The business cycle: dynamical coupling and chaotic fluctuations

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CHAPTER 6

CHAOTIC FLUCTUATIONS: BETWEEN MONEY AND LABOUR

"There is widespread agreement that it is necessary to introduce into economics both dynamical relations and general interdependence."

(Goodwin(1947), 181)

1 Introduction

In the previous chapter, the consequences of a coupling between the labour and the credit (money) market were analysed. It was shown that the coupling made the fluctuations of the aggregate economy deviate from the fluctuations in the separate markets. However, given the appearance of fluctuations in the partial sectors, it is no surprise to encounter aggregate fluctuations.

In this chapter, the economy modelled consists of households, firms and banks (paragraph two). As in chapter five, the interaction between the real and the monetary sector (the coupling) is analysed. Contrary to the mechanisms in the former chapter, the behavioural relationships in this chapter are assumed to be linear. Furthermore, capital accumulation is determined endogenously. The dynamics of the aggregate economy can be described by the dynamics of capital accumulation, reducing the mathematics of the model compared to the mathematics in chapter five.

When the goods market interacts with only the credit market (paragraph three) or with only the labour market (paragraph four), the dynamics of capital accumulation can be represented by a logistic equation. Chaotic fluctuations in the partial markets are possible, but the conditions for chaos are unlikely to be met in reality. The partial markets will move towards a non-fluctuating steady state equilibrium. It will be shown that for different conditions, chaotic fluctuations appear in the aggregate economy, even in the absence of fluctuations in the sectors. In this case, however, the dynamics of capital accumulation are represented by a fourth-degree polynomial. Some analytical results can be obtained, but results rely on simulations and bifurcation diagrams.

The same kind of questions are asked again:
1. When the partial markets interact, can this cause persistent fluctuations in the aggregate?

1 An earlier version appeared as Langen(1993) and was presented at the “International Symposium on Economic Modelling” 1993, Athens, Greece.
2. Is it possible to derive the behaviour of the aggregate economy, looking at the partial markets?
3. If the economy shows fluctuations, is it possible to stabilise the economy by government interventions?

Production, again, depends on the capital stock. However, firms invest in real capital, so changes in production are the result of changes in the capital stock, not because of changes in the rate of utilisation of a constant capital stock.

The capital stock determines the demand for labour. In the labour market, the demand for labour is equal to the supply of labour by immediate adjustments of the wage share, in contrast to the modified Goodwin(1967)-cycle in the former chapter. Wage-income is assumed to be consumed, total income is either consumed or invested. In this model, all variables are real (prices and inflation are introduced in paragraph six).

The banks are supplied with an exogenous amount of money. The interest rate is determined by the difference between the demand for credits (financing the wage funds and the costs of other inputs) and the amount of base money.

The costs of production (wage and interest costs) influence the profit share, which determines capital accumulation. The dynamics of the total economy depend on the combined dynamics of both markets. The aggregate dynamics can be reduced to an equation of capital accumulation, which depends on the behavioural parameters of the labour and credit market (paragraph 5). The occurrence of persistent fluctuations in the aggregate economy depends on market behaviour, but also on the ratio between the different levels of the capital stock, necessary for market clearing.

Two situations can be distinguished. Firstly, the situation in which the equilibria of the labour and credit market coincide. In this case, the capital stock necessary for equilibrium in the aggregate economy is equal to the level of capital necessary for market equilibrium in the partial markets. This situation is analysed in paragraph 5.3.

Secondly, it is also possible that the level of capital that is necessary for market equilibrium in the labour market is different from the level that is necessary for equilibrium in the credit market. In this case, the capital stock for which the aggregate economy is in equilibrium deviates from the level of both stocks, necessary for market equilibrium in the partial markets. This is the case in paragraph 5.4.

In general, given assumptions on the parameters and initial values, the aggregate economy can exhibit persistent (chaotic) fluctuations, which differ from the dynamical adjustments, that are expected from partial analyses of the credit and labour market.
Inflation and government interventions are discussed in paragraph six. To analyse inflation, additional assumptions are required, whereas the government adds another sector to the aggregate economy.

The method used in this chapter is to derive the conditions for (chaotic) fluctuations. Then these conditions are changed in such a way that chaos disappears, analysing the influence of changes in other variables and parameters. When the total economy is presented in section five, both mathematical methods as simulations (using MS-Excel) and bifurcation diagrams (using DMC) are used to analyse the dynamics. This method can be criticised for the subjective choice of start values and parameters, but can be justified by the complexity of the model which makes it impossible to achieve overall analytical results. The method used for solving some key-variables is described in the appendix.

2 The model

2.1 The production sector

The economy consists of small firms. Producers own the capital stock. Production takes place using factors of production, as raw materials, intermediary products, labour and capital goods. The level of production is determined by the stock of capital. The ratio of production to the capital stock is assumed to be constant:

\[ q_t = \beta k_t \]  

\( q_t \) = production  
\( k_t \) = working capital  
\( \beta \) = capital productivity

The producers hire labour to operate the capital goods and to process the other inputs. The supply of labour is -by assumption- not a constraint on production. The demand for labour is determined by the capital stock:

\[ l_t = a k_t \]  

\( l_t \) = demand for labour  
\( a \) = labour intensity of capital

Again, the wage fund\(^2\) is assumed to be paid for in advance. The firms borrow money from the banking sector, to finance the production factors

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\(^2\)When spoken of the wage-fund or the wages, these are defined as \( w_t \), the wage rate is represented by \( w \), the labour share of income:  
\[ wa = \frac{w_t}{q} = \frac{wa'k}{\beta k} \]; taking \( a = a'/\beta \) (\( \beta \) is assumed

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other than capital. In total, this is assumed to be proportional to the wage fund. The demand for credit is:

\[ c_t = \chi w_t l_t = \chi w a_t k_t \]  

\[ c_t = \text{demand for credit} \]
\[ w_t = \text{wage rate} \]
\[ \chi = \text{ratio of credits to wage fund} \]

The banks provide the amount of credit demanded. They charge an interest over the principal. As in chapter five, profits\(^3\) are determined as a residue. So, profits can be calculated by subtracting the costs of labour and credit from production:

\[ p_t = q_t - l_t w_t - l_t c_t \]  

\[ p_t = \text{profits} \]
\[ i_t = \text{interest rate} \]

Substituting (6.2.1), (6.2.2) and (6.2.3) into (6.2.4) gives profits in terms of the capital stock, as a declining function of the costs of production \((wa, i)\) and an increasing function of the output-capital ratio:

\[ \frac{p_t}{k_t} = \beta - w a_t [1 + \chi c_t] \]  

\[ \frac{p_t}{k_t} = \text{function of capital stock} \]

The producers are assumed to have a desired share of profits in income, an institutional and historical determined level that is seen as the fair reimbursement for entrepreneurial capital:

\[ p_t^* = m_l \]  

\[ \pi = \text{desired share of profit in income} \]

By assumption, both the wage income and the interest payments are used for consumption, non is saved in contrast to the assumption in chapter five. When the profits \((p_t)\) are in excess of the desired profits \((p_t^*)\), the producers decide to enlarge the production in the next period, using more capital. This constant in this model). In the following the labour share is taken as the relevant variable \((wa_t)\), so, \(w a_t \) is written as \(wa\) and their equilibrium level \(wa^*\) as \(wa^*\). Also see paragraph 2.3.2.

\(^3\)Since the income is taken net of the interest payments, "net profits" or "net returns" is more appropriate, but the terms "profits" or profit share for short, are used to indicate the income which accrues to the producers. In deviation with the former chapters, \(p\) represents profits, not the price level. In turn, \(\pi\) gives the desired profit share in income.
addition to the capital stock is financed internally, using the excess profits \( \sigma(p_t-p_t^*) \), with \( \sigma \leq 1 \). The remainder of the profits is consumed. When \( p_t-p_t^* < 0 \), the entrepreneurs ‘consume’ part of the existing capital stock. This gives the following dynamic equation:

\[
k_{t+1} = k_t + \sigma(p_t - p_t^*)
\]

(6.2.7)

\( \sigma \) = propensity to invest out of excess profits

Given the wages and the interest rate, the dynamics of the production sector are:

\[
\frac{k_{t+1}}{k_t} = 1 + \sigma(\beta(1 - \pi) - w_t[1 + \chi_t])
\]

(6.2.8)

Rising costs of production, as the interest rate (\( i \)) and the wage share (\( wa \)), lower capital accumulation. The same applies for a rise in the desired profit share (\( \pi \)). A rise in the productivity of capital (\( \beta \)) stimulates accumulation. If \( \sigma(\beta(1 - \pi) - w_t[1 + \chi_t]) \) is negative the capital stock declines steadily to zero. Producers realise a smaller return on their investment than desired, so they invest less, reducing production. There is a natural lower limit, because a negative production is not possible. The capital stock rises continuously, when \( \sigma(\beta(1 - \pi) - w_t[1 + \chi_t]) \) is positive: a constant excess return on investment induces continuous expansion, this is not a feasible solution. A rising capital stock implies an ever rising demand for labour and credit, raising the interest rate and wage-share, thereby decreasing profits.

Given the interest rate and the wage share, a non-changing equilibrium on the goods market is defined as a non changing level of production, using the same amount of capital in each period \( (k_t = k^*, (k_{t-1} - k_t/k_t) = 0) \). Only when \( \sigma(\beta(1 - \pi) - w_t[1 + \chi_t]) \) is zero, the capital stock does not change.

In the next paragraphs, the behaviour of bankers on the credit market and the behaviour of labour on the labour market are shown to respond to changes in the capital stock. After the analyses of the partial dynamic mechanisms (credit-output and labour-output), the dynamics of the aggregate economy will be analysed.

### 2.2 The credit market

The banking sector is (exogenous) furnished with an amount of (base) money \( (d') \), at a constant interest rate. The banks can provide credit above or below the amount of money supplied. It is assumed they meet the amount of credit demanded by the firms. When the demand for credits rises above \( d' \), the interest rate rises above \( i^* \) (the interest rate on \( d' \) plus the profit rate of the banks). When demand declines, so does the interest rate, depressing
the profit rate of the banks. When the demand for credit equals the supply of money, the interest rate equals $i^*$. The supply of credit ($c^s$) is determined by the excess of the interest rate above the desired equilibrium rate $i^*$:

$$c^s = \eta(i_t - i^*) + d'$$  \hspace{1cm} (6.2.9)

As stated above, the firms demand credit to fund production costs. Demand for funds ($c^d$) is given by:

$$c^d = x\omega_k k_t$$  \hspace{1cm} (6.2.10)

Assuming market clearing, in which the firms demand an amount of funds, supplied by the banks, whereas the banks determine the rate of interest on these funds. The interest rate equation which follows from the equilibrium condition ($c^d=c^s$), (6.2.9) and (6.2.10) is:

$$i_t = i^* + \gamma(\omega_k k_t - d)$$  \hspace{1cm} (6.2.11)

- $i_t$ = the interest rate
- $i^*$ = the interest rate when $c^d=d$
- $\gamma = \chi/\eta$ = speed of adjustment\(^4\)
- $d$ = $d'/\chi$ = the adjusted money supply

When the wage fund ($\omega_k$) rises, so does the demand for funds. Banks supply credit above the (exogenous) supply of base money ($d$) and increase the interest rate. When the demand for funds is below $d$, the interest rate declines below its equilibrium value ($i^*$). The received interest payments, $i_t c_t$, are paid to the owners of the banks and used for consumption purposes.

### 2.3 The labour market.

#### 2.3.1 Introduction

The labour market is modelled in the same way as the credit market. Excess demand and supply cause the wage share to change. The production sector is the same, as characterised in paragraph 2.1, equation (6.2.1) to (6.2.8). Fluctuations originating in the labour market can be described using the Goodwin(1967)-"profit squeeze model". Funke(1985) uses a variation of this model to generate chaotic fluctuations. Following Funke(1985, 63), the dynamics in the level of wage rate and employment can be described by a logistic equation (also see Pohjola(1981)). An alternative approach was followed by Goodwin(1990), who uses the techniques of the Rössler Band to

\(^4\)Since $\chi$ is constant, the other constants can be written as: $\gamma = \chi/\eta$; $d = d'/\chi$. 

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describe comparable chaotic behaviour in a model with three dimensions (see chapter four).

Below, a model of the labour market is developed in which demand for labour determines actual employment. The wage share is set so that the labour supply equals demand. As in the case of the production-credit interaction, the resulting profit squeeze influences the capital accumulation. To concentrate on the labour market dynamics, the interest rate is assumed constant, at $i^*$.  

2.3.2 The wage-profit relationship

Firms demand labour, determined by planned production. The labour market is assumed to be in equilibrium: $l^d = l^s$. The wage fund is determined by the demand for labour.

Given equilibrium in the product market ($k_i = k_{i+1} = k^*$) and in the credit market ($i_i = i^*$) there is a wage share, at which both bankers and producers earn the desired reward for their factor of production (credit and capital). Using (6.2.8) this wage share can be calculated and is defined as $w a^*$:

$$w a^* = \frac{\beta(1 - \pi)}{(1 + \chi^*)}$$  \hspace{1cm} (6.2.12)

Firstly, the demand for labour ($l^d$) results from the capital stock (variables as defined before):

$$l^d_i = a_i k_i$$  \hspace{1cm} (6.2.13)

Furthermore, it is assumed that the demand for labour at $k^*$ equals $n$, so $w a^*$ and $n$ define an equilibrium at the combined goods and labour market.

To generate the required amount of labour, firms have to set a wage rate inducing the supply of labour to adjust to the needs of production. Labour supply ($l^s$) is sensitive to the wage rate:

$$l^s_i = \theta w - \omega$$  \hspace{1cm} (6.2.14)

With:

$w$ = wage rate  
$\theta$, $\omega$ = constants

There is a physical upper limit to the amount of labour that can be supplied ($\bar{l}$). In the following paragraphs, it is assumed that this constraint is not binding. Yet, assume firms to keep the labour-capital ratio constant until
this constraint is binding. After that the effective labour supply rises through labour saving techniques.

The rise in capital above \( \bar{k} \left( \frac{1}{a} \right) \) requires \( a \) to decline. When \( ak (= l^d) \) rises, the wage rate rises too, to generate the necessary supply of labour \( (l^s) \). When the physical constraint on labour is reached, the supply of labour equals \( \bar{l} \). The rise in labour productivity is passed on to the labourers in the wage rate.

Using labour demand \( (l^d) \) and labour supply \( (l^s) \) in equation (6.2.13) and (6.2.14) a wage share-equation can be derived:\(^5\)

\[
wa_t = wa^* + \epsilon(k_t - n) \quad (6.2.15)
\]

With:

\( n = \) amount of labour for which \( wa_t = wa^* \)

As stated above, the labour share is taken as the relevant variable \( (wa_t) \). The division of the labour share over the wage rate \( (w) \) and the labour intensity \( (a) \) is analysed in section six.

The equilibrium level of \( n \) is independent of concepts of full employment or unemployment (see below). At \((wa^*, n)\), all sectors are in equilibrium. In the goods market, \( k=k^* \), in the credit market, \( i=i^* \). Demand for labour \( (l^d(k)) \) equals supply \( (l^s(w)) \), both \( n \) at \( wa^* \). Is labour demand above \( n \), the wage share will be above \( wa^* \). So, either the interest rate has to be below \( i^* \) (the banks receive less than desired), or the profit share is below \( p^* \) (the firms earn less than desired). The resulting dynamics will be analysed in paragraph 5.

### 2.4 The dynamics of the economy.

The link between the present and the future capital stock is the capital accumulation as described in equation (6.2.8). Disequilibrium in the labour- and credit market is transferred to the production sector through their effect on the profits. The profits can be calculated from (6.2.5), in which the interest rate equation (6.2.11) and the wage share-equation (6.2.15) are substituted, symbols as defined before:

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\(^5\)Taking \( \theta w - \omega = ak \), this gives the labour supply function: \( l^d = a \left[ \frac{wa - wa^*}{\epsilon} \right] + an \). Labour supply has a positive relation with the wage rate, but it is defined compared with the labour share necessary for equilibrium at the product market and \( \theta = (a^2 / \epsilon) \), \( \omega = a(wa^*/\epsilon) - an \).
Taking (6.2.16) and (6.2.7) gives the dynamic adjustments for the economy, using the time path of the working capital:

\[
p_t = \left[ \beta - (w_t - \varepsilon n)(1 + \chi[i^* - \gamma d]) \right] k_t
\]

\[
- s [\chi r (w_t - \varepsilon n)^2 + e (1 + \chi[i^* - \gamma d])] k_t^2
\]

\[
- 2 \sigma \chi r (w_t - \varepsilon n) k_t^3 - \sigma \chi r e^2 k_t^4
\]

(6.2.16)

A rise in the present stock of capital \((k_t)\) raises income for the firm \((\partial k_t / \partial k_t = \beta)\). This rise in production is realised through hiring labour and borrowing money, so costs have to be deducted \((-1 + \chi_d) w_t k_t\). A rise in the present hiring of labour raises the present wage share \((\partial wa_t / \partial k_t = \varepsilon)\) and the interest rate \((\partial i_t / \partial k_t = \gamma)\). The rise in the interest rate has no direct effect on the wage share \((\partial wa_t / \partial i_t = 0)\), but the wage share influences the interest rate \((\partial k_t / \partial wa_t = \gamma k_t)\). The influence of a rise in the present capital stock on accumulation \((\partial (k_{t+1} - k_t) / \partial k_t)\) can either be positive or negative, depending on the pressure on the labour- and credit market. The increase in production is countered by a rise in costs. The profits can rise (when costs rise less than production) or decline (when costs rise more than production). The interdependency between profit share, wage share and the interest rate is the cause of the complexity of equation (6.2.17).

Equation (6.2.17) is of the fourth order, so it is not possible to derive straightforward analytic conclusions from this equation. The equation can be reduced to a third order equation for some key-variables of \(k\). The rules of Cardano can be used to give a solution for these values, such as the equilibrium value \(k^*\) (see the appendix and figure 6.4 in paragraph five).

Before turning to the dynamics of the aggregate economy (paragraph five), the dynamics of the interaction between the monetary sector and the
production sector (paragraph three) and the labour market-goods market relationship (paragraph four) are analysed.

To determine the range of $\beta$, investment can be seen as a proxy for the replacements of capital and material used. In a growing economy, investment is larger than replacement, whereas the ratio is not constant in time. Taking the output/investment relationship for Germany (1970-1990 (data Sysifo)) and for the Netherlands (Central Economic Plan, several years), and taking in account the over-estimation of the relationship between capital and output, $\beta$ can be approximated by the investment-output ratio. The average output-investment ratio is about 5, implying a desired profit ratio of 0.68. This seems very high. Taking a more realistic desired profit-capital ratio of 0.25 gives a $\beta$ of 2.13 which suggests a net growth of the capital stock of 3%, which is on average equal to growth in this period. In the following simulations, $\beta(1-\pi)$ is taken to be 1.6.

3 The production-credit economy

3.1 The model

To trace the dynamics of this production-credit economy, the labour market is assumed to be stationary, with $wa$ constant at $wa^*$. Substituting $\varepsilon=0$ in (6.2.17) shows that the capital stock remains constant when the demand for money equals the supply of money (no credit is provided above the amount of base money). The interest rate adjusts to changes in the level of the capital stock: every level of the capital stock results in a different demand for credit. This interest rate can be used to calculate the profits, which in turn determines capital accumulation, giving the next-of-period capital stock.

The dynamics of the model are given by (6.2.17) and $\varepsilon=0$:

$$k_{t+1} = [1 + \sigma(\beta(1-\pi)-(wa^*)(1+\chi(i^*+\gamma_1)))]k_t - \sigma\chi(wa^*)^2k_t^2$$  \hspace{1cm} (6.3.1)

Equation (6.3.1) can be written as:

$$k_{t+1} = \zeta k_t(1-\frac{\sigma}{k_t})$$  \hspace{1cm} (6.3.2)

Where:

6In the following $a=1$ is assumed.
7The profits of the banks, $p_b$, are assumed to be used to cover the expenses of the banks, whereas the remainder, $r^*p_b$, is used by the monetary authorities to cover their expenses: all interest payments are used for consumptive purposes.

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\[ \eta = \sigma \chi \eta d \ \frac{\beta (1 - \pi)}{1 + \chi i^*} \]

\[ \omega = \sigma \chi \left( \frac{\beta (1 - \pi)}{1 + \chi i^*} \right)^2 \]

Equation (6.3.2) is also known as the logistic equation. The logistic is known to have the following properties for \( 0 < k < \zeta/\omega \) (see chapter 3):

1. There exist two equilibria:
   \[ k_1^* = 0 \]
   \[ k_2^* = \frac{(\zeta - 1)}{\omega} = \frac{d}{wa^*} \]

2. Dynamics:

<table>
<thead>
<tr>
<th>( \zeta )</th>
<th>Dynamics</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 &lt; ( \zeta ) &lt; 1</td>
<td>monotone motion toward 0</td>
</tr>
<tr>
<td>1 &lt; ( \zeta ) &lt; 2</td>
<td>monotone motion toward ( k^* )</td>
</tr>
<tr>
<td>2 &lt; ( \zeta ) &lt; 3</td>
<td>fluctuating motion toward ( k^* )</td>
</tr>
<tr>
<td>3 &lt; ( \zeta ) &lt; 3.6</td>
<td>stable periodic fluctuations</td>
</tr>
<tr>
<td>3.6 &lt; ( \zeta ) &lt; 4</td>
<td>aperiodic fluctuations</td>
</tr>
</tbody>
</table>

The effects of changes in the parameters on \( \zeta \) are summarised in table 6.1. A rise in the sensitivity of capital accumulation to deviations between actual and desired profits (\( \sigma \)) enlarges the possibility of oscillations in the economy. The proportion of the production factors pre-financed (\( \chi \)) raises the influence of changes in the interest rate on the amount of working capital used, so the possibility of fluctuations rises too. However, a rise in \( \chi \) lowers the equilibrium wage share, \( wa^* \). This lowers the sensitivity of future capital to present capital, \( \zeta \). In the aggregate, a rise in \( \chi \) raises \( \zeta \). A higher sensitivity of the interest rate to market disequilibrium (\( \gamma \)) leads to a higher possibility of fluctuations. The same goes for the supply of base money (\( d \)), which raises the possibility of disequilibrium. A higher capital productivity (\( \beta \)) raises the profit rate, and so the deviation between actual and desired profits. A decline in the desired profit rate (\( \pi \)) lowers this deviation and thereby lowers the possibility of fluctuations. A lower equilibrium rate of interest (\( i^* \)) lowers the equilibrium wage rate and raises profits towards the desired level, reducing fluctuations.

<table>
<thead>
<tr>
<th>( \sigma )</th>
<th>( \chi )</th>
<th>( \gamma )</th>
<th>( d )</th>
<th>( \beta )</th>
<th>( \pi )</th>
<th>( i^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 6.1: The influence of behavioural parameters on the dynamics.

\[ 8 \frac{\partial^2 \zeta}{\partial \chi} = \frac{\sigma \beta (1 - \pi) d (1 + \chi i^*) - \sigma \beta (1 - \pi) d \chi i^*}{(1 + \chi i^*)^2} = \frac{\sigma \beta (1 - \pi) d}{(1 + \chi i^*)^2} > 0. \]
3.2 The dynamics

Some restrictions follow from equation (6.3.3) in the choice of the parameters and the initial values. First, $\xi$ should be below 4, otherwise the model loses its stability. Second, the initial value of $k$ should be between 0 and $\xi/\sigma^9$. Assuming the economy to behave properly, the parameters are chosen accordingly. The chosen constants are $i^*=0.1; d=0.8, \sigma=1, \beta(1-\pi)=1.6$ and $\chi=10$, using $\gamma$ as the control parameter. Given $i^*, \beta(1-\pi)$ and $\chi, wa^* =0.80$, and the equilibrium value of $k, k^*=1$.

Figure 6.1: The bifurcation diagram for $\gamma>0.15$, for $0 > \gamma > 0.15, k=k^*$. 

Starting from a (relative) low level of the capital stock, the demand for credits is below the supply of base money. The banks do supply the required amount of credit, but at a level of the interest rate below its equilibrium rate. The profit rate is above the equilibrium rate of profit: accumulation of capital rises. Because of this rise, the demand for credits increases and so does the interest rate: the interest rate and capital stock (and production) display a positive linear relationship.

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9 The restriction of a non-negative capital stock is evident, but a maximum capital stock should be explained. Those restrictions can be justified by the partial character of this model. Here, part of reality is modelled: the part that is relevant to the expose. There are other important economic relationships (as the relation with the foreign sector) which are not specified here, but also no attention is given to other social relationships, which could influence the economic relationships. An ever-expanding capital stock will disrupt the social relations in a way, which could lead to an adjustment of the parameters within the stable limits. On the other hand, they could lead to a crisis, which destroys a major part of the productive capital (see for example Rosser(1991) for the relation between bifurcation's, catastrophes and the decline of political power).
The larger the sensitivity of the interest rate (the possibility of the banks to raise \( i \)): \( \gamma \), the larger the adjustment in the interest rate. The bifurcation diagram of \( \gamma \) is given in figure 6.1.

As \( k_t \) approaches zero, (6.2.11) gives \( i_t = i^* - \gamma d \). For \( i_t > 0 \), it is assumed that \( i^* > \gamma d \), or that there is a maximum level of \( \gamma \): \( \gamma_{\text{max}} = i^*/d \). Given the parameter values above, this gives \( \gamma_{\text{max}} = 0.1/0.8 = 0.125 \). As can be seen in (6.3.3) and figure 6.1, the credit market moves towards its equilibrium for \( \gamma = 0.125 \) This value of \( \gamma \) results in \( \xi = 1.8 \) (given the value of the other parameters). Starting from a low level of capital, the interest rate is below \( i^* \). This raises profits and capital accumulation. The rise in production and interest rate move profits and the interest rate towards their equilibrium value.

Irregular and chaotic fluctuations can only appear for \( \gamma > 0.3125 \), so these fluctuations occur for values of \( \gamma \) for which \( i_t \) can be negative when \( k \) approaches zero.

![Figure 6.2: The bifurcation diagram for \( \beta(1-\eta) \).](image)

Substituting \( \gamma = \gamma_{\text{max}} = i^*/d \) in equation (6.3.2) gives: \( \xi = 1 + \sigma i^* \frac{\beta(1-\pi)}{1 + \chi i^*} \).

Manipulating this equation and taking \( \sigma = 1 \) and \( 1/\gamma i^* \) very small gives: \( \sigma \beta(1-\pi) > \xi - 1.6 \). Aperiodic solutions (\( \xi > 3.6 \)) can only appear for \( \sigma \beta(1-\pi) > 2.6 \). Given a productivity of 1.6 and a non-negative interest rate, the maximum value for \( \xi \) is slightly below 2.6: a cyclical movement towards the equilibrium level of \( k \). Assuming \( \xi \) to be 2.59, \( \chi i^* \) has to be 159, or when \( i^* = 0.1 \), a ratio of labour to working capital (\( \chi \)) of 1590!

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An increasing adjusted capital productivity ($\beta(1-\pi)$) leads to chaotic fluctuations, as can be seen in the bifurcation diagram (figure 6.2). This causes the equilibrium level to decline as can be derived from $k^*$ in (6.3.3i):

<table>
<thead>
<tr>
<th>$\zeta$</th>
<th>$\beta(1-\pi)$</th>
<th>$k^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>0.80</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
<td>0.40</td>
</tr>
<tr>
<td>3.6</td>
<td>5.2</td>
<td>0.31</td>
</tr>
<tr>
<td>4.0</td>
<td>6.0</td>
<td>0.27</td>
</tr>
</tbody>
</table>

In paragraph two, the value of $\beta$ was estimated to be between 2 and 5, so the appearance of irregular and chaotic fluctuations because of the value of $\beta(1-\pi)$ is unlikely.

In the former $wa$ is assumed to be constant at its equilibrium value. In the next paragraph the labour market is analysed, taking the interest rate as given.

4 The wage-profit model

4.1 The model

This section concentrates on the partial dynamics, which result of the behaviour on the labour market. Capital accumulation will raise the demand for labour, which increases the wage share as seen in (6.2.15). The rise in the costs of production lowers the profit rate, which has a negative effect on the accumulation of capital. By taking $\gamma=0$ in equation (6.2.11) the interest rate remains constant at $i^*=i^{10}$. Equation (6.2.17) can be used to determine the effect of the labour market dynamics on the capital accumulation ($\gamma=0$):

$$k_{t+1} = \left[1 + \sigma(1-\pi)\beta - \left(wa^* - \sigma e(1 + \chi^*)\right)\right]k_t - \sigma e(1 + \chi^*)k_t^2$$

Substituting the value for $wa^*$ (see 6.2.12) gives:

$$k_{t+1} = (1 + \sigma e(1 + \chi^*)^*)\sigma e(1 + \chi^*)k_t - \sigma e(1 + \chi^*)k_t^2$$

The first that can be deducted from equation (6.4.1) is the absence of the adjusted capital productivity ($\beta(1-\pi)$). This is the consequence of the assumed value of $wa^*$. A rise in productivity leads to a proportional rise in $wa^*$, and so in $wa$. The possibility of the occurrence of (irregular) fluctuations given low values of $\varepsilon$ will increase when the labour supply is large, the equilibrium interest rate is large and $\sigma$ is high. From (6.4.1), follows that $\zeta_{\text{wa}} = 1+\sigma e(1+\chi^*)$. The influence of the parameters on $\zeta_{\text{wa}}$ is given in table 6.2.

10In the simulations $\chi=1/i^*$ is assumed, so $\chi^*=1$.
Secondly, from the known properties of the logistic follows the interval of $\varepsilon$, over which the model is globally stable. The value of $\varepsilon$ has to fall in the interval: \[-1 \frac{1}{[1 + \chi^*]n} < \varepsilon < \frac{3}{[1 + \chi^*]n};\] given $0 < k < \frac{1}{[1 + \chi^*]\sigma} = n$, and $k^* = n$.

### 4.2 The dynamics

Since (6.4.1) is similar to (6.3.2), there is no need to elaborate on the cycles, which can result from different parametric values. Figure 6.3 gives the bifurcation diagrams for $\chi$ and $\varepsilon$. The other parameters are: $n=1, \beta(1-\pi)=1.6, \sigma=1, \chi=10, i^*=0.1$, which again gives $wa^*=0.8$.

![Bifurcation Diagrams](image)

Figure 6.3: The bifurcation diagrams of $\varepsilon$ and $\chi$.

Taking $k_t=0$, the wage share is determined by $wa_t = wa^*-\varepsilon n$ as can be seen in (6.2.11). For the wage share to be above zero, $\varepsilon > wa^*/n$, giving $\varepsilon =0.8$ in the simulations above. When $\varepsilon$ is taken to be maximal $wa^*/n$, this gives $[1+\sigma n(1+\chi^*)]=2.6$. As known from the characteristics of the logistic equation, to generate persistent fluctuations, $\zeta_{wa}>3$. When fluctuations occur, for $wa_t>0$, these are dampened fluctuations towards the equilibrium value of the capital stock, $k^*$.

As $\chi$ rises, the maximum value of $\varepsilon$ declines as $wa^* (=\beta(1-\pi)/1+\chi^*)$ declines ((given the desired profit share, a rise in the inputs which have to be financed with credit increases interest payments and lowers the wage share at which the labour market and the goods market clear simultaneously).
For \( \chi > 10 \), \((wa^* - \varepsilon n)\) becomes negative. As is evident from figure 6.3, for values of \( \chi \) below 14 (given \( \varepsilon = 0.8 \)), the capital stock moves towards its equilibrium value, \( k^* \).

Not shown is the bifurcation diagram of \( \beta(1-\pi) \). Rewriting (6.4.1), taking \( \varepsilon = \varepsilon_{\text{max}} \), the control parameter becomes: \( \zeta_{w} = 1 + \alpha \beta(1-\pi) \). Aperiodic dynamics appear for \( \beta(1-\pi) > 2.25 \). This value of \( \beta(1-\pi) \) contradicts the definition of \( wa^* \) as the equilibrium wage share (\( wa^* < 1 \)); given the parameters this requires \( \beta(1-\pi) < 2 \), so no chaos can occur.

The emergence of cycles does not only depend on the parameters, but also on the value of \( \iota \), as determined within the monetary sector. When the last deviates from its equilibrium value, the parameter \( [1 + \sigma \varepsilon n(1 + \chi \iota)] \) of equation (6.4.1) also changes. This brings us to the main subject of this chapter: the interaction of the three sectors together.

5 The total economy: the labour-market-credit market interaction

5.1 Introduction

In the paragraphs above, a model was developed, consisting of three markets. The model consists of six equations, which are repeated below. The dynamics of the credit market are given by (6.2.9):

\[
i_t = i^* + \gamma (wa_i k_i - d)
\]

Demand and supply on the labour market are equated by changes in the labour share, as seen in (6.2.15):

\[
wa_t = wa^* + \varepsilon (k_t - n)
\]

Profits are equal to production minus costs of the factors of production, (6.2.5):

\[
p_t = q_t - (1 + \chi_i)wa_i k_t
\]

Production is depending on the capital stock, (6.2.1):

\[
q_t = \beta k_t
\]

Desired profits are assumed to be a constant part of production, (6.2.6):

\[
p_t^* = \pi q_t
\]
Capital accumulation depends on the difference between actual and desired profits, (6.2.7):

\[ k_{t+1} = k_t + \sigma(p_t - p^*_t) \]

Solving the model, the dynamics can be reduced to an equation for the capital accumulation. Substituting (6.2.9), (6.2.15) and (6.2.1) into (6.2.5) gives an equation for the actual profits. Using this solution, and (6.2.6) in (6.2.7) gives the capital accumulation in terms of the behavioural parameters (as in (6.2.17)):

\[
k_{t+1} = \left[ 1 + \sigma \left[ \beta(1 - \pi) - (wa^* - \varepsilon n)(1 + \chi[i^* - \gamma]) \right] \right] k_t \\
- \sigma \left[ \gamma \chi (wa^* - \varepsilon n)^2 + \varepsilon (1 + \chi[i^* - \gamma]) \right] k_t^2 \\
- 2 \sigma \gamma \chi e (wa^* - \varepsilon n) k_t^3 - \sigma \gamma \chi e^2 k_t^4
\]

The partial analyses in the former paragraphs show that the credit-production economy \((e=0)\) will display chaotic fluctuations for:
- high levels of \(\gamma\), but in this case the interest rate becomes negative;
- high values of \(\beta(1-\pi)\), but the required value of \(\beta(1-\pi)\) is too large, compared with the empirical values; and,

Chaotic fluctuations appear in the labour-production model \((\gamma=0)\) for:
- high values of \(\varepsilon\), resulting in a negative wage share; and
- high values of \(\beta(1-\pi)\), resulting in a wage share \(wa^*\) above 1.

Given the restriction of non-negativity on the wage share and the interest rate and realistic values of the parameters, the partial dynamics of the production-credit economy and the production-labour economy move towards their partial equilibria. In deviation with the conclusions with respect to the non-linear markets in chapter five, a partial analysis of the labour and the credit market reveals parameters that justify the conclusion that the economy is inherently stable.

In this paragraph the two markets are combined (coupled). The credit market and the labour market interact with the production sector through the profit share equation. To begin, the model is described in 5.2. After this the several sources of (chaotic) dynamics are analysed.
5.2 The credit-labour model

From the paragraphs above, some simplifying assumptions can be derived. Remember, the wage share for the goods market equilibrium ($w^*$) and the maximum value for the parameters $\gamma$ ($i_0>0$) and $\varepsilon$ ($wa_1>0$) were defined as:

$$w^* = \beta(1-\pi)$$

$$\gamma_{\text{max}} = \frac{i^*}{d} \quad \rightarrow \quad i = \gamma wa_1 \gamma$$

$$\varepsilon_{\text{max}} = \frac{wa^*}{n} \quad \rightarrow \quad wa = \varepsilon \gamma$$

Using (6.5.1), equation (6.2.17) can be rewritten for the economy as an aggregate of the three markets as:

$$k_{t+1} = w(k_{t+1} + \sigma \beta (1-\pi))(1 + \gamma (i^* - \gamma d) - \sigma k^2_t - \sigma \gamma \varepsilon^2 k^4_t)$$

(6.5.2)

Analyses show equation (6.5.2) to be a hill shaped function, for the relevant range of $k \geq 0$ (see figure 6.4). Stability is assured (see Li and Yorke (1975) and chapter two) for $f(k_m) < k_0$, so after reaching its maximum value, $k$ remains inside the domain of the function $(0, k_0)$. Abstracting from the maximum parameters in (6.5.1), the slope in the equilibrium point ($k^*$) is given by:

$$\delta = \frac{\partial f(k_{t+1})}{\partial k_t} \bigg|_{k=k^*} = 1 + \sigma \beta (1-\pi) - (w^* - \varepsilon n)(1 + \chi (i^* - \gamma d)) - 2\sigma \gamma (w^* - \varepsilon n)^2 + \varepsilon (1 + \chi (i^* - \gamma d)) k^2 - 6\sigma \chi \varepsilon (w^* - \varepsilon n) k^4$$

Substituting the assumptions in (6.5.1) gives:

$$\delta_{\text{max}} = \frac{\partial f(k_{t+1})}{\partial k_t} \bigg|_{k=k^*} = 1 + \sigma \beta (1-\pi) - 2\sigma e k^* - 4\sigma \chi \varepsilon^2 k^3$$

As equation (6.5.2) is of the fourth order, it is not possible to present further analytic results. The equation can be reduced to a third order equation for the key-values of $k$, so some statements can be made regarding $k_0$ (the

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11 Take $k_{t+1}=f(k_t)$, then $f'(0)>0$, $f''(0)<0$, $f'(k)<0$ for $k$ large and $f''(k^*)<0$. 

The Business Cycle
intersection of (6.5.2) with the $k$-axis), $k_m$ (the maximum value of (6.5.2)) and $k^*$ (for which $k_{t+1}=k_t$, in (6.5.2)). The derivation of these key-values is shown in the appendix\textsuperscript{12}. Numerical analyses and simulations show, in general, that the model loses its stability around $\delta \gtrsim -2.25$\textsuperscript{13}. In the following numerical solutions, a value of $|\delta|=2.2$ is taken as the maximal value for stability.

![Graph showing $k_{t+1}$ vs $k_t$ with $k^*$ highlighted]

Figure 6.4: The $k_t$-$k_{t+1}$ curve with $k^*=1$.

There are three potential sources of chaotic fluctuations:

1. **Market behaviour**: $\gamma$, $\varepsilon$, $\beta(1-\pi)$.
   Chaos can emerge as the result of the behaviour on the two markets. In the former paragraph, it was concluded that under realistic assumptions, the behaviour on partial markets alone couldn’t be responsible for the occurrence of macro-economic fluctuations. When coupled, the market forces that move the partial markets towards equilibrium have a destabilising effect on the aggregate economy, as shown below.

2. **The coupling between markets**: $\chi$, $\sigma$.
   The labour market influences the equilibrium at the credit market through the wage fund, where $\chi$ can be seen as a weight for the amount of working capital (labour and other inputs) to be financed. Both markets together determine the costs of production. The remaining profits determine the next-period capital stock, in which the sensitivity of investment to $(p-p^*)$, $\sigma$, plays a role.

3. **The market equilibria**: $\rho$.
   The labour market equilibrium is realised when $k=n=k_n$. When $k$ is above (below) $n$, the wage share rises above (falls below) $w_a^*$. The credit-market

\textsuperscript{12}I am in debt to Thijis Jansen, who brought the rules of Cardano to my attention and Leo van Veldhuizen who helped solving the mathematics. See Teller(1965, 22).

\textsuperscript{13}The dynamics are given by (approximate):
   - Monotone and cyclical movement to $k^*$: $\delta \gtrsim -1$
   - Persistent cycles around $k^*$: $-1 \gtrsim \delta \gtrsim -1.96$
   - Chaotic fluctuations: $-1.96 \gtrsim \delta \gtrsim -2.25$

The Business Cycle
reaches its equilibrium when \( k = d / w a^* = k_d \). If \( k \) exceeds (is less than) \( k_d \), the interest rate rises above (falls below) \( i^* \). The possible combinations in the aggregate economy are summarised in Table 6.3. The relationship between \( k_n \) and \( k_d \) can be described by:

\[
d = \rho(wa^* n)
\]  

(6.5.3)

This assumption is economically irrelevant, but is used as shorthand for the difference between the two levels of capital stock. A deviation of \( \rho \) from 1 indicates that the equilibrium level of capital of the credit market and the labour/production market do deviate. Substituting (6.5.3), using the definitions in (6.5.1), into (6.5.2) gives the dynamics of the capital stock in terms of the basic parameters:

\[
k_{t+1} = k_t \left[ 1 + \sigma \beta (1 - \pi) - \frac{\beta (1 - \pi)}{n (1 + \chi^*)} k_t \left[ 1 + \frac{\chi^*}{\rho n^2} k_t^2 \right] \right]
\]  

(6.5.4)

<table>
<thead>
<tr>
<th>labour market</th>
<th>credit market</th>
<th>( k &gt; k_n )</th>
<th>( k = k_n )</th>
<th>( k &lt; k_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k &gt; k_d )</td>
<td>I: shortage of labour: wage share above ( wa^* ); shortage of money: interest rates above ( i^* ); profits and capital stock low.</td>
<td>II: labour market in equilibrium: ( wa = wa^* ); shortage of money: interest rates above ( i^* ); profits and capital stock low.</td>
<td>III: abundant labour supply: wage share below ( wa^* ); shortage of money: interest rates above ( i^* ); reaction of profits and capital stock depends on the relative influence of both markets.</td>
<td></td>
</tr>
<tr>
<td>( k = k_d )</td>
<td>IV: shortage of labour: the wage share above ( wa^* ); credit market is in equilibrium: ( i = i^* ); profits and capital stock low.</td>
<td>V: labour and credit market in equilibrium: ( wa = wa^* ); ( i = i^* ); profits and capital stock remain constant: ( p^<em>, k^</em> ).</td>
<td>VI: abundant labour supply: wage share below ( wa^* ); credit market is in equilibrium; ( i = i^* ); profits and capital stock high.</td>
<td></td>
</tr>
<tr>
<td>( k &lt; k_d )</td>
<td>VII: shortage of labour: the wage share above ( wa^* ); excess supply of money: interest rates below ( i^* ); reaction of profits and capital stock depends on the relative influence of both markets.</td>
<td>VIII: labour market in equilibrium: ( wa = wa^* ); excess supply of money: interest rates below ( i^* ); profits and capital stock high.</td>
<td>IX: abundant labour supply: wage share below ( wa^* ); excess supply of money: interest rates below ( i^* ); profits and capital stock high.</td>
<td></td>
</tr>
</tbody>
</table>

Table 6.3: Taxonomy of the nine possible situations in the total economy.

### 5.3 Dynamics with a coinciding equilibrium in the labour and credit markets

In the labour market, there is a level of \( k \), for which \( k_t = n \) (say \( k_n \)), so \( wa_t = wa^* \). On the credit market, the demand for credit is equal to the supply when \( k_t = d / wa_t \) (say \( k_{d_t} \), with \( i = i^* \). The relationship between the two levels of capital is given by (6.5.3): \( k_d = \rho k_n \), so \( k_d = d / wa^* \). When the markets interact, as described by (6.5.2), there is an aggregate level of capital (\( k^* \)), for which \( (k_{t+1} - k_t) / k_t = 0 \). In this paragraph, it assumed that the two levels of capital, \( k_d \) and \( k_n \) coincide: \( \rho = 1 \). Since the equilibrium capital stock is the same in both markets, only situation I, V and IX on the diagonal in Table 6.3 are possible.
Keeping the parameters the same as before (taking \( n=1 \)), the aggregate equilibrium level of capital is then equal to \( k_d = k_n = 1 = k^* \). The slope at the intersection \( k^* \) is determined by:

\[
\delta = \frac{\partial k_{t+1}}{\partial k_t} \bigg|_{k^*} = 1 + \sigma \left[ \beta (1-\pi) - (wa^*-en)(1 + \chi[i^*-yd]) \right] - 2\sigma \gamma (wa^*-en)^2 + \varepsilon (1 + \chi[i^*-yd]) \right] - 6\sigma \gamma \varepsilon (wa^*-en) - 4\sigma \gamma \varepsilon^2
\]

or,

\[
\delta_{\text{max}} = 1 - \sigma \frac{\beta (1-\pi)}{(1 + \chi i^*)} \left[ 1 + 3\chi i^* \right] \tag{6.5.5}
\]

**Market behaviour**

Using the parameters from the former section, \( \beta (1-\pi)=1.6, \sigma=1, \chi=10, i^*=0.1, wa^* = \frac{\beta (1-\pi)}{1 + \chi i^*} = 0.80, d=wa^*n=0.80, \gamma = \frac{i^*}{d} = 0.125 \), gives the following results (see the appendix) \( k^*=1; k_m=0.7571; l(k_m)=1.2471 \) and \( k_0=1.2582 \). The simulations show a capital stock (and production) that is on average 25% below the equilibrium capital stock and equilibrium production\(^{14} \) \( k_{av}=0.74 \) over 3000 periods). Using (6.5.5) to calculate the slope gives: \( \delta=-2.2 \).

The combination of a stable labour market and a stable credit market results in chaotic fluctuations for the economy as a total. Banks, workers and firms behave in a stabilising way. When the capital stock is above its equilibrium level, negotiations on the labour market raise the wage share, as the demand for labour is above \( n \). The amount of credit necessary (working capital) is above \( d \), which raises the interest rate. As the costs of production rise, profit declines, decreasing the growth in capital and, in time, resulting in a declining capital stock: the market forces move the partial markets towards their equilibrium position \( (k_d, k_n) \). As the capital stock approaches this equilibrium level \( (k_d=k_n=k^*) \), the combined reaction of the wage share and interest rate causes \( k \) to 'overshoot' its equilibrium level, so the actual capital stock lies below the equilibrium level, as shown in figure 6.5. Because bankers, firms and labourers behave in ‘the right way’ in reaction to excess demand and supply, the aggregate economy exhibits fluctuations. A slower reaction of all groups to market disturbances (lower behavioural parameters, \( \varepsilon \) and \( \gamma \)) enhances macroeconomic stability.

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\(^{14}\)This seems to be a characteristic of chaotic fluctuations, although not (yet) analytical proven (C. Hommes, e-mail, 1995).
Figure 6.5: The $w_a - p$ relationship, the $i - p$ relationship and the time path of $k$ for $\gamma = 0.125$; $\varepsilon = 0.80$.

Figure 6.6 gives the bifurcation diagrams for $\varepsilon$, $\gamma$ and $\beta(1 - \pi)$, calculated keeping the other variables constant. Substituting the values of the parameters used in the simulations in the first equation of (6.5.5), with the exception of $\varepsilon$ and $\gamma$: $\delta = 1 - 6.4\gamma - 2\varepsilon - 8\gamma\varepsilon$. When the results from the simulations are used to solve this equation, the aperiodic area appears for $1.6 < |\delta| < 2.2$.

To vary $\beta(1 - \pi)$ and still guarantee a non-negative wage share, $\varepsilon$ is taken to be at its maximum value: $\varepsilon = \varepsilon_{\text{max}} = \frac{w_a^*}{n} = \frac{\beta(1 - \pi)/1 + \chi i^*}{n}$.

Ongoing fluctuations can result in a situation in which an analysis of the partial market reveals only a tendency towards equilibrium: the stronger the adjustment towards equilibrium (the larger the behavioural parameter), the larger the chance of chaos. Contrary to the conclusion in the former paragraphs (fluctuations require negative rewards of factors of production) (chaotic) fluctuations will occur for positive values of the interest rate and the wage share in the coupled model.
The coupling between markets: $\chi, \sigma$
These parameters determine the influence of the two markets on each other. Parameter $\chi$ gives the amount of credit firms need, given the wage share and the capital stock. The overall effect of a change in $\chi$ is undetermined. As can be seen in the first equation in (6.5.5), there is a direct positive effect on $\delta$, and so on the occurrence of fluctuations. Indirect, a rise in $\chi$ has a negative effect on $\delta$ because $wa^*$ declines. If this decline in $wa^*(=\varepsilon_{max}^n)$ is ignored, the wage share becomes negative when $wa^*(\chi) \leq \varepsilon$ or $\chi \leq \frac{\beta(1-\pi)}{n(i^* \varepsilon)} - \frac{1}{i^*}$ (which gives $\chi \leq 10$ for the parameters used here). This case is depicted in the first diagram of figure 6.7. In this diagram, it can be seen that fluctuations occur for $3 < \chi < 10$. Taking the equilibrium wage share to be below one and given the other parameters, the minimal value of $\chi$ is 6 ($wa^* < 1, \chi > [\beta(1-\pi)-1]/i^*$). Assuming $\varepsilon = 0.8$ (as above), the relevant range of $\chi$ is $[6, 10]$, which gives $-2.2 \leq \delta \leq -2.1$: when the combination of partial markets causes the aggregate economy to exhibit...
fluctuations, changes in the coupling through \( \chi \) (given the restrictions on \( wa^* \)) will not change the dynamics.

When the distribution of income through the market forces takes account of the necessary amount of working capital, the wage share can be made to depend on \( \chi \): \( \eta n(\chi) = \frac{\beta(1-\pi)}{1+\chi d^*} \). A lower value of \( \chi \) increases the equilibrium wage share (and \( \varepsilon \)): lower interest payments increase the wage share. The bifurcation diagram (second diagram in figure 6.7) is the mirror of the usual diagrams: the economy is stable for high values of \( \chi \), whereas chaos occurs for small values: a low value of \( \chi \) gives a high value of \( \varepsilon \). The fluctuations in the labour market are amplified by the dynamics in the credit market, resulting in chaotic fluctuations for the aggregate economy.

Figure 6.7: The bifurcation diagram for \( \chi \) (different assumptions) and the \( \chi-\delta \) relationship for different \( \gamma \).

The dependency of the influence of \( \chi \) on the other parameters is illustrated more strongly by comparing the second and the third diagram in figure 6.7. The second diagram was drawn assuming \( \gamma=0.125 \) (\( \varepsilon=0.8 \)), the third takes \( \gamma=0.05 \). In this last case, the maximal possible type of fluctuations is a period-2 cycle. This is conformed by the last diagram, which shows the relationship
between $\delta$ and $\chi$ for the two different values of $\gamma$. Given the parameters $(\beta(1-\pi)=1.6, r^*=0.1$ and $\sigma=1)$ $-2.2 \leq \delta \leq -1.6$ for $0.125 \leq \gamma \leq 0.078125$.

Concluding: the influence of $\chi$ depends on the behavioural parameters. If the economy potentially displays chaotic fluctuations, chaos will also occur for different values of $\chi$. When the behavioural parameters are such that chaotic fluctuations will not occur, a higher value of $\chi$ cannot change this.

The influence of the savings- or investment quote, $\sigma$, on the dynamics can be summarised by: $\delta = 1 - \sigma X$, in which the value of $X$ is determined by the value of the other variables. Figure 6.8 gives the relationship between $\sigma$ and $\beta(1-\pi)$ for different values of $\gamma$, assuming $\omega^* = \frac{\beta(1-\pi)}{1+\chi^*} = \epsilon = \epsilon_{\text{max}}$ and chaotic fluctuations ($\delta=-2.2$). As external financing of the capital stock is ignored, $\sigma \leq 1$. When reactions on the credit market are slow ($\gamma$ small), the adjusted capital productivity ($\beta(1-\pi)$) has to be large to generate chaotic fluctuations. The same can be concluded for the rate of investment: given $\gamma$ and $\epsilon$, a smaller investment quote requires a higher adjusted capital productivity if chaos is to occur. Taking $\sigma=\sigma_{\text{max}}=1$, from the analyses of $\chi$, it is known that the economy will not exhibit irregular fluctuations for $\gamma<0.078125$ ($\epsilon=\epsilon_{\text{max}}$).

Figure 6.8 further gives the relationship between $\sigma$ and $\gamma$, in the second diagram, for $\beta(1-\pi)=1.6$ and different values of $\delta$.

---

15 For $\epsilon=\epsilon_{\text{max}}$ and $d=aw*n$, $\delta$ can be calculated using:

$$\delta = 1 - \sigma \beta(1-\pi) \left[ 1 + 2\beta(1-\pi)\gamma \frac{\chi}{(1+\chi^*)^2} \right].$$

16 $\delta = 1 - \sigma \beta(1-\pi) \left[ 1 + 2\gamma \beta(1-\pi) \frac{\chi}{(1+i^* \chi)^2} \right]$ or $\sigma = \frac{1-\delta}{\beta(1-\pi) \left[ 1 + 2\gamma \beta(1-\pi) \frac{\chi}{(1+i^* \chi)^2} \right]}.$
Conclusion:
In the former paragraph, fluctuations were shown to be highly unlikely because of the assumed non-negativity of the interest rate and the wage share. Coupling does change this conclusion. Using the same parameters ($\varepsilon_{\text{max}}, \gamma_{\text{max}}$) or even smaller ones, the aggregate economy exhibits all kind of dynamic behaviour, including chaotic fluctuations. When the behaviour on both markets is such that they move towards their equilibrium level, the combined effect of such stabilising behaviour results in fluctuations on a macroeconomic level.

Using the definition of the slope in $k^*$, the observation of $k^* = \pi = 1$ and several restrictions based on the assumption of $0 \leq wa^* \leq 1$, it was shown that the parameters determining the coupling ($\gamma$ and $\sigma$) do only play a minor role in the occurrence of irregular fluctuations.

In economic terms: the economic subjects react to excess supply and demand, moving the partial markets towards their equilibrium position. The employers and employees move towards a distribution of income that satisfies both the desired profit rate and a market-clearing wage (share). On the credit market, bankers and firms negotiate an interest rate at which the desired working capital can and will be supplied. The combined actions on the labour and credit market make the capital stock overshoot its macroeconomic equilibrium position, $k^*$. The resulting fluctuations are neither the result of market failure nor a desired outcome of the economic process. The magnitude of the coupling between the partial actions is shown to be of less importance compared to the speed of adjustment of these partial actions.

Only the three situations of the diagonal in table 6.3 are possible as the equilibrium in the credit market and the labour market coincide. These are simultaneous equilibrium (V), labour/credit shortage (IX), with a low interest rate and low wages, and labour/credit abundance (I), which is characterised by high wages and high interest rates. Both markets move in the same direction when the capital stock deviates from its equilibrium value. When the equilibrium positions in the two markets differ ($k_\pi \neq k_d$), the dynamics in the two markets can differ. This influences the conclusions in this chapter in several ways, as described in the next section.

5.4 Dynamics in a model with different equilibria in the labour and credit market

Again the same assumptions made in the former paragraph are used, resulting in equation (6.5.2). However, there is no a priori reason to assume $\rho$ to be equal to 1. There is no mechanism, which ensures that the level of the capital stock, for which demand and supply of credit are equal and $i_l = i^*$, is equal to the capital stock at which the labour market clears at $wa^*$. 
In the former chapter, it was assumed that the banking sector aims at a positive desired net credit creation, above the amount of money supplied by the monetary authorities: the banks derive their income from this gap between government supply and private demand. Its therefore logical to assume the credit market to clear at a level of base money below the level at which $w_a = w_a^*$: $\rho < 1$. There are also reasons to investigate the dynamics for $\rho > 1$. If the government uses the supply of base money as an instrument for their monetary policy, they can overshoot the target level. Furthermore, if there is an exogenous negative shock to production, the reaction of the government (adjustments in the supply of base money) probably lags behind the changes in economic reality.

In the following analyses some assumptions are made to reduce the complexity of the model. Firstly, the equilibrium wage share ($w_a^*$) determines the reaction parameter in the labour market and $n$ is taken to be one: $\varepsilon = \varepsilon_{\max} = w_a^*$. Secondly, the equilibrium level of credit is described as: $d = \rho w_a^* = \rho e$. This gives the following dynamical equation\(^{17}\):

$$k_{t+1} = [1 + \beta(1-\pi)]k_t - [1 + \chi(i^* - \gamma e)]e k_t^2 - \chi e^2 k_t^4 \quad (6.5.6)$$

The equilibrium value for $k, k^*$, can be determined using (6.5.6), the rules of Cardano (see the appendix) and:

$$0 = \beta(1-\pi) - [1 + \chi(i^* - \gamma e)]\varepsilon k^* - k^*^2 \quad (6.5.7)$$

Using the solution of $k^*$ from (6.5.7),

$$\frac{\partial k_{t+1}}{\partial k_t} = 1 + \beta(1-\pi) - 2[1 + \chi(i^* - \gamma e)]e k_t - 4\chi e^2 k_t^3$$

and the solution for $k^*^3$

from the former equation, the slope in $k^*$ is given by:

$$\delta = 1 - 3\beta(1-\pi) + 2[1 + \chi(i^* - \gamma e)]e k^* \quad (6.5.8)$$

There is a negative relationship between $\rho$ and $\gamma_{\max}$:

$$\gamma_{\max} = \frac{i^*}{d} = \frac{i^*}{w_a^* \rho} \quad (6.5.9)$$

This relationship is shown in the first diagram of figure 6.9. This equation can be read two-sided: it gives the maximum value for $\gamma$, given $\rho$, but also the maximal value of $\rho$, given $\gamma$, for which $i_t \geq 0$ for $k_t = 0$.

---

\(^{17}\) As base-situation, the parameters are: $\beta = 1.6, \chi = 10, i^* = 0.1$, giving $w_a^* = 0.8$ and $\varepsilon_{\max} = 0.8$ ($n = 1$).
Assuming $\rho$ to be below 1, $k_d$ is below $k_n$. When the simulation starts with a very low level of $k$, the interest rate is below $i^*$ and the wage share is below $wa^*$. The profit share is above its desired level, so capital is accumulated. There is a range of capital for which the interest rate is above its equilibrium value, depressing the profit share, but because the wage share is still below $wa^*$: capital continues to grow (the total effect of the costs on the profit share is positive). When both interest rate and wage share are above their equilibrium value, capital accumulation falls.

Taking different values for $\gamma$ and $\rho$, the bifurcation diagrams in figure 6.9 are drawn. The second diagram shows the bifurcation diagram for $\rho$, taking $\gamma=\gamma_{\text{max}}(\rho)$. For small values of $\rho$, $\gamma$ is large, resulting in chaotic fluctuations. As $\rho$ rises, $\gamma$ decreases, so the model becomes stable. The third diagram shows the relationship between $k^*$, the slope $\delta$ and the ratio between $k_d$ and $k_n$, $\rho$. As $\rho$ rises, $k^*$ rises. The increasing stability is seen as a rise in $\delta$.

The following diagrams give the result for $\gamma=0.125$ and $\gamma=0.05$. This gives $i_d(k_t=0)>0$ for $\rho_{\gamma=0.125}|1$ and $\rho_{\gamma=0.05}$2.5. Both bifurcation diagrams show the possibility of chaotic fluctuations. In the first case, these appear for $\rho<1$, when the capital stock required for equilibrium in the labour market is below the one required for equilibrium in the credit market. When $\gamma$ is lower, the chaos results for values of $\rho>1$, for positive interest rates even when $k_t$ approaches zero.
Figure 6.9: Different bifurcation diagrams for $\rho$ and the $k^*\delta\rho$ relationship all for different values of $\gamma$.

Using equations (6.5.7) and (6.5.8), table 6.4 is calculated. In this table, different values of $\gamma$ are used to calculate

1) the $\rho(\delta=-2.2$ gives the $\rho$, at which $\delta=-2.2$ (the appearance of chaos) with the accompanying equilibrium value of $k$ in parentheses,

2) the maximal value of $\rho$ for which $i(k=0)\geq0$ ($\rho_{\text{max}}$) and

3) the $\delta$ which can maximal be attained for $\rho=\rho_{\text{max}}$ ($\delta_{\text{max}}$).
If $\rho_{-2.2}\delta>\rho_{\text{max}}$, chaos is not possible, give $i \geq 0$. The kind of dynamics possible can be found by comparing $\delta_{\text{max}}$ with the estimated values for $\delta$ (monotone and cyclical movement to $k^*$: $\delta \geq -1$; persistent cycles around $k^*$: $-1 \geq \delta \geq -1.96$; irregular fluctuations: $-1.96 \geq \delta \geq -2.25$ (approximate)).

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>$\rho_{-2.2}$ ($k^*$)</th>
<th>$\rho_{\text{max}}$</th>
<th>$\delta_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>19.6 (2.3)</td>
<td>12.5</td>
<td>-1.2</td>
</tr>
<tr>
<td>0.02</td>
<td>9.1 (1.8)</td>
<td>6.25</td>
<td>-1.4</td>
</tr>
<tr>
<td>0.05</td>
<td>3.2 (1.4)</td>
<td>2.5</td>
<td>-1.8</td>
</tr>
<tr>
<td>0.07</td>
<td>2.1 (1.2)</td>
<td>1.8</td>
<td>-2.0</td>
</tr>
<tr>
<td>0.09</td>
<td>1.5 (1.1)</td>
<td>1.4</td>
<td>-2.1</td>
</tr>
<tr>
<td>0.1</td>
<td>1.35 (1.1)</td>
<td>1.25</td>
<td>-2.1</td>
</tr>
<tr>
<td>0.125</td>
<td>1 (1)</td>
<td>1</td>
<td>-2.2</td>
</tr>
<tr>
<td>0.15</td>
<td>0.78 (0.94)</td>
<td>0.83</td>
<td>-2.3</td>
</tr>
<tr>
<td>0.2</td>
<td>0.53 (0.86)</td>
<td>0.625</td>
<td>-2.4</td>
</tr>
<tr>
<td>0.5</td>
<td>0.11 (0.63)</td>
<td>0.25</td>
<td>-2.7</td>
</tr>
<tr>
<td>0.75</td>
<td>0.03 (0.55)</td>
<td>0.17</td>
<td>-2.8</td>
</tr>
<tr>
<td>0.9</td>
<td>0.01 (0.52)</td>
<td>0.14</td>
<td>-2.9</td>
</tr>
</tbody>
</table>

Table 6.4: $\rho$ for $\delta=-2.2$, $\rho_{\text{max}}$ and $\delta_{\text{max}}$ for different values of $\gamma$.

The slope in $k^*$ indicates the possible appearance of chaotic fluctuations at $\gamma \geq 0.05$ ($\rho_{\text{max}}>1.9$). For $\gamma \leq 0.125$, the model losses its stability before chaos is reached, because the interest rate becomes negative: $\rho_{\text{max}}<\rho_{-2.2}$.

For $\gamma>0.125$, the situation changes. For $\rho<1$, chaotic and irregular fluctuations can occur as $\rho_{-2.2}<\rho_{\text{max}}$ is feasible. Since $\rho_{-2.2}<\rho_{\text{max}}$, chaos will occur for positive values of $i_t$, even when $k$ approaches zero.

When $k^*<1$, as required for the occurrence of irregular and chaotic fluctuations, the equilibrium for the labour-goods market is superior to the equilibrium at the credit market: $k_d < k_n$ ($\rho<1$).

For example, when $\gamma=0.5$, table 6.4 shows that $\rho=0.11$ and $k^*=0.63$. given $k_n=1$, the equilibrium at the credit market is given by $k_d = 0.11$. When $k$ approaches $k^*$ from below, the upward labour market forces outweigh the downward forces of the credit market, so the aggregate economy 'overshoots' its equilibrium level. Only in the neighbourhood of $k_n$, the equilibrium forces in the credit market outweigh the influence of the labour market.

When $\gamma>0.125$, the interest rate remains positive but the model losses stability because $\rho_{-2.2}<\rho_{\text{max}}$. For $k^*<1$, the appearance of chaos requires the equilibrium at the labour market to be inferior to the equilibrium at the credit market. Take $\gamma=0.05$, then $d(\rho_{\text{max}})=2.5$, so $k_d=2.5$ and $k_n=1$. When the speed of adjustment in the credit market increases, the equilibrium in the labour market is superior to that in the credit market: for $\gamma=0.75$, $d=0.024$ and $k_d=0.03$ ($k_n=1$).

Conclusion:
There is no a priori reason to assume that the capital stock which clears the credit market ($i_t=i^*$) is the same at which $w_t = w^*$. Taking the capital stock for which $w_t = w^*$ in the labour market to be above that in the credit
market ($\rho < 1$), the reaction of the firms and households in the labour market moves the capital stock towards $k_n$, which enlarges disequilibrium in the credit market. When $wa_t$ comes nearer to $wa^*$, these forces are counterbalanced by the forces on the credit market, reducing the capital stock. The aggregate economy will exhibit persistent fluctuations because of those two opposing potentials.

The opposite case ($\rho > 1$) confirms the findings of the secondary nature of the influence of fluctuations in the credit market. The dynamics in the labour market are not counteracted by the movements in the credit market. The aggregate economy moves towards its equilibrium position. In equilibrium, there is a high level of employment (as compared to $n$), and an excess supply of base money ($d$). Assuming $i^*$ to be a 'fair' reimbursement of the costs from the banks and the monetary authorities, this can not be a long run equilibrium because $i(k^*) < i^*$.

6 Inflation and the coupling between the production and the monetary sector

6.1 Inflation and rigidities

The model above concentrated on the real properties of the economy. Because real profits are determined as a residual, real demand equals real supply in the goods market. To introduce price changes and inflation, nominal rigidities have to be introduced.

Rigidities
Real supply is given by $q_t$, and divided over different kinds of income (wages, interest income and profits): $l_t w_t + i_t c_t + p_t$. Total nominal income is spend in the same period: $l_t W_t + i_t C_t + P_r T$. Capitals give nominal variables, whereas $P_r$ gives nominal profits, to distinguish from the price level ($P$).

There are several ways to introduce rigidities, three of them are considered below.

First, transactions require money, which is supplied free as 'manna' by a government agency. Nominal demand equals the supply of money: $M_t = l_t W_t + i_t C_t + P_r T$. The price level is determined as the ratio between real supply and nominal demand: $P_t = \frac{M_t}{q_t}$. Inflation is determined exogenous by the difference between rate of growth in the money supply and the growth in production, as result of changes in the capital stock: $\dot{P} = \dot{M} - \dot{q}$, so the model predicts a negative correlation between inflation and production. Given perfect foresight the nominal variables are adjusted for the rate of inflation: $W = wP; P_r = pP$.

18Remember $q_t = \beta k_t \rightarrow \dot{q} = \dot{\beta} + \dot{k} = \dot{k}$,
A second rigidity can be the investment behaviour of the firms. Suppose, the firms aim at a constant nominal level of investment: $\bar{K} = P_k$. Nominal demand is then given by: $D = P_i [\omega_i + i_c] + \bar{K}$. Equating nominal demand with real income\(^{19}\), gives the price level: $P = \frac{\beta}{(1 + \chi'_i) w a + 1}$; $P_{wa} < 0$; $P_i < 0$. A decline in the costs of production ($wa$, $i$), raises the share of profits. This raises the funds available for investments more than the demand for physical capital goods. The price level of investment goods rises relative to consumption goods.

Lastly, nominal wage determination can deviate from the real wage determination, for example, because of the power of trade unions. A higher nominal wage share also influences the demand for nominal credits. Nominal demand equals $D = W_l + i_C + P_p$. Setting demand equal to supply gives the price level: $P = 1 + \frac{(1 + \chi'_i)(w - W)}{(\beta - (1 + \chi'_i)wa)k}$. Assuming the power of the trade unions to rise with production (the gap between $w$ and $W$ to widen), this version predicts a negative relationship between production and inflation: a rise in production raises nominal demand for consumption goods, but depresses nominal demand for investment goods. Nominal demand rises less than production, so the price level declines. However, in this analysis changes in the interest rate and labour productivity were ignored.

In the next paragraph an economy is modelled in which firms have some freedom to determine their labour demand, and allow a deviation between the real and the nominal wage determination. Conclusions are shown to be less straightforward than above.

### 6.2 Nominal wages, prices and employment

In the analyses above, a relationship for the real wage share was derived, without going into the division between the labour intensity of capital ($a$) and the real wage rate ($w$). In this paragraph this model is extended to capture the possible influence of these dynamics\(^{20}\) and their relationship to inflation.

Employment was given by:

$$l_t = a_t k_t$$

---

\(^{19}\)Wage and interest income is assumed to be spend on consumption goods.

\(^{20}\)This extension of the model has only illustrative use, in the sense that the actual model specification is sensitive to the value of the parameters and to the definition of the equations.
6. Chaotic Fluctuations: Between Money and Labour

$l_t = \text{employment}$

$a_t = \text{labour intensity of capital}$

$k_t = \text{capital stock}$

Firms, together with trade unions, determine the nominal wages at the beginning of the period. They base the present nominal wage rate ($W_{t+1}$) on the disequilibrium in the product- and the labour market in the former period as measured by the difference between demand for labour ($a_t k_t$) and the equilibrium level ($n$). This gives the relative changes in the nominal wage rate ($W_{t+1}/W_t$):

$$W_{t+1} = W_t \left(1 + \phi [a_t k_t - n] \right)$$

(6.6.1)

$$\phi = \phi' / \varepsilon$$

The actual real wage share ($wa$) in this period is determined as before with the labour demand determined by both present capital stock and labour intensity of capital:

$$wa_{t+1} = wa^* + \varepsilon (a_{t+1} k_{t+1} - n)$$

(6.6.2)

When the nominal wage rate is set, firms have some freedom in determining the labour intensity. A higher nominal wage rate (as compared to the desired wage rate) depresses the usage of labour, the labour intensity of production is adjusted downwards:

$$a_{t+1} = a_t \left(1 + \psi [w^* - W_{t+1}] \right)$$

(6.6.3)

$$w^* = wa^* / a^* = \text{the desired wage rate}, \text{as } W^* = w^* \text{ and } P=1 \text{ in equilibrium.}$$

The accumulations of capital is again determined by the deviation between actual and desired profit share, $(p_t - p_t^*)$, so the capital stock is given by:

$$k_{t+1} = k_t \left(1 + \beta (1 - \pi) - (1 + \chi_t)wa_t \right)$$

(6.6.4)

With the interest rate also determined as before:

$$i_{t+1} = i^* + \gamma (wa_{t+1} k_{t+1} - d)$$

(6.6.5)

The price level is determined by the nominal equilibrium, assuming the real economy to be in equilibrium as in the former paragraph. Nominal demand ($E$) is:

$$E_{t+1} = P_{t+1} P_{t+1} + (1 + \rho_{t+1}) W_{t+1} a_{t+1} k_{t+1}$$

The Business Cycle
\[ P_t = \text{price level} \]

The distribution of real supply \((S)\) is given by:

\[ S_{t+1} = q_{t+1} = p_{t+1} + (1 + \chi_{t+1})w_{a_{t+1}}k_{t+1} \]

Setting real demand \((E/P)\) equal to supply \((S)\) gives:

\[ P_{t+1} = \frac{W_{t+1}q_{t+1}}{wa_{t+1}} \quad (6.6.6) \]

The \(W-a\) dynamics, which follow from (6.6.1) and (6.6.3) resemble the cycles analysed in chapter five:

\[ W_{t+1} = W_t(1 + \phi[a_tk_t - n]) \]

\[ a_{t+1} = a_t(1 + \psi[w^* - W_t(1 + \phi[a_tk_t - n])]) \]

From equation (6.6.1), the equilibrium value of the labour intensity can be determined, by setting \(\Delta W=0\): \(a_tk_t=n\). Assuming (for simplicity) the credit and labour market to clear at the same value of \(k\) (\(\rho=1\)), this implies \(k^*=n\), so \(a^*=1\).

Similar, by setting \(\Delta a=0\), in (6.6.3), gives \(W^*=w^*\), whereas \(w^*\) can be determined by \(wa^*/a^*\). Substituting the equilibrium in (6.6.6) gives the equilibrium price level: \(P^* = \frac{w^*a^*}{wa^*} = 1\). The analyses in chapter five show the amplitude and periodicity of the \(W-a\) cycle to depend on the parameters and the initial deviations of their equilibrium value.

**Partial equilibrium**

To concentrate on the relationship between the labour intensity and the nominal wage rate (the partial \(W-a\) dynamics), the capital stock is assumed to remain at its equilibrium value \((k=k^*)\) and the coupling through the real wage share is ignored \((e=0; wa_t=wa^*=0.8)\). To simplify the model, the equilibrium in the labour market and the credit market are assumed to coincide at the same capital stock \((k_n=k_d=k^*; \rho=1)\) and the profit-investment ratio \((\sigma)\) is set at one.

Figure 6.10 shows the nominal wage-labour intensity dynamics. Changes in the labour intensity and price level reflect changes in the nominal wage rate, taking \(\phi=1, \psi=1\) and as initial variables \(a_0=a^*=1\) and \(W_0=0.75\) \((W^*=0.80)\). The third diagram shows the inflation-employment relationship, indicating on average a high level of employment when inflation is low. Demand for labour rises as the nominal wage rate (inflation) is below its equilibrium level. Although the capital stock, and all other real variables are constant,
employment exhibits fluctuations as the result of savings in labour, because of changes in the nominal wage rate.

\[ dP = \frac{\partial k_{t+1}}{\partial k_t} \bigg|_{k^*=1} = \left[ 1 + \beta(1 - \pi) - \gamma a^* - \epsilon(1 + \chi(i^* - \gamma d)) \right] 
- 2 \left[ \chi \gamma (wa^* - \epsilon a^*) + \epsilon a_t (1 + \chi(i^* - \gamma d)) \right]
- 6 \chi \gamma (wa^* - \epsilon a^*) a_t - 4 \chi \gamma e^2 a_t^2 \]  

(6.6.7)
The slope $\delta$ is labelled with $t$ as a reminder of the time-dependency of $\delta$, because of $a_t$, in contrast with $\delta$. Simulations showed irregular fluctuations for $\delta<2.2$. From (6.6.7) it can be derive that\[21\] :

$$
\begin{align*}
\delta_t &< \delta \quad \text{if:} \quad a_t > 1 \\
\delta_t &\geq \delta \quad \text{if:} \quad a_t = 1 \\
\delta_t &> \delta \quad \text{if:} \quad a_t < 1
\end{align*}
$$

For larger values of $a$, given the other parameters, the likelihood of irregular fluctuations is higher.

A rise in the capital stock does increase the nominal and real wage share. The rise in the real wage share depresses next period capital stock through the decline in profits. The increasing nominal wage rate depresses the labour intensity, $a$, which lowers next-period-labour demand, but also lowers the fluctuations in $k$ as $\delta$ declines.

Figure 6.11 gives the dynamics of the model described above. The simulation has been made assuming for the real sectors: $\beta(1-\pi)=1.6$, $\chi=10$, $i^*=0.1$, $n=1$, $d=0.8$ giving $k^*=1$ and $wa^*=0.8$. Furthermore, $\varepsilon=0.6$ and $\gamma=0.06$, so $\delta=1.16$ (given the parameters above: $\delta = 2.248-2.544a-0.864a^2$; for $a=1.24$, $\delta=-2.2$). In absence of nominal disturbances, real distortions cause a cyclical adjustment towards $k^*$. The nominal sector is given by: $\psi=1$ and $\phi=1$, as above, with $a^*=1$ and $W^*=0.8$. Initial values are $k_0=k^*$, $a_0=a^*$ and $W_0=0.75$.

As shown in the first diagram of figure 6.11, a nominal mistake in setting the wage rate ($W_0=W^*$), causes both the real and the nominal economy to fluctuate. The time series of the capital stock are characterised by a long-term cycle, around which short-term fluctuations are seen. The long-term wave is caused by the adjustment of the labour intensity to changes in the nominal wage rate, whereas the short-term fluctuations arise from the equilibrium seeking adjustments in the capital stock. During the long-term upswing, firms do react less severe as the production is less labour extensive and the capital stock moves towards its equilibrium value ($a_t<1$, $\delta_t>\delta$). However, when the economy approaches its equilibrium position, the labour intensity increases, which causes the labour market to 'overshoot' its real equilibrium wage rate. The labour intensity rises and the profit share declines drastically. The simulations show some periods for which $a>1.24$, so the fluctuations become explosive. However, because the capital stock declines fast, $a$ declines before the aggregate economy becomes explosive.

\[21\delta = [1 + \beta(1-\pi)-(wa^*-en)(1 + \chi(i^*-\gamma d))][-2\chi\varepsilon(wa^* - en)^2 + \varepsilon(1 + \chi(i^*-\gamma d))] - 6\chi\varepsilon(wa^*-en) - 4\chi^2\varepsilon^2

so: $\delta_t - \delta = [2\varepsilon(1 + \chi(i^*-\gamma d)) + 6\chi\varepsilon(wa^*-en)](a_t - 1) - 4\chi^2\varepsilon^2(a_t^2 - 1)$
The recovery in the nominal sector takes more time because of the assumed lag between the capital stock adjustments and the nominal wage rate. As the long-term wave moves the capital stock away from its equilibrium position during the downswing and the labour intensity rises, adjustments are more hectic ($a_t>1; \delta_t<\delta$). The resulting dynamics show an asymmetric cycle: a smooth upswing followed by a distorted downswing: short-term fluctuations are mild during a long-term boom, compared by the short-term fluctuations during the long-term downswing.

The long and short-term fluctuations are also visible in the $W-a$ dynamics: the economy moves between the two cycles shown as the capital stock moves up and downwards. The high values of the capital stock (see the first diagram of figure 6.11) give the upper cycle, whereas the low values of $k$ are responsible for the lower cycle. The cycles themselves (the long-term waves) represent the $W-a$ dynamics.

This duo-cycle is even more evident in the case of the $dP-l$ relationship as shown in the last diagram of figure 6.11. The economy, again, moves from cycle towards cycle, between a state of high inflation and low employment versus a state of deflation and high employment. Within each state, there is a clockwise movement, as the price level rises and employment declines when the nominal wage rate rises.

The occurrence of hectic fluctuations depends on the value of the parameters in the real and in the nominal sector. In figure 6.12, the bifurcation diagrams for $\psi$, $\phi$, $\varepsilon$ and $\gamma$ are shown, calculated using DMC, assuming one parameter to vary, with the other parameters set at $\psi=1$, $\phi=1$, $\varepsilon=0.6$ and $\gamma=0.06$. It is evident from these diagrams that chaotic fluctuations can result if one of the four parameters is large enough. The two-cycle
character of the capital stock is only visible in the $\psi$-diagram, but is present in the others too.

Figure 6.12: The bifurcation diagrams for $\psi$, $\phi$, $\gamma$ and $\varepsilon$.

### 6.3 Government policy

Government policies are left implicit in the analyses above, for two reasons. Firstly, the occurrence of fluctuations is based on the (stabilising) behaviour of economic subjects as the firms, the labourers and the bankers. Introduction of the government requires additional assumptions on the objects and the behaviour of government agencies. This would introduce another sector in the model. Not only would this raise the complexity of the model, but also it could enlarge the possibility of chaos if the government’s goals differ from those of the private sector.

Secondly, as before, by emphasising the dynamically adjustments within the private sector, it becomes clear that to stabilise the economy, the government should influence the behaviour of the economic subjects. Incidental measurements, once removed, only enlarge fluctuations in a chaotic environment. Within the present setting, government policy should take the form of an automatic stabiliser. In terms of chaotic control, only by removing the essential non-linear mechanism (for example, the nominal wage share-labour intensity mechanism) less fluctuations will result.
Intuitively, a rise in the interest on base-money, as provided to the banks, rises \( \gamma \), lowering \( \delta \) and increasing the possibilities of the occurrence of fluctuations (see figure 6.12). When the monetary authorities change the interest rate, this would cause the aggregate economy to display persistent fluctuations, even when the credit market initially is in equilibrium. As before the government only has a lasting influence if they set a variable for a long time, or by changing behavioural parameters of the model, changing the behaviour of the private sector. In the presence of erratic fluctuations, however, the government can increase aggregate welfare by moving the economy towards its equilibrium (since \( k_{av} < k^* \)). On the other hand, policies have to be financed, either by a tax on profits (depressing accumulation), wages (where the effect depends on the compensation labour can realise) or lending (raising the interest rate). Furthermore, when \( n \) is taken as a proxy for full employment, the equilibrium position of the aggregate economy can be below that necessary for full employment. There will be political dissatisfaction, giving the government an incentive to stimulate the economy beyond the equilibrium point. This causes the economy to fluctuate, with an average employment below \( ak^* \). If the political business cycle is taken into account, there is an additional endogenous mechanism to doubt the emerging of the steady state of a market economy: governing parties have an additional incentive to intervene in an otherwise non-fluctuating economy.

By adding an additional mechanism to a stable, but potential chaotic, economy, the possibility of hectic fluctuations increases, even when the sectors of the economy are ‘equilibrium-seeking’ in their own right. The nominal sphere above can be described as the engine of the fluctuations. The real sphere, however, provides the fuel for the chaotic dynamics as the movements towards their partial equilibrium provide new disturbances in the nominal sphere.

In reality, an economy exists of a manifold of relationships between economic subjects, each balancing and counterbalancing actions of others. Each subject is striving to realise an optimal situation. When the individual goals are equal, economic transactions will move the partial market towards equilibrium. There is, however, no guarantee that all markets move towards the same aggregate equilibrium. Even so, the combined effect can generate (chaotic) fluctuations when adjustment speed differs over markets. When failures and mistakes are added, the complexity and connections cause fluctuations to be the representative economical state, not the exception.
Summary and conclusions

The purpose of this chapter was to analyse the possibilities of the occurrence of fluctuations in an economy in which the partial markets display a strong tendency towards market clearing. Several conclusions can be drawn from the analyses and simulations in this chapter. Fluctuations occur in the labour-production model as the result of the reaction of the wage share to disequilibrium in the goods and labour market. In the credit-production model, fluctuations are the result of the reaction of the interest rate to disequilibrium between the demand and supply of credit. However, given the linearity in the price functions (wages, interest rate), the occurrence of irregular (chaotic) fluctuations is unlikely, taking into account the non-negativity of wage share and interest rate and the magnitude of the other parameters. The profit rate does not rise enough above the desired level to generate lasting fluctuations: the workings of the market enforce the equilibrating forces, so the markets move towards equilibrium.

This conclusion changes when the two sectors were to interact through the profit equation. Given a strong correlation between the wage share and the amount of credit, the combined forces of the two markets have a destabilising effect on the aggregate economy. The combination of the drive towards partial equilibrium in each market causes the aggregate economy to overshoot its equilibrium position, resulting in persistent fluctuations. If a lesser correlation was assumed, fluctuations vanish. A low speed of adjustment in the credit or labour market requires a high output-capital ratio, to generate chaotic fluctuations.

Another potential source of (chaotic) fluctuations lies in the magnitude of the coupling between sectors. The influence of these couplings depends strongly on the partial dynamics. When the coupling of markets does not lead to chaotic fluctuations, a higher value of the coupling parameter will not change this.

When the two levels of capital stock, necessary for equilibrium in the labour-goods market \(k_n\) and the credit market \(k_d\) do deviate, the likelihood of irregular fluctuations rises.

This conclusion, however, depends on the relative strength of the market-clearing forces in each market. Simulations show that irregular and chaotic fluctuations are possible for positive interest rates when equilibrium level of capital in the credit market is below that of the aggregate economy, which is below the level required by the labour-goods market \(k_d<k^*<k_n\).

Then the possibility of ‘money illusion’ was introduced. The labour intensity and the nominal wage rate were assumed to interact with each other. Firms readjust their organisations, saving on ‘overhead labour’ as the nominal wage is high. When the nominal wages are sensitive to changes in employment, this recursive relationship results in persistent fluctuations in
employment, nominal wages and the price level, even as the capital stock is constant. The changes in the labour intensity influence the demand for labour. When there is a relationship between the wage share and the demand for labour, the nominal cycle disturbs equilibrium in the real sector, so the dynamics of the capital stock add to the nominal fluctuations.

Simulations show fluctuations, which are the result of a long wave in labour intensity and a short wave, resulting from changes in the capital stock. These interactions are also found in the inflation-employment relationship, but (again) not representing a causal relationship, but a shared determination by the wage and utilisation dynamics. Chaos can occur, as seen in the bifurcation diagrams, for both nominal and real reasons. The behaviour of the aggregate economy can not -even in the long run- be used to derive conclusions on the behaviour of partial markets, but furthermore the analyses of partial markets do not give enough information on the behaviour of the total economy.

Government action was not explicitly modelled in this chapter because of the complexity of the model. Some analyses were made, indicating that if the government strives after objectives that are not compatible with the equilibria in the other markets, this does add to the possibility of the occurrence of fluctuations in the aggregate economy. Furthermore, government action requires an almost perfect knowledge of the economic mechanism, the absence of Friedman's inside and outside lag and a paternalistic view on the world.

The main conclusion is that chaos can occur because of the interaction between several stable sectors in the economy (credit market, labour market, goods market, government). This is especially true as the existence of different markets increases the possibility of different requirements for local equilibrium. Although price and quantity adjustments on the partial markets move the markets towards their equilibrium position, the combination of these forces causes the economy to fluctuate. When these local market mechanisms move in the same direction, this causes the economy to 'overshoot' its equilibrium position. Even when the market-forces move in different directions, the dominant market causes the other markets to be out of equilibrium (outweighing the local mechanisms) and the aggregate economy will not arrive at its equilibrium position. Lastly, the general equilibrium can be at a level of employment or credit, which is undesirable from a 'social welfare' point of view. Careful government interventions can increase social welfare in this situation.

Several extensions and versions of the model(s) developed in this chapter are possible and desirable. The augmentation of other sectors or more realistic features raise the complexity of the composed aggregate equilibrium capital stock and increase the amount of coupling parameters. Both factors
raise the potential instability as shown by the addition of the nominal wage determination in the analyses above.
Appendix

In section five some simulations were presented, coupling the monetary sector and the labour market. The model consists of six equations:

\[ i_t = i^* + \gamma (wa_t k_t - d) \]  \hspace{1cm} (6.A.1)

\[ wa_t = wa^* + \varepsilon (k_t - n) \]  \hspace{1cm} (6.A.2)

\[ p_t = q_t - (1 + \chi_t) wa_t k_t \]  \hspace{1cm} (6.A.3)

\[ q_t = \beta k_t \]  \hspace{1cm} (6.A.4)

\[ p_t^* = m_t \]  \hspace{1cm} (6.A.5)

\[ k_{t+1} = k_t + \sigma (p_t - p_t^*) \]  \hspace{1cm} (6.A.6)

Substituting (6.A.1) to (6.A.5) into (6.A.6) gives the dynamics of the model in terms of the capital stock:

\[ k_{t+1} = [1 + \sigma ((1 - \pi) \beta - (wa^* - \varepsilon n)(1 + \chi [i^* - \mu d]))] k_t - \sigma (\gamma (wa^* - \varepsilon n)^2 + \varepsilon (1 + \chi [i^* - \mu d])) k_t^2 - 2 \sigma \gamma \varepsilon (wa^* - \varepsilon n) k_t^3 - \sigma \chi \gamma \varepsilon k_t^4 \] \hspace{1cm} (6.A.7)

This can be written as:

\[ k_{t+1} = k_t [c - bk_t - ak_t^2 - dk_t^3] \] \hspace{1cm} (6.A.8)

With:

\[ c = [1 + \sigma ((1 - \pi) \beta - (wa^* - \varepsilon n)(1 + \chi [i^* - \mu d]))] \]
\[ b = \sigma (\gamma (wa^* - \varepsilon n)^2 + \varepsilon (1 + \chi [i^* - \mu d])) \]
\[ a = 2 \sigma \gamma \varepsilon (wa^* - \varepsilon n) \]
\[ d = \sigma \chi \gamma \varepsilon \]

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Substituting the assumptions in (6.5.1) gives for the parameters:

\[ c = 1 + \sigma \beta (1 - \pi) \]
\[ b = \sigma \varepsilon = \frac{\sigma \beta (1 - \pi)}{\pi (1 + \chi^* \beta)} \]
\[ a = 0 \]
\[ d = \sigma \varepsilon^2 = \frac{\sigma \beta (1 - \pi)}{\rho \pi^2 (1 + \chi^* \beta)} \]

Since (6.A.7) and (6.A.8) are equations of fourth-order, it is complicated to solve them explicitly for the equilibrium value of \( k \), its maximum and the slope at \( k^* \). When the parameters and \( k \) are assumed to be non-negative, it follows from (6.A.8) there is an interval at which it is a hump shaped function, with a maximum and two intersections with the \( k \)-axis at \( k=0 \) and \( k=k_0 \). Equation (6.A.8) gives \( k_{t+1} \) as a combination of a third order function and \( k_t \). From a third order equation the intersection with the \( x \)-axis can be determined by using the rules of Cardano (see Teller(1965, 22)). The rules of Cardano states, if a function has the form of:

\[ x^3 + \Theta x^2 + \Phi x + \Omega \]

It can be written as:

\[ z^3 + pz + q \]

With:

\[ z = x + \frac{\Theta}{3} \]
\[ p = -\frac{\Theta^2}{3} + \Phi \]
\[ q = \frac{2}{27} \Theta^3 - \frac{\Theta \Phi}{3} + \Omega \]

If its determinant \( D \) is larger than zero, one real intersection exists:

\[ z_0 = u + v \]
With:

\[
\begin{align*}
\mu &= \sqrt[3]{-\frac{q}{2} + \sqrt{D}} \\
\nu &= \sqrt[3]{-\frac{q}{2} - \sqrt{D}} \\
D &= \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3
\end{align*}
\]

Using these rules, it is possible to determine some of the characteristics of (6.A.8). Firstly, the intersection of (6.A.8) with the \(k_t\)-axis; \(k_0\) can be calculated:

\[0 = \left[c - bk_t - ak_t^2 - dk_t^3\right] \quad (6.A.9)\]

Giving:

\[
\begin{align*}
\Theta_0 &= a/d \\
\Phi_0 &= b/d \\
\Omega_0 &= -c/d
\end{align*}
\]

Secondly, the maximum, \(k_m\) follows from:

\[
\frac{\partial k_{t+1}}{\partial k_t} = c - 2bk - 3ak^2 - 4dk^3 = 0 \quad (6.A.10)
\]

Giving the transformation:

\[
\begin{align*}
\Theta_m &= 3a/4d \\
\Phi_m &= 2b/4d \\
\Omega_m &= -c/4d
\end{align*}
\]

Lastly, the intersection between (6.A.8) and the 45-degree line, at \(k^*\) has to be determined. This follows from:

\[k - \left(c - bk - ak^2 - dk^3\right)k = 0 \quad (6.A.11)\]

This leads to:

\[
\begin{align*}
\Theta^* &= a/d \\
\Phi^* &= b/d \\
\Omega^* &= \{1 - c\}/d
\end{align*}
\]