The business cycle: dynamical coupling and chaotic fluctuations

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CHAPTER 5

FINANCIAL CRISSES AND THE LABOUR MARKET

"A nonlinear economic model may explain a significant part of both the regularity and irregularity observed in economic time series"  
(Hommes(1991), 1)

1 Introduction

Goodwin(1947) analysed the effects of two sectors interacting with each other (or as Goodwin(1947, 181) is cited: "are interdependent in some way (coupled, we may say")). Goodwin(1947) concludes that the coupled sectors become less stable than the least stable independent sector, and that the fluctuations in the aggregate economy differ from the fluctuations in the uncoupled sectors. Morishima(1992) examines the relationship between the monetary sector and the real sector, finding evidence that the stability of the aggregate economy depends positively on the collaboration between bankers and entrepreneurs: "bridging of these two (the real sector and the monetary sector) is crucially important, in order for the economy to work smoothly and efficiently" (Morishima(1992), 186). Co-ordination of decisions between financiers and investors reduces the possible occurrence of fluctuations.

This chapter examines the consequences of a coupling between the monetary sector and the real sector. To do so, a model is developed which captures the interaction between the monetary sector and the labour market1. Contrary to Goodwin(1947), the markets do not interact directly through prices, but through the influence of costs of production on profits, and thereby on the rate of utilisation of the existing stock of capital. Morishima(1992) sees the investment-decision as the connection between the two sectors. Here, the capital stock is assumed to be constant. The reason for the connection between the real and the monetary sector is the need for credit, which is explained by Morishima(1992) by the time-lag between investment and sales. As Morishima's relationship between investment and credit is ineffective, Negishii(1989)'s theory on the wage fund is followed here. The economy can be characterised as a 'Hawtrey-Hayek-economy', augmented with the predator-prey mechanism of Goodwin(1967), both described in chapter two, and aspects of the Foley(1987)-model, described in chapter four.

Paragraph one presents the structure of the model. Paragraph two discusses the behavioural relationships for the monetary sector. It specifies the

1 An earlier version of this model can be found in Langen(2000a), which also analyses the individual sectors.
demand for credit by the firms, households’ savings and money creation by banks. The interest rate is determined by demand for and supply of funds. How the interest rate adjusts to demand for and supply of funds can be described by a modified predator-prey model, with persistent fluctuations. The dynamics of the labour market (paragraph three) follow the Goodwin (1967) tradition, and are based on the Phillips curve. Greater demand for labour leads to an increasing wage share. The rise in the wage share lowers profits, which depresses the rate of utilisation, and thereby again reduces demand for labour. The labour market thus displays a wage share-utilisation cycle.

The dynamics in both markets influence each other. They are ‘coupled’. The interest rate influences production costs, which determine the rate of profit and so utilisation of the capital stock and production. Production determines the demand for credit, one of the determinants of the interest rate. The cyclical behaviour in each sector influences the dynamics in the other sector, so the aggregate economy will also display fluctuations. Section four explores the consequences of this coupling on both sectors and on the aggregate economy. Like Morishima (1992), this chapter concentrates on the propagation mechanism and the results for the time path of the economy, without giving much attention to the original impulse causing the disequilibrium.

Two questions have to be answered in this chapter:
- How do partial fluctuations influence the behaviour of the aggregate economy (the coupling)? Or, given the occurrence of modified discrete Goodwin-Lotka-Volterra cycles in the two markets, how does this influence the dynamics of the aggregate economy?
- How does the occurrence of the coupling influence the possibilities of an active government policy regarding inflation and unemployment?

The dynamics of the model used in this chapter are - when possible - determined using two techniques, the bifurcation diagram and the Lyapunov characteristic exponent. To simulate the model for a large number of periods (100,000 or more), the calculation of bifurcation diagrams and of the LCE’s, a program designed by Medio and Gallio, named DMC (see Medio (1992)) is used, other simulations are made using MS-Excel. The technique of calculating a bifurcation diagram and the largest Lyapunov characteristic exponent were introduced in chapter three.

1.1 The economy

The economy under consideration is a highly stylised one. Firms produce only one good. Production is time consuming (see figure 5.1). At the beginning of period (at \(t-1\) in figure 5.1), the firms hire labour to produce the good. The time of production is labelled \(\tau\), i.e. the time between the purchase of the factors of production and the realisation of the finished
product. After production, the goods are sold, so at the end of the period, at \( t \), the value of production is realised.

\[
\begin{array}{c}
\text{production} \\
\tau \\
\text{payment of the wages} \\
\hline
\text{sales} \\
1-\tau \\
\text{payment of interest; realisation of profit}
\end{array}
\]

Figure 5.1: The timing of production and sales.

It is assumed that the physical capital stock is constant; the utilisation of this stock can vary. The degree of utilisation determines the costs of production. Firstly, it determines the demand for labour and so the labour costs. Secondly, the firms have to borrow money from the banks to finance the wages. This is called the wage fund. In this we follow Negishi (1989), who (defending the wage-fund theory) observes that in modern capitalism it is custom to pay wages after work (production) is done, but before the money value of production is realised through sales. Negishi (1989, 74/75) states "they [the wages] are often paid after the consumption of the labour power". However "[T]he fact that wages are paid after the use of labour power does not... deny the advancement by capitalist to labourers in the sense that wages are paid before the realisation of value of output -the labour embodied in the output".

When the finished product is sold, the firms receive the monetary value of production. To cover the costs in the period between the payment of wages and the realisation of the sales, firms borrow from the banks. The principal and interest is (re)paid to the banks, from the revenues of the sales, at the end of the period, at \( t \). The remaining value accrues to the owners of the firms as profit income.

In the following, the exact length of the production time (\( \tau \)) and the sales of products (1-\( \tau \)) is ignored, it is assumed that credit is demanded at the beginning of the period and repaid at the end.

Firms have an amount of capital, \( (K) \), which is assumed to be constant and non-perishable. There is no net-investment; \( K \) can be interpreted as a fixed input given to the entrepreneurs. At the beginning of the production period, the firms choose a level of production, which depends on the capital stock present. This production leads to a demand for credit and labour. The demand for funds, \( (D_t) \), is assumed to be equal to the wage fund, \( W_t \). The reserves of the banks consist of the deposits (savings of the households) from the former period, \( S_t \). The difference between the reserves and the demand for funds is called the net credit creation of the banks, \( V_t \):
Initially this amount of 'money' appears on the account of the firms as cash reserve, \( W \). The initial accounts at the start of period, at \( t \), after the loans from the banks to the firms, are:

<table>
<thead>
<tr>
<th>A. Households</th>
<th>Wealth (savings)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deposits</td>
<td>( S_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Banks</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Loans to firms</td>
<td>( D_t )</td>
</tr>
<tr>
<td>Deposits</td>
<td>( S_t )</td>
</tr>
<tr>
<td>Credit creation</td>
<td>( V_t )</td>
</tr>
<tr>
<td>( D_t )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Firms</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash reserve</td>
<td>( W_t )</td>
</tr>
<tr>
<td>Wage fund</td>
<td></td>
</tr>
<tr>
<td>Capital</td>
<td>( K )</td>
</tr>
<tr>
<td>Net wealth</td>
<td>( D_t + K )</td>
</tr>
</tbody>
</table>

The firms hire labour and produce the consumption good. Labour is paid their wages, \( W_t \), using credit provided by the banks. After sales, real income \( (Y_t) \) is used to repay the credit, \( D_t = W_t \), for interest payment to the banks, \( i_t D_t \), and paid as profits to the owners of the firms, \( \Pi_t \), which are part of the households:

\[
Y_t = W_t + i_t D_t + \Pi_t 
\]  

The government is not modelled explicitly because government policy is not at the centre of this analysis. When government action is introduced, it is assumed to influence income through its tax policy, whereas government spending is adjusted accordingly (balanced budget).

Since the amount of credit equals the wage fund (see (5.1.1)), the profit share can be written as (using (5.1.2) and divide by income\(^2\)):

\[
\pi_t = 1 - (1 + i_t) w_t 
\]  

The profit share depends on the interest rate and on the share of wages, which depends -as will be discussed later- on the wage rate and the rate of utilisation. The influence of production depends on the effects on the distribution of income over banks, wage earners and the firms.

\(^2\)In the following, upper case symbols are used for levels, lower case symbols indicate ratios to income, so \( w_t = \frac{W_t}{Y_t} \); \( d_t = \frac{D_t}{Y_t} \); \( \pi_t = \frac{\Pi_t}{Y_t} \); \( s_t = \frac{S_t}{Y_t} \); only \( i_t \) gives the interest rate.

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Banks receive interest from the firms, \( i_t D_t \), and pay interest to the households, \( i_t S_t \). In the end, all income is paid as income to the households, who use income to save (as deposits with the banks) or to consume (buy the consumption goods of the firms). Savings are defined as the level of deposits, \( S_t \). The difference between the level of deposits at the end of the period and the initial level is called net-savings, \( (S_{t+1} - S_t) \). At the end of the period (at time \( t+1 \)) savings, \( (S_{t+1}) \) equal:

\[
S_{t+1} = s_{t+1} Y_t
\]  

(5.1.4)

During the period from \( t-1 \) to \( t \), households withdraw their savings \( (S_t) \) and receive an income (which equals production, \( Y_t \)). They determine the level of their next-period savings \( (S_{t+1}) \). Nominal expenditure on consumption goods, \( (E_t) \), is present income \( (Y_t) \) minus net-savings, \( (S_{t+1} - S_t) \):

\[
E_t = Y_t - (S_{t+1} - S_t)
\]

\[
= (1 - s_{t+1}) Y_t + s_t Y_{t-1}
\]  

(5.1.5)

Flexible prices, \( (p_t) \), are assumed to take care of equilibrium in the goods market. Setting \( E_t \) equal to nominal production, \( p_t Y_t \), in (5.1.5) gives the dynamics of the price level \( (p_t) \):

\[
p_t = 1 - s_{t+1} + s_t \frac{Y_{t-1}}{Y_t}
\]  

(5.1.6)

The present price level, \( (p_t \) in (5.1.6)), is lower when the present rate of saving is higher \( (dp_t/ds_{t+1} < 0) \) because demand declines. A higher saving rate or income in the former period increases present period demand \( (dp_t/ds_t > 0, dp_t/dY_{t+1} > 0) \), a rise in supply depresses the price level \( (dp_t/dY_t < 0) \). To simplify the model, prices are assumed to equate demand and supply in the goods market, but have no influence on the savings and production decision process.

The mutations of the accounts following the process in period \( (t \) to \( t+1 \)) described above are:

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3 For simplicity, the two rates of interest are assumed to be the same. The difference between those interest payments is paid out as profit income to the owners of the banks, again part of the household sector. Different rates of interest for firms and households does complicate the analyses, but does not influence the conclusions drawn in this chapter.

4 The price level does act as a flexible propensity to consume equating real demand and supply. Here, however, consumption and production are set independent of each other and equilibrated in real terms by the price mechanism.
### A. Households

<table>
<thead>
<tr>
<th>Net savings</th>
<th>Wages</th>
<th>Profits of the firms</th>
<th>Interest received</th>
<th>Profits of the banks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(S_{t+1} - S_t)$</td>
<td>$W_t$</td>
<td>$\Pi_t$</td>
<td>$i_S$</td>
<td>$i(D_f - S_f)$</td>
</tr>
<tr>
<td>Consumption</td>
<td>$Y_t$</td>
<td>$i_D_t$</td>
<td>$i_D_t$</td>
<td>$Y_t$</td>
</tr>
</tbody>
</table>

### B. Banks

<table>
<thead>
<tr>
<th>Interest paid</th>
<th>Interest received</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_S$</td>
<td>$i_D_t$</td>
</tr>
</tbody>
</table>

### C. Firms

<table>
<thead>
<tr>
<th>Wages</th>
<th>Sales</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_D_t$</td>
<td>$E_t/p_t$</td>
</tr>
</tbody>
</table>

These mutations give the new accounts, at $t+1$, at the beginning of the next period, before borrowing of the firms:

### A. Households

<table>
<thead>
<tr>
<th>Deposits</th>
<th>Wealth (savings)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{t+1}$</td>
<td>$S_{t+1}$</td>
</tr>
</tbody>
</table>

### B. Banks

<table>
<thead>
<tr>
<th>Deposits</th>
<th>Net wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{t+1}$</td>
<td>$S_{t+1}$</td>
</tr>
</tbody>
</table>

### C. Firms

<table>
<thead>
<tr>
<th>Capital</th>
<th>Net wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>$K$</td>
</tr>
</tbody>
</table>

So far, the basic dynamic identities of the economy were described. The next step is to introduce the behavioural relationships and the adjustment processes in the real and monetary sector of the economy, to determine the dynamics for this economy. The behaviour on the two markets will be discussed separately. In the end the dynamics of the economy will be described by a set of four equations:

The monetary sector:

1. $i_{t+1} = f(i_t, s_t, w_t, u_t)$
2. $s_{t+1} = g(i_t, s_t, w_t, u_t)$

The real sector:

1. $w_{t+1} = h(i_t, s_t, w_t, u_t)$
2. $u_{t+1} = z(i_t, s_t, w_t, u_t)$

With:

- $i_t$ = rate of interest
- $s_t$ = savings rate
- $w_t$ = wage share
- $u_t$ = rate of utilisation of the capital stock

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To simplify the exposition, first neglect the role of other sector's variables on the other market (which is assumed to be constant). In paragraph two, the monetary sector will be analysed, determining the dynamics of the saving rate and the interest rate, the first two equations. The real variables ($w$ and $u$) are for the moment taken as given:

\[
i_{t+1} = f\left(i_t, s_t, \bar{w}, \bar{u}\right)
\]

\[
s_{t+1} = g\left(i_t, s_t, \bar{w}, \bar{u}\right)
\]

Paragraph three focuses on the labour market. Then the interest rate and the saving rate are assumed to be constant. Here, the dynamics are governed by the interaction between the utilisation of the stock of capital (which determines the demand for labour) and the wage share in income:

\[
w_{t+1} = h\left(w_t, u_t, \bar{w}, \bar{s}\right)
\]

\[
u_{t+1} = z\left(w_t, u_t, \bar{w}, \bar{s}\right)
\]

In section four, the model is elaborated to allow for a linkage between sectors. Here, the net credit creation will depend on income too, so the interest rate will depend on the utilisation rate. The utilisation of capital is influenced by the costs of financing the wage fund and therefore by the interest rate. The chapter is concluded with section five, giving a summary and drawing some final conclusions.

2 The monetary sector

This section concentrates on the monetary sector. To do so, production and the wage share in income are assumed to be constant. The demand for funds is, therefore, also constant. Money is assumed to be endogenous: the banks provide all the funds needed by the firms, at the given interest rate. Supply of credit equals demand for credits. The amount of net credit supplied by the banks, the net credit creation, depends on the difference between the demand for funds and the deposits, which are given by the savings out of former income. As in the monetary business cycle of Hawtrey and Hayek, the banks have a desired level of net credit creation. In deviation of the Hawtrey/Hayek-approach, credit is not restricted when the actual amount of net credit creation is above the desired level, only the interest rate is changed. When the actual net credit creation is above the desired level, the banks set a higher interest rate for the next period, inducing savings to rise and thereby lowering the actual net credit creation.

Income is assumed to be constant, so changes in the level of deposits are determined by changes in the saving rate. The determination of the saving rate is taken up in sub-section 2.2. The dynamics of the monetary sector are discussed in 2.3 and in 2.4.
2.1 The demand for loanable funds

Firms demand funds to finance spending on outlays (labour and other inputs, capital outlays in the terminology of Foley(1987)). As noted by Negishi(1989) the demand for funds is the result of a lag between physical production and the realisation of the value of production through trade. In the short-run the influence of the labour market on the demand for funds is ignored, so the demand for funds is assumed to be exogenous:

\[ d_t = \bar{d} \]  

(5.2.1)

\[ d_t = \text{demand for funds in at } t \]

2.2 The supply of loanable funds

The capital stock is assumed to be constant: private households do not invest directly in capital or other real assets; savings are held in the form of deposits at the banks.

Households decide to consume part of their present income and to save a portion, s, of their income. They are assumed to change their net-savings \((S_{t+1} - S_t)\) when the present interest rate differs from an (exogenous) level of the interest rate \((\alpha/\beta)\). Given the interest rate, this decision results in a new level of deposits, \(S_{t+1} = S_{t+1}Y_t\), available on the credit market at \(t+1\). A rise in the interest rate above the desired level increases the saving ratio, relative to its former value. Given income, this increases the supply of deposits. In the aggregate\(^5\), net savings \((S_{t+1} - S_t)\) are positive when \(i_t\) is above \(\alpha/\beta\):

\[ \frac{S_{t+1} - S_t}{s_t} = -\alpha + \beta i_t \]  

(5.2.2)

\[ s_{t+1} = \text{savings (rate) in at } t, \text{ deposits available at } t+1 \]

\[ i_t = \text{interest rate in at } t \]

The demand for credits by the firms equals \(d\). It is assumed that the supply of credits equals the demand for credit. Part of the supply of credits is backed by the available deposits \((S_{t+1})\), part has to be supplied by the banks. The difference \((v)\) is called the net-credit-creation. Following the monetary theories in chapter two (Hawtrey, Hayek), the banks are assumed to have a desired level of net-credit creation \((\varepsilon/\chi)\). The banks set the interest rate in response to deviations between the desired and the actual net-credit-creation. When the actual net-credit-creation is above the desired level, they raise the interest rate: this raises savings (deposits), lowering the net-credit-

\(^5\)Attention is paid to the macro-dynamics. On an individual level households could want to loan against an interest rate above \(\alpha/\beta\), but those are assumed to be a minority in the total of households. Also there are more firms demanding money, than offering deposits.

\(^6\)All parameters are taken to be positive, unless stated otherwise.
creation. It is assumed, that $\varepsilon$ and $\chi$ generate a positive desired net-credit-creation, so it covers the costs of banking, including a 'reasonable' level of profits for the owners of the banks. These assumptions give:

\[ v_{t+1} = d_{t+1} - s_{t+1} \]
\[ \frac{i_{t+1} - i_t}{i_t} = -\varepsilon + \chi v_{t+1} \]  \hspace{1cm} (5.2.3)

$\nu_t = \text{net-credit-creation}$

The first equation in (5.2.3) gives the net-credit-creation necessary because of the funding of the wage fund, minus the deposits (the savings from the last period). The second equation determines relative changes in the interest rate as result of the net-credit-creation. Solving (5.2.2) and (5.2.3), using (5.2.1), gives the dynamics of the credit market, describing the present variables as a function of their past values\(^7\):

\[ i_{t+1} = (1 - \varepsilon + \chi d - \chi s_t \{1 - \alpha\} - \chi \beta_v i_t) i_t \]
\[ s_{t+1} = (1 - \alpha + \beta_i) s_t \]  \hspace{1cm} (5.2.4)

Equation (5.2.4) gives the evolution of the interest rate and the savings rate. The supply of deposits in $t+1$ ($s_{t+1}$, the savings in $t$) is a function of the level of deposits in $t$ and the interest rate in $t$, $i_t$. The net-credit-creation is determined by the supply of deposits in $t+1$, so the interest rate in $t+1$ depends negatively on the level of deposits in the former period ($s_t$), whereas the effect of the interest rate in $t$ ($i_t$) on $i_{t+1}$ is ambiguous.

### 2.3 The market for loanable funds

In this section the conditions for the steady state\(^8\) and the dynamics outside the steady state are analysed\(^9\). Appendix A gives a mathematical analysis of the uncoupled systems and their steady states. There are two fixed points, ($i=0$, $s=0$) and ($s=s^*, i=i^*$). When the net-credit-creation equals its desired

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\(^7\)Although this model looks similar to the familiar Lotka-Volterra model, it deviates from it through the dependency of $i_{t+1}$ on $s_{t+1}$ instead of $s_t$.

\(^8\)A steady state (an equilibrium or a fixed point) of map $f$ is a point $x^*$ for which $f(x^*)=x^*$, at which the dynamical system is at rest.

\(^9\)Equation (5.2.4) is comparable with the logistic:

\[ i_{t+1} = A(s)i_t(1 - B(s)i_t); \quad A(s) = 1 - \varepsilon + \chi d - \chi s_t \{1 - \alpha\}; \quad B(s) = \frac{\chi \beta_v}{1 - \varepsilon + \chi d - \chi s_t \{1 - \alpha\}} \]

There is, given the savings in $t$, a level of the interest rate above which the deposit's in $t+1$ rise to such a level where the actual net-credit-creation declines below its desired level, causing the interest rate to decline in $t+1$.

This is not analysed further here, because the influence of the coupling is the main theme of this chapter (see below). Due to the formulation above it is already evident that the dynamics of $i$ will depend on $A(s, u)$ in the models below.
level, the interest rate remains constant. The desired net-credit-creation was given by:

\[ v^* = \frac{\epsilon}{\chi} \]

The desired level of \( v \) can be substituted in (5.2.3) to determine the equilibrium level of savings:

\[ s^* = \frac{\bar{d}}{\chi} - \frac{\epsilon}{\chi} \]

To generate the required savings, the necessary interest rate is given by (5.2.2):

\[ i^* = \frac{\alpha}{\beta} \]

At \( i^* \), net savings are zero \( (s_{t+1} - s_t = 0) \), so the level of savings is constant \( (S_t = S_{t+1}) \). In appendix A both the equilibria \((i^*, s^*)\) and \((0, 0)\) are shown to be 'neutrally stable'. In this case, it is not possible to make general statements about the stability of the system. Simulations and the phase-diagram have to be used to show the occurrence of cycles in this model.

Using (5.2.4), the phase-diagram, in figure 5.2, is drawn by depicting the locus for \( \frac{i_{t+1} - i_t}{i_t} = 0 \) and for \( \frac{s_{t+1} - s_t}{s_t} = 0 \):

\[ \frac{i_{t+1} - i_t}{i_t} = 0 \rightarrow s = \frac{\bar{d} - \epsilon}{\chi[1 - \alpha + \beta i]} \]

\[ \frac{s_{t+1} - s_t}{s_t} = 0 \rightarrow i = \frac{\alpha}{\beta} \]

The 'non-changing' level of the interest rate depends on both the level of savings and the interest rate itself. Given the level of the interest rate, savings above the equilibrium level of savings \( (s^*) \) lower the interest rate. Savings below this level result in a higher level of net credit creation, which raises the interest rate. The directions of interest rate and savings are drawn in figure 5.2\textsuperscript{10}.

\textsuperscript{10} Assuming \( \alpha < 1 \), so the second branch of \( i_{t+1} - i_t/i_t = 0 \) is assumed away. Of course, the real interest rate can be negative. In that case the households would also like to borrow from the banks, so no backing of the loans exists anymore. The monetary authorities, because of the fear of bankruptcy, will resist this. This situation is assumed away in this model. The occurrence of high inflation, low nominal interest rates and a society going into debt would, by non...
Assume savings and the net creation of credit initially to be at their equilibrium level. When the interest rate is below $i^*$, savers decide to lower their savings (consume more) in the present period. Expenditure increases and given production, the price level rises. The decline in savings increases the credit creation in the next period above the desired level of credit, so the banks raise the interest rate to generate more savings. The rise in savings lowers expenditure on goods, decreasing the price level. The model predicts a (imperfect) negative relationship between the interest rate and changes in price level\(^{11}\). When the interest rate reaches its equilibrium value, the net-credit-creation is beneath its desired level. This causes a decline in the interest rate and the accompanying decline in savings and rise of credit creation.

### 2.4 The dynamics

#### 2.4.1 The bifurcation diagrams

Several bifurcation diagrams are calculated, for the parameters, $\beta$ and $\chi$. Each time another parameter is changed. The remaining parameters are held constant. The total period of the simulation is taken to be 5,000 iterations and there are 500 steps between the minimal and maximal value of the parameter. It is, therefore, possible for the model to exhibit a different behaviour for parameter values, which are not observed here.

\(^{11}\)This is confirmed by the survey of Blanchard and Fischer(1989, 19-20), who find the real interest rate to behave (slightly) pro-cyclical, whereas inflation has a (small) negative correlation with changes in GNP.

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The following results are found:

<table>
<thead>
<tr>
<th>Range of $\beta$</th>
<th>Dynamics</th>
<th>Range of $\chi$</th>
<th>Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01-29</td>
<td>Cycles</td>
<td>0.01-30</td>
<td>Cycles</td>
</tr>
<tr>
<td>29-30</td>
<td>Unbounded</td>
<td>30-31</td>
<td>Unbounded</td>
</tr>
<tr>
<td>30-118</td>
<td>Cycles</td>
<td>31-45</td>
<td>Cycles</td>
</tr>
<tr>
<td>118-.....</td>
<td>Unbounded</td>
<td>45-.....</td>
<td>Unbounded</td>
</tr>
</tbody>
</table>

For each of the parameters the bifurcation diagram for the first stable period is shown below in figure 5.3 ($\beta$: 3a, $\chi$: 3b).

![Bifurcation Diagrams](image)

Figure 5.3: The bifurcation diagrams for $\beta$ and $\chi$.

### 2.4.2 The Lyapunov characteristic exponents

For the parameters reported above $LCE$'s are calculated using the DMC-program of Medio and Gallio.

$\beta$: In the case of $\beta$, the $LCE$'s show fluctuations around different trends: for $0.01<\beta<99$, the trend is zero.

$\chi$: For $0.01<\chi<15$, the $LCE$'s are negative and show dampened fluctuations towards zero. Between 20 and 41, the fluctuations are around zero. The model, however, loses its stability around $\chi=30$. Effectively, taking the estimation errors of the simulations in account the resulting $LCE$'s cannot be distinguished from zero: the dynamics are characterised as quasi-periodic and periodic behaviour.

### 3 The labour market

#### 3.1 Employment and wages

Goods are produced using labour as input. After production, wages are paid, production is sold, credit redeemed and interest is paid. The remaining value of sales is distributed to the owners of the firms, part of the households, as profits. Part of the income is spent on consumption, part is

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saved. Since here only the dynamics of the labour market are analysed, the monetary sector is assumed to be in equilibrium; \( s_t = s^*, \ i_t = i^* \)

The existing capital stock is assumed constant and non-perishable in the short run. Changes in production are proportional to changes in the utilisation of the capital stock. There is a large number of small firms, each owning one unit of capital. Firms are indifferent if they stay or leave the production sector, at an average level of profit, \( \pi^* \). Below this level of \( \pi \), firms start to leave the production sector. They store the capital good, until profitability rises above \( \pi^* \). Above this level of profit, firms re-enter. The rate of utilisation gives the total number of active firms (capital goods) to the number of firms active when the rate of profit equals the 'normal' level of profit, \( \pi^* \). The adjustment is continuous in the sense that not all firms simultaneously enter (leave) the production sector for \( \pi > \pi^* \) (\( \pi < \pi^* \)). The level of utilisation rises proportionally to the ratio of actual to 'normal' or desired profits:

\[
\frac{u_{t+1}}{u_t} = \frac{\pi_t}{\pi^*} \tag{5.3.1}
\]

\( u_t \) = utilisation of the capital stock
\( \pi_t \) = share of profits in income
\( \pi^* \) = desired share of profits in income

Using equation (5.1.3), (5.3.1) gives the rate of utilisation as a function of the wage share in income and the interest rate:

\[
u_{t+1} = u_t \lambda (1 - (1 + i^*)w_t) \tag{5.3.2}
\]

\( \lambda = \frac{1}{\pi^*} \) = the inverse of the desired share of profits
\( i^* \) = equilibrium interest rate
\( w_t = \frac{W_t}{Y_t} \) = wage share in income

A rise in the rate of utilisation of the existing capital stock increases the demand for labour to operate the active stock of capital goods. Given the wage rate, this raises the wage fund through its effect on employment. The rise in employment, in turn, raises the wage rate, again a rise in the wage fund. This is most certainly the effect when the supply of labour reaches some kind of absolute maximum (\( l^* \)). A higher capital stock requires more labour, which is -then- realised by a rise in labour productivity that raises the wage rate12. Here, a rise in the rate of utilisation is assumed to increase

\[12\text{A rise in productivity can be realised by a decrease in shirking, or by inducing labour to work harder or longer. These actions raise costs either through control-costs or higher wages. Here the latter is assumed (also see Phelps(1994), 20).} \]
the wage share, ignoring the division between a rise in employment and in
the wage rate. Secondly, the wage share behaves smoothly: the rate of
utilisation determines changes in the wage share, not only the level.

The labour market is illustrated in figure 5.4. The demand for labour is
predetermined by the profit rate in the former period \((ld(\pi_t))\). The wage
share in \(t+1\) is determined by the labour supply, which depends on the wage
share in the former period, and the demand for labour. This can be
modelled as a linear relationship \((Is(w^1_t))\) in figure 5.4) or as a non-linear one
\((Is(w^2_t))\) in figure 5.4), depending on the reaction of the wages to the changes
in demand.

To model this mechanism, relative changes in the wage share are assumed
to depend on the deviations of the rate of utilisation from its ‘normal’ rate
\((u^*-u_t; u^*=1)\). This determines the second dynamic equation for the labour
market:

\[
w_{t+1} = w_t (1 - \phi (1 - u_{t+1}))
\] (5.3.3)

Equations (5.3.2) and (5.3.3) describe the dynamics of the labour market and
production (as represented by utilisation). Starting from a high level of
profit, utilisation increases in the next period. This increases the demand for
labour and the wage share. The higher the utilisation of capital, the more
labour is used in production. The higher labour demand increases the wage
share in production. A higher labour share depresses the profit share, so

---

13 The model gives no explanation for unemployment because the labour market always clears.
This is a simplification to reduce the complexity of the model. A further extension of this
analysis should account for the division between changes in the labour productivity and the
demand for labour.

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next-period utilisation and employment decline. This feedback generates an 
utilisation-wage share cycle, comparable to Goodwin(1967)\(^{14}\).

Furthermore, it is assumed that if the wage share falls towards zero, 
utilisation rises with a definite amount, instead of going to infinity. This is 
due to the time consuming adjustments: firms cannot enter 
instantaneously. When the rate of utilisation goes to zero, it also takes time 
for the wage share to fall to zero.

Equation (5.3.2) and (5.3.3) give the following \(u-w\) model:

\[
\begin{align*}
    u_{t+1} &= u_t \lambda \left(1 - (1 + i^*)w_t\right) \\
    w_{t+1} &= w_t \left(1 - \phi \left(1 - u_t \lambda \left(1 - (1 + i^*)w_t\right)\right)\right)
\end{align*}
\]

The steady state values for the rate of utilisation and the wage share are (for 
the analyses of the stability in the equilibrium, see appendix A):

\[
\begin{align*}
    u^* &= 1 \\
    w^* &= \frac{\lambda - 1}{\lambda (1 + i^*)}
\end{align*}
\]

Solving (5.3.2) and (5.3.3) for \(\frac{u_{t+1} - u_t}{u_t} = 0\) and \(\frac{w_{t+1} - w_t}{w_t} = 0\) gives:

\[
\begin{align*}
    \frac{u_{t+1} - u_t}{u_t} = 0 \rightarrow w = \frac{\lambda - 1}{\lambda (1 + i^*)} \quad (5.3.4) \\
    \frac{w_{t+1} - w_t}{w_t} = 0 \rightarrow u = \frac{u^*}{\lambda (1 - (1 + i^*)w)} \quad (5.3.5)
\end{align*}
\]

Equation (5.3.4) states that, for the rate of utilisation to remain constant, the 
wage share has to be at a level at which the firms receive the desired profit 
share and the savers the desired interest rate. In equation (5.3.5) the wage 
share is constant when the rate of utilisation is such that all participants in 
the production process receive their desired income: a higher rate of interest 
has to be compensated by a higher rate of utilisation so income of the other 
participants does not change. Equations (5.3.4) and (5.3.5) are shown in a

\(^{14}\)Compare the discrete time version of the Goodwin-Lotka-Volterra-model in Lorenz(1993). 
The differences with the standard solution to the Goodwin-Lotka-Volterra (GLV)-model are: 
1-the timing of the reactions. In the GLV-model employment and the wage share are 
dependent upon the variables in the last period. Here we assume the wage share to react 
directly to the level of employment; 2-in the GLV-model, the influence of the profits on 
employment is indirect, through actual changes in the capital stock. Here, a more direct 
influence is assumed, in the form of entry and exit.
phase diagram (figure 5.5). When utilisation is above (below) \( \frac{w_{t+1} - w_t}{w_t} = 0 \), the wage share rises (declines). The level of the wage share (adjusted for the interest rate\(^\text{15} \)) is between 0 and 1. A level of \((1+i^*)w=1\), is only sustainable when the rate of utilisation goes to infinity, otherwise, such a high wage share in income lowers utilisation. When the wage share is to the right (left) of \( \frac{u_{t+1} - u_t}{u_t} = 0 \), utilisation declines (rises) because the wage share is too high (low) for an equilibrium situation. The combined movements in the four possible situations are drawn in figure 5.5.

![Phase diagram](image)

Figure 5.5: The phase diagram for \( u \) and \( w \).

### 3.2 The dynamics

#### 3.2.1 The bifurcation diagrams

A bifurcation diagram is calculated, for different values of parameter \( \phi \) and shown in figure 5.6. The other parameters are held constant at the value used in the base-simulation. The total period of the simulation is taken to be 5,000 iterations and there are 500 steps between the minimal and maximal value of the parameter. It is, therefore, possible for the model to exhibit a different behaviour for parameter values that are not observed here.

The following results are found:

<table>
<thead>
<tr>
<th>Range of ( \phi )</th>
<th>Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01-1.1</td>
<td>Cycles</td>
</tr>
<tr>
<td>1.1-1.15</td>
<td>Unbounded</td>
</tr>
<tr>
<td>1.15-1.5</td>
<td>Cycles</td>
</tr>
<tr>
<td>1.5-...</td>
<td>Unbounded</td>
</tr>
</tbody>
</table>

\(^{15}\)When there is mentioning of the wage share in income in this section, the wage share adjusted for the costs of financing the wage fund, \( w(1+i^*) \), is meant.
3.2.2 The Lyapunov characteristic exponents

For the parameters reported above $LCE's$ are calculated using the DMC-program of Medio and Gallio. Again the $LCE's$ show dampened fluctuations, tending towards zero from below, for $0.1<\phi<1$. For $1.1<\phi<1.5$, there is a stable cycle for which the trend rises slowly above zero. Taking the estimation errors of the simulations in account the resulting $LCE's$ cannot be distinguished from zero: the resulting dynamics are characterised as quasi-periodic and periodic behaviour.

4 Coupling of the labour market and the monetary sector

In this section the influence of the two sectors on each other is traced. By combining the markets, the dynamics of the two-sector economy (labour and money) can be determined. Independent of each other, the sectors exhibit (quasi-) periodic behaviour (as shown by analysing the bifurcation diagrams and the $LCE's$). Here, it will be shown that an interaction between the two sectors changes the dynamics of both the aggregate economy and that of the individual sectors. Questions are whether the coupling increases the aggregate fluctuations and if these fluctuations deviate from those in the partial sectors?

4.1 The influence of labour market fluctuations on the monetary sector

To determine the effect of fluctuations in the labour market on the monetary sector, the model has to be adjusted to allow for the link between the labour market to the monetary sector and vice versa.
The first effect to consider is the demand for funds. In section one the demand for funds was defined as the amount of money needed by the firms to pay the wage fund, and normalised to \( d=1 \). If the demand for funds is determined in the labour market, it will depend positively on the wage share: \( d_t = d(w_t); \quad d' > 0 \).

Secondly, the supply of deposits is determined by the former-period-savings. These depend positively on income in the former period: \( S_t = S(s_t, u_{t-1}); \quad S_{tt} > 0 \).

Given these two relationships, the net-credit-creation depends positively on the wage share and the former rate of utilisation: \( V_t = D_t - S_t = V(w_t, s_t, u_{t-1}) \).

The present rate of utilisation depends negatively on the former wage share, whereas it has a positive relationship with the present wage share: \( \Delta u_t / \Delta w_{t-1} < 0 \) (see 5.3.3), \( \Delta w_t / \Delta u_t > 0 \) (see 5.3.2). So a relative high present rate of utilisation indicates a low wage share in the former period and precludes a high wage share in the present period.

There are three effects on the net-credit-creation:
1. the rise in demand for funds, when the rate of utilisation rises;
2. the rise in the supply of funds because of a rise in utilisation;
3. the rise in the supply of funds because of a rise in the rate of saving.

The rate of utilisation is taken as the link between the labour market and the monetary sector, a high utilisation rate indicates low savings and a high demand for money, thereby raising the net-credit-creation. Replacing \( d_t \) with \( u_t \) summarises the influence of the factors following changes in the wages and utilisation. The net-credit-creation then depends on the savings rate \( (s_t) \) and on the rate of utilisation:

\[
V_t = u_t - s_t \tag{5.4.1}
\]

Substituting (5.4.1) in (5.2.3) gives a new equation for the interest rate:

\[
i_{t+1} = i_t \left(1 - \epsilon - \chi u_{t+1}\right) \\
= i_t \left(1 - \epsilon - \chi(u_{t+1} - s_{t+1})\right) \\
= i_t \left(1 - \epsilon + \chi u_{t+1} - \chi s_t [1 - \alpha] - \chi \beta s_i t\right) \tag{5.4.2}
\]

The equilibrium level of \( i \) does not change because of the coupling \( (i^* = a / \beta) \), as can be seen by taking \( s_{t+1} - s_t = 0 \) in (5.2.4). The level of \( s \) for which the money market is in equilibrium, depends on the state of \( u \). Setting \( \frac{i_{t+1} - i_t}{i_t} \) to zero in (5.4.2) gives the following equation for \( s \):

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\[ s_t = \frac{1}{[(1-\alpha) + \beta i_t]} u_{t+1} - \frac{\epsilon}{\chi (1-\alpha + \beta i_t)} \]

Setting \( i \) to \( i^* \) gives for \( s^* \) (the value of \( s \) necessary for local money market equilibrium):

\[ s^* = u_{t+1} - \frac{\epsilon}{\chi} \]

The equilibrium level of savings changes because of the changes in utilisation. A rise in utilisation increases the demand for funds. To keep the net-credit-creation and the interest rate unchanged, the savings rate has to rise with the rise in the rate of utilisation. Theoretically there is a continuum of \( \frac{i_{t+1} - i_t}{i_t} = 0 \)-lines, one for every value of \( u \). The equilibrium \((s^*(u_t), i^*)\) lies between those values, depending on the level of \( u_t \).

### 4.2 The influence of the monetary sector on the labour market

In section 3, it was assumed that changes in the utilisation rate depend only on the wage share in the former period. When the financing of the wage-fund through the financial market is taken into account, those costs also influence the decision on the utilisation rate. Equation (5.3.2) has to be adjusted accordingly:

\[ u_{t+1} = u_t \lambda (1 - [1 + i_t] w_t) \]  \hspace{1cm} (5.4.3)

A rise in the wage costs lowers the rate of utilisation, as before, due to its effect upon actual profits. In this case, a higher interest rate also raises costs of production, depressing utilisation. The determination of the wage share remains the same. The interest rate, however, has a negative effect on the wage share through its effect on present utilisation.

The interaction changes the level of the wage share, at which the utilisation rate equals its 'normal' level. Keeping the desired profit share \((\pi^* = 1/\lambda)\) equal, a higher level of the interest rate means there has to be a lower level of the wage share to reach equilibrium. So when the costs of financing are taken into account, this lowers the value of the wage share, necessary for labour market equilibrium. Using (5.3.3) and (5.4.3), this gives:

\[ \frac{w_{t+1} - w_t}{w_t} = 0 \rightarrow u^* = 1 \]

\[ \frac{u_{t+1} - u_t}{u_t} = 0 \rightarrow w^* = \frac{\lambda - 1}{\lambda [1 + i_t]} \]
As with the influence of $u$ on $i$, the steady state of one market relies on the state of the other market. A rising interest rate requires a lower wage share to keep the profit rate at its desired level: the total of adjustment falls on the wage share (in equilibrium), keeping $u^*$ constant. So, the curve $\frac{w_{t+1} - w_t}{w_t} = 0$ shifts.

4.3 The two-sided feedback between the labour market and financial sector

In the monetary sector, the savings rate and income (as approximated by the rate of utilisation) determine the interest rate. The present savings rate (part of former income, which at the present is held as deposits) is determined by the distribution of income between present and future consumption, based on the interest rate:

$$i_{t+1} = \left(1 - \varepsilon - x\beta i_t [1 - \alpha] - x\beta s_t i_t + x[u_t \lambda (1 - [1 + i_t]w_t)]\right)i_t$$

$$s_{t+1} = (1 - \alpha + \beta i_t) s_t$$

In the labour market, present production (again represented by the rate of utilisation of the existing stock of capital) is determined by the former share of profits in income, which is in turn the result of the costs of production (the wage fund and interest paid). The demand for labour, following from production, determines the wage share:

$$u_{t+1} = u_t \lambda (1 - [1 + i_t]w_t)$$

$$w_{t+1} = w_t \left(1 - \phi (1 - u_{t+1})\right)$$

General equilibrium gives the familiar equations: $i^* = \frac{\alpha}{\beta}$, $s^* = 1 - \frac{\varepsilon}{\chi}$; $u^* = 1$ and $w^* = \frac{\lambda - 1}{\lambda \left(1 + \frac{\alpha}{\beta}\right)}$, in which $x^*$ gives the value of the variable $x$, $(x = i, s, u, w)$ necessary for equilibrium on the money market and the labour

---

16Langen(2000a) analyses the partial effects of the one-sided-couplings: keeping $i$ constant at $i^*$ in the labour market, investigating the influence of the wage-utilisation dynamics on the monetary sector. The effect of the monetary cycle on the labour market can be modelled by taking the actual interest rate, $i_t$, when determining costs, without a feedback from the labour market towards the money market. Using Medio's DMC, these partial versions of the model analysed were shown to be truly chaotic for large ranges of the parameters.
market simultaneously \((\{x_{t+1} - x_t\}/x_t = 0;\) other variables taken at their equilibrium value).

Figure 5.7: The coupled cycle in a flow diagram (+/- gives the changes in the period, as result of changes within the period and the former period).

Figure 5.7 gives the flow chart of the dynamics. If \(u\) rises as result of an exogenous change, the firms demand more labour, so the wage share rises as in the uncoupled case. The interest rate rises also because the coupling of the two sectors: the increased demand for funds stimulates the net credit creation, given the savings in the former period. The rise in costs of production has a negative effect on the current profit rate. This depresses the rate of utilisation in the next period: depression is setting in, accompanied by a decline in the wage share. The savings rate, however, rises, as the interest rate has risen. The decline in the demand for funds, combined with the rise in deposits lowers the net credit creation of the banks. This reduces the present interest rate.

The production costs (the interest rate and the wage share) decline. The rate of profit on capital rises because of this decline. The rate of utilisation will increase in the next period, endogenously returning to the start of the cycle.

This model highlights one of the functions of the banks: banks are firms created to generate income, at least to cover labour and managerial costs, by setting the interest rate as high as possible. The banks’ goal is contrary to the goal of savers and firms (respectively receiving a large amount of interest and minimising interest costs). An action on the part of the banks induces a counteraction of savers, counterbalancing the effect on income of the banks: a raise in the interest rate because of the extra income, brings about a rise in savings annulling the extra income and bringing down the interest rate because of the excess supply of funds. The firms are also led by their profits. As they take into account the real costs of financing the wage fund, they respond more to changes in the wage share, when both the interest rate and the wage share influence the profit share in the same direction.
Two observations
The costs of production (interest payments and the wage fund) move together. High labour costs are enhanced by high financial costs. This conclusion depends on the reaction of the rate of savings. In this model, the wage share has no independent direct effect on the savings as its effect is captured by the present rate of utilisation.

The chart also shows a negative co-movement between the rate of savings and utilisation. The savings rate rises because the interest rate is high, which causes the rate of utilisation (in combination of the rising wage share) to decline. Therefore, the model predicts a positive relationship between consumption and production. A lower rate of savings is the mirror side of a rise in consumption. During a depression both production and consumption decrease. These changes, however, are independent of each other: there is no direct relationship between consumption and production.

4.4 Simulating the coupled economy

The coupling of both sectors causes the model to become explosive (see figure 5.8). A negative deviation from $i$ of $i^*$ leads to a positive deviation. After time a high savings rate (a high supply of deposits) will coincide with a low level of utilisation (a low demand for money). Firms demand funds below the level of available deposits, so banks' income becomes negative. The interest rate becomes negative and the model becomes explosive. To restrict the interest rate above zero, the central bank is assumed to guarantee a minimum level of interest on the deposits, for example by giving out bonds at $i_{min}$. As the interest rate approaches $i_{min}$, part of the savings is kept as bonds, lowering the supply of deposits. This ensures stability as the two sectors are coupled. The minimal interest rate, however, is chosen to be at a level that does not influence the uncoupled dynamics. A drawback of this added non-linearity, which is necessary for stability, is the impossibility to use DMC to determine the LCE's.

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17In an earlier version of this chapter it was also assumed that the coupled model was stable when $\phi = \alpha' (xe - e)/(\lambda - 1)$. Very long-term simulations (up to 10,000,000 periods), using DMC, show that even for small deviations between the initial variables and their equilibrium values, the model becomes unstable. The unstable dynamics of the monetary sector is confirmed by the work of Morishima (1992, 23-24).
There is a difference between the short and the long run, as can be seen by simulating the model. In the short-run (assuming a moderate difference between the initial value of the variables and their equilibrium value) the cycles appear to be complex as shown in figure 5.9A. The short-run dynamics are roughly similar to the one-coupled dynamics in Langen (2000a). Both sectors show a tendency to ‘fill’ the surface within the original uncoupled cycle. The shifting ‘8-shaped’ labour market dynamics reflect the higher periodicity of the money market: there is a high state equilibrium \((1, w_{\text{high}})\) and a low state equilibrium \((1, w_{\text{low}})\), whereas those equilibria move slowly up and down as interest rate changes (with the old \((u^*, w^*)\) in the middle). The dynamics in the labour market are reflected by an up- and downward change in \(s^*\) where the cycles are interrupted before closing. The actual appearance of the cycles depends on the periodicity of the cycles in each market (see below).

\[\begin{align*}
\text{Figure 5.8: The unrestricted model: long-term instability.}
\end{align*}\]
In the long run, fluctuations in the money market become unbounded. A combination of a high savings rate and a low level of utilisation give the monetary authorities reason to intervene. So, the restriction of the minimal interest rate becomes effective. The resulting cycles are depicted in figure 5.9B: the $is$-dynamics are simplified into an a-symmetric cycle; the '8-shaped' $uw$-dynamics becomes a double cycle. The long-run dynamics are insensitive to initial values: in time a similar deviance between $i_{\text{min}}$ and $i^*$ is encountered in each case.

The backbending of the $u-w$ curve is the result of uncompleted cycles around different equilibrium values when the variables in the other market move around. Compared with the original independent cycles, variability becomes larger, but the periodicity is largely maintained. When the rate of utilisation declines, so does the wage rate. This decline compensates for the slow reaction of the monetary sector (the interest rate does not decline fast enough). So although the interest rate continues to rise, the rate of utilisation rises too, but only to a relative maximum: the boom is frustrated by the rising interest rate. Only when both the wage share and the interest rate decline does the rate of utilisation rise above 100%, towards an absolute maximum.
In the time series (not shown), this difference in periodicity between sectors suggests the appearance of a 'long wave' which is disturbed by 'short-term' fluctuations. Both, however, are the result of the same mechanism.

Figure 5.10 gives the i-w and u-s cycles. It shows the existence of an anomaly between the simulations and the theoretical observations in figure 5.7. Theoretical, the costs of production should be correlated positively, whereas the correlation between the rate of savings and the rate of utilisation is negative. For the simulation in figure 5.10 the calculated correlation coefficient for the interest rate and the wage share is -0.96 and the correlation coefficient of the rate of savings and the rate of utilisation is 0.93, contrary to expectations. This difference is explained by the difference in periodicity. The 'own' dynamics in the labour market dominate the influence of the money market on the rate of utilisation and the wage share. As the rate of savings rises or declines, fluctuations in the rate of utilisation are more frequent as the influence of labour is larger (φ is higher). The theoretical positive correlation returns if a moving average is taken from the actual movements in the rate of utilisation: the short-term (labour market) dynamics destroy the theoretical relationship.

If the parameters are adjusted, so that the periodicity in each sector is the same (\( \phi = \frac{\alpha(x_i - \epsilon)}{(\lambda - 1)} = 0.037 \)), the calculated correlation coefficients become 0.78 for the i-w relationship and -0.96 for the u-s relationship. The influence of \( w \) and \( i \) on \( u \) are equally important, because of the weakened bargaining strength of labour, so the fluctuations in the labour market are moderated.

Inflation is larger than in the uncoupled case. As before, demand follows the time path of utilisation, but lies above supply when the rate of savings declines (and vice versa). The gap between demand and supply of consumption goods results from the difference in timing between utilisation and the rate of savings. This gap causes inflation to be very high during a period of absolute boom. A rise in present utilisation increases the present interest rate. This stimulates the next period savings, depressing
expenditure, but depressing supply even more: excess demand rises. The model predicts a high rate of inflation following the upper turning point in supply because the production declines faster than demand.

**Conclusion:**
When firms take the real costs of funds into account and influence the money market through their demand as result of the financing of the wage fund, the fluctuations in both sectors rise in amplitude, but the periodicity remains the same as in the uncoupled case. The monetary authorities can ensure stability by guaranteeing a minimum interest rate.

Although the economy always moves towards the long-term cycle, the transition period is very long. Compared to the dynamics of the uncoupled sectors, complexity rises (especially in the short term). But, compared to the one-sided coupling in Langen(2000a) the complexity of the time paths declines. Distortions cause adjustments in both markets, instead of in one market. Disequilibrium in one of the two sectors causes both sectors and the aggregate economy to fluctuate in a persistent way. Inflation rises because the gap between demand and supply is widened, as expenditure changes when the interest rate changes, but the interest rate and the wage share both influence production.

The periodicity of the original cycles in each of the markets can deviate. In that case:
1. data from simulations show a long wave, distorted by short-term fluctuations;
2. simulations show a negative correlation between prices of inputs (credit, labour) and a positive correlation between savings and utilisation, whereas -on theoretical grounds- the reverse correlations are expected. The behaviour of the aggregate economy differs from the behaviour expected, based both on a study of the behaviour on each market and on an analysis of the theoretical model.

The short-term dynamics are ignored in the following analyses of government policy. For actual policy-making they remain important as the transition period is long: during simulations, it took between 10,000-1,000,000 periods for the model to lose its stability. Even when the wages are assumed to be paid on a daily basis, this is roughly equivalent to 27 to 2739 real-time years. The period in which these short-term cycles exist depends on -the deviation between the initial value of the variables and their equilibrium value, and -the magnitude of the parameters. When the government decides to intervene by changing the level of a parameter, it could take a long period before the economy returns towards its long-run equilibrium. Simulations show that during the transition period, fluctuations are more severe. In addition, the average employment is lower and average inflation is higher after the intervention. In the next
sub-section, the attention is turned to the possibility of influencing the parameters, ignoring this interference with levels.

4.5 Government policy

As stated above, changes in the initial level of variables do not change the long-term dynamics of the economy. Only in transition, the variance is lower when the government moves the actual variable towards the neighbourhood of the equilibrium. When the government wants to change the long-term dynamics, it should influence the parameters of the economic system.

Assume the government to aim at a stable price level (low inflation) and a high stable level of employment. The price level is determined by the confrontation between supply ($Y_t = u_t$) and demand, which consists of present income minus present savings and former period savings ($[1 - s_{t+1}]u_t + s_t u_{t-1}$). The price level ($p$) and inflation ($dp$) are given by:

$$p_t = 1 - s_{t+1} + s_t \left( \frac{u_{t-1}}{u_t} \right)$$

$$dp_t = \frac{p_t - p_{t-1}}{p_{t-1}} \times 100\%$$

Equation (5.4.4) shows the price level to rise when present consumption is above present production. When the level of last period savings were large compared to the level of present desired savings, present consumption is high; if the level of present savings is high relative to the level of former savings, it depresses present consumption. Utilisation is assumed to represent production: a high level of present utilisation lowers the price level. A high level of utilisation in the former period is followed by a high level of savings in the following period: this enhances present consumption and the price level.

Employment varies proportionally with the product of the rate of utilisation ($u$) and the labour share ($w$)\(^{19}\):

\[^{19}\text{In order to determine the level of employment, the real wage rate} (ω) \text{is taken as given (the following can be adjusted for a distribution of} \ w \text{between the wage rate and employment. Changes in employment are smaller if the wage rate increases when the wage share rises). Using the definition of the wage share and the assumptions above, it follows}

\text{that}: \quad ω_t = \frac{W_t}{Y_t} = \frac{ωd_t}{Y(u_t)} \text{ and } l_t = \theta y_t u_t.

\text{With:}

\begin{align*}
l_t &= \text{employment}
\end{align*}
The time path of inflation and employment of the simulation used before \((\alpha=0.2; \beta=4; \chi=1.5; \varepsilon=1; \lambda=1/0.27; \phi=0.2)\) are shown in figure 5.11.

![Time path of inflation and employment](image)

Figure 5.11: The time path of inflation \((dp)\) and employment \((l)\).

The government expenditures are assumed to be neutral, in the sense that expenditures have no independent effect on total demand. Government policy involves a redistribution of income. Income that is taxed away from households and firms is spent in the same period on additional income for public servants, consumed and saved in the same proportions as the other households.

Using the coupled model, simulations show the impossibility of the government to bring about permanent change by a temporary intervention (see Langen(2000a)). The government can, in one period, supply bonds at an interest rate above the going market rate. The rise in the savings rate depresses the price level. When followed by a decline in production, this raises inflation in future periods. The initial rise in the interest rate leads to overshooting but in time the economy returns to its initial fluctuations. The same goes for a one-time lowering of the entry costs of the firms. This will result in higher costs of production and lower utilisation. Yet, when the intervention is removed, the economy returns to its former fluctuations.

An example of reducing the fluctuations by structural changes is given by Morishima(1992, 23-24). In his view, the dynamics are caused by both the behaviour of the banker and the creative entrepreneur: "We .. conclude that the economy is likely to be monetarily unstable .. Besides .. the real economy will be unstable with respect to entrepreneurs' choice of investment projects. It is obvious that their decisions on investment will be abortive unless they are approved and supported by bankers, so that the economy will perform better if they work in collaboration with each other".

\[
\begin{align*}
I_{t+1} &= \theta \omega_{t+1} u_{t+1} \\
\theta &= \theta(Y, \omega)
\end{align*}
\]

\[\omega = \text{wage rate}\]

\[\theta = (Y^{-1}/\omega) = \text{(assumed) constant}\]
An example of such a collaboration would be the sharing of profits between banks and firms, so the costs of production fluctuate less. Structural changes and automatic stabilisers are ignored; the aim of this chapter is to analyse the effects of two fluctuating sectors. Such an intervention should remove the existing non-linearities. By doing so, the objective of analyses in this chapter is removed.

In the following part of this section, the effect of exogenous changes in the parameters on inflation and employment are analysed, ignoring the precise way the government could implement these policies. Paragraph 4.6 explicitly treats the relationship between inflation and employment: the Phillips curve.

**Equilibrium values**

Changing the parameters can change the equilibrium level of the variables. These equilibrium levels, giving equilibrium in both markets simultaneously, are:

\[
\begin{align*}
   u^{**} &= 1 \\
   w^{**} &= \frac{\lambda - 1}{\lambda(1 + \alpha/\beta)} \\
   i^{**} &= \frac{\alpha}{\beta} \\
   s^{**} &= 1 - \frac{\varepsilon}{\chi} \\
   v^{**} &= \frac{\varepsilon}{\chi}
\end{align*}
\]

The equilibrium level of the rate of utilisation is assumed be at a ‘normal’ usage of the capital stock, here normalised to one, independent of the parameters. The equilibrium wage share depends on the expected rate of profit (\(\lambda\)) and the equilibrium rate of interest (\(\alpha/\beta\)). The interest rate is determined by the savings behaviour. The equilibrium level of the savings rate and the net-credit-creation depend on the reaction of the banks to disequilibrium in the money market (\(\chi\)) and the level of maximum credit creation (\(\varepsilon\)).

Three different effects can be distinguished (see appendix B). Firstly, as a change of the parameter moves the equilibrium level towards the actual value within a sector, the partial fluctuations are reduced, so the influence on the other sector is smaller. The fluctuations of the system as a total are dampened (partial policies). Secondly, a change in the parameter can reduce the fluctuations in one sector (see Langen(2000a)), but also influence the equilibrium value of one of the variables in the other sector. The total effect
depends on whether the difference between the variable and its equilibrium is enlarged or reduced (shifting markets).
Lastly, a policy aimed at the change in a parameter can cause an (un)expected change in another parameter. The total effect is again undetermined (simultaneous policies).

**Partial policies**
Both in the monetary and real sphere, policies that only affect the parameters of one market are possible.

The desired net credit creation ($n^{**}$) is given by $\epsilon/\chi$. The government could restrict the desired net credit creation by reducing $\epsilon$. Also, as in the Hawtrey-Hayek model (in chapter two), a negative change in the psychological environment can induce the banks to charge a higher rate of interest for each level of credit creation (raising $\chi$).

A decline in the desired net credit creation raises the savings necessary for equilibrium ($s^{**}$). Given $s_t$ and $u_t$, this raises the interest rate $i_{t+1}$, and fluctuations in the credit market rise.

Simulations show the $i$-$s$ cycle to rise upwards (in the $s$-$i$ space as depicted in figure 5.9B) when $\epsilon$ declines. Because credit becomes more expensive firms react by contracting utilisation, causing the wage share to decline:

fluctuations in the labour market also become more severe.

As the costs of production show larger fluctuations, so will utilisation. This is reflected in larger fluctuations in employment, around the same equilibrium level ($w^{**}$). Fluctuations in inflation rise too. Both the rate of savings and the rate of utilisation fluctuate more, periods of excess demand and excess supply are more frequent when $\epsilon$ declines. A rise in $\chi$ has similar effects as the decline in $\epsilon$.

A parameter that affects none of market equilibria is the bargaining power of labour: $\phi$. Changing $\phi$ influences the dynamics of the labour market. A rise in the demand for labour because of a rise in $u_t$, will increase the wage share more than before, when the bargaining power of the trade-unions or the individual workers has risen.

The larger reaction of the wage share influences the amount of credit demanded by the firms. The changes in the interest rate become more severe and so will those in the rate of saving.

The larger the reaction of labour on utilisation, the smaller the variance in employment. The reaction of inflation shows an ambiguity: first the variance and the average level decline as $\phi$ rises, but later both statistics rise again. When the cycles in both sectors are asynchronous, the fluctuations in the savings rate and the rate of utilisation cancel each other out: demand (negatively correlated with savings) and supply (positively correlated with utilisation) rise and fall together. If $\phi$ passes a certain level the bargaining process between labour and firms dominates the dynamics of the aggregate economy. The sectors fluctuate in a more synchronised way, so demand and supply move away from each other and inflation is larger.

The Business Cycle
A more direct intervention is the setting of the minimum interest rate by the monetary authorities. For $0<i_{\text{min}}<i^*$, the fluctuations become smaller as $i_{\text{min}}$ is increased. It is even possible, when $i_{\text{min}}=i^*$, to force the money market towards its equilibrium position. The lower fluctuations in the money market cause the labour market to behave as if there is no coupling. Inflation and employment are minimal and are determined only by the labour market dynamics.

Another (partial) policy is to redistribute income between labour and profit income, using a tax rate, $\tau$. Workers receive $w_t$ from the firms, and have an after-tax income of $(1-\tau)w_t$. The rate of utilisation is determined by the costs of production, minus the subsidy: the firms pay the wage fund, $w_t$, and the interest, $w_t\tau$, and receive $\tau w_t$, so production costs become: $(1+i_t-\tau)w_t$. The rate of utilisation can be written as:

$$u_{t+1} = u_t \lambda (1 - (1 + i_t - \tau)w_t)$$

The actual profit rate rises because of the subsidy. For as long as $\pi_t + \tau w_t > \pi^*$, firms enter and demand for labour increases; the subsidy is used to enlarge employment until $\pi_t + \tau w_t \leq \pi^*$. As the rate of utilisation on average equals one, average employment is equal to this new equilibrium wage share. The new equilibrium value of the wage share lies above the old wage share:

$$w_{\text{new}}^{**} = \frac{\lambda - 1}{1 + \frac{\alpha}{\beta} - \tau} = \frac{w_{\text{old}}^{**}}{(1 - \tau)} > w_{\text{old}}^{**}$$

The workers in this model exhibit 'tax-illusion'. The after-tax wage share is equal to the old wage share, $w_{\text{after-tax}} = w_{\text{new}}^{**} (1 - \tau) = w_{\text{old}}^{**}$. Per worker income has declined: employment can only be raised by lowering the wage costs.

*Shifting markets*

The two-sided coupling between the money market and the labour market is caused by the interaction between the rate of utilisation and the interest rate. The equilibrium value of rate of utilisation is taken to be independent of the parameters of the model. The equilibrium value of the interest rate determines not only the equilibrium in the monetary sector, but also the equilibrium in the labour market, through its effect on the equilibrium wage share.

The equilibrium level of the rate of interest ($\alpha/\beta$) can rise by direct intervention of the government, influencing savings behaviour. To do so, the government has to raise the 'deterioration' of savings ($\alpha$) or has to reduce the sensitivity of the savings rate to the interest rate ($\beta$). Again, an
external impulse, for example a decline in the confidence of the public in the banking sector, can have the similar effects.

In the following, a rise in the equilibrium level of the interest rate is modelled as a rise in $\alpha$. However, a decline in $\beta$ has the same effects.

The outcomes of several simulations are given in table 5.1. As $\alpha$ rises, so does the equilibrium interest rate. To realise the desired level of profit ($1/\lambda = 0.27$), the equilibrium wage share has to decline, so $w*(1+i**)$ remains equal ($0.73 = 1-0.27$). The average level of employment equals the equilibrium wage share ($u** = 1$) and is declining when $\alpha$ rises. The average inflation is rising with $\alpha$. The variance of both inflation and employment increases as $\alpha$ rises.

<table>
<thead>
<tr>
<th>$\alpha$ ((\beta=4))</th>
<th>Variance of inflation</th>
<th>Average inflation</th>
<th>Variance of employment</th>
<th>Average employment ((w**))</th>
<th>$i**$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.1</td>
<td>0.01</td>
<td>0.0090</td>
<td>0.72</td>
<td>0.01</td>
</tr>
<tr>
<td>0.05</td>
<td>1.9</td>
<td>0.03</td>
<td>0.0010</td>
<td>0.72</td>
<td>0.0125</td>
</tr>
<tr>
<td>0.10</td>
<td>1.9</td>
<td>0.01</td>
<td>0.0003</td>
<td>0.71</td>
<td>0.025</td>
</tr>
<tr>
<td>0.20</td>
<td>1.9</td>
<td>0.01</td>
<td>0.0010</td>
<td>0.69</td>
<td>0.05</td>
</tr>
<tr>
<td>0.30</td>
<td>103.5</td>
<td>0.42</td>
<td>0.0200</td>
<td>0.68</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Table 5.1: Simulations using different values for $\alpha$.

As the desired rate of interest rises, firms reduce the wage share. The struggle over income reduces employment and increases inflation.

Simultaneous policies
To stimulate capital utilisation, the government can subsidise firms, to reduce the desired rate of profit ($\pi^*$). Such a policy can be financed by creating money ($m$). The creation of money also reduces the equilibrium rate of savings. Assuming the profit income to remain within the firm, the liquidity of the firms increases. With each level of utilisation and the accompanying labour demand, less credit is needed (modelled as a rise in $e$).

This gives:

$$
\begin{align*}
\lambda_{new} &= \frac{1}{\pi^* - m} \\
e_{new} &= 1 + m \\
w_{new} &= \lambda_{new} \left( 1 + i** \right)^{-1}
\end{align*}
$$

Both the equilibrium positions of $s$ and $w$ shift. Assuming $m>0$, the savings rate necessary for equilibrium in the money market decreases: fewer savings are required for the interest rate to remain unchanged. As the desired rate of profit declines, $\lambda$ increases ($d\lambda/dm>0$). This increases the equilibrium rate of the wage share.

The Business Cycle
Intuitively, this can be interpreted as follows. The decline in the savings rate causes the demand for present goods to rise. To supply more goods, given a ‘normal’ utilisation of the existing capital stock, labour input rises. A higher demand for labour causes the wage share to rise. This raise in the equilibrium wage share is only sustainable when the desired profit rate declines as result of the government intervention, which is realised by subsidising the profit share.

The results of several simulations are presented in table 5.2. In these simulations, \( m \) rises so \( \pi^* \) declines, \( s^{**} \) declines (as the additional money supply rises) and \( w^{**} \) rises. The variance in inflation and employment decline as \( m \) rises (\( \pi^* \) declines) whereas the average inflation declines too.

<table>
<thead>
<tr>
<th>( m )</th>
<th>( \pi^* )</th>
<th>( s^{**} )</th>
<th>Average employment (( \omega^{**} ))</th>
<th>Variance in employment</th>
<th>Average inflation</th>
<th>Variance of inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.12</td>
<td>0.15</td>
<td>0.25</td>
<td>0.8095</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.03</td>
</tr>
<tr>
<td>0.07</td>
<td>0.20</td>
<td>0.29</td>
<td>0.7619</td>
<td>0.002</td>
<td>0.007</td>
<td>1.30</td>
</tr>
<tr>
<td>0.02</td>
<td>0.25</td>
<td>0.32</td>
<td>0.7143</td>
<td>0.006</td>
<td>0.01</td>
<td>2.85</td>
</tr>
<tr>
<td>0</td>
<td>0.27</td>
<td>0.33</td>
<td>0.6952</td>
<td>0.003</td>
<td>0.02</td>
<td>3.98</td>
</tr>
<tr>
<td>-0.03</td>
<td>0.30</td>
<td>0.35</td>
<td>0.6667</td>
<td>0.003</td>
<td>0.02</td>
<td>4.25</td>
</tr>
<tr>
<td>-0.23</td>
<td>0.50</td>
<td>0.48</td>
<td>0.4762</td>
<td>0.004</td>
<td>0.06</td>
<td>12.2</td>
</tr>
</tbody>
</table>

Table 5.2: Outcome of the simulations, using different \( r \)’s (with \( s_0 = 0.9s^{**}_0 \) and the initial value of the other variables at their equilibrium value).

As \( m \) rises, from below zero (a negative rate of subsidy on profit income equals a tax on profit income), the required profit rate declines and the amount of base money rises (assuming the government to raise the money supply proportionally). The lower desired rate of profit raises utilisation and so production and employment.

This rise in the money supply raises the liquidity of the firms. Given the desired net credit creation, the rate of savings in equilibrium declines. The fluctuations in the credit market become less volatile, because the banks have to generate less savings to realise their desired net credit creation. Because of the lower equilibrium level of the profit share, the equilibrium wage share can rise. Both the higher wage share and the lower savings rate stimulate consumption. However, simulations show the fluctuations and the average level of inflation to decrease, indicating that the gap between supply and demand of goods is smaller than before. The average level of employment increases, whereas the variance declines.

4.6 The inflation-employment relationship

Above the effect government policies on inflation and employment is analysed, which involved changes in the behaviour of firms, households and banks. It was concluded that it was possible for the government to stimulate employment and reduce inflation, given the possibility to change the behavioural parameters. This sub-section concentrates on the empirically established positive relationship between employment and
inflation, the so-called Phillips curve, which took a central place in the discussion on the effectiveness of government intervention. The standard Phillips curve, as described in the macro-economic literature (see for example Romer(1996, 225-232)), is defined as:

$$dp = f(l) + E(dp)$$

With:
- $dp = \text{inflation}$
- $l = \text{employment}$; $f' > 0$
- $E(dp) = \text{expected inflation}$; $E' > 0$

The long-term stability of this relationship, the Phillips curve, is still subject to discussion. The existence of a trade off between (un)employment and inflation is both an empirical and theoretical discussion. In this section, a negative and a positive relationship between inflation and employment are shown to exist, abstracting from the role of expectations ($Edp=0$).

Remember, the price level is given by (5.4.4):

$$p_t = 1 - s_{t+1} + s_t \left( \frac{u_{t-1}}{u_t} \right)$$

Taking inflation:

$$dp_t = 100 \times \frac{p_t - p_{t-1}}{p_{t-1}} = \frac{1 - s_{t+1} + s_t \left( \frac{u_{t-1}}{u_t} \right)}{1 - s_t + s_{t-1} \left( \frac{u_{t-2}}{u_{t-1}} \right)} - 1$$

and using the definition of employment (5.4.5): $l_t = w_t u_t$ gives the following relationship between inflation and employment:

$$dp_t = V_t \frac{1}{l_t} + W_t$$ (5.4.6)

With:
- $$V_t = \frac{s_t u_{t-1} w_t}{1 - s_t + s_{t-1} \left( \frac{u_{t-2}}{u_{t-1}} \right)}$$
- $$W_t = \frac{1 - s_{t+1}}{1 - s_t + s_{t-1} \left( \frac{u_{t-2}}{u_{t-1}} \right)} - 1$$

If is abstracted from the influence of changes in the wage share, a positive shock in the rate of utilisation lowers the price level because production
rises more than demand (part of the additional income is saved). The positive shock raises employment. Equation (5.4.6) gives an inverse relationship between employment and inflation: high inflation is accompanied by low employment (high unemployment). The model predicts a theoretical relationship between inflation and (un)employment that conflicts with the standard Phillips curve.

As can be seen from (5.4.6), this relationship is not stable. The parameters $V$ and $W$ shift in time because the change in utilisation brings about changes in the rate of interest and the wage share. The rise in the interest rate increases the savings rate, whereas changes in the wage share have a direct influence on the parameters.

The influence of shifts in the parameters can be seen in figure 5.12. The curve $(dps, Is)$ shows the outcome of a simulation, taking the parameters as before. The observed inflation-employment relationship is upward sloping but with enough variance to make an estimation of a linear relationship very untrustworthy\textsuperscript{20}. Once the system is out of equilibrium, there seems to be a positive relationship between actual employment and actual inflation: the estimated correlation coefficient between $dp$ and $l$ is 0.35. Simulations show the same relationship as the standard Phillips curve.

When equation (5.4.6) is estimated, with $V=V_{\text{average}}$ and $W=W_{\text{average}}$, where the averages are calculated from the simulation, it results in the downward sloped curve $(dpt, lt)$\textsuperscript{21}. The theoretical negative relationship is confirmed by the average actual outcome of the simulation, where a direct regression on the realised inflation-employment relationship shows a positive correlation.

This contradiction is caused by the shifts of the negatively sloped curve as the parameters change: each realised value of $dps$ and $Is$ is a point on the negatively sloped $dp-l$ curve. This conclusion resembles the natural rate hypothesis of the shifting short-term Phillips curve. However, the realised inflation and employment do not converge towards a natural rate, but move in a circular way.

The actual positive relationship depends heavily on the local periodicity. When the uncoupled cycles are synchronised ($\phi=0.037$), the estimated correlation coefficient becomes -0.74: the correlation between inflation and employment is determined by the way the both sectors interact, more specific the extent to which the periodicities harmonise. In this case, actual

\textsuperscript{20}Comparing figure 12 with figure 5.17 in Romer(1996, 227), the same circular movement for 1961-1994 (USA) can be seen. Also see Thio(1992).

\textsuperscript{21}Equation (4.6) becomes, in the standard simulation on average:

$$dp_t = V_{\text{avg}} \frac{1}{l_t} + W_{\text{avg}} = 0.24 \frac{1}{l_t} - 0.33, \text{ giving } \frac{\partial dp_t}{\partial l_t} = -0.24 \frac{1}{l_t} < 0.$$
outcome displays a negative relationship, as predicted by the theoretical analyses of the model.

![Diagram](image)

Figure 5.12: The employment-inflation relationship \((dpt, lt)\) = theoretical relationship between inflation and employment with \(V = V_{\text{average}}\), \(W = W_{\text{average}}\) \((dps, ls)\) = outcome of simulation.

The next question to ask whether if it is possible for the government to exploit the inflation-employment trade-off. The simulations show:
- a rise in the costs of central bank money (proportional change in \(\varepsilon\) and \(\chi\)) reduces inflation, whereas employment is not affected;
- reducing the effect of the interest rate on the savings (proportional change in \(\alpha\) and \(\beta\)) also reduces inflation, without an effect on employment (as \(i^{**}\) remains equal);
- a decline in the desired rate of profit (a larger \(\lambda\)) increases both employment and inflation;
- a rise in the reaction of the wage share to market pressure (\(\phi\) larger) also raises employment and inflation.

From these simulations, it can be concluded that it is possible to raise average employment or to reduce the variation in employment but this causes inflation to rise. On the other side it is possible to reduce inflation by monetary interventions without affecting employment. Yet the rise in inflation when enlarging employment is a side-effect not a causal relationship. It is not possible for the government to exploit this relationship: a higher level of inflation does not automatically imply a higher level of employment.

**Conclusion:**
The model above suggests an explanation of stagflation: the occurrence of both inflation and unemployment in the 1970s. A negative shock to production (rate of utilisation) lowers employment directly. Demand is less sensitive to the rate of utilisation than production as it is partly determined by former period savings, which is unaffected by the downward supply shock. The excess demand causes prices to rise with declining employment. In the following periods, production rises as the wages and the interest rate react to disequilibrium. Consumption rises, but less because of the decline in savings in the former period: inflation declines, whereas employment rises.
For the data in the seventies to show the negative relationship between inflation and employment, the monetary and real sector have to move with the same periodicity.

The \( dp-l \) relationship as sketched above is not observed in the simulation because the relationship shifts as the wages and the interest rate react to the changes in the rate of utilisation. The fluctuations suggest a positive relationship, but there is no causal relationship as given by the Phillips curve, nor the reoccurrence of stability as unemployment moves towards a ‘natural level’. There is a common causation causing inflation to react to disequilibrium in the goods market and employment to change with production.

5 Summary and conclusion

Central to this model on a market level is the movement of prices and quantities in response to excess supply and demand. Specific for the economy modelled above is the coupling of markets: because prices of the factors of production are determined in different markets, movements in one market influences the dynamics in the other market.

In the first sections, partial analysis shows the separate sectors of the economy to exhibit endogenous fluctuations. Banks determine the interest rate, based on the savings in the former period. The model stresses one of the functions of the banks: banks are firms, which have to generate income. As banks try to further their income by increasing the interest rate on outstanding credits, this action induces a reaction of savers, which counterbalances the rise of income of the banks. An increase in the interest rate brings about a rise in savings, annulling the extra income and bringing down the interest rate because of the excess supply of funds. Producers determine utilisation, depending on, the realised profit rate. A higher rate of utilisation raises demand for labour. The accompanying rise in labour costs depresses the profit rate, which lowers the rate of utilisation. This struggle over income results in an utilisation rate-labour costs cycle. The fact that sluggish adjustment is a source of economic dynamics is consistent with other research.

When firms take into account the real costs of financing the wage fund, their response to changes in the wage share will be more severe when both costs of production influence the profit share in the same direction as to the case in which either they don’t take the costs of financing in account or when the costs of production move in opposite directions.

The monetary sector is influenced by the labour market because savings and the demand for credit depend on real production (real income). The fluctuations in both sectors rise in amplitude, but the periodicity remains the same as in the uncoupled case. The rise in amplitude made the model
unstable in the long run. An exogenous floor in the rate of interest is important to guarantee stability (the minimum interest rate). Because of the minimum interest rate, the complexity of the time paths decline in the long run, compared to the complex dynamics in the short run.

The first question asked was: *How do partial fluctuations influence the behaviour of the aggregate economy?*

Firstly, disequilibrium in one of the two sectors causes both sectors and the aggregate economy to show persistent fluctuations. Secondly, the coupled case is characterised by the occurrence of a long wave. The amplitude of the fluctuations rises in periods of co-movement of the variables, whereas it is moderated in the following periods of conflicting dynamics in the two sectors.

Thirdly, the appearance of the theoretical correlations in the data depends on the periodicity of the fluctuations in both markets. The actual aggregate fluctuations can deviate strongly from the expected fluctuations, based on theoretical research and analyses of the markets.

The next question asked was whether *the government has the ability to lower inflation or raise employment by intervention in the markets?* A one-time intervention will not change the long-term behaviour of the economy. In general, it is only possible to change the long-run dynamics of the economy with a policy that reduces the distance between the actual variable and its equilibrium level. Given the model, in which the variables are determined by the behaviour of economic subjects, this can only be reached by influencing this behaviour.

Three possible policies are analysed:

a. partial politics;
b. shifting markets;
c. simultaneous policies.

The government has opportunities to increase employment, by raising the wage share and lowering the labour costs. Such a policy (generally) raises the variance of inflation. Other interventions reduce inflation, with mixed effects on employment.

The theoretical inflation-employment trade-off in this model showed a reverse correlation between inflation and employment, when compared to correlation of inflation and employment of the empirically derived Phillips curve. Data from simulations, however, exhibit the same positive correlation between inflation and employment as the empirically derived Phillips curve. The sign of the correlation between inflation and employment in the data from the simulation depends on the periodicity of the cycles in the credit and labour market. It is not possible for the government to take advantage of this correlation as it is not a casual relationship, but a co-movement, driven by changes in production and demand.

The Business Cycle
On the whole, fluctuations are caused by the struggles over the distribution of income:
1. the struggle over income between workers and employers;
2. the struggle over income between savers, banks and firms in the money market;

None of the parties involved desires the economy to fluctuate. Their conflict over the distribution of income causes fluctuations to appear. Market power shifts as targets are realised, conflicting interests drive the dynamics.

Since the two markets are modelled in a way which guarantees the emerging of cycles, it is not surprising that the aggregate economy exhibits fluctuations. The aggregate fluctuations differ from those in the markets and can differ (depending on the parameters) from the analytically derived correlations, based on the theoretical model.

In the next chapter the distribution of income is maintained as the driving force. Banks, firms and workers still struggle over their share of income. By modelling their behaviour in a linear way, the markets converge towards an equilibrium position. The only cause of dynamical behaviour of the aggregate economy is the coupling. Central question in the next chapter is: In which case do fluctuations arise? Which behaviour is essential for the occurrence of cycles and chaos when the individual markets are stable?
Appendices

A. The dynamics in the uncoupled and coupled systems.

In this appendix a general system identical to the market models used in this chapter is analysed. By solving the general model, it is simple to resolve the dynamics of the specific market models. The following system is defined:

\[
\begin{align*}
\frac{x_{t+1} - x_t}{x_t} &= A + By_t, \\
\frac{y_{t+1} - y_t}{y_t} &= X + \Delta x_{t+1}
\end{align*}
\] (5.A1.1)

Which can be written as:

\[
\begin{align*}
f(x_t, y_t) &= x_t(1 + A + By_t) \\
g(x_t, y_t) &= y_t(1 + X + \Delta x_t[1 + A + By_t])
\end{align*}
\] (5.A1.3)

The following derivatives are calculated from (5.A1.3):

\[
\begin{align*}
f_x &= (1 + A) + By_t \\
f_y &= Bx_t \\
g_x &= \Delta(1 + A)y_t + B\Delta y_t^2 \\
g_y &= 1 + X + \Delta(1 + A)x_t + 2\Delta Bx_t y_t
\end{align*}
\] (5.A1.4)

This system of equations in (5.A1.4) can be summarised in the following Jacobian matrix:\[a1\]:

\[
J = \begin{bmatrix}
f_x & f_y \\
g_x & g_y
\end{bmatrix} = \begin{bmatrix}
(1 + A) + By_t & Bx_t \\
\Delta(1 + A)y_t + B\Delta y_t^2 & 1 + X + \Delta(1 + A)x_t + 2\Delta Bx_t y_t
\end{bmatrix}
\] (5.A1.5)

Evaluating the Jacobian at the equilibrium point (0,0) gives the following trace \(\text{tr}(J^0)\) and determinant \(\text{det}(J^0)\):
\[ J^0 = \begin{bmatrix} 1 + A & 0 \\ 0 & 1 + X \end{bmatrix} \]

\[ trJ^0 = 2 + A + X \]  \hspace{1cm} (5.A1.6)

\[ DJ^0 = 1 + A + AX + X \]

The characteristic roots are given by:

\[ \lambda_1^0 = 1 + A \]

\[ \lambda_2^0 = 1 + X \]  \hspace{1cm} (5.A1.7)

For the second equilibrium \((x^*, y^*)\) the characteristic roots are again calculated, using the Jacobian \((J^*)\). Firstly, the equilibrium values have to be determined:

\[ \frac{x_{t+1} - x_t}{x_t} = A + By_t = 0 \rightarrow y^* = -\frac{A}{B} \]

\[ \frac{y_{t+1} - y_t}{y_t} = X + (1 + A)\Delta x_t + ABx_t y_t = 0 \rightarrow x^* = \frac{-X/\Delta}{1 + A + By^*} = -\frac{X}{\Delta} \]  \hspace{1cm} (5.A1.8)

Substituting the equilibria of (5.A1.8) in the Jacobian of the system gives:

\[ J^* = \begin{bmatrix} 1 & -BX \\ \frac{-\Delta A}{B} & 1 + XA \end{bmatrix} \]

\[ trJ^* = 2 + XA \]  \hspace{1cm} (5.A1.9)

\[ DJ^* = 1 \]

Solving for the characteristic roots gives:

\[ \lambda_{1,2} = 1 + \frac{1}{2} XA \pm \sqrt{XA\left(1 + \frac{1}{4} XA\right)} \]  \hspace{1cm} (5.A1.10)

The determinant of the Jacobian, evaluated in the equilibrium of the system, \((x^*, y^*)\) is equal to 1. This means that the system is so-called 'neutrally stable'. Because of this, the stability of the system cannot be determined. Simulations have to be used to show the existence of the cycles.
The uncoupled dynamics of the model.

The credit market was described by the following equations:

\[
\begin{align*}
    i_{t+1} &= i_t \left[1 - \varepsilon + \chi(\bar{d} - s_{t+1})\right] \\
    s_{t+1} &= s_t \left[1 - \alpha + \beta i_t\right]
\end{align*}
\]

Equation (5.A1.11)

Using this system with \(x=s, y=i\) gives for the characteristic roots (\(A=-\alpha; \lambda=\chi d - \varepsilon\)):

\[
\begin{align*}
    \lambda_1^0 &= 1 - \alpha < 1 \quad (0 < \alpha < 1) \\
    \lambda_2^0 &= 1 + \chi d - \varepsilon > 0 \quad (\varepsilon < 1; \chi d > 0) \\
    \lambda_{1,2}^* &= 1 - \frac{1}{2} \alpha(\chi d - \varepsilon) \pm \sqrt{\alpha(\chi d - \varepsilon) \left[1 - \frac{\alpha(\chi d - \varepsilon)}{4}\right]} \quad (\epsilon < 1; \chi d > 0)
\end{align*}
\]

Equation (5.A1.12)

In the simulations in this chapter, the following values for the parameters were used: \(\alpha = 0.2\) and \(\chi d - \varepsilon = 0.2\). Using this to calculate the characteristic roots, as defined in (5.A1.12), gives:

\[
\begin{align*}
    \lambda_1^0 &= 0.8; \lambda_2^0 = 1.5 \\
    \lambda_{1,2}^* &= 0.95 \pm 0.32
\end{align*}
\]

Simulations reveal existence of cycles, as shown in figure 5.A-1, for \(\alpha = 0.2, \beta = 4, \chi = 1.5\) and \(\varepsilon = 1\) and for \(\alpha = 0.5, \beta = 10, \chi = 3\) and \(\varepsilon = 1\), using DMC, with the initial values of \(s\) and \(i\) in the neighbourhood of their equilibrium values.

![Figure 5.A-1](image-url)

Figure 5.A-1: A: \(\alpha = 0.2, \beta = 4, \chi = 1.5\) and \(\varepsilon = 1\); B: \(\alpha = 0.5, \beta = 10, \chi = 3\) and \(\varepsilon = 1\)
The system of the labour market was described by:

\[ u_{t+1} = u_t \lambda \left[ 1 - (1 + i_t) w_t \right] \]
\[ w_{t+1} = w_t \left[ 1 - \phi + \phi u_{t+1} \right] \]  
(5.A1.13)

Again the characteristic roots in (5.A1.7) and (5.A1.10) can be solved, now taking \( u = x \), \( w = y \), \( \lambda = \lambda -1 \), \( B = -\lambda (1+i) \), \( X = -\phi = -\Delta \) and \( i = i^* \).

\[ \lambda_1^0 = 1 + (\lambda - 1) < 1 \]  
\[ (1 < \lambda) \]
\[ \lambda_2^0 = 1 - \phi < 1 \]  
\[ (0 < \phi < 1) \]  
(5.A1.14)

\[ \lambda_{1,2}^* = 1 - \frac{1}{2} (\lambda - 1) \pm \sqrt{-(\lambda - 1) \phi \left[ 1 - \frac{(\lambda - 1) \phi}{4} \right]} \]

In the simulation in this chapter the following parameters are used: \( \lambda = 4 \) and \( \phi = 0.2 \). Using these values gives the following values for the characteristic roots (using (5.A1.13)) for the labour market:

\[ \lambda_1^0 = 4; \lambda_2^0 = 0.8 \]
\[ \lambda_{1,2}^* = 0.7 \pm 0.714 \]

As in the case of the money market, simulations show the occurrence of stable cycles. Figure 5.A-2 gives two possible cycles for \( \lambda = 1/0.27 \) in both simulations, \( \phi = 0.2 \) and \( \phi = 0.8 \) and with initial values of \( u \) and \( w \) in the neighbourhood of their equilibrium level.

![Figure 5.A-2: A: \( \phi = 0.2, \lambda = 1/0.27 \); B: \( \phi = 0.8, \lambda = 1/0.27 \)](image-url)
The coupled model

The model was given by:

\[ i_{t+1} = (1 - \varepsilon - \varphi_t[1 - \alpha] - \varphi_t \lambda_t + \alpha[\nu_1 \lambda_t(1 - [1 + i_t] w_t)]/i_t \]
\[ s_{t+1} = (1 - \alpha + \beta_t)s_t \]
\[ w_{t+1} = w_t(1 - \phi(1 - u_{t+1})) \]
\[ u_{t+1} = u_t \lambda_t(1 - [1 + i_t] w_t) \]

(5.1.15)

The Jacobian of (5.1.15), evaluated in the equilibrium, is:

\[
\begin{pmatrix}
1 - \chi - \alpha(\chi - \varepsilon) + \chi \left( \frac{1 - \alpha}{\beta} \left( \frac{\lambda - 2}{\lambda} \right) \right) & -\chi \left( \frac{\alpha}{\beta} \right) & -\chi \lambda \left( \frac{\alpha}{\beta} \right) \left( 1 + \left( \frac{\alpha}{\beta} \right) \right) & \chi \left( \frac{\alpha}{\beta} \right) \\
\beta \left( 1 - \left( \frac{\varepsilon}{\chi} \right) \right) & 1 & 0 & 0 \\
-\phi \lambda \left( \frac{\lambda - 1}{\lambda + \left( \frac{\alpha}{\beta} \right)} \right) & 0 & 1 - \phi(\lambda - 1) & \phi \left( \frac{\lambda - 1}{\lambda + \left( \frac{\alpha}{\beta} \right)} \right) \\
-\lambda \left( \frac{\lambda - 1}{1 + \left( \frac{\alpha}{\beta} \right)} \right) & 0 & -\lambda \left( 1 + \left( \frac{\alpha}{\beta} \right) \right) & 1
\end{pmatrix}
\]

Manipulation of the determinant of the Jacobian, shows again \( D(J^{**}) = 1 \), so again, the model is 'neutrally stable'. In this case, however, simulations show that the dynamics of the coupled system become unbounded, so an additional non-linearity (the minimal interest rate) was introduced. Because of this, the dynamics can not be determined using the Jacobian.
B. Control

Although this system is not chaotic, the methods of control can be divided in the categories as described in chapter three (also see Langen(2000a)):

1- by using discrete actions, in this case changing the level of variables. The effect of a change in the level of a variable depends on the magnitude of the change, as can be seen in figure 5.A2. Suppose the variables $x$ and $y$ to move along the orbit $AB$. A decline in $y$ from its initial point $y_A$ towards $A'$ causes $x$ and $y$ to move along $A'A''$. The same effect will be reached by increasing $y$ towards $A''$. Only when $y$ is raised within $AB$ a smaller orbit will result.

![Figure 5.A2: A change in the level of $x$.](image)

2- by adjusting the structure of the system to remove the crucially non-linearity. The removal of the non-linearities will stabilise the system, but also removes our interest in this economy.

3- by changing parameters, which changes the equilibrium, whereas the essential non-linearity remain in tact.

This does change the equilibrium value of the variables of the system. In figure 5.A3 the equilibrium value of $x$ is lowered from $x^*_0$ towards $x^*_2$. The initial point $(x_t, y_t)$, at which time the variable is, determines the new cycle. In figure 5.A3, it is assumed changes (lowering $x_0^*$ towards $x_1^*$, then towards $x_2^*$) take place when $x_1 < x^*$ and $y = y_1$. 

The Business Cycle
Figure 5.A3: A change in the equilibrium value of $x^*$. 