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Resonant excess quantum noise in focused-gain lasers

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We demonstrate that the transverse eigenmodes in a waveguide that combines a parabolic index guide with a Gaussian gain guide can be highly nonorthogonal. The excess-noise factor K that arises from this nonorthogonality exhibits resonant features with maximum values that can easily reach $K \approx 400$. This simple model applies directly to stable-cavity microchip lasers with focused gain. © 2001 Optical Society of America

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It is well known that in the absence of losses the eigenmodes of an optical resonator form an orthogonal set. Any real laser is an open system, however, in which losses are unavoidable and even desirable if one wishes to obtain an output signal. That this is so opens up the possibility that the resonator eigenmodes can become nonorthogonal. Siegman[1,2] pointed out that this nonorthogonality has important general consequences for the quantum-limited noise performance of the laser output, specifically for the excess-noise factor K. This factor, $K \geq 1$, first introduced by Petermann[3] for gain-guided laser systems, describes the enhancement of the quantum-limited laser linewidth over the standard Schawlow–Townes value.

The interest in the excess-noise factor K has surged in recent years, sparked by the demonstration[4,5] that large K values ($\geq$100) can be reached experimentally in hard-edged unstable resonators, in which the transverse modes are nonorthogonal. Although excess quantum noise has also been demonstrated for nonorthogonal longitudinal and polarization modes, and for stable-cavity lasers with a small aperture (see Ref. 6 and references therein), the hard-edged unstable resonator remains the achetypal system for studies of excess quantum noise. This system has so far been of special interest not only because of the size of the K factors but also because of the intriguing resonances that occur in K for relatively small changes in the resonator parameters.[5] In contrast, for unstable resonator lasers with a Gaussian variable-reflectivity mirror (acting as a soft aperture), the eigenmodes can be readily calculated, and no such resonances are found.[2] Hard-edged unstable resonators have complicated transverse-mode profiles whose calculation requires significant numerical effort; the profiles can even exhibit fractal behavior.[7]

This discussion raises the question of whether hard-edge unstable resonators are indeed unique in their excess-noise behavior. In this Letter we show that this is not the case. We demonstrate that strong resonances in the K factor can also be achieved in another common system, namely, that of a waveguide with a combined parabolic index guide and a Gaussian gain guide. This model is widely applicable: As has been shown elsewhere, it applies directly to a stable-cavity microchip laser with a focused longitudinal pump.[8,9] The model that we study is relatively simple and exhibits large excess-noise factors for modest losses, whereas the modal profiles remain smooth and well behaved. These results suggest that large and resonant K factors are much more general than previously thought, and the simplicity of the model should stimulate and facilitate further studies.

We start by describing the model. The eigenmode equation for a waveguide with parabolic index guiding and Gaussian gain guiding can be transformed into

$$\left[-\frac{1}{2}\,\nabla_\rho{}^2 + 2\rho^2 + ig(\rho)\right]u = \mu u\,, \tag{1}$$

where $\rho = r/w_0$ is the transverse coordinate r scaled to the waist size w_0 of the parabolic index guide (the latter represented by the term $2\rho^2$), $\nabla_\rho{}^2$ is the transverse Laplacian describing diffraction, and

$$g(\rho) = g_0 \exp(-2\rho^2/\rho_g{}^2) \tag{2}$$

is the gain function, with a Gaussian transverse profile with scaled width ρ_g and gain g_0 per Rayleigh range $\pi w_0{}^2/\lambda$. The eigenfunctions $u_n(\rho, \phi)$ of Eq. (1) represent the transverse-mode amplitude profile, and the real and imaginary parts of the eigenvalues μ_n represent the modal phase shift and the gain per Rayleigh range, respectively.

Serrat *et al.* discussed the application of this model to a stable-cavity microchip laser with focused gain[8]; the model was recently extended to include the effect of detuning away from gain maximum and favorably compared with experimental results.[9] Specifically, when the cavity mode is detuned from maximum gain, one needs to account for the dispersion of the gain medium (gain-related change in refractive index). This can be done if g_0 in Eq. (2) is taken to be complex valued.

For simplicity we limit ourselves here to the case of maximum gain and take g_0 to be real valued.

Because of the cylindrical symmetry of Eq. (1), we can assign each eigenmode a well-defined angular momentum l [angular dependence $\exp(il\phi)$]. Since eigenmodes with $l \neq 0$ have zero intensity on axis, their overlap with the gain of Eq. (2) is significantly reduced. Our interest is in low-loss (high-modal-gain) modes, and we can thus limit the discussion to modes with zero angular momentum. Considering just these modes eliminates the ϕ dependence and simplifies the situation further.

To obtain the excess-noise factor, one generally needs to find the adjoint modes v_n.[1] The set v_n is biorthogonal to the set u_n, and the excess-noise factor of mode n is $K_n = \langle v_n|v_n\rangle\langle u_n|u_n\rangle/\langle v_n|u_n\rangle|^2$, where the inner product $\langle|\rangle$ is defined as

$$\langle a(\rho)|b(\rho)\rangle \equiv 2\pi \int_0^\infty \rho a^*(\rho) b(\rho) \mathrm{d}\rho\,. \tag{3}$$

For Eq. (1) the excess-noise factor K can be obtained more simply: In Eq. (1) both the diffraction operator and the parabolic (real-valued) index guide are self-adjoint, and the Hermitian adjoint of the gain guide is its complex conjugate. As a result, the adjoint mode v_n of each eigenmode u_n is simply the complex conjugate $u_n{}^*$, and the K factor can be given in terms of u_n alone as $K = |\langle u_n|u_n\rangle/\langle u_n{}^*|u_n\rangle|^2$.

The model contains only two parameters, namely, the scaled gain width ρ_g and the relative strength of the gain g_0. We have numerically calculated the eigenmodes, eigenvalues, and K factors of Eq. (1) for a range of these parameters. At specific values of ρ_g and g_0, resonances in K occur. An example is shown in Fig. 1. It shows the eigenvalues and K factors of the two lowest-loss modes (both with angular momentum $l = 0$) in the range $g_0 = 12-16$ for $\rho_g = 0.315$. Note how in Fig. 1 the resonance in K coincides with a near degeneracy of the two eigenvalues: The modal gains Im μ cross, while the modal phase shifts Re μ avoid crossing. In fact, we find that each resonance in K in this system is accompanied by such a near degeneracy of eigenvalues. This is very similar to the case of the hard-edged unstable resonator, in which each resonance in K is also accompanied by a near degeneracy of eigenvalues.[5]

The example illustrated in Fig. 1 is readily achievable experimentally; in fact, the situation is quite close to that of some of our own experimental work.[9] In that experiment, the waist size of the Gaussian gain was $w_g = 14\ \mu$m, and the waist size of the parabolic index guide was $w_0 = 46\ \mu$m, yielding $\rho_g = w_g/w_0 = 0.3$. For the cavity geometry in that experiment, the modal gain of Im $\mu \approx 3$ at the resonance in Fig. 1 would correspond to the mode's being at threshold for mirror reflectivity $R_m \approx 85\%$, which is within the range studied experimentally.[9] The difference in the modal phase shift Re μ of 0.5 at the resonance in Fig. 1 corresponds to a frequency difference of 0.7 GHz in the experiment (neglecting mode pulling).

The occurrence of a series of these mode crossings in our model is illustrated in Fig. 2. It shows the trajectories of the eigenvalues for a fixed value of the gain width ρ_g and various values of the gain strength g_0. For instance, the solid curves labeled with the value $\rho_g = 0.315$ in Fig. 2 correspond to the mode crossing shown in Fig. 1; the maximum in K occurs where the two curves are closest to each other.

Figure 2 exhibits a number of features of the combined gain and index guide considered here. We now discuss some of them. For $g_0 \ll 1$ the gain guide becomes negligibly weak, the parabolic index guide dominates, and we have Im $\mu \ll 1$. The eigenmodes u_n are usual Laguerre–Gaussian modes, with eigenvalues $\mu_n \approx 2(2n+1)$, with $n = 0, 1, 2, \ldots$. These

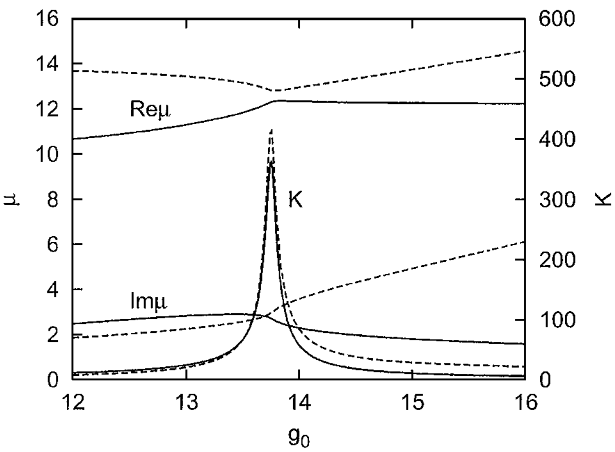

Fig. 1. Eigenvalues μ and excess-noise factors K of the two lowest-loss transverse modes for $\rho_g = 0.315$. The real part of the eigenvalue corresponds to the phase shift per Rayleigh range; the imaginary part, to the modal gain. For fixed gain width ρ_g and gain strength $g_0 \to 0$, the modes are adiabatically connected to mode u_2 (solid curves) and u_3 (dashed curves) of the parabolic index guide; see also Fig. 2.

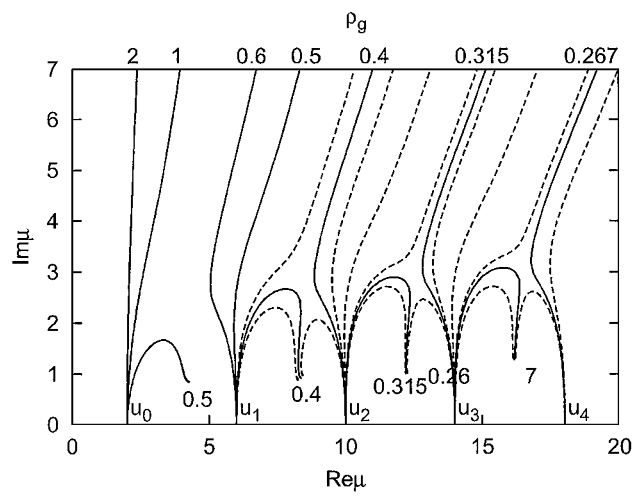

Fig. 2. Adiabatic trajectories in eigenvalue space for fixed values of the gain width ρ_g and various values of gain strength g_0. The solid curves have been labeled with the corresponding values of ρ_g. The dashed curves correspond to intermediate values. For those connected to mode u_1 of the purely parabolic index guide at $g_0 = 0\,(\mu = 6)$, $\rho_g = 0.42, 0.38$; for mode $u_2\,(\mu = 10)$, $\rho_g = 0.42, 0.38, 0.35, 0.32, 0.31$; for mode $u_3(\mu = 14)$, $\rho_g = 0.32, 0.31, 0.29, 0.27$; and for mode $u_4(\mu = 18)\,\rho_g = 0.27, 0.26$.

eigenvalues are independent of ρ_g and can be seen in the bottom part of Fig. 2 as the convergence points of the trajectories near Im $\mu = 0$.

For $g_0 \gg 1$ the gain guide dominates, and a single gain-guided mode emerges with significantly more gain, Im $\mu \gg 1$, than for any of the other eigenmodes. The modal phase shift Re μ now depends on ρ_g, as the labeling of the various curves near the top of Fig. 2 illustrates.

The transition between the two regimes occurs near $g_0 \approx 1 + 1/{\rho_g}^2$. Going from the high- to the low-gain regime by varying g_0 and keeping ρ_g fixed (i.e., following one of the curves in Fig. 2), we find that the single high-gain mode adiabatically connects to the index-guided mode with a similar value of the modal phase shift Re μ, typically, $n \approx 1/\rho_g$. For instance, for $\rho_g = 0.315$ the high-gain mode transforms into the index-guided Laguerre–Gaussian mode u_3 (Re $\mu = 14$), as the solid curve labeled 0.315 at the top of Fig. 2 shows. For this value of ρ_g, the curve connected to index-guided mode u_2 (Re $\mu = 10$) also reaches significant modal gain (up to Im $\mu \approx 3$) as g_0 is increased from zero, before the modal gain drops again to lower values as g_0 is further increased (see also Fig. 1).

For a slightly higher value of ρ_g, 0.32, the high-gain mode has jumped and adiabatically connects to index-guided mode u_2. When ρ_g is varied so that the eigenvalues at the crossing are brought closer to degeneracy, the peak value of the K factor increases and the width of the resonance (when plotted as a function of g_0) decreases. For values of ρ_g from 0.315 to 0.32, there is a critical value (which we have found to be near $\rho_g = 0.3162$), where the curves in Fig. 2 touch and where the peak K factor really diverges for a single very specific gain strength g_0 (found to be near $g_0 = 13.70$). Similar jumps occur between all adjacent index-guided modes, as Fig. 2 illustrates. For each of these jumps there is a near degeneracy of two modal eigenvalues (in the transition regime between gain and index guiding) and a resonance in K.

It is interesting to compare the present results with the initial analysis of Petermann,[3] who considered a waveguide with both the index and the gain guide having a parabolic shape. The same situation can be considered by replacement of the term $2\rho^2 + ig(\rho)$ in Eq. (1) with $2\rho^2(a_r - ia_i)$, with $a_i > 0$ (i.e., maximum gain on axis). The only difference from Ref. 3 is then that we consider two transverse directions (and cylindrical symmetry), whereas Petermann considered a waveguide in one transverse direction. Both cases can be solved analytically and yield a direct relation between the ratio a_r/a_i of index and gain guiding and the K factor of the lowest-loss mode. For our case of two transverse directions the result is

$$\frac{a_r}{a_i} = \frac{2 - K}{2\sqrt{K - 1}}\,. \tag{4}$$

This is directly analogous to Eq. (39) of Ref. 3, the result for parabolic guiding in one transverse direction. The important points to note from Eq. (4) are that K varies monotonically with the ratio a_r/a_i (i.e., no resonances in K) and that large K can be achieved only for $a_r < 0$, i.e., index antiguiding. This result is very similar to the case of an (un)stable resonator with a Gaussian variable-reflectivity mirror.[2] In contrast, our system does show resonances and does not contain a form of antiguiding.

A natural extension of the present model is to include the effect of dispersion of the gain medium.[9] This extension yields additional index guiding for positive detuning and index antiguiding for negative detuning. We have found that the inclusion of this dispersion leads to additional resonances at nonzero detuning. A discussion of these effects is beyond the scope of this Letter. It can also be shown[10] that this model applies as well to free-space amplifiers with focused Gaussian gain, such as Raman amplifiers.[10]

In conclusion, we have shown that large resonances in the excess-noise factor can be achieved in a waveguide that combines a parabolic index guide and a Gaussian gain guide. This model applies directly to a range of lasers and optical amplifiers, most notably to Nd-doped microchip lasers.[9] This should allow more-convenient experimental studies of laser systems with strong nonorthogonalities.

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References

1. A. E. Siegman, Phys. Rev. A **39,** 1253 (1989).
2. A. E. Siegman, Phys. Rev. A **39,** 1264 (1989).
3. K. Petermann, IEEE J. Quantum Electron. **QE-15,** 566 (1979).
4. Y.-J. Cheng, C. G. Fanning, and A. E. Siegman, Phys. Rev. Lett. **77,** 627 (1996).
5. M. A. van Eijkelenborg, Å. M. Lindberg, M. S. Thijssen, and J. P. Woerdman, Phys. Rev. Lett. **77,** 4314 (1996).
6. M. P. van Exter, N. J. van Druten, A. M. van der Lee, S. M. Dutra, G. Nienhuis, and J. P. Woerdman, Phys. Rev. A **63,** 043801 (2001).
7. G. P. Karman, G. S. McDonald, G. H. C. New, and J. P. Woerdman, Nature **402,** 138 (1999).
8. C. Serrat, M. P. van Exter, N. J. van Druten, and J. P. Woerdman, IEEE J. Quantum Electron. **35,** 1314 (1999).
9. N. J. van Druten, S. S. Oemrawsingh, Y. Lien, C. Serrat, M. P. van Exter, and J. P. Woerdman submitted a paper called "Observation of transverse modes in a microchip laser with combined gain and index guiding" to J. Opt. Soc. B.
10. N. J. van Druten, M. P. van Exter, and J. P. Woerdman, "Equivalence of transverse modes in Raman amplifiers and microchip lasers," submitted to Opt. Commun.
11. B. N. Perry, P. Rabinowitz, and M. Newstein, Phys. Rev. Lett. **49,** 1921 (1982).