cdot (0, 0, 0, \sqrt{s}) \quad \& \quad \text{for this pairing } M_{ij} = M_{kl}$$

(6.3)

**Why a 5C(a)-fit:** To find the equal mass (pairing) in the event

The pairing in the event with the best $\chi^2$, once fitted with constraints given by equation (6.3) is defined as the equal mass pairing. Once this pairing is chosen, the *equal mass* is defined as the di-jet mass in that pairing after the fit. The equal mass distribution for ZZ events and the two physics background processes at a centre-of-mass energy of 200 GeV can be seen in figure 6.3. For ZZ events a Breit-Wigner fit indicates the peak position and width of the distribution.

![Equal mass distribution plots](image)

Figure 6.3: The number of expected events for various event types as a function of the equal mass at a centre-of-mass energy of 200 GeV. The left plot shows the distribution for ZZ events and the right plot for $W^+W^-$ and $q\bar{q}(\gamma)$ events.

Figure 6.3 shows a few important features. The fact that the equal mass cannot exceed $\frac{1}{2} \sqrt{s}$ is clear from energy conservation, but to understand the rising distribution of the $q\bar{q}(\gamma)$ background...
The distribution for $W^+W^-$ events has in addition a large tail towards higher masses\(^3\). This is due to the fact that the ISR is neglected in the constrained fit, resulting in an overestimation of the jet energies as explained in section 4.3.3. A fit to the central part of the distribution yields slightly higher values than the pole mass as is shown in the left plot of figure 6.3 for ZZ events (Z boson mass is 91.2 GeV/$c^2$). Even though the W and the Z mass are more than 10 GeV/$c^2$ apart there is a significant fraction of $W^+W^-$ events with an equal mass close to $M_Z$.

### 6.3 ZZ-probability distributions

To obtain optimal sensitivity for the ZZ signal the information extracted from the measured event is compared with the SM expectation following the procedure described in section 5.4 and an event-by-event ZZ compatibility ($P_{ZZ}$) is computed. Relative probabilities are obtained by normalising the compatibilities to the sum of the three SM hypotheses. In the remainder of this section the ZZ-probability is therefore defined as:

$$P_{ZZ} = \frac{P_{ZZ}}{P_{ZZ} + P_{WW} + P_{QCD}} \quad (6.4)$$

The normalised distributions of $\ln(P_{ZZ})$ for ZZ events and the combined background for the $W^+W^-$ and $q\bar{q}(\gamma)$ processes at a centre-of-mass energy of 200 GeV are shown in figure 6.4.

The double peak structure in the ZZ-probability distribution for ZZ events reflects the two distinct final states (with and without a $b$-quarks pair). From the last two sections it is clear that the separation strength of various variables is different and this is reflected in the structure of the distribution. The real separation power is obtained when both the signal and backgrounds are scaled to equal luminosity (the expected number of background events is about 20 times larger than the number of ZZ events). In the left plot of figure 6.5, the number of expected events at a centre-of-mass energy of 200 GeV is shown as a function of $P_{ZZ}$ for all SM processes separately.

To indicate the contribution of the ZZ events, the ZZ purity (per bin) is calculated as a function of $P_{ZZ}$ and shown in the right plot of the figure. As can be seen, the calculated probability is almost a real purity, a property that is shared by all centre-of-mass energies. The advantage of this is clear when combining different centre-of-mass energies since distributions can be simply added without losing sensitivity. It is this property that is used in section 6.4 to add the distributions from the 183-207 GeV data set to create a 'LEP2' ZZ-probability distribution. This allows to obtain good control of both the data/Monte Carlo agreement and a statistically more significant understanding and estimation of the systematic uncertainties.

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\(^3\) For ZZ events as well when they are produced well above threshold.
NC02 (ZZ) cross section measurement

Figure 6.4: Normalised distributions of the logarithm of the ZZ-probability for ZZ events and for the combined $W^+ W^-$ and $q\bar{q}(\gamma)$ background at a centre-of-mass energy of 200 GeV.

Figure 6.5: The distribution of the ZZ-probability at a centre-of-mass energy of 200 GeV. The data are represented by the points and SM expectations by the solid histograms. In the right plot the ZZ purity (per bin) is calculated as a function of the computed ZZ-probability.
The agreement between data and expectation over the full range of ZZ probabilities is indicated by plotting the number of events (data and that expected from the SM) at increasingly stringent levels of purity (for ZZ events with a 4-quark final state). This plot, as shown in figure 6.6, allows in addition to study the background composition at any given efficiency (purity). Note that neighbouring bins are fully correlated.

![Number of expected events left for various processes as a function of the efficiency for ZZ-events with a 4-quark final state at a centre-of-mass energy of 200 GeV. The number of events observed in the data is represented by the points.](image)

**Figure 6.6:** Number of expected events left for various processes as a function of the efficiency for ZZ-events with a 4-quark final state at a centre-of-mass energy of 200 GeV. The number of events observed in the data is represented by the points.

### 6.4 Extraction of the ZZ cross section

The ZZ cross section is extracted by means of a binned maximum likelihood fit to the ZZ-probability distribution, thereby exploiting the differences in the shape of the distribution for signal and background. In the fit only the contribution from ZZ events is varied.

First, the effect of the dominant systematic uncertainty on the extracted cross section (the uncertainty on the background $q\bar{q}(\gamma)$ cross section) is evaluated to define the optimal region of the ZZ-probability distribution to be used in the fit. Then, the method used to extract the ZZ cross section is defined and the results are given for all centre-of-mass energies. After an evaluation of the dominant systematic errors the ZZ cross sections are finally (section 6.5) translated into a pure NC02 cross section and compared to the SM prediction.
6.4.1 Selecting the fit-region

The distribution of the purity (per bin) of ZZ events as a function of the computed ZZ-probability (figure 6.5) is identical for all centre-of-mass energies. This property allows to add the histograms of different energies (adding bins of equal purity) and construct a 'LEP2' ZZ-probability distribution as shown in figure 6.7. The increase in statistics will allow a more precise determination of the optimal region to be used for the ZZ cross section fit.

![Distribution of ZZ-probability](image)

Figure 6.7: The left plot shows the distribution of the combined ZZ-probability distribution for the full 183-207 GeV data set. The data are represented by the points and SM expectations by the solid histograms. The right plot shows the distribution of the two main relative uncertainties on the extracted ZZ cross section as a function of a cut on the ZZ-probability: the statistical uncertainty and the effect from the uncertainty on the background estimation.

The statistical error is minimal when using the full distribution and increases when a more pure sample is used. This is expected and illustrated by the dashed line in the right plot of figure 6.7. The dominant systematic error on the extracted ZZ cross section is the uncertainty on the background estimation and this has a reverse behaviour: the effect on the extracted ZZ cross section is maximal when the full distribution is used (using also the large region where the background dominates) and decreases when going towards a more pure sample. The optimal region of the ZZ-probability distribution to use for the cross section fit is the one in which the quadratic sum of the statistical and systematic error is minimal. From figure 6.7 it is clear that the optimal region is the one above $P_{ZZ} = 0.25$ since at that point the value of the combined uncertainty is minimal.

6.4.2 Performance

The number of expected ZZ events after a cut on $P_{ZZ}$ at 0.25 and the efficiency for hadronic ZZ events is shown in table 6.1. For most centre-of-mass energies this cut corresponds roughly to an efficiency just above 40% for the ZZ signal. As can be seen in figure 6.6 (for a centre-of-mass energy of 200 GeV), the contribution of the expected $W^+W^-$ and $q\bar{q}(\gamma)$ events to the combined background is almost similar at that efficiency and are also comparable to the ZZ expectation.
NC02 (ZZ) cross section measurement

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$\epsilon$(ZZ) (4 quarks)</th>
<th>$Z\bar{Z}$ (4 quarks)</th>
<th>SM (all)</th>
<th>observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>183</td>
<td>13.7</td>
<td>0.95</td>
<td>2.14</td>
<td>2</td>
</tr>
<tr>
<td>189</td>
<td>28.3</td>
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<td>35.51</td>
<td>36</td>
</tr>
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<td>192</td>
<td>36.3</td>
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<td>9.98</td>
<td>14</td>
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<td>38.66</td>
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</tr>
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<td>25.76</td>
<td>28</td>
</tr>
<tr>
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<td>50.30</td>
<td>53</td>
</tr>
<tr>
<td>207</td>
<td>44.7</td>
<td>33.40</td>
<td>95.65</td>
<td>85</td>
</tr>
<tr>
<td>total</td>
<td>40.1</td>
<td>108.46</td>
<td>306.53</td>
<td>316</td>
</tr>
</tbody>
</table>

Table 6.1: The expected number of ZZ events and the full expectation from the SM above $P_{Z\bar{Z}} = 0.25$ at various centre-of-mass energies. As an indication of the performance also the efficiency (in %) for fully hadronic ZZ events is given. The number of expected events is compared to the number of events observed in the data.

As mentioned before, the double peak structure for ZZ events in figure 6.4 is caused by the two classes of ZZ fully hadronic final states: with a pair of b-quarks (the right most signal like peak, due to their more unique signature) and without a pair of b-quarks. Using a region above $P_{Z\bar{Z}} = 0.25$, a large fraction of the ZZ events in the fit contain a pair of b-quarks. Their fraction is somewhere between 55 and 60% implying that there is also a non-negligible contribution from ZZ(light quarks) via the mass information.

As an illustration of the obtained sensitivity and the data/Monte Carlo agreement for a very pure sample of ZZ events, the distribution of the reconstructed equal mass is shown in figure 6.8 for events with $P_{Z\bar{Z}} > 0.50$. At this level the purity of the signal is 64% to be compared with a purity of 35% for the sample selected with $P_{Z\bar{Z}} > 0.25$.

![Figure 6.8: Reconstructed equal mass distribution for ZZ-like events ($P_{Z\bar{Z}} > 0.50$).](image-url)
6.4.3 Extraction of the ZZ cross section: the rules

In this section a quick overview is given of the procedure to obtain information on the ZZ cross section using the ZZ-probability distribution. As an illustration of the definitions, the results for the measurement at a centre-of-mass energy of 200 GeV are also given.

Constructing the likelihood function

When small numbers of events are expected (as is the case in the fit of the ZZ-probability distribution) the probability to observe \( n \) events when \( \lambda \) are expected is given by the Poisson distribution:

\[
P(n|\lambda) = \frac{\lambda^n e^{-\lambda}}{n!}
\]  

(6.5)

In each bin of the distribution the probability that \( n_i \) events were observed while \( \lambda_i \) were expected can now be computed. Since the background cross sections from \( W^+ W^- \) and \( q\bar{q}(\gamma) \) are fixed, the expected number of events is directly related to the ZZ cross section. In the rest of the section the number of ZZ events is varied and the cross section in the fit relative to the expected cross section from the SM is defined as \( \sigma_{zz}^{\text{fit}} \):

\[
\sigma_{zz}^{\text{fit}} = \sigma_{zz}^{\text{SM}}
\]  

(6.6)

For the full measurement \( M \), given by the expected distribution of \( P_{ZZ} \) and the distribution of the observed events in the data, the total likelihood \( \mathcal{L} \) is defined as the product of the individual likelihoods per bin:

\[
\mathcal{L}(n_i|\zeta_{zz}) = \frac{\lambda_i(\zeta_{zz})^{n_i} e^{-\lambda_i(\zeta_{zz})}}{n_i!}
\]  

and

\[
\mathcal{L}(M|\zeta_{zz}) = \prod_i \mathcal{L}_i
\]  

(6.7)

In the expression for the full likelihood the product runs only over the bins of the distribution selected for the fit. By varying \( \zeta_{zz} \), a likelihood curve can be constructed that describes the relative likelihoods of this measurement for different values of \( \zeta_{zz} \). As an example, the likelihood curve (relative to the maximum likelihood: \(-\Delta \ln \mathcal{L}(M|\zeta_{zz})\)) corresponding to the measurement at a centre-of-mass energy of 200 GeV is given in the left plot of figure 6.9. The likelihood distributions for all centre-of mass energies are given in appendix A.1.

Extracting information on \( \zeta_{zz} \)

The likelihood curve provides only information on the likelihood of this measurement given a certain value of \( \zeta_{zz} \). This is not what is needed, since it does not say anything about \( \zeta_{zz} \) itself. To determine the ZZ cross section the exact opposite is needed: “Given the measurement \( M \), what inference can be made on the value of \( \zeta_{zz} ? \)” This question can be answered by constructing the probability density function for \( \zeta_{zz} \) using Bayes’ Theorem [72]:

\[
P(\zeta_{zz}|M) = \alpha_0 P(M|\zeta_{zz}) \cdot P(\zeta_{zz})
\]  

(6.8)

In this expression \( P(M|\zeta_{zz}) \) is the likelihood function from equation (6.7), \( \alpha_0 \) is a normalisation constant with \( \alpha_0^{-1} \) explicitly given by \( \int_0^{+\infty} P(M|\zeta_{zz}) P(\zeta_{zz}) d\zeta_{zz} \) and \( P(\zeta_{zz}) \) is the \( a \ priori \) distribution of the possible values of \( \zeta_{zz} \) and is called the Bayesian prior function. Since the NC02
cross section is a quantity that is positive definite, the (normalised) prior function is chosen as:

\[ P(\zeta_{zz}) = \begin{cases} 
0. & \zeta_{zz} < 0, \\
1./\zeta_{\text{max}} & 0. \leq \zeta_{zz} \leq \zeta_{\text{max}} 
\end{cases} \quad (6.9) \]

If \( \zeta_{\text{max}} \) is chosen to be large it will not introduce a bias in the determination of the extracted values for \( \zeta_{zz} \). In this definition the (normalised) likelihood function can be directly interpreted as a probability density function for \( \zeta_{zz} \). Since there is no unique procedure to summarise the measurement (given by the likelihood function) in a central value and a confidence interval the recommendations of the workshop on confidence levels [73] should be followed:

"An experiment should provide for each measurement the full likelihood function and a detailed description of the procedure used to extract the central value and the confidence intervals."

In this thesis the 'rules' are defined as follows:

- **Central value**: The median value of \( \zeta_{zz} (\zeta_{zz}^{\text{med}}) \) defines the central value.

- **Upper and lower value for the CL interval**: The lower(upper) value of the CL interval is determined by the value of \( \zeta_{zz} \) for which 15.87\% of the integrated probability distribution is contained below(above) this value.

- **Upper limit**: If less than 31.74\% of the probability density function is located below the value of \( \zeta_{zz} \) for which the probability density function is maximal (\( \zeta_{zz}^{\text{max}} \)) an upper limit is given. The upper limit is defined as the value of \( \zeta_{zz} \) for which 5\% of the integrated probability distribution is contained above this value (95\% CL).

Using these definitions the median value for \( \zeta_{zz} \) extracted from the measurement is always above \( \zeta_{zz}^{\text{max}} \). This can be understood from an interesting general feature of a Poisson distribution given a number of observed events \( n \):

- Most likely value of \( \lambda \): \( \lambda_{\text{max}} = n \)
- Expectation value of \( \lambda \): \( < \lambda > = n + 1 \)

In the next section the results from the cross section measurements at the different centre-of-mass energies are given.
Figure 6.9: The left plot shows the (delta)-likelihood distribution of the measurement as a function of the ZZ cross section in the fit at a centre-of-mass energy of 200 GeV. The right plot shows the corresponding probability density function of $\zeta_{zz}$. The dotted line indicates the value of $\zeta_{zz}$ for which the likelihood was maximum and the solid line indicates the median value ($\zeta_{zz}^m$). The two white areas under the distribution each represent 15.87% of the full integral.

### 6.4.4 Results from the fit

The (delta) likelihood distributions as a function of $\zeta_{zz}$ are shown, together with the corresponding probability density functions for $\zeta_{zz}$ in appendix A.1 for each centre-of-mass energy. In table 6.2 the fitted values for $\zeta_{zz}$ obtained at all centre-of-mass energies in the full 183-207 GeV data set are presented. The measurement at 183 GeV allows only an upper limit to be given.

<table>
<thead>
<tr>
<th>$\sqrt{s}$</th>
<th>$\zeta_{zz}^m$</th>
<th>$\delta_-$</th>
<th>$\delta_+$</th>
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</thead>
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<td>183 GeV</td>
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<td></td>
</tr>
<tr>
<td>189 GeV</td>
<td>1.06</td>
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</tr>
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</tr>
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<td>0.22</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Table 6.2: Results from the ZZ(4 quarks) cross section fit at all centre-of-mass energies in the 183-207 GeV data set. The confidence interval is given by: $\zeta_{zz}^m - \delta_- \leq \zeta_{zz} \leq \zeta_{zz}^m + \delta_+$. 
6.4.5 Systematic uncertainties

From table 6.2 it is clear that the statistical errors on the extracted ZZ cross sections at each centre-of-mass energy are large. The limited number of 4-jet events at LEP2 does not allow a precise determination of the systematic errors associated to this measurement. But since the systematic effects are not (strongly) energy dependent some of the systematic errors can be evaluated to a much better precision by using the combined statistics from the full 183-207 GeV data set.

- **background cross sections**
  The uncertainty on the $q\bar{q}(\gamma)$ 4-jet cross section is conservatively taken to be in the order of 10%. The effect of the uncertainty on the $q\bar{q}(\gamma)$ background cross section on the extracted value of the ZZ cross section has been evaluated in section 6.4.1. Varying the $q\bar{q}(\gamma)$ cross section in the fit with $\pm 10\%$ results in a variation of the ZZ(NC02) cross section of $\mp 4.6\%$ (as can be seen in the right plot of figure 6.7 at $P_{ZZ}=0.25$). A similar exercise varying the more precisely known $W^+W^-$ cross section within $\pm 2\%$ has an effect of $\mp 0.9\%$. The combined uncertainty on the extracted ZZ(NC02) cross section is therefore 4.7%.

- **gluon splitting**
  The effect of the uncertainty on the gluon splitting rate into $b\bar{b}$, as explained in section 5.1.1, on the determination of the ZZ cross section is mainly caused by the increase in the $b\bar{b}$ component of the $q\bar{q}(\gamma)$ cross section. A 4.6% effect is observed when the ZZ cross section is extracted without the improved gluon splitting. Since the uncertainty on the correction to the splitting rate is estimated to be about half as large as the effect itself [70], the uncertainty on the extracted ZZ cross section is estimated to be $0.5\cdot4.6\%=2.3\%$.

- **conversion factor**
  Due to the uncertainty on the theoretical prediction from YFSZZ ($\pm 1\%$) and the limited 4-fermion Monte Carlo statistics the conversion factor to translate the ZZ cross section into a pure NC02 cross section has an uncertainty in the order of 2% (depending on the centre-of-mass energy).

- **b-tag likelihood ratios**
  The parametrisation of the separation between b-jets and non-b jets as defined in section 5.1.2 is varied within the uncertainties associated to the b-tag per jet. The b/non-b likelihood ratio is modified such that for b-like jets (initial likelihood ratio $>1$) the likelihood ratio is lowered by 7.5%. The likelihood ratio for non-b like jets on the other hand is enhanced. The effect on the extracted ZZ-cross section is found to be small and in the order of 1-2%.

A conservative estimate of the total systematic uncertainty is given by the quadratic sum of the individual contributions and amounts to 6%.
6.5 NC02 cross section results

The measured ZZ cross sections can be translated into a pure NC02 cross section using the conversion factors as shown in the right plot of figure 6.1. The corrections are of the order of a few % (with a typical uncertainty of around 2%). The quoted uncertainties on the NC02 cross sections are first statistical and then systematic. In a graphical representation (figure 6.10) the measured fully hadronic NC02 cross sections are compared to the SM expectation.

\[ \sigma_{\text{NC02}}(183 \text{ GeV}) < 0.71 \text{ pb} \quad (95\% \text{ CL}) \]
\[ \sigma_{\text{NC02}}(189 \text{ GeV}) = 0.35^{+0.13}_{-0.11} \pm 0.02 \text{ pb} \]
\[ \sigma_{\text{NC02}}(192 \text{ GeV}) = 0.69^{+0.43}_{-0.34} \pm 0.04 \text{ pb} \]
\[ \sigma_{\text{NC02}}(196 \text{ GeV}) = 0.78^{+0.24}_{-0.22} \pm 0.05 \text{ pb} \]
\[ \sigma_{\text{NC02}}(200 \text{ GeV}) = 0.39^{+0.17}_{-0.15} \pm 0.02 \text{ pb} \]
\[ \sigma_{\text{NC02}}(202 \text{ GeV}) = 0.57^{+0.28}_{-0.24} \pm 0.03 \text{ pb} \]
\[ \sigma_{\text{NC02}}(205 \text{ GeV}) = 0.62^{+0.20}_{-0.17} \pm 0.04 \text{ pb} \]
\[ \sigma_{\text{NC02}}(207 \text{ GeV}) = 0.54^{+0.13}_{-0.12} \pm 0.03 \text{ pb} \]

Figure 6.10: Measured NC02 (4 quarks) cross section as a function of \( \sqrt{s} \).
6.6 \( ZZ(b\overline{b}q\overline{q}) \) cross section measurement

For events in which one of the Z bosons decays into a \( b\overline{b} \) quark-pair, the final state is almost identical to that of a Higgs signal: a heavy object decaying into a \( b\overline{b} \) pair recoiling against a Z boson. The probability computation for this process will be performed following the procedure defined in section 5.4. The event topology and mass information is identical to that of the full ZZ process; the only difference being the possible final state b-tag configurations and the cross section (38.7% of the fully hadronic ZZ cross section). Investigating this process requires more emphasis on the b-content of the event where the use of the b-tag likelihood ratios (section 5.1) incorporates the knowledge on the fraction of light-quark jets that are reconstructed with a high value of the combined b-tag probability per jet.

The double peak structure for ZZ events (with and without b-quarks in the final state), is also present in the \( ZZ(b\overline{b}q\overline{q}) \) probability distribution. In this case the peaks are further separated as can be seen in figure 6.11: the normalised distributions of \( \ln(P_{ZZ(b\overline{b}q\overline{q})}) \) for \( ZZ(b\overline{b}q\overline{q}) \) events, for the background from ZZ(light quarks) and the combined background for the \( W^+W^- \) and \( q\overline{q}(\gamma) \) processes at a centre-of-mass energy of 200 GeV.

![Figure 6.11](image)

Figure 6.11: Normalised distributions of the logarithm of the ZZ-probability for \( ZZ(b\overline{b}q\overline{q}) \) events, ZZ(light quarks) and combined background separately at a centre-of-mass energy of 200 GeV.

The distribution of the \( ZZ(b\overline{b}q\overline{q}) \)-probability for each of the centre-of-mass energies has the property that in each bin of \( P_{ZZ(b\overline{b}q\overline{q})} \), the probability is equal to the fraction of \( ZZ(b\overline{b}q\overline{q}) \) in that bin. The agreement between data and expectation is shown in the left plot of figure 6.12 where the probability distributions from all energies in the 183-207 GeV data set have been combined (as was done for the full ZZ cross section). A good agreement between the observed and expected number of events is observed as is the case in the cumulative distribution (right plot of figure 6.12). In that plot the numbers of events (data and expected from the SM) at decreasing efficiencies for \( ZZ(b\overline{b}q\overline{q}) \) are shown, allowing in addition the evaluation of the background composition.
at any given efficiency (purity). In both plots the ZZ(bbq̄q̄) signal is not given separately, but only the full ZZ expectation is given. In the right plot the efficiencies are explicitly correct for the ZZ(bbq̄q̄) signal only. As expected, the most important background is q̄q(γ) (with b-quarks) while the W^+W^- background is quickly eliminated.

Figure 6.12: The left plot shows the distribution of the ZZ(bbq̄q̄) probability for the full 183-207 GeV data set. The data are represented by the points and SM expectations by the solid histograms. The right plot shows the number of expected events left (data and SM expectation) as a function of the efficiency for selecting ZZ(bbq̄q̄) events.

### 6.6.1 Performance and fit results

To extract the ZZ(bbq̄q̄) cross section, a procedure is used that is similar to the one used for the full ZZ cross section. In the fit the region above \( P_{ZZ}(b\overline{b}q\overline{q}) = 0.20 \) is used. The number of ZZ(bbq̄q̄) events and the total number of expected events from the SM at that point are given in table 6.3, where they are compared to the number of observed events. The region \( P_{ZZ}(b\overline{b}q\overline{q}) > 0.20 \) corresponds to an efficiency of almost 60% (for ZZ(bbq̄q̄)) and the fraction of ZZ(bbq̄q̄) compared to the total number of ZZ events is well above 90%. The (delta) likelihood distributions as a function of \( \zeta_{zz} \) are shown, together with the corresponding probability density functions for \( \zeta_{zz} \) in appendix A.2 for each centre-of-mass energy. As a summary, the expectation values for \( \zeta_{zz} \) obtained at all centre-of-mass energies in the full 183-207 GeV data set are presented in table 6.4.

These results are transformed into a pure NC02 cross section using for each centre-of-mass energy the conversion factors from figure 6.1. The extracted cross sections are compared to the SM expectation in figure 6.13, where a good agreement between the SM prediction and measurement is observed. Note that the measurements of the cross section for the full ZZ and ZZ(bbq̄q̄) are strongly correlated.

---

4 Meaning: \( \zeta_{bbq\overline{q}} \), since the contribution from ZZ(bbq̄q̄) events is varied.
NC02 (ZZ) cross section measurement

<table>
<thead>
<tr>
<th>( \sqrt{s} ) (GeV)</th>
<th>( \epsilon(\text{ZZ}) ) (bbq\bar{q})</th>
<th>ZZ (bbq\bar{q})</th>
<th>SM (all)</th>
<th>observed</th>
</tr>
</thead>
<tbody>
<tr>
<td>183</td>
<td>37.0</td>
<td>0.96</td>
<td>2.55</td>
<td>2</td>
</tr>
<tr>
<td>189</td>
<td>51.3</td>
<td>9.81</td>
<td>22.24</td>
<td>24</td>
</tr>
<tr>
<td>192</td>
<td>59.4</td>
<td>2.28</td>
<td>5.43</td>
<td>6</td>
</tr>
<tr>
<td>196</td>
<td>56.7</td>
<td>7.40</td>
<td>16.95</td>
<td>18</td>
</tr>
<tr>
<td>200</td>
<td>58.0</td>
<td>9.20</td>
<td>20.80</td>
<td>26</td>
</tr>
<tr>
<td>202</td>
<td>59.4</td>
<td>4.58</td>
<td>10.65</td>
<td>8</td>
</tr>
<tr>
<td>205</td>
<td>59.0</td>
<td>8.66</td>
<td>20.46</td>
<td>17</td>
</tr>
<tr>
<td>207</td>
<td>60.0</td>
<td>17.50</td>
<td>38.89</td>
<td>37</td>
</tr>
<tr>
<td>total</td>
<td>57.7</td>
<td>60.39</td>
<td>137.98</td>
<td>138</td>
</tr>
</tbody>
</table>

Table 6.3: The expected number of events above \( P_{ZZ} \) (bbq\bar{q}) = 0.20 at all centre-of-mass energies. The total number of expected events is compared to the observed number of events in the data.

**Fit results**

<table>
<thead>
<tr>
<th>( \sqrt{s} ) (GeV)</th>
<th>( \epsilon_{zz} )</th>
<th>( \delta_- )</th>
<th>( \delta_+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>183</td>
<td>&lt;4.83</td>
<td>0.39</td>
<td>0.47</td>
</tr>
<tr>
<td>189</td>
<td>1.18</td>
<td>0.41</td>
<td>0.53</td>
</tr>
<tr>
<td>192</td>
<td>&lt;3.12</td>
<td>0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>196</td>
<td>0.92</td>
<td>0.44</td>
<td>0.59</td>
</tr>
<tr>
<td>200</td>
<td>1.26</td>
<td>0.43</td>
<td>0.52</td>
</tr>
<tr>
<td>202</td>
<td>0.86</td>
<td>0.44</td>
<td>0.59</td>
</tr>
<tr>
<td>205</td>
<td>1.14</td>
<td>0.38</td>
<td>0.45</td>
</tr>
<tr>
<td>207</td>
<td>1.18</td>
<td>0.28</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Table 6.4: Results from the ZZ(bbq\bar{q}) cross section fit at all centre-of-mass energies in the 183-207 GeV data set

Looking at the errors associated with these measurements one can argue that with these large statistical errors a possible SM Higgs signal (which has a similar cross section and efficiency) can never be clearly identified. One should realise that in ‘measuring’ the Higgs cross section the full luminosity (all centre-of-mass energies) is used. As an example, one can interpret the ZZ(bbq\bar{q}) measurements as the ‘search’ for on-shell Z boson pair production where one Z boson always decays into a b\bar{b}-pair. Using the data in the full 183-207 GeV data set a ‘discovery’ can be claimed:

\[
\sigma_{\text{NC02}}^{\text{measured}} (\text{bbq\bar{q}}) / \sigma_{\text{NC02}}^{\text{SM}} (\text{bbq\bar{q}}) = 1.05^{+0.17}_{-0.15} (\text{stat})
\]  

(6.10)

Figure 6.13: Measured NC02(bbq\bar{q}) cross section as a function of \( \sqrt{s} \).


6.7 Possible further improvements

The method of computing event-by-event probabilities allows additional information extracted from the event (in the form of likelihood ratios) to be easily included in the ZZ-probability computation. The main improvements that can be made to enhance the sensitivity of the analysis to the ZZ signal are the following:

- **Improved rejection of q̅q(γ) events:**
  The pre-selection efficiency for q̅q(γ) events is at the level of 2%, but due to its b̅b component this is the dominant background for both the ZZ and ZZ(b̅bq̅q̅) cross section measurements. An improved separation between 4-fermion and 2-fermion events could be achieved by using a more sophisticated (topological) separating variable than D_{pur} (see equation (4.12)), the variable used in this analysis.

- **(Charged) angular distributions of the bosons:**
  Another improvement, at the level of jet-pairings, could be achieved by using the predicted angular distribution (the NC02 diagrams have only t-channel exchange of an electron) of the produced Z bosons in ZZ final states. The (differential) cross section results from this analysis have been used [74, 75] to put limits on neutral TGC's (through possible s-channel contributions), but could be used directly in the ZZ-probability distribution as well.

  An improvement connected to this is the use of jet-charge information (a momentum-weighted sum of the charge of all particles in a jet) to obtain an estimate of the charge of the heavy object defined by the jet-pairing. This would allow pairings where the bosons have both non-zero charge (like W^+W^- events in which the difference in charge is 2) to be disfavoured compared to a jet-pairing in which both are compatible with being neutral.

- **B-(flavour) tagging:**
  Using improved (flavour) b-tag information, jets originating from b-jet fragmentation can be separated from b̅-quark fragmentation jets. This information can help in solving ambiguities on the level of jet-pairings, especially in events with 4 b-quarks (~5% of the fully hadronic ZZ cross section), since (b̅bb̅) pairings can then be favoured over combinations that resemble situations not allowed in the SM: (b̅bb̅).

6.8 Combined results, summary and conclusion

The results from the DELPHI experiment on the extracted NC02 cross sections (in case both Z bosons decay hadronically) have been presented. The cross section results are in good agreement with the SM prediction as is shown in figure 6.10.

The DELPHI combined results on the NC02 cross section as a function of the centre-of-mass energy, using all ZZ decay modes can be found in the DELPHI reports on the NC02 cross section measurements [76, 77, 78]. In the combination, the weight of the three most sensitive channels to the final precision are the following: q̅q̅q̅ (~43%), q̅q̅ν̅ν (~35%) and q̅q̅l^+l^- (~22%). The weight of the last two channels is enhanced (compared to their fraction of the full NC02 cross section) as expected, since q̅q̅ν̅ν is 28% of the cross section, but has no pairing ambiguity and
$q\bar{q}l^+l^-$ is only 9% of the full cross section ($l=\mu$ or e), but there is no pairing ambiguity and in addition there is no significant background.

Also the results from the 4 LEP experiments are combined [79]. The combined measured NC02 cross sections for all centre-of-mass energies at LEP2 are shown in figure 6.14. A good agreement with the SM prediction is observed.

![LEP combined results](image)

Figure 6.14: Measured LEP combined NC02 cross section as a function of $\sqrt{s}$.

Finally, the results on the combined ZZ($b\bar{b}q\bar{q}$) cross section measurement performed in this thesis show that a Higgs boson, if the cross section is comparable to the ZZ($b\bar{b}q\bar{q}$) cross section, will not escape detection.