Strings and necklaces: on learning and browsing medical image segmentations
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Segmentation of the spinal column directly from three-dimensional image data is desirable to accurately capture its morphological properties. We describe a method that allows true three-dimensional spinal image segmentation using a deformable integral spine model. The appearance of vertebrae is learned from multiple continuous features recorded along vertebra boundaries in a given training set of images. Important summary statistics are encoded into a necklace model on which landmarks are differentiated on their free dimensions. The landmarks are used within a priority segmentation scheme to reduce the complexity of the segmentation problem. Necklace models are coupled by string models. The string models describe in detail the natural variability in appearance of spinal curvatures from multiple continuous features recorded in the training set. Strings are used for restricting segmentation within feasible solutions. In the segmentation phase, the necklace and string models are used to find the spinal column in unknown image data via elastic deformations. The driving application in this work is analysis of CT scans of the human lumbar spine. A segmentation illustration shows the method is promising for assessing morphological properties of the spinal column.

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6.1 Introduction

Segmentation of three-dimensional spinal images is an essential task for analyzing the morphology of the spinal column. This is difficult to achieve fully automatically with current computer vision methods due to the articulated structure of vertebrae and their dense context with ribs and other organs. Vertebrae exhibit many predicaments, violating the smoothness assumption under which many segmentation methods operate. Apart from this their image appearance is mostly far from evident. Insufficient image contrast, interfering anatomical structures, and other local irregularities lead to an inhomogeneous gray-level appearance that is hard to capture when assuming the image structure is homogeneous. Spinal image segmentation may encounter serious problems when it does not appraise the sophisticated image and shape appearance of the spinal column. It is therefore desirable to construct a segmentation model that exploits vertebral inhomogeneities rather than being hampered by them.

Commonly, segmentation of spinal images is done on the basis of geometrical models (e.g. [62], [124], [116], [49], [60]). The models capture the shape of vertebrae. They often also utilize their spatial inter-relationships, reducing image segmentation to fitting geometrical and spatial models to highlighted edges in the image data. There are two important shortcomings to this approach. Geometrical models often lack expressiveness to capture the full range of feasible vertebral shapes and constellations that can be expected in an image. And, geometrical models are targeted at exploiting a priori shape and spatial information, while in spinal images the spinal column is not only defined by these properties but also by its image structure. There is a need to also model image structure [96]. In order to construct apt segmentation models for the spinal column it is natural to observe multiple spinal properties and to address their natural variability.

The lack of segmentation models that exploit in full the many features defining the spinal column has incited us to tackle the segmentation problem by learning from example images. When subjected to statistical analysis a set of example images might reveal new and relevant features, conceivably enriching segmentation models. That is, if a good description of the spinal column is not possible from a priori geometrical knowledge, it may be learned from multiple features observed in a significant set of example images. Learning in this context helps focusing on the most relevant information in a potentially overwhelming quantity of image data full of boundaries of various scales and spatial configurations. Hence, we concentrate on learning multiple features of the spinal column rather than attempting to improve segmentation models by fixing on more a priori geometrical knowledge.

This chapter presents a segmentation method that exploits salient and variational information deduced from multiple continuous features as observed in a given training set of spinal images, under the assumption that the appearance of vertebral structures can be well captured in a statistical sense [2], [78], [7]. It is organized as follows. In section 2 related work on segmentation of spinal images is discussed. Section 3 briefly describes the image material used in this work and introduces the proposed method. The following issues are addressed: the necklace model for capturing vertebral structures, the string model for expressing spinal curvatures and the spine model
6.2 Related Work

In literature a number of two- and three-dimensional model-based approaches are proposed for segmentation of spinal images. The majority exploits vertebral landmarks to reduce the complexity of the segmentation problem.

Kauffmann et al. [62] first detect the axis of the spinal column by manually placing points along it and fitting a curve through them. The fitted curve is used to initialize and rigidly match templates of the vertebral body with the image data to obtain vertebral outlines. Landmarks are extracted from the best fitting contours and subsequently used for three-dimensional reconstruction. Verdonck et al. [124] manually indicate specific landmarks in the image and find others using an interpolation technique. The landmarks, together with a manually indicated axis of the spinal column, are used to automatically compute endplates on vertebrae and the global outline of the spine. A rigid spine model is then fitted to two sets of landmarks, one in frontal and the other in lateral view, to obtain a three-dimensional description. In [2], Aykroyd and Mardia use wavelets to model one-dimensional deformation functions of spinal curvatures from front and side view X-ray images, treating each vertebra as a well defined landmark. They statistically analyze spinal shape and shape differences for the two views.

These methods not always catch the three-dimensional properties of the spine as they rely on reconstruction from two-dimensional images using biplanar or stereographic techniques [116]. They yield three-dimensional descriptions from manually marked landmark points on multiple image views, resulting in inaccurate and reproducible measurements. Apart from this, the sparse set of landmark points is often connected by straight lines in order to give a wire frame representation [62], yielding non-realistic geometrical shapes and often lacking expression to capture local morphology, such as for vertebral deformities. This and the steady increase of patient-friendly three-dimensional imaging methods have motivated the development of true three-dimensional landmark-based methods.

The approach developed by Dansereau et al. [26] uses a three-dimensional personalized parametric model of the spine. Simple geometric primitives like elliptic cylinders and prism are adapted to fit corresponding landmarks in the image, giving a rough impression of the three-dimensional character of the spinal column. In [92] a model of the cervical spine is used consisting of a finite-element model augmented by additional structures used to locate landmarks, contours, surfaces, and regions. The surfaces and volumes are used by statistical estimation modules to automatically extract landmarks, using the finite-element model as a road map. Once the landmarks are found they are employed to refine the vertebra models. In [74], a point-based statistical model is proposed for segmentation of single vertebrae. Manually selected points associated with anatomical landmarks on the vertebra boundary are used for
point-to-point correspondence between model and image data, resulting in a close but not exact fit of the model. A mesh relaxation method moves ordinary model points to the target boundary for an exact match. The point distribution model is easily extended to capture multiple vertebrae but without the capacity to explicitly quantize properties of the entire spinal column.

Three-dimensional methods are a necessity to capture the truly intricate characteristics of the vertebral column. The reported methods are capable of doing that. However, they do this on the basis of a priori geometric models only, while adscititious exploitation of image models would enhance segmentation. Apart from this, most of the three-dimensional methods rely on manual localization of anatomical landmarks, which is subjective and labor intensive due to the huge amount of image data.

We take a different approach and aim at segmenting the spine by a learned model of continuous image and shape features on which landmarks are automatically localized on their free dimensions and applied accordingly.

6.3 Materials and Method

In this work we make use of an image data set consisting of CT scans of the abdomen part of a group of 18 elderly people. All subjects have been scanned with a Philips SR 700 CT at 140 KV (Philips Medical Systems, Best, The Netherlands) at a resolution of approximately $225 \times 225 \, \text{Wm}$ and slice thickness of 0.5 mm. The images are originally taken for the purpose of investigating the aorta but most of them also display the lumbar part of the spinal column. Concentrating on the lower four lumbar vertebrae (L2, L3, L4, L5), we demonstrate our method on 6 CT images of subjects with minimal spinal and vertebral deformities. In figure 1 transversal, sagittal and coronal slices of an image from the data set are shown, exhibiting normal and abnormal vertebrae.

For segmentation of these images we use deformable model methods (e.g. [15], [115], [79]). First introduced by Kass et al. [61] and Staib et al. [114], the idea behind them is to treat segmentation as an optimization problem, typically by minimizing a model fitting function that rewards locally smooth boundaries that pass through high-gradient image regions. A model is deformed in the image trying to find a compromise between features derived from the image and features obtained from a shape model. The deformation stops when an equilibrium is reached. The deformable model is then assumed to lie on the target boundary in the image.

We adopt the deformable model approach with the difference that we aim at learning vertebral features rather than defining them on the basis of a priori knowledge. In addition, to exploit salient and variational information as observed in a given training set of spinal images we construct a) models of the lumbar vertebrae using necklaces, and b) models of the spinal curvatures using strings and c) an integral spine model using coupled necklace and string models. In the following section we describe the models one by one.
6.3. Materials and Method

![Images of CT scans](a) (b) (c)

**Figure 6.1**: Axial, sagittal and coronal cross sections of a spinal CT image. The images reveal an infrarenal aortic aneurism, renal and osteoporotic fracture of the plate only obvious on the sagittal reconstruction.

### 6.3.1 Necklace Model for Vertebral Structures

A visual inspection of the vertebra surface suggests that the surface is bended in ways significantly different from a flat surface, i.e. it has many concave and convex surfaces differing from weakly to strongly curved. Analogous to diversity in shape, the gray-level appearance of the vertebra boundary exhibits many different structures. At some parts the vertebra boundary has well-defined intensity discontinuities, while at other parts there is vague pictorial evidence or none at all due to bad image quality or interfering structures in the neighborhood.

To appropriately capture and exploit the locally sophisticated geometrical and image appearance of the vertebra surface we need to observe multiple features along its boundary. To that end we employ necklaces. As discussed in the previous chapter, the necklace model allows for the analysis of inhomogeneous boundaries by recording a repertoire of image and shape features. Specifically it allows for exploitation of salient features as landmarks for segmentation.

**Feature Definition**

The appearance of the spinal column is learned from a training set of $M$ examples consisting of three-dimensional spinal images $I_m : x \in \mathbb{R}^3 \rightarrow \mathbb{R}, m = 1, ..., M$ and true vertebral outlines as edited in the volume data by a medical expert in three two-dimensional orthogonal slices of the volume data. We represent each vertebral outline $\vartheta = 1, ..., V$ in the $M$ training images by B-spline surfaces $s_m^\vartheta : u \in \mathbb{R}^2 \rightarrow \mathbb{R}^3$ [93]. After manual alignment of the surfaces, image and shape features are extracted at the surface and conveniently captured as a manifold in an $N$-dimensional feature space, adopting the multi-feature object representation of necklaces. Considering the
\( \vartheta \)th vertebrae the set of multivariate continuous features is deduced from the training data by the mapping \( F^\vartheta : u \in \mathbb{R}^2 \rightarrow \mathcal{F}^\vartheta \)

\[
F^\vartheta(u) = [f_1^\vartheta(u), ..., f_M^\vartheta(u)].
\] (6.1)

Here, we have chosen to compute the location, rotation and scale invariant mean curvature as the local shape feature to be recorded along the surface outlines. We also compute the principal curvatures to simplify landmark definition in subsequent steps. The feature values are analytically computed from the B-spline surfaces [93] \( s_m^\vartheta(u) \). The feature values are used to construct feature functions \( f_m^\vartheta(u) \) by interpolating a smooth surface through them.

We also compute three image features described in the previous chapter. To recapitulate, the first feature has a large value when it concerns a point on a flat structure, e.g. the upper surface of the vertebral body. The second feature has a large value when it concerns a point on a tubular structure, e.g. where the upper and lower surface merge with the anterior surface of the vertebral body. The third feature has a large value when it concerns a point on a tip-like structure, e.g. the tip of the spinal process. The features are extracted from the training images along the training surfaces, i.e. at image positions \( I_m(s_m^\vartheta(u)) \)

The extraction of \( N = 6 \) features for each vertebra yields \( V \) sets of \( M \) surfaces in a 6-dimensional feature space, to be analyzed statistically for model construction.

**Landmark Selection**

We aim at exploiting landmarks that are defined by the multiple features recorded along the continuous vertebra boundary. Vertebral landmark definition reduces to localizing peaks in feature function values \( f_m^\vartheta(u) \). However, rather then separately investigating each training instance \( m \) for landmarks, we first compute the statistics of the training sets. Then we try to obtain robust landmarks from the population average function values. After alignment of the feature functions, the population average for vertebra \( \vartheta \) is computed as

\[
\bar{f}^\vartheta(u) = \frac{1}{M} \sum_{m=1}^{M} f_m^\vartheta(u|I_m, s_m). \] (6.2)

This two-dimensional surface in the \( N \)-dimensional feature space is obtained by averaging each training surface \( f_m^\vartheta(u|I_m, s_m) \) in each dimension. The variance in shape feature values is

\[
\sigma_{f^\vartheta(u)} = \left( \frac{1}{M} \sum_{m=1}^{M} \|f_m^\vartheta(u|I_m, s_m) - \bar{f}^\vartheta(u)\|^2 \right)^{1/2}. \] (6.3)

The average feature function \( \bar{f}^\vartheta(u) \) is searched for high curvature points on the basis of its local second order properties [65], [117]. These properties are obtained from the infinite set of planes passing through and containing the normal at a specific point
on the population average $f^\theta(u)$. For example, when the only features considered are the $x,y$ and $z$-coordinates of the vertebral surfaces $s^\theta_m(u)$, each of the normal planes intersects the surface by a planar curve. The curvature at the point of interest is an identifying curvature measure for the surface. The pair of directions $v$ and $w$ are defined such that these curvatures reach their maximum and minimum curvatures $\kappa_1$ and $\kappa_2$ as illustrated in figure 6.2. We use these principle curvatures and locally associated directions to define landmarks.

![Figure 6.2: Surface point properties](image)

**Figure 6.2:** Surface point properties are derived from the curves defined by the intersection of the surface with the two orthogonal planes that go through that point and contain the normal vector $n$. The curves with minimal and maximal principal curvatures in corresponding directions $v$ and $w$ define the type point: a) sheet points have low bending and freedom to move in two directions, b) curve landmark have a fixed position in all but one dimension c) point landmark have no free dimensions and are precisely localized.

We make distinction between point landmarks, curve landmarks and sheet points by evaluating the principal curvature values at each point of $f^\theta(u)$. Point landmarks are surface points $U_A$ where both principle curvatures have an extreme absolute value. They are precisely localized in three dimensions. Surface points where the absolute value of one of the principal curvatures is extreme, are curve landmarks, denoted by $U_B$. They are precisely localized in two dimensions. At sheet points $U_C$ both values of the absolute principle curvature is low. Typically they are well-defined in only one dimension. The sets $U_A, U_B, U_C$ together contain all relevant path positions. A threshold for the principal curvature values may be chosen such that the definition of these sets, i.e. the distribution of geometrical landmarks, largely coincides with anatomical landmarks.

At this point we have $V = 4$ necklace models: one for each vertebra captured. In the segmentation step, the information contained in $f^\theta(u)$ and $\sigma_{T^\theta}(u)$ is used as a reference model for feature selection and qualification. The sets $U_A, U_B, U_C$ are used for landmark-based segmentation.
6.3.2 String Model for Spinal Curvatures

At the level of the spinal column we combine each of the vertebra models to use knowledge about the spinal curvatures. The cervical and lumbar curvatures are characterized by a convex shape, while the thoracic and sacral curvatures are characterized a convex forward shape. Knowing that these spinal curvatures are almost always present with some variation across and among subjects, this information can assist image segmentation.

We aim at capturing the spinal curvatures using string models. As discussed in chapter 3, the string model has the capacity to build a detailed underlying statistical model of open and closed boundaries from multiple continuous shape and image features, in contrast to other similar approaches (e.g. [19], [20], [35], [43], [85]). Here, we use strings to capture the common appearance of spinal curvatures in our training data and the main modes of variation therein, in ways similar to [2].

Feature Definition

The shape of the spinal curvatures is learned from the same training set of $M$ examples images and $V$ outlines associated with each vertebra. Assuming the landmarks on the lumbar vertebrae occur at approximately the same position, we select point landmarks $u_\ell \in \mathcal{U}_A$ for $\ell = 1, \ldots, L$ and learn the appearance of the $L$ curves that pass through the $V$ surface points $s_{m}^{-1}(u_\ell), \ldots, s_{m}^{-1}(u_\ell)$. Each curve captures the spatial relation between corresponding point landmarks on adjacent vertebrae, derived automatically from the vertebral outlines. We represent the $\ell$th the curve in the $m$th training image, by the B-spline curve $c_{m}^{\ell} : u \in \mathbb{R} \rightarrow \mathbb{R}^3$. Multiple continuous image and shape features are extracted along the curves in the training set and captured by space curves in feature space $\mathcal{F}^{\ell}$, yielding feature function

$$F^{\ell}(u) = [f_{1}^{\ell}(u), \ldots, f_{M}^{\ell}(u)].$$

(6.4)

We observe two features along the curve $c_{m}^{\ell}(u)$. The first feature is the bending of the curve, capturing shape statistics. We have chosen this feature because of its invariance properties and because it might reveal new and interesting anatomical knowledge. For example, the value and location of the maximum curvature along the spinal column is a relevant clinical measure for spinal deformities [2]. The second feature measures the image gradient magnitude, supporting the definition of spinal curvature by means of image evidence, which is mainly confined at tips of the vertebral structures.

The extraction of 2 features for each spinal curvature yields $L$ sets of $M$ curves in a two-dimensional feature space, to be analyzed statistically for model construction.

Variational Information

We aim at statistically modeling the natural variability of the spinal curvatures in terms of shape and gray-level features. This differs from the mainstream methods in that we do not confine the definition of spinal curvatures based on a priori geometrical knowledge such as smoothness on intra- and inter-curve properties of the spinal column [126]. Rather by learning what the common appearance of the spinal curvatures
is, the search criteria can be based on natural variations. Our training data can be summarized as

$$\tilde{f}^\ell(u) = \frac{1}{M} \sum_{m=1}^{M} f^\ell_{m}(u|I_m, c_m).$$

(6.5)

The one-dimensional curve $\tilde{f}^\ell(u)$ in the multi-dimensional feature space is obtained by averaging each training curve $f^\ell_{m}(u|I_m, c_m)$ in each dimension separately. The variance is

$$\sigma_{f^\ell}(u) = \left( \frac{1}{M} \sum_{m=1}^{M} ||f^\ell_{m}(u|I_m, c_m) - \tilde{f}^\ell(u)||^2 \right)^{1/2}

(6.6)$$

The population average feature function $\tilde{f}^\ell(u)$ and variation $\sigma_{f^\ell}(u)$ contain important evidence in the training data. Variational information is captured in more detail by functional data analysis [95], producing string models of the spinal curvatures. The string models allow to observe multiple features along one-dimensional curves, to weight the features according to the most important natural modes of variation and to explain unknown instances by a statistically determined feature weighting procedure. Strings resemble necklaces in that multiple continuous features are recorded and statistically analyzed, however, the emphasis lies on weighting features according to their natural variations, whereas necklaces mainly focus on selecting features for defining landmarks.

**Figure 6.3:** Three string models found as an average over 6 normal patients viewed from three different perspectives: a) axial b) sagittal and b) coronal.

At this point we have a detailed statistical description of the spinal curvatures. For simplicity, we assume all the relevant information is contained in $\tilde{f}^\ell(u)$, and $\sigma_{f^\ell}(u)$. 
We proceed with only these two quantities. Figure 6.3 illustrates the population average shape of the spinal curvatures of the lumbar part of the spinal column.

### 6.3.3 Integral Model for Spinal Column

To form an integral model of the spinal column we couple the necklace models of vertebrae and the string models of the spinal curvatures. This is accomplished by stacking the $V$ necklace models at positions that are statistically determined from the strings models. This is illustrated in figure 6.4.

![Figure 6.4](image)

**Figure 6.4:** The integral spine model consists of multiple necklace models for vertebrae coupled by string models as in a marionette from three different perspectives: a) axial b) sagittal and b) coronal.

The spine model is deformed onto to the data of a new unknown image such that it fits best a recorded spinal column.

### Qualification

A *deformable integral spine model* consists of deformable surfaces $s_v^\theta(u)$ and deformable curves $c_{\ell}^\theta(u)$, which account for variability among vertebral structures and their interrelationships. For all $V$ vertebra models, the initial surface $s_{l=0}^\theta(u)$ and curve $c_{\ell=0}^\theta(u)$ are the population average. That is, assuming models $s_m^\theta(u)$, $\theta = 1, ..., V$ and $c_m^\ell(u)$, $\ell = 1, ..., L$ are properly aligned and uniformly parameterized to
establish point correspondence they are defined as

\[ s_{t=0}(u) = \frac{1}{M} \sum_{m=1}^{M} s_m^\theta(u) \]  

(6.7)

\[ c_{t=0}(u) = \frac{1}{M} \sum_{m=1}^{M} c_m^\ell(u). \]  

(6.8)

The fit quality is determined on the basis of features \( f_t^\theta(u) \) and \( f_t^\ell(u) \) emanating from \( s_{t=0}^\theta(u) \) and \( c_{t=0}^\ell(u) \) respectively. The model fitting function makes a compromise between the fit quality of the necklace models and the string models:

\[ \Theta_{\text{spine}} = \frac{1}{V} \sum_{v=1}^{V} \omega^\theta \cdot \theta^\theta(\tilde{f}^\theta, \sigma_{f^\theta}, f_t^\theta) + \frac{1}{L} \sum_{l=1}^{L} \omega^\ell \cdot \theta^\ell(\tilde{f}^\ell, \sigma_{f^\ell}, f_t^\ell). \]  

(6.9)

The first term measures the distance between the expected and the sampled values for each vertebra \( \vartheta \). To ensure a controllable distance measure, the Mahalanobis distance \([34]\) is computed using mean and variation information obtained from learning. For the \( \vartheta \)th vertebra this means

\[ \theta^\theta(\tilde{f}^\theta, \sigma_{f^\theta}, f_t^\theta) = \int_u v(u) \|rac{\tilde{f}^\theta(u) - f^\theta(u)}{\sigma_{f^\theta}(u)}\|^2 du. \]  

(6.10)

The fit is controlled by means of the function \( v(u) \) which weights the fit at each point of the deformable surface \( s_t^\theta(u) \). Weighting is done according to the type of surface point under consideration: for point landmarks there is a predefined weight \( v_A \), for curve landmarks \( v_B \) and for sheet points \( v_C \). For example, the search for point landmarks is performed using settings \( v_A = 1, v_B = 0, v_C = 0 \). The weights may also be set such that features along the entire surface contribute by setting all values larger than 0, but constrained to add up to 1.

The second component in equation 6.9 measures the distance between the expected and the sampled values for each string model. This way it seeks at all times resemblance between the reference spinal curvature \( f^\ell \) and the deformed curve \( f_t^\ell \) normalized by the common modes of variation. For the \( \ell \)th string model the fit quality is formulated as

\[ \theta^\ell(\tilde{f}^\ell, \sigma_{f^\ell}, f_t^\ell) = \int_u \|rac{\tilde{f}^\ell(u) - f^\ell(u)}{\sigma_{f^\ell}(u)}\|^2 du \]  

(6.11)

The model fitting function is regulated by means of weights \( \omega^\theta \) and \( \omega^\ell \), which are positive and add up to 1. Their value is defined by the user and generally tuned such that they emphasize either the fit of the necklace models or the fit of the strings models. The fit quality forms the basis for deformation.

**Optimization**

Having specified the model fitting function, we must choose how to optimize the degrees of freedom of the spine model. Optimization involves two main steps for
each point on the deformable necklace model. In the first step a newly suggested position is calculated based on the fit, followed by deformation of the necklace model to move the point into the newly preferred position. Optimization only affects the geometry of the deformable surfaces: in our implementation the curves constituting the string models are automatically derived from them. The aim is to find the optimal deformable surfaces for all \( s_i^v(\mathbf{u}) \), \( v = 1, ..., V \), such that

\[
\Theta_{spine}^{optimal} = \arg\min_{s_i^v} \Theta_{spine}.
\] (6.12)

As the deformable surfaces know the type of point they search for, movement of surface points can be explicitly made dependent on that type, contradicting the usual way of repositioning all points in a three-dimensional space regardless of their dimensionality. Sheet points, curve landmarks and point landmarks are searched for in one, two and three dimensions respectively (see figure 6.5). This allows a better control of the segmentation as movement of surface points is restricted to well-defined directions.

Figure 6.5: For each type of surface point the search area is specified in terms of local surface properties: a) sheet points are optimized in the normal direction (small stripes indicate the direction at each sheet point) perpendicular to the surface, b) curve landmarks in a two-dimensional area spanned by the normal and first principal direction and c) point-landmarks in a three-dimensional area specified by the normal and principal directions.

A priority scheme is used when optimizing the deformable surface \( s_i^v(\mathbf{u}) \). For each deformation of a surface onto the image data the following scheme is employed:

1. Optimize point landmarks of the vertebra in a three-dimensional area. The result is a rough estimate of the position of the vertebra boundary by its point landmarks.
2. Optimize curve landmarks in a two-dimensional area departing from 1). The result is the location of surface curve points determining the outline of the vertebrae.

3. Optimize sheet points in a one dimensional area departing from 2). The result is the location of all boundary points.

4. Optimize all points one more time in their respective dimensions departing from 3), to fine tune the result and to obtain a global optimum.

In searching for a specific vertebra we also estimate the position of other vertebrae. We do the estimation only when searching for point landmarks so that movement of one point landmark affects the entire spine model. We accomplish this by distributing the force that works on a single point to all other points, in this case, weighted according to distance. For example, if there is a drive \( d(u_i) \) working on surface point \( s^0_i(u_i), u_i \in U_A \), which is also the connection point \( c^i_t(u_i) \) for a string model, this yields the following estimation for the discrete points \( s^0_i(u_j), j = 1, ..., J \) on all vertebra models \( \vartheta = 1...V \)

\[
s^0_{i+1}(u_j) = s^0_i(u_j) + d(u_i)e^{-(\delta_c/c_d)}. \tag{6.13}
\]

The distance \( \delta_c = D(c^i_t(u_i), c^j_t(u_j)) \) between points \( c^i_t(u_i) \) and \( c^j_t(u_j) \) is used to determine the extent of the distribution. A small value for the distribution constant \( c_d \) influences the shape of the deformable necklace model above or beneath the one being optimized, while a large value also affects the shape of the surfaces at large distances. This way segmentation of a single vertebral structure influences the entire spine model.

### 6.4 Illustration

We give an illustrations of segmentation of vertebral structures from CT images with help of the deformable integral spine model. First the 4th lumbar vertebra is segmented using a necklace model, then part of the lumbar spine is segmented with help of the spine model.

For construction of the vertebra surfaces, a total of 144 points are used, with articulated structures such as the spinal process and transverse processes requiring more points than flat parts such as the vertebral body. These points are interpolated by a B-spline surface to obtain a continuous representation. For each vertebra surface in each training instance, image and shape features are recorded at 400 sample points. Feature functions are constructed from them and statistically analyzed to build the necklace models according to equations 6.1-6.3. Landmarks on the learned necklace models are defined on the basis of curvature threshold \( c_t = 0.05 \). The following point landmarks are selected to form string models according to equations 6.4-6.6: two points corresponding to the two tips on the spinal process, and the two tips at the two transverse processes. These six points are also used for human computer interaction.
Figure 6.6: The segmentation scene for three different perspectives. The image data is rendered transparently and the deformable model in it opaquely. First row: the initial necklace model. Second row: the deformation result.

For initialization of the spine model in the image data, a fixed point is selected to enable quick correspondence between the model and the target boundary. The deformable spine model is interactively bootstrapped by pointing and clicking at the corresponding point in the image. In this segmentation session no translation, rotation or scaling is required as the first guess is acceptable. The first row in figure 6.6 shows the condition after initialization from three different perspectives, with the image data rendered with opacity 0.5 and the model in it with opacity 1. The local fit quality at a number of control points is indicated with colored spheres. The color varies from green, indicating a good fit to red, expressing a bad fit.

Following the priority scheme described in the previous section, we optimize the initial deformable model in four steps. In the first step point landmarks are auto-
matically fit to the image data after marking of the landmarks. This produces a preliminary solution which is closer to the target boundary than in the starting position. In the second step, curve landmarks, in particular at the lower part of the vertebral body, move towards the target boundary. In the third step, the lower and the upper planes of the vertebral body are reasonably found by deformation of the surface in one dimension. The result after optimizing all surface points once again in their respective dimension is illustrated in the second row of figure 6.6. The deformable surface has found an optimal solution, weighting both image and shape features along the entire surface. The majority of the surface points was fitted well to the target boundary. At some parts the necklace model moves away due to attraction by neighboring structures or due to locally too much deviation of the target boundary from the population average.

![Figure 6.7: Landmark-based segmentation of the 3rd and 4th lumbar vertebrae. The spine model is visualized together with gray-level planes through the original image and transparent renderings of the high intensity objects: a) initial condition, b) condition after fitting the 3rd lumbar vertebra model and c) condition after fitting the 4th lumbar vertebra model.](image)

To illustrate how the spinal column is extracted using the spine model, segmentation of part of the lumbar spine is performed. First the L3 is segmented by deformation of the corresponding deformable surface as before. Simultaneously the position of the L4 is estimated by changing the geometry of the corresponding deformable surface according to the solution for the L3. On the basis of the preliminary solution for the L4, point landmarks on the L4 are sought under the constraints that the expected spatial relation is maintained as much as possible, i.e. spinal curvatures comply to the statistics. No interaction is required as the spine model is accurate enough. Then curve landmarks and sheet points are sought in the image data. The position of the initial deformable surfaces and curves and their position after optimizing are
illustrated in figure 6.7. The step-by-step automatic segmentation of the $L3$ and $L4$ succeeds despite the articulated vertebral structures and their constellation.

6.5 Discussion and Conclusion

The segmentation of vertebral structures in the previous section is of exemplary nature. The primary goal is to illustrate how the spinal column can be captured using multiple continuous features that are averaged and weighted to exploit salient information and patient-to-patient variation learned from a set of images of normal patients. We have used image data from 6 patients to train vertebra models for the $L2, L3, L4$ and $L5$, six models of the spinal curvatures and one integral spine model. Clearly a larger image data is required to assess the accuracy of the method and its usefulness for clinical use. However, we have demonstrated that the application of multiple continuous features along vertebral surfaces and spinal curves is promising for segmentation of unknown spinal images. For further validation a particular interest is in the long term goal of analyzing average and variation characteristics of normal and pathological spinal columns from longitudinal studies [96], in ways similar to [2] and [7], for the purpose of calculation of local and global deformity quantifying parameters.

We accentuate some important items of our method in comparison with [2], [7], [96]. In the first place, our method works completely in three dimensions, allowing to measure truly three-dimensional properties of vertebral structures and spinal curvatures, rather than relying on two-dimensional features. In the second place, we capture not only shape properties but also image intensity properties as they also define the appearance of vertebral structures. Multiple continuous features are extracted, then statistically analyzed by multivariate functional techniques [95] to obtain important population statistics. This is done a) to exploit natural variation in appearance for constraining the deformation of the spine model, rather restricting it on a priori geometrical constraints and b) to exploit salient information, defined as differential geometrical landmarks by multiple features, for reducing the complexity of the segmentation problem. Furthermore we aim at a step-by-step automatic segmentation departing from geometrically well-defined landmarks on a particular vertebra rather than at aiming for finding a one shot integral solution for the spinal column using manually marked anatomical landmarks.

As concerns the implementation, in our method we used continuous curves and surfaces in the form of B-splines to represent the spinal column and features derived thereof. This in contrast to most other methods that usually use point distribution models, finite element models or geometric primitives. The advantage of a continuous representation is that quantitative information can be analytically computed, allowing for more complete and accurate measurements, especially in the clinically relevant differences in curvature and torsion of the spinal column. No manual or other additional heuristic techniques are required to compute the positions of landmarks, contrary to e.g. [62]. We found that the continuous curves and surfaces provide a compact representation in the form of control points. This is beneficiary because a
restricted number of points are required to define the spinal column and to control the spine model when segmenting images.

A drawback of the reported method is an inherent difficulty of deformable models often getting trapped in local minima of optimization. It has been acknowledged previously that proper initialization is required to guarantee satisfying result in view of the presence of disturbing attractors in the image [83]. This is particularly true for complex images such as of the spine. As automatic initialization of deformable models is still an open problem [45], we deal with these problem by minimal human-computer interaction. The user simply points and clicks in the image to make one-to-one correspondence between model and image by means of six point landmarks. The use of point landmarks alleviates the problem of interaction in three-dimensional space [45] due to their zero-dimensional property. During segmentation the user controls the entire spine model as a marionette by interaction with a few point landmarks and propagation of landmark solutions to other parts of the spinal column.

We have only considered CT images of normally appearing spinal columns. When dealing with spinal deformities such as sciotic spines that exhibit lateral curvatures and vertebral rotations, segmentation using the integral spine model may face problems due to considerable deviations from the normal spinal column. In this context it is essential to select invariant features in order to capture a broad range of natural variations and simultaneously minimizing the effects of non-essential variations in ground-truth delineations. Also, it important to carefully handle the alignment problem has this will be difficult, if not possible, when dealing with much variation. A possible approach is construct statistical spine models for different classes of conditions. This already has been proposed in [2] where, similar to our approach, a large collection of X-ray images from a longitudinal study of idiopathic scoliosis is examined from side and front view to obtain important summary statistics such as curvature. However, as stated by Aykroyd and Mardia, even when different spine models are constructed for different spinal conditions, there would still be substantial variability within these conditions to obtain good statistical summary. This remains an open problem.