Absorption of immigrants in European labour markets. The Netherlands, United Kingdom and Norway
Zorlu, A.

Citation for published version (APA):
Zorlu, A. (2002). Absorption of immigrants in European labour markets. The Netherlands, United Kingdom and Norway Amsterdam: Thela Thesis

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Chapter 3

THE EFFECT OF IMMIGRATION ON EARNINGS:
THEORY

3.1 Introduction

While Western Europe has become one of the magnet fields for international migration flows in the post-war period, immigration policies have become increasingly restrictive. The restrictive immigration policies are designed particularly to limit immigrant flows from developing countries by new legislation or indirectly by tighter application of existing policy instruments. The policies are based on an implicit assumption that newly entering immigrants would have an adverse effect on wages and employment of natives in addition to causing a further deterioration of the labour market position of settled immigrants in these countries. These assumptions are closely related to qualifications of immigrants and state of the host country labour market. Up to now, research on immigrants has focussed on the supply-side of immigrant labour in Europe and, in particular, the Netherlands. However, it is unclear what the reaction of the host country labour market is to an entering immigrant labour force in terms of its capability to absorb new immigrants and the adjustment of these immigrants to host country labour markets. In other words, little is known about the impact of immigration on the host country labour market.

Since the paper of Grossman (1982), a number of studies has been conducted. Most of these studies concern traditional immigration countries like the United States, Canada and Australia (see surveys: Friedberg and Hunt, 1995; Borjas, 1994). In Europe, the number of studies is still limited but increasing in the last few years (Winkelmann and Zimmermann, 1993; De New and Zimmermann, 1994; Gang and Rivera-Batiz, 1994; Hatzius, 1994; Pischke and Velling, 1997; Venturini, 1999). This may be due to the lack of appropriate data sets for in-depth analysis. However, recently an increasing number of questions pertaining to the ethnic origin are included in data sets. In addition, the emergence of data sets
including a reasonable number of observations about ethnic minority workers has stimulated research on this group of individuals.

Economic theory predicts that the impact of immigration on the host country labour market manifests itself in two main forms: on wages and (un-)employment. In flexible economies like the US, the adjustment in the labour market after immigration is thought to be through wages and regional mobility. On the other hand, immigration is supposed to cause unemployment when wages are less flexible, like in European countries. Our preference to study the effect of immigration on wages instead of employment is mainly dictated by the availability/accessibility of appropriate data. However, possible reactions of native wages on immigration provide information about employment indirectly since changes in wages and employment are a simultaneous process in the labour market. A possible impact on the unemployment level can not be examined in this study.

This chapter presents the underlying theoretical framework that will be applied to the empirical research presented in the next three chapters on the Netherlands, UK and Norway. Section 3.2 starts with discussing economic theory of immigration. Section 3.3 presents a comprehensive labour market model to examine the impact of immigration on wages across gender and skill categories. In Section 3.4 two empirical methods used to estimate wages elasticities are discussed. Firstly, a simple model is presented which facilitates the estimation of wage elasticities for various types of labour in a single market approach. Secondly, a multi-factor model with translog technology is applied to obtain elasticities of complementarity among various types of native and immigrant labour, from which wage elasticities can be derived. Section 3.5 summarises and concludes.

3.2 Theory

Economic theory interprets immigration flows as a rise in labour supply in the host country. An immigration flow will potentially have consequences on the level and distribution of welfare among natives. The impact of immigration is illustrated by a simple labour market model in Figure 3.1. The labour market is assumed to be competitive. The wage level, $W$, is measured on the vertical axis and employment level, $N$, is given on the horizontal axis. Demand for labour, labelled $D$, is a
decreasing function of wages, and labour supply, labelled $S$, is perfectly inelastic curve (exogenous with respect to wages). Suppose that the labour market is initially in equilibrium at $E$ with wage and employment levels $W$ and $N$ respectively. An immigration flow leads to a supply shock, expressed as an outward-shift in the labour supply curve from $S$ to $S_1$, which leads to a decrease in wages from $W$ to $W_I$ and an increase in employment level from $N$ to $N_I$, given labour demand. This basic model provides us with a powerful analytical tool to test the predictions of theory about the effect of immigration on the labour market outcomes of natives. However, the real world with its numerous imperfections strongly differs from this basic model. Scmidt et al. (1994) and Ortega (2000) show that immigration does not necessarily lead to a decline in wages in the case of equilibrium unemployment when low skilled wages are set by a monopoly union that takes into account both the complementarity of skilled workers and unskilled workers and, when endogenous job creation due to increased immigration is considered. Therefore, predictions based on such a highly simplified model need to be tested for every economy with different institutions that would likely lead to cross-country differences in wage and employment outcomes.

![Figure 3.1. The impact of immigration](image)

In a less flexible labour market, a rise in labour supply due to immigration is not easily adjusted by changes in wages but may result in unemployment for natives. In addition, immigrants themselves face difficulties to participate in the labour market. They frequently end up in unemployment and correspondingly, many immigrants become dependent on governmental welfare support. This typical Eu-
ropean problem is possibly an outcome of the organisation of the welfare state, discriminating behaviour of employers and the motivation of immigration to Europe. Relatively small differences between unemployment benefits and minimum wages may generate small incentives to search for a job for lower qualified workers, which results in high (long-term) unemployment in many European countries. On the other hand, the (un)conscious resistance of employers and other labour market institutions may discourage or prevent immigrant workers to participate.

Immigrants who entered European countries after the formal end of the active recruitment in 1973, are mostly family members of earlier immigrants and recently asylum seekers, in contrast to the traditional immigration countries where an overwhelmingly large part of immigrants are labour immigrants. This type of immigration process generates a high level of economically inactive population. The labour market participation among ethnic minorities is relatively low and their unemployment rate is extremely high in most European countries. The high degree of inactivity coincides with a high dependence degree of immigrants on social welfare programs. On the other hand, many immigrants choose self-employment as an alternative to wage and salary employment, which may have a large multiplier effect on the economy. The effect of immigration is dampened over a long time horizon by the assimilation of their children and grandchildren in the labour market.

On the aggregate level, rising employment as a result of immigration as demonstrated in Figure 3.1, leads to an increase in the national income in the host country. However, gains are not evenly distributed over the native population and immigration redistributes income among factors of production. Who gains and who loses is crucially determined by the skill distribution of immigrants (Borjas 1983, 1987 and 1999). Therefore, the effect of immigration on labour market outcomes should be examined with a more complex model than that of Figure 3.1, allowing for substitutability and complementarity between natives and immigrants in production. Natives and settled immigrants who have productive endowments that are complementary to those of newly entering immigrants will gain while domestic labour who have productive endowments that are easily substituted with the endowments of entering immigrants will lose. The total gains from immigration will be larger, the larger the degree of complementarity in productive endowments
between natives and immigrants.

Furthermore, immigration has an effect not only on employment and wages in the host country but also on the nature of employment, education, training opportunities and incentives. Although all these channels of impact are theoretically and empirically important, we study only the reduced-form impact of immigration on the wages of natives. This partial approach is imposed by both the lack of appropriate data to perform these analyses and the lack of comprehensive theoretical and empirical tools to measure long-term and multiplier (spill-over) effects.

The methodology of estimating the effect of immigration on wages and employment rates of natives and on immigrants themselves is straightforward: correlating both the level and increase in the ratio of immigrants in the regional labour markets with native wages and (un-)employment rates (for the US see Grossman, 1982; Card, 1990; Altonji and Card, 1991; Butcher and Card, 1991; Borjas, 1994; Borjas, Freeman and Katz, 1996 and 1997; Card, 2001 and for Europe, Gang and Rivera-Batiz, 1994; De New and Zimmermann, 1994; Pischke and Velling, 1997; Venturini, 1999). A increasing number of these type of studies finds a modest negative effect on the wages of natives in the US, French and Germany.¹

Nevertheless, this strategy has been criticised in the 1990s (Borjas, 1999; Card, 2000). The criticism is motivated by several conceptual problems. Firstly, the spatial correlation-approach ignores possible responses of the native labour force and capital to an increasing immigration by which the effect of immigration will be diffused over the whole economy. In other words, since natives may move to non-immigrant intensive areas, immigration flows are not necessarily an extension of labour supply in the local labour market. Secondly, in the long run the effect of immigration flows can be diffused across the economy by inter-regional trade and capital movements. Thirdly, a correlation of native wages and immigration flows can be affected by demand shocks that raise wages and attract immigrants. Finally, immigration flows to a particular area can be influenced by ethnic social

networks rather than higher wages in that area.

Butcher and Card (1991), Card and DiNardo (2000) and Kritz and Gurak (2001) find no evidence that increases in immigrant population in specific skill groups lead to the migration of native-born population from the same skill group in the US labour market. They conclude that the small measured effect of immigration on the labour market outcomes of native-born can hardly be explained by systematic outflow of natives from immigrant-intensive areas. Card (2001) examines the effect of immigration flows on the wages of natives and earlier immigrants taking into account responses of native population to immigration flows. Nevertheless, in conformity with the existing studies, he finds a modest adverse effect of immigration on native wages and employment rates. He concludes that the impacts of immigration on local labour markets are mitigated by other adjustment channels, such as endogenous shifts in industry structure. These studies that consider labour markets in the United States are confirmed in the European context by Hunt (1992) and Pischke and Velling (1997) who study the impact of Algerian repatriates on the French labour market in 1962 and the impact of immigration on the West-German labour market, respectively.

As mentioned above, a more sophisticated model is needed to understand the impact of immigration on wages in local labour markets. In the next section, we discuss a comprehensive model used first by Altonji and Card (1991) to analyse the impact of immigration on unskilled wages in an economy with two labour inputs, i.e. unskilled and high skilled workers. We apply their model firstly to an economy in which male and female workers are employed in place of low and high skilled workers, and second, extend the model for a three-factor case, i.e. low, medium and high skilled workers.

3.3 A System of Connected Local Labour Markets

The theoretical framework discussed here is based on a model suggested by Altonji and Card (1991) to study the effect of immigration on the wages of unskilled and skilled labour. We consider an equilibrium concept of two- and three-sector labour markets in which the various types of labour serve as production factors in local labour markets. The demand for each type of labour is assumed to be
a decreasing function of its wages and the prices of capital and other inputs are assumed to be exogenous in the local labour markets (cities). An increase in the supply of a certain category labour is regarded as an outward shift in the supply curve of this labour category, which by itself implies a decrease in the wages and an increase in the employment of this labour-type. An increase in the supply of one labour category affects wages and employment of others through cross-elasticities. Increases in the labour supply may due to both native and immigrant labour. Newly arriving labour also generates demand for locally produced goods so that labour and production markets move to a new equilibrium at new wage and employment rates.

This model is applied in two alternative ways: native labour force is disaggregated by gender and also by three skill categories. Firstly, the host country labour force is assumed to consist of males and females. In an alternative case, the labour force is composed by low, medium and high skilled workers. Consequently, we modify the model of Altonji and Card for these alternative cases in the next two sub-sections. Inferring the impact of immigration in two alternative ways will provide more information about the degree of labour market competition between immigrants and native workers by skill and gender categories. This information may indirectly give indications why various native groups have different attitudes with regard to immigrants and immigration policies. Unfortunately, simultaneously applying the gender and skill decomposition is not feasible due to data limitations.

Altonji and Card (1991) analyse the effect of immigration with a partial equilibrium model for local labour markets. A local economy is assumed to produce one good that is consumed both locally and exported to other cities and import another good. The quantity $Y$ of the local good is produced by a single competitive industry with constant returns to scale using labour inputs and other inputs with exogenously fixed prices.

### 3.3.1 Heterogeneous Labour by Gender

In this section, the model of Altonji and Card is applied to gender categories in place of skill categories. The disaggregation of native labour force into gender
categories may be justified by the following two considerations. Firstly, immigration flows to European countries are mainly composed by male immigrants, i.e. 'guest workers'. The chain migration in the form of family-reunification or -formation has barely changed the gender composition of immigrant labour supply since the participation rate of women among many immigrant groups is relatively low. Secondly, immigration in the 1960s and 1970s has coincided with the rapid increase in female participation rate in European labour markets. These two developments suggest that this male-biased character of immigration would affect especially the wages of native women. However, the impact of immigration depends on substitutability between immigrant males and native females as well as substitution elasticities between male and female workers. Indeed, earlier studies provide some evidence that immigrants may compete with natives who have comparable labour market characteristics (Borjas, 1983 and 1987). Since employment and wage structure differ for men and women, the effect of immigration on the wages of native men and women is considered separately in this section. It must be noted that male and female labour force are strongly interrelated at the household level, suggesting that (fe)male labour supply is not independent from partner’s labour supply. We simply ignore this and treat individual men and women as separate 'households'. A categorisation of labour by education, age or gender is never without complications, implying that interpretations must be taken carefully.

Suppose that a single competitive industry in a local economy produces $Y$ units of goods by a homogenous production function with constant returns to scale employing male and female workers in addition to other inputs. Total industry cost, in units of the imported good, is described by

$$C(w_M, w_F, Y) = Y c(w_M, w_F)$$

where $w_M$ and $w_F$ are real wages of male and female labour respectively, in units of imported good and $c(.)$ is the unit labour cost function, $c' = \frac{\partial C}{\partial w_i}$. The unit price of the local good, in units of the imported good, is denoted $p$. Then, with perfect competition and constant returns, we have $p = c(w_M, w_F)$. Suppose, the total population consists of male and female workers, labelled $L_M$, and $L_F$, respectively. Suppose per capita demand functions of the male and female populations for the local good are $Y_i(.)$, where $i = M, F$. Then local product market equilibrium requires


\[ Y = L_M Y_M (p, w_M) + L_F Y_F (p, w_F) + Y_x (p) \] (3.2)

Where \( Y_x (p) \) is demand from the rest of the economy, i.e. total export to other local economies. Denote per capita labour supply functions of male and female workers by \( S_i(w_i, p), \) where \( i = M, F. \) Then, local labour market equilibrium requires

\[ L_M S_M (w_M, p) = Y c_M (w_M, w_F) \] (3.3)

\[ L_F S_F (w_F, p) = Y c_F (w_M, w_F) \] (3.4)

where \( c_i \) denotes the derivative of the cost function to \( i, \) \( i = M, F. \) In initial equilibrium, the population share of males and females in a local labour market are given as \( M_N = L_M / L \) and \( F_N = L_F / L, \) where \( L = L_M + L_F. \) The local economy is shocked by an immigrant flow \( \Delta I. \) The share of males in the immigration flow is denoted by \( M_I, \) and the share of female workers is \( 1 - M_I. \) The effect of the immigration flow on equilibrium wages can then be found from differentiating equations (3.2), (3.3) and (3.4), using the fact that the price change \( \Delta p / p \) equals the share-weighted sum of proportional changes in factor prices. Altonji and Card assume, for simplicity, that cross-elasticities of output demand \( (\partial Y_i(p, w_i)/\partial w_i) \) and of labour supply \( (\partial S_i(w_i, p)/\partial p) \) are zero\(^2\). Hence, the local product demand of a given gender group is not sensitive to its wage rate (implying that expenditures on the imported good adjust) and a gender category's labour supply is not sensitive to the relative price of the locally produced and the imported good. Then, changes in wage rates have to be solved from the following equations

\[ \lambda_M \left( \frac{M_I}{M_N} \right) \frac{\Delta I}{L} = (\eta_{MM} - \varepsilon_M) \Delta \log w_M + \eta_{MF} \Delta \log w_F \] (3.5)

\[ \lambda_F \left( \frac{1 - M_I}{1 - M_N} \right) \frac{\Delta I}{L} = \eta_{FM} \Delta \log w_M + (\eta_{FF} - \varepsilon_F) \Delta \log w_F \] (3.6)

where \( \varepsilon_i \) is the own-wage labour supply elasticity of gender \( i, \) and \( \eta_{ij} \) are the Marshall-Hicks labour demand elasticities.

\(^2\)These are obviously restrictive assumptions, but remember we deal with local labour markets. Hence, we assume that a local wage increase does not raise demand for the local product and that a price increase of the local product does not affect local labour supply.
\[ \eta_{ij} = \theta_i(\sigma_{ij} - \psi) \]  

where \( \theta_j \) is the wage bill of group \( j \) relative to output value, \( \sigma_{ij} \) is the partial elasticity of substitution\(^3\) of gender group \( i \) with respect to group \( j \) and \( \psi \) is the elasticity of local output to its relative price \( p \) (a weighted average of elasticities of local consumers and export). The \( \lambda_i \) are defined as\(^4\)

\[ \lambda_M = (1 - k_1) \frac{Y_M}{Y} + \frac{Y_e}{Y}, k_1 = \frac{M_N(1 - M_I)}{M_I(1 - M_N)} \]  

\[ \lambda_F = (1 - k_2) \frac{Y_F}{Y} + \frac{Y_e}{Y}, k_2 = \frac{M_I(1 - M_N)}{M_N(1 - M_I)} \]  

The \( \lambda_i \)'s range numbers between zero and one and represent the gross changes in labour supply resulting from for the net increases in demand for goods generated by the inflow of new immigrants.

In equations (3.5) and (3.6), the right-hand side gives the response of gender group's local excess demand (i.e. total demand change in labour demand net of local supply) to the changes in the wage rates. The left-hand side is the new supply of a gender group made up of immigrants, relative to the existing population and adjusted for their share in product demand. Thus, in (3.5), the new male supply is \( M_I \Delta I \), which is related to the existing male population \( M_N \). \( \lambda_M \) is the adjustment for product demand composition with respect to the gender groups (natives and immigrants have the same demand function for the same gender categories but the change in gender composition may affect the product demand composition by gender groups). If the gender composition of the immigrants and natives is identical \( (M_N = M_I) \), \( k_1 = k_2 = 1 \). Then, the impact of the supply shock is restricted to the share of export demand, \( \lambda_M = \lambda_F = \frac{Y_e}{Y} \), from equations (3.8) and (3.9). Hence, if all output is consumed locally, the \( \lambda \)'s are equal to zero,

\(^3\)If there are only two inputs, the partial elasticity of substitution between two inputs \( (\sigma) \) is dual to the Hicksian partial elasticity of complementarity \( (c) \), \( \sigma = 1/c \) from the duality of production and cost functions. If there are more than two inputs, then \( \sigma \neq 1/c \) (Hamermesh, 1993). See appendix for discussion about the relationship between the elasticities of substitution and complementarity.

\(^4\)They are slightly rewritten from Altonji and Card's specification to bring out the role of the export share in demand.
\( \lambda_M = \lambda_F = 0 \), and so are the changes in the wage rates. Immigration does not affect gender composition or product demand composition (i.e. gender intensity of total product demand) and wage rates are unaffected by immigration. This is due to constant returns-to-scale: the local economy simply blows up in size but all proportions are maintained. If the export share is not equal to zero, this proportionality is disturbed: the factor endowment of the local economy grows relative to the national economy and the output price (i.e. the terms of trade) will fall. This entails a fall in wage rates.

Using equations (3.5) and (3.6), changes in male and female wage rates can be related to changes in the share of immigrants in the local population. These changes can be inferred in two alternative ways:

1) **single market case**: measuring the effect of immigration on the wages of native males and females, assuming that demand for male and female labour inputs are independent from each other in production,

2) **spill-over case**: measuring the effect of immigration on the wages of native labour force, when the demand and wages for these labour inputs are interrelated.

In the **single market** case, the cross-wage elasticities of labour demand are neglected. The demand for a gender group of labour is assumed to be independent of the wage rate of another gender group, i.e. \( \eta_{MF} = \eta_{FM} = 0 \). In this case, solving equations (3.5) and (3.6) yields changes in log wages of each gender

\[
\Delta \log w_M = \gamma_M \left( \frac{\Delta I}{L} \right) \tag{3.10}
\]

\[
\Delta \log w_F = \gamma_F \left( \frac{\Delta I}{L} \right) \tag{3.11}
\]

where \( \gamma_M \), and \( \gamma_F \) are in fact equilibrium restoring wage elasticities: the relative change in the male and female wages if the population grows by 1 percent due to immigration. They are equal to

\[
\gamma_M = \frac{-\lambda_M}{(\varepsilon_M - \eta_{MM})} \left( \frac{M_I}{M_N} \right) \tag{3.12}
\]
Clearly, with positive supply elasticities, \( \varepsilon_i \), the supply shock affects both wages negatively. Suppose the gender composition of immigration equals the gender composition of the native labour force. Then, \( \lambda_M = \lambda_F = \frac{\lambda}{2} \) (note that \( k_1 = k_2 = 1 \) from equations (3.8) and (3.9)). Suppose, \( \frac{\lambda}{2} = 1 \), all production is exported, or, stated otherwise, there are no demand effects from local labour. Then, \( \gamma_i \) reduces to \( \frac{-1}{(\varepsilon_i - \eta_{\text{FF}})} \), for \( i = M, F \), which is the standard result of comparative statics in a single market. With zero supply elasticities \( \varepsilon_i \), we have the formalisation of the model in Figure 3.1. Alternatively, if \( Y_e = 0 \), the proportionality from constant-returns-to-scale eliminates all effects so that wages do not change at all.

Equations (3.10) and (3.11) represent the effect of the proportional increase of immigrant labour on the wages of natives. \( \gamma_i \) can be interpreted as the resulting wage effect for group \( i \) from a supply shock due to immigration. The impact of immigration, in terms of the coefficients \( \gamma_i \), may be measured by the reduced-form estimation of separate earnings functions for native male and female workers in which the percentage of immigrants in the local labour market is included in the earnings functions, as presented in Section 3.4.1.

Secondly, in a more realistic spill-over case, both linear supply and demand curves of labour are assumed to be elastic, and further, we assume that demand for a certain gender is related to wages of the other gender, in addition to its own wage rate, i.e. the cross-elasticities of factor demand differ from zero (\( \eta_{ij} \neq 0 \)). In this case, changes in male and female wage rates can be related to changes in the share of immigrants in the local population. Solving equations 3.5 and 3.6 for the changes in the log wage rates \( \Delta \log w_M \) and \( \Delta \log w_F \) yields

\[
\Delta \log w_M = \gamma^*_M \left( \frac{\Delta I}{L} \right) 
\]

\[
\Delta \log w_F = \gamma^*_F \left( \frac{\Delta I}{L} \right) 
\]

where the elasticities \( \gamma^*_M \), and \( \gamma^*_F \) are equal to

\[
\gamma^*_M = \frac{\lambda_F \left( 1 - M_F \right)}{\eta_{\text{FF}} - \varepsilon_F} - \frac{\eta_{\text{MF}}}{\eta_{\text{FF}} - \varepsilon_F} = \frac{\lambda_M M_M}{\eta_{\text{MF}} M_N - \left( \eta_{\text{MM}} - \varepsilon_M \right)}
\]
The Effect of Immigration on Earnings: Theory

Equations (3.16) and (3.17) are obvious generalisations: setting $\eta_{MF} = \eta_{FM} = 0$ brings back equations (3.12) and (3.13). The twisting of scale proportionality through exports is also maintained: when $M_I = M_N$ and $Y_e = 0$, wages do not respond at all to immigration, whereas with $Y_e/Y = 1$, the standard single-market comparative static result reappears. Most interesting is of course the effect on the signs of the relationships. In the single market case, immigration always reduces wages. However, here substitution effects can potentially reverse the sign. For illustration, suppose $M_I = M_N$ and $Y_e/Y = 1$, then

$$\gamma_F^* = \frac{\eta_{ME} M_I}{\eta_{MM} M_N M_M} - \frac{\eta_{FE} M_I}{\eta_{FM} M_M} = (\eta_{FF} - \varepsilon_F)$$

Equations (3.16) and (3.17) are obvious generalisations: setting $\eta_{MF} = \eta_{FM} = 0$ brings back equations (3.12) and (3.13). The twisting of scale proportionality through exports is also maintained: when $M_I = M_N$ and $Y_e = 0$, wages do not respond at all to immigration, whereas with $Y_e/Y = 1$, the standard single-market comparative static result reappears. Most interesting is of course the effect on the signs of the relationships. In the single market case, immigration always reduces wages. However, here substitution effects can potentially reverse the sign. For illustration, suppose $M_I = M_N$ and $Y_e/Y = 1$, then

$$\gamma_F^* = \frac{\eta_{ME} M_I}{\eta_{MM} M_M} - \frac{\eta_{FE} M_I}{\eta_{FM} M_M} = (\eta_{FF} - \varepsilon_F)$$

Equations (3.16) and (3.17) are obvious generalisations: setting $\eta_{MF} = \eta_{FM} = 0$ brings back equations (3.12) and (3.13). The twisting of scale proportionality through exports is also maintained: when $M_I = M_N$ and $Y_e = 0$, wages do not respond at all to immigration, whereas with $Y_e/Y = 1$, the standard single-market comparative static result reappears. Most interesting is of course the effect on the signs of the relationships. In the single market case, immigration always reduces wages. However, here substitution effects can potentially reverse the sign. For illustration, suppose $M_I = M_N$ and $Y_e/Y = 1$, then

$$\gamma_F^* = \frac{\eta_{ME} M_I}{\eta_{MM} M_M} - \frac{\eta_{FE} M_I}{\eta_{FM} M_M} = (\eta_{FF} - \varepsilon_F)$$

In case of complements ($\eta_{MF}, \eta_{FM} < 0$), $\gamma_M^* < 0$ just as before. Yet with strong enough substitutability ($\eta_{MF}, \eta_{FM} > 0$), $\gamma_M^* < 0$ is possible. Note, this requires $\eta_{MF} > \eta_{FF} - \varepsilon_F$.

For regression purposes, the simple model (3.12)-(3.13) and the spill-over case (3.16)-(3.17) can not be distinguished. Identification requires more information than regressing wages on supply shifts. However, using some calculations pertaining to substitution elasticities, shares of wage bill, product demand and employment, we can perform some calculations to illustrate the magnitude of the effect predicted by the two models. In what follows, the impact of an immigration flow of 10% of native population is predicted along four scenarios, using equations (3.14) and (3.15). The substitution elasticities between males and females are assumed to be .5 and the share of females in wage bill and product demand is assumed to be .4 while their employment share is equal to that of males, $M_N = .5$. Additionally, 30% of the local output is exported. The first two scenarios concern a male-biased immigration flow while scenarios III and IV a balanced immigration flow. Both main immigration scenarios are simulated for low and high value for supply elasticities ($\varepsilon_M = \varepsilon_F = .1$ in scenarios I and III and $\varepsilon_M = \varepsilon_F = .5$ in scenarios II and IV).

The results of the scenario predictions are displayed in Table 3.1. The outcomes of scenario I and II indicate that a male-biased immigration flow has a negative
impact on the wages of native males and a positive impact on the wages of native females. As the supply elasticities of labour becomes larger, this impact becomes smaller (scenario II). A balanced immigration flow leads to different wage effects depending on the assumed supply elasticities. Note that this impact is due to the export demand. Without export demand, the impact of immigration is eliminated by scale proportionality. When the supply elasticities are low, the impact on the female wages is negative (scenario III). If the supply elasticities are high, the impact on the female wages is positive (scenario IV).

These outcomes imply that also supply elasticities as well as the substitution elasticities play an important role in determining the impact of immigration. High supply elasticities cushion immigration by reducing native labour supply when immigration reduces wages.

Table 3.1. Predicted effect of immigration

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$\gamma_M^*$</th>
<th>$\gamma_F^*$</th>
<th>$\Delta \log w_M$</th>
<th>$\Delta \log w_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario I</td>
<td>-.83</td>
<td>1.04</td>
<td>-.083</td>
<td>.104</td>
</tr>
<tr>
<td>Scenario II</td>
<td>-.18</td>
<td>.45</td>
<td>-.018</td>
<td>.045</td>
</tr>
<tr>
<td>Scenario III</td>
<td>.42</td>
<td>-.15</td>
<td>.042</td>
<td>-.015</td>
</tr>
<tr>
<td>Scenario IV</td>
<td>.28</td>
<td>.039</td>
<td>.028</td>
<td>.003</td>
</tr>
</tbody>
</table>

**SCENARIO I**: *male-biased immigration with low supply elasticities.* Males are overrepresented in immigration, $M_I = .6 > M_N = .5$ and labour supply elasticities are $\varepsilon_M = \varepsilon_F = .1$

**SCENARIO II**: *male-biased immigration with high supply elasticities.* Males are overrepresented in immigration, $M_I = .6 > M_N = .5$ and labour supply elasticities are $\varepsilon_M = \varepsilon_F = .5$

**SCENARIO III**: *gender-neutral immigration with low supply elasticities,* $M_I = M_N = .5$ and labour supply elasticities are relatively low, $\varepsilon_M = \varepsilon_F = .1$

**SCENARIO IV**: *gender-neutral immigration with high supply elasticities.* $M_I = M_N = .5$ and labour supply elasticities are relatively high, $\varepsilon_M = \varepsilon_F = .5$
3.3.2 Heterogenous Labour by Skill Level

Alternatively, we extend the model of Altonji and Card by adding medium skilled labour. Now, we consider an equilibrium concept of a three-sector labour market in which low, medium and high skilled labour are production factors in local labour markets. Again, a homogenous production function is assumed. The technology is characterised by constant returns to scale. Production is realised by employing low, medium and high skilled workers given other inputs. The total labour force, $L$, consists of low skilled $L_U$, medium skilled, $L_C$ and high skilled, $L_H$; $L = L_U + L_C + L_H$. The respective proportions of low, medium and high skilled labour are $U_N = L_U/L$, $C_N = L_C/L$ and $H_N = L_H/L$ with $U_N + C_N + H_N = 1$.

The effects of the immigration flow on the wages of native workers can be defined in an identical way as before (see appendix A.2). Suppose that an immigrant flow of size $\Delta I$ occurs: in an initial equilibrium, a fraction of immigrants, $U_I$, is low skilled, $C_I$, is medium skilled and the rest is high skilled workers, $H_I$. The skill distribution of this immigration flow is given as low skilled, $U_N$, medium skilled $C_N$ and high skilled workers $H_N$. The proportional changes in wage rates of each labour type now must satisfy

$$\lambda_U \left( \frac{U_I}{U_N} \right) \frac{\Delta I}{L} = (\eta_{U_U} - \varepsilon_U) \Delta \log w_U + \eta_{U_C} \Delta \log w_C + \eta_{U_H} \Delta \log w_H \quad (3.19)$$

$$\lambda_C \left( \frac{C_I}{C_N} \right) \frac{\Delta I}{L} = \eta_{CU} \Delta \log w_U + (\eta_{CC} - \varepsilon_C) \Delta \log w_C + \eta_{CH} \Delta \log w_H \quad (3.20)$$

$$\lambda_H \left( \frac{1 - U_I - C_I}{1 - U_N - C_N} \right) \frac{\Delta I}{L} = \eta_{HU} \Delta \log w_U + \eta_{HC} \Delta \log w_C + (\eta_{HH} - \varepsilon_H) \Delta \log w_H \quad (3.21)$$

where, as in (3.7), $\eta_{ij}$ is the elasticity of labour demand for skill group $i$ with respect to the wage of group $j$, $\varepsilon_i$ indicates the elasticity of labour supply of skill group $i$ and $\lambda_i$ has same function and is defined over the same range as before, $0 < \lambda_i < 1$. The adjustment coefficients may be defined for the three factor cases as

$^5$As the subscript $m$ is no longer available, we use $c$, for 'central' skill level.
\[\lambda_U = (1 - k_1) \frac{Y_C + Y_H}{Y} + \frac{Y_e}{Y}, \quad k_1 = \frac{U_N(1 - U_I) + C_I H_I}{U_I(1 - U_N) + C_I H_I} \quad (3.22)\]

\[\lambda_C = (1 - k_2) \frac{Y_U + Y_H}{Y} + \frac{Y_e}{Y}, \quad k_2 = \frac{C_N(1 - C_I) + U_I H_I}{C_I(1 - C_N) + U_I H_I} \quad (3.23)\]

\[\lambda_H = (1 - k_3) \frac{Y_U + Y_C}{Y} + \frac{Y_e}{Y}, \quad k_3 = \frac{H_N(1 - H_I) + U_I C_I}{H_I(1 - H_N) + U_I C_I} \quad (3.24)\]

where \(H_N\) is the share of high skilled labour in the local labour market, i.e. \(H_N = 1 - U_N - C_N\) and \(H_I\) is the share of high skilled labour in the new immigration flow, i.e. \(H_I = 1 - U_I - C_I\).

The left-hand side of the equations (3.19)-(3.21) indicate the effective proportional increase in the supply of labour for the respective skill groups as a result of the immigration flow. The right-hand side gives the response of skill groups' local excess demand to the changes in the wage rates.

Suppose that some part of output is consumed locally while a fraction of output is exported, \(\frac{Y_e}{Y}\), and the skill composition of the new immigrants is identical to the skill composition of workers in the native population. Then, linear homogeneity of the production function implies that relative wages of skill groups will not change as a result of immigration so that \(\lambda_U = \lambda_C = \lambda_H = \frac{Y_e}{Y}\). If workers in the immigration flow are less (more) skilled than the skill composition of the existing population, i.e. \(U_I > U_N, C_I = C_N\) and \(H_I < H_N\), then \(\lambda_U > \lambda_C > \frac{Y_e}{Y} > \lambda_H\), immigration increases (decreases) the skilled (unskilled) wage. If the immigration flow is higher skilled than the existing population, i.e. \(U_I < U_N, C_I = C_N\) and \(H_I > H_N\), then the opposite holds \(\lambda_U < \frac{Y_e}{Y} < \lambda_C < \lambda_H\).

Again, two approaches are applied to examine the impact of immigration, as the former section. Suppose, for the case of an isolated single market, that the wage and employment of heterogeneous labour inputs are unrelated. Therefore, the demand for a certain skill group of labour is independent of the wage rate of another skill group, i.e. cross-demand elasticities are zero (\(\eta_{ij} = 0\) for \(i = U, C, H\) and \(i \neq j\)). Then, the solution in terms of changes in log wages of each skill group is

\[\Delta \log w_U = \gamma_U \left(\frac{\Delta I}{L}\right) \quad (3.25)\]
\[ \Delta \log w_C = \gamma_C \left( \frac{\Delta I}{L} \right) \]  

\[ \Delta \log w_H = \gamma_H \left( \frac{\Delta I}{L} \right) \]  

The coefficients \( \gamma_i \) are equilibrium restoring wage elasticities of labour from the corresponding skill levels. They are equal to

\[ \gamma_U = -\frac{\lambda_U}{(\varepsilon_U - \eta_{UU})} \left( \frac{U_I}{U_N} \right) \]  

\[ \gamma_C = -\frac{\lambda_C}{(\varepsilon_C - \eta_{CC})} \left( \frac{C_I}{C_N} \right) \]  

\[ \gamma_H = -\frac{\lambda_H}{(\varepsilon_H - \eta_{HH})} \left( \frac{1 - U_I - C_I}{1 - U_N - C_N} \right) \]  

These wage elasticities can be approximated by the estimation of earnings functions in which percentage of immigrants are included, as suggested in section 3.1.

It is immediately clear that the three-factor skill model is a straight generalisation of the two-factor gender model. All wage responses to the immigration shock are negative. Immigration with the same skill composition as the native labour force has no effects if exports are zero and has the standard comparative static effect if all output is exported. Suppose that the new immigrants are all low skilled, \( U_I = 1, C_I = 0 \) and \( H_I = 0^6 \). The effect of immigration on the log wages of low skilled is then expressed as

\[ \gamma_U = -\frac{1}{\varepsilon_U - \eta_{UU}} \frac{1}{U_N} \]  

This is in fact the standard *single market* comparative static result. Wages of other skill groups are unaffected.

Similarly, if all new immigrants are medium skilled, the effect of an increase in immigration with respect to medium skilled labour on the wages can be expressed as

\[ 6\text{Note that } \lambda_C \text{ and } \lambda_h \text{ are undefined at the same time since } k_2 \text{ and } k_3 \text{ are undefined. This can be generalised as follows: when immigration is composed of only one type of workers, the wages of other workers remain undefined. However, cross-effects are assumed to be zero here, implying that other wages can not be affected. Thus, definitions of other wage changes are not needed.} \]
\[ \gamma_C = -\frac{1}{\varepsilon_C - \eta_{CC} U_C} \]  \hspace{1cm} (3.32)

In another extreme case, should the whole immigration flow be composed of high skilled workers, the effect on the wages of high skilled natives is given by

\[ \gamma_H = -\frac{1}{\varepsilon_H - \eta_{HH} U_H} \]  \hspace{1cm} (3.33)

Let’s now construct the general case with spill-overs (non-zero cross-elasticities). If demand for labour for a skill group is related to wages of other skill groups, in addition to its own wage rate, i.e. \( \eta_{jh} \neq 0 \), the reduced-form impact of immigration on the wages follows from solving the equation system (3.19)-(3.21) for \( \Delta \log w_U, \Delta \log w_C \) and \( \Delta \log w_H \). The solutions are very complicated equations, but in matrix notation they are quite simple. Write equations (3.19)-(3.21) as

\[ Bd = (N - E) W \]  \hspace{1cm} (3.34)

where

\[ d = \frac{\Delta l}{L}, \text{ a scalar} \]

\[ B = \lambda_K \frac{K_L}{K_N}, \quad K = U, C, H, \text{ a column vector} \]

\[ N = [\eta_{ij}] \text{, a square matrix} \]

\[ E = \begin{bmatrix} \varepsilon_U & 0 & 0 \\ 0 & \varepsilon_C & 0 \\ 0 & 0 & \varepsilon_H \end{bmatrix} \]

and \( W \) is a column vector with log wage changes \( \Delta \log w_i \).

We can solve equation (34) as

\[ W = \gamma^* d \]  \hspace{1cm} (3.35)

where \( \gamma^* = (N - E)^{-1} B. \)
Equation system (3.35) indicates the effect of immigration on the wages of low, medium and high skilled natives. To get some idea of the change in $\gamma^*$, related to demand and supply parameters as well as skill distribution of immigration, the impact of an immigration flow as 10 percent of total labour force is simulated using equation system (3.35), assuming some values for parameters in $\gamma^*$. After experimentation with various values for parameters and sensitivity analysis, four scenarios are formulated to assess the impact of balanced and skill-biased immigration flows. Note that extreme values are assumed for some parameters which results in extraordinary changes in the wages (especially in Scenario II). Moreover, the simulations show the impact of immigration shock without allowing for endogenous adjustment, resulting in enormous wage changes. As implicitly assumed, labour within each skill group is homogeneous. It is also assumed that the product demand of skill groups is equal to their share in the total wage bill, and 30% of the local product is exported. The results are displayed in Table 3.2. The first four scenarios depict an immigration flow characterised by mainly low skilled workers, relative to the skill composition of natives, $U_I = .4 > U_N = .2$. It is notable that a less skilled immigration flow has a substantial impact on the wages, presented by scenarios I-VI. For scenarios I, III, V and VI, the cross-elasticities of substitution between the labour types are assumed to be positive, i.e. all labour inputs are substitutes\(^7\). The impact is then straightforward: immigration has an adverse effect on less skilled wages and a positive effect on high skilled wages. Scenarios II and IV show a case in which low and high skilled workers are complements, i.e. the elasticity of substitution between low and high skilled labour is negative. In this case, immigration has a diverse impact on the wages of the various groups, depending on their respective labour supply elasticities. The impact is positive on low skilled wages and negative on high skilled wages (scenario II), which is exactly the opposite outcome of scenario I. As labour supply is less elastic (scenario IV), the impact is strengthened with respect to scenario III. The last two scenarios give the outcomes of a balanced immigration flow (the skill composition of immigration and natives is identical). Such an immigration flow leads to a small adverse effect on the low skilled wages and positive effect on the medium and high skilled wages. Note that immigration still has a small effect on the wages, despite

\(^7\) All inputs may be substitutes but not complements. If there are $n$ inputs, there must be at least $n - 1$ substitutes (see appendix).
the skill composition is balanced. This is due to the export demand. Without export demand, the impact would be diffused throughout the economy via the cross-elasticities.

Table 3.2. Predicted effect of immigration

\[
\frac{\Delta L}{L} = .1, \sigma_i = .25, \theta = .5, \nu = -2, \frac{\nu_i}{\nu} = .3
\]

<table>
<thead>
<tr>
<th>Scenario</th>
<th>( \gamma_u^* )</th>
<th>( \gamma_c^* )</th>
<th>( \gamma_H^* )</th>
<th>( \Delta \log w_U )</th>
<th>( \Delta \log w_C )</th>
<th>( \Delta \log w_H )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>5.00</td>
<td>2.40</td>
<td>3.40</td>
<td>.500</td>
<td>.240</td>
<td>.340</td>
</tr>
<tr>
<td>II</td>
<td>5.95</td>
<td>2.24</td>
<td>7.40</td>
<td>.595</td>
<td>.224</td>
<td>.740</td>
</tr>
<tr>
<td>III</td>
<td>-1.66</td>
<td>1.16</td>
<td>1.35</td>
<td>-1.16</td>
<td>.116</td>
<td>.135</td>
</tr>
<tr>
<td>IV</td>
<td>-3.68</td>
<td>.70</td>
<td>3.58</td>
<td>-3.68</td>
<td>.070</td>
<td>.358</td>
</tr>
<tr>
<td>V</td>
<td>-4.1</td>
<td>.70</td>
<td>.11</td>
<td>-0.41</td>
<td>.070</td>
<td>.011</td>
</tr>
<tr>
<td>VI</td>
<td>-0.08</td>
<td>.43</td>
<td>.11</td>
<td>-0.008</td>
<td>.043</td>
<td>.011</td>
</tr>
</tbody>
</table>

**Scenario I:** low skilled people are overrepresented in immigration while the share of medium skilled remains the same, \( U_I = .4 > U_N = .2 \), \( C_I = C_N = .5 \), \( H_I = .1 < H_N = .3 \). Labour supply elasticities are relatively low, \( \varepsilon_i = 1 \) and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \).

**Scenario II:** low skilled people are overrepresented in immigration while the share of medium skilled remains the same, \( U_I = .4 > U_N = .2 \), \( C_I = C_N = .5 \), \( H_I = .1 < H_N = .3 \). Labour supply elasticities are relatively low, \( \varepsilon_i = 1 \) and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \) (low and high skilled labour are complements) but \( \sigma_{ij} = \sigma_{ji} = .5 \).

**Scenario III:** low skilled people are overrepresented in immigration while the share of medium skilled remains the same, \( U_I = .4 > U_N = .2 \), \( C_I = C_N = .5 \), \( H_I = .1 < H_N = .3 \). Labour supply elasticities are relatively high, \( \varepsilon_i = 5.5 \) and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \).

**Scenario IV:** low skilled people are overrepresented in immigration while the share of medium skilled remains the same, \( U_I = .4 > U_N = .2 \), \( C_I = C_N = .5 \), \( H_I = .1 < H_N = .3 \). Labour supply elasticities are relatively high, \( \varepsilon_i = .5 \) and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \) (low and high skilled labour are complements) but \( \sigma_{ij} = \sigma_{ji} = .75 \).
SCENARIO V: the skill distribution of immigrants and natives are identical, \( U_I = U_N = .2, C_I = C_N = .5, H_I = H_N = .3 \), labour supply elasticities are low, \( \varepsilon_i = .1 \), and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \).

SCENARIO IV: the skill distribution of immigrants and natives are identical, \( U_I = U_N = .2, C_I = C_N = .5, H_I = H_N = .3 \) and labour supply elasticities are high, \( \varepsilon_i = .5 \) and substitution elasticities are \( \sigma_{ij} = \sigma_{ji} = .75 \).

Immediately, one notices that the outcomes of the model vary strongly with parameter values included in \( \gamma^* \). It is extremely difficult to predict possible results of immigration flows since the impact is not only determined by the sign and extent of substitution elasticities, but also by all other parameters in gamma. Given these sensitivities, it is a hazardous venture to assess labour market effects from information on underlying elasticities.

The Altonji and Card model has two important limitations: Firstly, it is assumed that the existing native population is immobile between local labour markets, suggesting that the short term impact is concerned. The findings of Card (2001) suggest that this assumption can be interpreted as reasonable since the reaction of local population on immigration is small in the US where the mobility of population is even high, compared to Europe. Secondly, the model assumes that local labour markets are perfectly competitive, suggesting that new immigrants will be absorbed by wage adjustments in the local labour markets. However, if there are serious barriers to wage adjustment, such as minimum wage legislation and welfare benefits, the impact of immigration is expected to be manifested in the (un)employment outcomes. European labour markets are supposed to be less flexible relative to the US. However, the impact of immigration on the native employment outcomes may be small or absent since the unemployment gap between natives and immigrants is extremely high. The impact, if any, could be on the (un)employment of immigrants that arrived earlier. These arguments suggest that the distributional effect of immigration may occur through wages thus making the assumptions more plausible.

The next section presents two alternative econometric specifications to estimate the wage elasticities (\( \gamma \) and \( \gamma^* \)): 1) estimating reduced-form impact of migration in a model where both labour supply and demand may be elastic; 2) estimating elasticities of complementarity and demand elasticities for a model in
which immigration is seen as an exogenous supply shock. These specifications will be empirically applied to data from the Netherlands, UK and Norway in the next three chapters.

3.4 Estimating Wage Elasticities

Empirically, there are two ways to estimate the wage elasticities: partial and general equilibrium approaches. Both have their own (dis)advantages by which the two approaches become complementary to each other. Since data in the case of the Netherlands is only available for 1997, the focus will be on a theoretical framework that enables us to study the cross-sectional impact of immigration in the Netherlands, the UK and Norway. The next section presents the theoretical framework that will be applied to empirical investigation in Chapters 4, 5 and 6. First, we analyse partial interactions between different types of labour by direct modelling and later take into account simultaneous interactions among labour inputs.

3.4.1 Reduced Form

It is well-known that immigrants are highly concentrated in certain geographical areas and/or industrial sectors, particularly in large cities and labour-intensive industries. The models discussed in Section 3.3 predict that the wages of natives with a productive endowment that substitutes for immigrants will be depressed in immigrant-intensive areas/industries in comparison to non-immigrant intensive cities or industries. In this case, wages elasticities labelled as $\gamma$ in Sections 3.3.1 and 3.3.2 will be negative. Alternatively, the wages of workers whose endowments are complementary to immigrants will increase, suggesting positive wage elasticities $\gamma$ for complementary native labour. This cross-market approach is empirically tested in a very simple way by estimating earnings functions.

However, the lack of appropriate data makes the estimation of the model suggested in Section 3.3 impossible. Since we use cross-section data from 1997, it is impossible to estimate the model in which an increase in log wages is related to an increase in immigrant population in local labour markets, given by equations (3.10) and (3.11) and (3.25-3.27). Recalling the function
The effect of immigration on earnings: theory

We are able to estimate earnings functions having the form

\[ \Delta \log w = \gamma \left( \frac{I}{L} \right) \]  \hspace{1cm} (3.36)

We are able to estimate earnings functions having the form

\[ \log w = \gamma \left( \frac{I}{L} \right) \]  \hspace{1cm} (3.37)

which is the same as

\[ \Delta \log w = \gamma \Delta \left( \frac{I}{L} \right) \]  \hspace{1cm} (3.38)

Changes in wages can be related to changes in the shares of immigrants in the local population \( \frac{I}{L} \), by using the equality \( \Delta \left( \frac{I}{L} \right) = (1 - \frac{I}{L}) \Delta \left( \frac{I}{L} \right) \) (Altonji and Card, 1991).

Substituting for \( \Delta \left( \frac{I}{L} \right) \) in (3.38), we can write earnings function

\[ \Delta \log w = \gamma \frac{1}{(1 - \frac{I}{L})} \Delta \left( \frac{I}{L} \right) \]  \hspace{1cm} (3.39)

Equation (3.38) is the same as

\[ \log w = \gamma \frac{1}{(1 - \frac{I}{L})} \left( \frac{I}{L} \right) \]  \hspace{1cm} (3.40)

As the share of immigrants \( \frac{I}{L} \) goes to zero, \( \frac{1}{(1 - \frac{I}{L})} \) approaches unity, suggesting that the estimation of log wages will provide the parameter \( \gamma \) which is the wage elasticity with respect to the portion of immigrants in the local population.

We estimate the wage elasticities by the estimation of earnings functions. Earnings functions to estimate are given in the following form:

\[ \log w_{ijr} = \beta_{ijr} X_{ijr} + \delta_{jr} P_{kr} + u_{ijr} \]  \hspace{1cm} (3.41)

\( w_{ijr} \) is the earnings of person \( i \) belonging to skill group \( j \) in area \( r \). \( X_{ijr} \) is a vector of potential explanatory variables such as those related to human capital, relevant individual and family characteristics, and other control variables for city, industry and job characteristics. \( P_{kr} \) is a vector of the share of immigrants from group \( k \) in local labour market \( r \), i.e. \( P_{kr} = \frac{I_{kr}}{L_r} \times 100 \). The estimation of this type earnings function for separate native groups yields possible effects of immigration on the
wages of groups concerned. The coefficient \( \delta \) is the marginal effect of \( P \) on the \( w \), and is an approximation of the \( \delta \)'s defined in Sections 3.3.1 and 3.3.2. A negative sign of the gamma coefficient indicates a substitution relationship between native workers and immigrants while a positive sign shows the complementarity. This effect can be written as the derivative of log \( w \) with respect to \( P \),

\[
\frac{d(\ln w)}{dP} = \frac{dw}{w} \frac{1}{dP} = \frac{dw}{dP} \frac{1}{w} \tag{3.42}
\]

Differentiating the earnings function with respect to \( P \)

\[
\frac{dw}{dP} \frac{1}{w} = \delta \tag{3.43}
\]

The marginal effect can be given by rewriting of this equation\(^8\)

\[
\frac{dw}{dP} = \delta w \tag{3.44}
\]

Since the wage elasticity of labour is given in general terms by the formula \( \gamma = \frac{\frac{dw}{w}}{\frac{dP}{P} w} \) the wage elasticity of an increase in labour supply in a local labour market can be written by multiplying both sides of equation (3.44) with \( P/w \)

\[
\gamma = \delta P \tag{3.45}
\]

This equation is the second term on the right-hand side of the earnings function given by equation (3.41). The wage elasticity \( \gamma \) is the same as the coefficients \( \gamma \) and/or \( \gamma^* \) in Section 3.3, (except for the base correction). Separate parameters in these coefficients can not be identified. Estimation of this simple earnings function by OLS technique will yield gamma coefficients for labour inputs distinguished. Estimating the effect of immigrants by using micro surveys allows us to control for differences in individual fixed effects that may determine productive skills in local labour markets. The individual fixed effects may be approximated by a

---

\(^8\)The coefficient \( \gamma \) is, in fact, the semi-elasticity here (\( \gamma = \frac{d\ln w}{dP} \)). We prefer to use the full-wage elasticities because a one percentage point increase in \( P \) gives a smaller proportional change when initial \( P \) is larger. For example, if \( P = 10 \), then a 1 percentage point increase means a 10% increase with respect to the initial value. Alternatively, if \( P = 5 \), then a 1 percentage point increase means a 20% increase. This skewness is eliminated when the full elasticity is used.
set of socioeconomic variables that are available in data. Earnings functions for various types of labour are estimated separately, implying that possible correlations between error terms are neglected.

Since we have only single cross-section data (in contrast to the traditional immigration countries), we are only modestly able to measure a reduced-form effect of immigration on native wages for the three countries in 1997. This implies that long-term (dynamic) effects cannot be measured. This methodology is easy to perform empirically. No additional restrictions are imposed on the model, except the usual statistical properties of stochastic error terms in the earnings functions. The most important disadvantage of this approach is that the separate parameters in $\delta$ (the individual elasticities) cannot be identified. The argument that native population can react to increasing immigration in local labour markets and as a result, the estimations will be unreliable, is less relevant for the Netherlands. The extremely small geographical size of the Netherlands and short distance between home and work lowers the probability that native population moves away from the local labour markets that are increasingly populated by new immigrants.

The estimation of earnings functions by OLS method regards all explanatory variables as exogenous, suggesting that these variables are uncorrelated with the error term. This estimation method neglects possible endogeneity between these variables. In other words, OLS views the concentration of immigrants in local labour markets as unaffected by wages, i.e. relative labour supplies as perfectly inelastic. In fact, wage differentials across labour markets can generate an incentive for workers to move from the labour markets awarding less to the labour markets providing higher wages. However, the presence of mobility costs and imperfect information suggest that the perfect equalisation of wage differentials cannot be realised in the short-run. If this so-called endogenity problem is present, OLS will not yield unbiased and consistent parameter estimators. One technique, which can solve this problem and other sorts of measurement problems, is instrumental variable estimation (IV). Since the coefficient of the percentage of immigrants in local labour markets, $\delta$, is of particular interest, we need one or more variables (instruments) that are highly correlated with the concentration of immigrants, $P$, but uncorrelated with the error term, $u$, to take into account the endogeneity of immigrant-concentration. The problem in practice is that such a variable is hard
to find. Moreover, the existence of the endogeneity is not always confirmed by the application of IV-technique to estimate the impact of immigration on native earnings. De New and Zimmermann (1994) find a clear evidence for endogeneity in the German Household Panel Survey (GSOEP) and validity of IV estimations, while Borjas (1986 and 1987) suggests that the results are not affected by estimation procedures in the US.

3.4.2 Production Function

An alternative methodology of measuring the effect of immigration on labour market outcomes is the estimation of labour demand functions. The empirical analysis of the competition between different categories of labour, which can be determined on the basis of skill, occupation, ethnic origin and so on is typically based on neo-classical input demand theory (Johnson, 1980; Grant and Hamermesh, 1981; Grossman, 1982; Borjas, 1985; Borjas, 1987; Altonji and Card, 1991; Lalonde and Topel, 1991). In this theory, changes in the use of a certain production factor affect the demand curves of other factors through elasticities of substitution and complementarity. The predictions of these theories are the same as before: if immigrants are substitutes for natives, the theory predicts that immigration will generally have an adverse effect on wages and employment of natives (Borjas et al. 1997).

In the literature on estimation methodologies of demand for types of production factors, discussion has been focussed on the choice between estimating cost function versus production functions, choice of functional form and sample, and disaggregation of labour force. Hamermesh and Grant (1979) argue that the production function approach is a more appropriate methodology when quantities of labour, rather than wages, are assumed to be exogenous.

Hamermesh (1993) argues that using capital as an input is often problematic because it is usually not available or includes a large measurement error. Therefore, excluding capital may be justified however the results should be interpreted carefully. Indeed, capital is excluded in the study on the UK and Norway because data about capital is not available (but available for the Netherlands). Hamermesh (1993) emphases that estimates of own-price elasticity will be biased toward
zero when a separability assumption is not satisfied. By the exclusion of input capital, the effect of capital on the substitution elasticities is ignored and a strong separability between labour and capital is implicitly assumed. Since we do not have data about capital, we are unable to test the separability and must assume that the separability assumption holds.

In previous studies, conducted overwhelmingly in the United States, there is some evidence of substitution between native and ethnic minority labour in line with theoretical predictions. Grant and Hamermesh (1981) analyse competition among youth, adult White male, White female and Black workers in manufacturing using translog production functions. They find a weak substitutability between Black and White men, and a strong substitutability between youth and White women workers. Using a comparable method, Grossman (1982) studies complementarity among native, foreign-born and second generation workers, and reports that both foreign-born and second generation workers are substitutes for native workers but second generation workers have a stronger degree of substitutability. Also, foreign-born workers are stronger substitutes for second generation than for native workers. Capital is complementary to all kinds of labour but the complementarity is the strongest for ethnic minorities. Using the generalised Leontief production function, Borjas (1983) finds no indication for substitutability between Black and Hispanic workers. In his work, Hispanic and White labour are complements while Black and White labour are neither substitutes nor complements. In his later work, Borjas (1986) finds competition between ethnic minority and White men as well as between women and White men but also finds complementarity between ethnic minority and Black males. Borjas (1987) disaggregates labour inputs further. White, Black, Hispanic, and Asian workers are grouped into two categories: native-born and foreign-born. Ethnic minorities as a group are substitutes for White native-born workers. Black native-born workers are weak substitutes for Black and Hispanic ethnic minorities while they are strongly complementary to White ethnic minorities. He finds no evidence of substitutability between Hispanic native-born and other native-born population groups. Griffin (1996) uses firm level data to detect substitutability relationships among various types of labour input and finds that White male and female workers are strong substitutes and so are minority male and White female workers. On the other hand, minority men and women seem to be closely complementary in production.
Akbari and DeVoretz (1992) estimate the degree of labour market competition between foreign- and native-born labour in labour markets by two-digit industries, rather than geographical area. They find no significant evidence that the post-war immigration had a adverse effect on native-born workers in Canada, and also no evidence that immigrants are complementary to capital. This study is a first attempt in European context to measure the degree of labour market competition between natives and immigrants.

Consider the following model: a local economy is assumed to produce $Y$ units of goods using as three-types of labour (unskilled, medium skilled and high skilled):

$$Y = F (L_U, L_C, L_H) \quad (3.46)$$

We assume that the production function satisfies standard neoclassical assumptions so that firms in the factor markets are price takers and production factors are awarded their marginal productivities.

$$\frac{\partial Y}{\partial L_i} = w_i, \quad i = u, c, h \quad (3.47)$$

The analysis of multifactor production function requires more specification of production technology. We assume that goods are produced by a transcendental logarithmic (translog) production technology. This type translog production function is a second-order approximation to a generalised production function in a local labour market (see Christensen et al., 1973; Berndt and Cristensen, 1973; Grant and Hamermesh, 1981; Grossman, 1982; and Gang and Rivera-Batiz, 1994. A standard translog production function has the form of

$$\ln Y = \ln \alpha_0 + \sum_i \alpha_i \ln L_i + \frac{1}{2} \sum_i \sum_j \beta_{ij} \ln L_i L_j, \quad i = u, c, h \quad (3.48)$$

The production function is characterised by constant returns to scale which implies that the production function is linearly homogeneous in $L$, imposing a constraint on the technology parameters $\sum_i \alpha_i = 1$ and $\sum_i \beta_{ij} = 0$.

The three factor share equations are derived from the three output elasticity equations, using equation (3.46).
THE EFFECT OF IMMIGRATION ON EARNINGS: THEORY

\[ \frac{\partial \ln Y}{\partial \ln L_i} = \frac{\partial Y}{\partial L_i} \frac{L_i}{Y} = \frac{w_i L_i}{Y} = \theta_i \quad \text{for all } j \]  

(3.49)

where \( w_i \) is the wage rate and \( \theta_i \) is the share of input value \( i \) in the value of output, with \( \theta_u + \theta_c + \theta_h = 1 \). The factor share equations can be derived from equation (3.48), imposing linear homogeneity.

\[ \theta_i = \frac{\partial \ln Y}{\partial \ln L_i} = \alpha_i + \sum \beta_{ij} \ln L_j + \nu_i \]  

(3.50)

where \( \nu_i \) is the error term and \( \beta_{ij} \) are the technology coefficients to estimate. Demand theory requires symmetry which implies cross-equation restrictions \( \beta_{ij} = \beta_{ji} \).

Using the observed mean wages \( w_i \) and employment levels of the three skill groups \( L_i \), for \( i = u, c, h \), in local labour markets, we can estimate the factor share equations, (3.50) to obtain the technology coefficients from which the partial elasticities of complementarity and correspondingly demand elasticities can be calculated.

Since perfect competition is assumed, output value may be set equal to the sum of income generated by the production factors employed, in this case \( Y = \sum_i w_i L_i \).

Then we may construct the factor share equations as follows:

\[ \theta_i = \frac{w_i L_i}{\sum_i w_i L_i} \]  

(3.51)

By choice of production function instead of the cost function, we assumed that factor quantities rather than factor prices are exogenous. In other words, the impact of an immigration flow, which is defined as a supply shock in Figure 3.1, is modelled here. The Hicks partial elasticities of complementarity are appropriate measures of factor substitutability. These elasticities are, in fact, the 'inverse' of the elasticities of substitution in Section 3.3, \( \sigma_{ij} \) for the two-factor cases\(^9\) since the elasticity of substitution and complementarity are the two sides of the same coin\(^10\).

Two factors are classified as substitutes and complements, depending on whether the partial elasticity of complementarity is positive or negative as presented below (Sato and Koizumi, 1973).

\(^9\)See Appendix A.2 for the multi-factor case.

\(^10\)Hamermesh (1986) shows that the elasticities of substitution can be obtained by the cost function while the elasticities of complementarity can be derived from the production function. He argues, based on the duality of cost and production function, that "having found one of them, the other is immediately available" (p.435).
Chapter 3

The Hicks partial elasticity of complementarity between factors $L_i$ and $L_j$, $\sigma_{ij}$, is defined as the proportional change in factor price $i$ relative to factor price $j$ as a result of exogenous changes in factor $j$’s supply relative to factor $i$, holding the output price and other input quantities constant.

$$c_{ij} = \frac{F_{ij}}{F_i F_j}$$  \hspace{1cm} (3.52)

where $F_i$ is the first derivative of the production function $F$ with respect to factor $i$, i.e. $F_i = \frac{\partial F}{\partial L_i}$, and $F_{ij}$ is the second derivative of the production function $F$, i.e. $F_{ij} = \frac{\partial^2 F}{\partial L_i \partial L_j}$

In terms of the translog share equations, the Hicks partial elasticity of complementarity is given by (Hamermesh, 1993) (see Appendix A.2):

$$c_{ij} = \frac{(\beta_{ij} + \theta_i \theta_j)}{\theta_i \theta_j}$$  \hspace{1cm} (3.53)

$$c_{ij} = \frac{(\beta_{ii} + \theta_i^2 - \theta_i)}{\theta_i^2}$$  \hspace{1cm} (3.54)

If an increase in the quantity of one factor has a positive (negative) effect on the price of another factor, these two factors are complements (substitutes). If an increase in input $j$ raises the price of $i$, i.e. $c_{ij} > 0$, factors $i$ and $j$ are q-complements. If an increase in input $j$ decreases the price of $i$, i.e. $c_{ij} < 0$, factors $i$ and $j$ are q-substitutes.

The factor price elasticities, associated with the $c_{ij}$, are given as (Hamermesh, 1986)

$$\delta_{ij} = \theta_j c_{ij}$$  \hspace{1cm} (3.55)

It shows the change in the price of factor $i$ as a result of a 1 percent change in the quantity of factor $j$, holding output price constant.\(^{11}\)

Equations (3.53) and (3.54) reveal that a proportional increase in the immigrant labour force will affect the wages of different types of labour through elasticities

\(^{11}\)Note the difference with equation (3.7), where the effect of a change in the wage rate on factor demand is specified. See Appendix A.2 for differences between elasticities of substitution and complementarity.
of substitution or complementarity between various types of labour. This effect is related not only to relative share of immigrants, but also the relative shares of other inputs in production, even in the absence of labour market discrimination.

This analysis implies that if a number of new low skill immigrant workers enters a certain local labour market, the wages of workers who are closer substitute for immigrants, in this case low skilled workers, will decrease, and consequently the wages of workers who are complements with immigrants will increase. Similarly, if the skill level of an immigration flow is high, wages of high skilled workers will decrease in the given local labour market.

The new immigrants are not only suppliers of labour but they also generate demand for goods produced locally. This can be taken into account when we assume that the labour demand elasticities are determined by the conventional Marshall-Hicks formulas given by equation (3.7) (Sato and Koizumi, 1973). These formulas make a link between the labour and product markets.

The factor shares can be simultaneously estimated for the three countries, using micro data aggregated for local labour markets to obtain the technology coefficients in equation (3.50). The elasticities of complementarity can be calculated using these technology coefficients. The main disadvantage of this strategy is that the disaggregation of labour force into sub-samples is limited. This limitation is especially important with respect to the labour force of ethnic minorities since their number is small in the surveys that are used for empirical work. Immigrants are highly concentrated in some geographical areas or industries while there are few immigrants in other areas where the likelihood of being sampled is close to zero. Indeed, there are many local labour markets for which the samples contain no immigrants. Therefore, many immigrant groups must be treated as a single labour input to satisfy the estimation requirements.

After the estimation of the partial elasticities of complementarity, the factor price elasticities given by equation (3.55) can be calculated. In the next three chapters, we estimate the elasticities of labour demand for the Netherlands, the UK and Norway.
3.5 Conclusions

In this chapter, we present a theoretical framework that will be used in the next three chapters to measure of the effect of immigration on wages in the Netherlands, the UK and Norway. Firstly, two main approaches are presented to estimate wage elasticities among different types of labour inputs: a simple regression analysis to determine reduced-form effects of immigrants on wages, and a labour demand model to take into account simultaneous interactions among labour inputs. The first model estimates the effect of immigration on wages using a reduced-form, where the reaction coefficient $\gamma$ is a complex formula in supply and demand elasticities that will not (and can not) be distinguished. In the second approach, we assume supply to be exogenous and identify production technology parameters from factor value shares from which the partial elasticities of complementarity and corresponding wage elasticities are derived. Both models have their own advantages and disadvantages, suggesting that these models are complementary to each other. Empirical application of the model discussed will be along two alternative classifications of native labour force: (1) an economy employing two production factors, i.e. male and female, (2) an economy employing three production factors: low, medium and high skilled workers. These alternative classifications will provide more information about the reaction of host country on immigration, for three countries separately.

The theoretical framework presented provides an important tool to perform empirical analyses. However, it should be noted that it has some restrictions that are mainly imposed by data problem (absence of data over capital, too few observations on immigrants etc.), but also by the lack of a comprehensive theoretical model in which relevant avenues of an adjustment process are integrated, such as domestic and international mobility of capital, labour and goods as well as unemployment. Moreover, an increase in the number of immigrants does not necessarily mean an extension of labour supply since the unemployment rate is four to five times higher among ethnic minorities than natives. In the light of these unmanageable difficulties, the theoretical framework is a modest attempt to investigate the impact of immigration on wages.
A Appendix

A.1 Connected Labour Markets by Skill Level

Suppose that a single competitive industry in a local economy produces \( Y \) units of goods as described by a homogenous production function with constant returns to scale employing low, medium and high skilled labour in addition to other inputs. The total labour force, \( L \), consists of low skilled \( L_U \), medium skilled, \( L_C \) and high skilled, \( L_H \); \( L = L_U + L_C + L_H \) and the proportions of low, medium and high skilled labour are \( u = L_U/L, c = L_C/L \) and \( h = L_H/L \), respectively so that \( u + c + h = 1 \). Total cost in industry is given as a function

\[
C(w_u, w_C, w_H, Y) = Yc(w_u, w_C, w_H) \tag{A.1}
\]

where \( w_u, w_C \) and \( w_H \) are wages of low, medium and high skilled labour respectively and \( c(w_u, w_C, w_H) \) is unit labour cost. Perfect competition implies that unit labour cost, in the absence of capital, is equal to the price of output, \( p = c(w_u, w_C, w_H) \).

Goods produced are consumed by low, medium and high skilled workers and some part of output is exported. Product market equilibrium is given as

\[
Y = L_U Y_U(p, w_U) + L_C Y_C(p, w_C) + L_H Y_H(p, w_H) + Y_x(p) \tag{A.2}
\]

where \( Y \) is the units of goods produced, \( Y_j(.) \) for \( j = u, c, h \), is the per capita demand functions each skill type and \( Y_x(p) \) is export demand.

Per capita labour demand and supply functions of low, medium and high skilled workers are given by

<table>
<thead>
<tr>
<th>Labour demand:</th>
<th>Labour Supply:</th>
</tr>
</thead>
<tbody>
<tr>
<td>low skilled labour: ( L_U(p, w_U) )</td>
<td>( S_u(w_U, p) )</td>
</tr>
<tr>
<td>medium skilled labour: ( L_C(p, w_C) )</td>
<td>( S_c(w_C, p) )</td>
</tr>
<tr>
<td>high skilled labour: ( L_H(p, w_H) )</td>
<td>( S_h(w_H, p) )</td>
</tr>
</tbody>
</table>

Equilibrium occurs in the local labour market when:

low skilled labour: \( L_U S_u(w_U, p) = Yc'_u(w_U, w_C, w_H) \)
medium skilled labour: \( L_C S_C(w_C, p) = Y c_C(w_V, w_C, w_H) \)

high skilled labour: \( L_H S_H(w_H, p) = Y c_H(w_V, w_C, w_H) \)

where \( c_i(.) = \frac{\partial c}{\partial w_i} \) are marginal labour costs of low, medium and high skilled labour. The unit product cost is equal to price of product, \( c(w_V, w_C, w_H) = p \).

A.2 Elasticities of Substitution Versus Elasticities of Complementarity

The partial elasticity of substitution (\( \sigma \)) was first introduced by Hicks (1932) and Robinson (1933). It measures the effect of a change in the price of one factor on the quantity demanded of another factor, holding output and other factor prices constant. The partial elasticity of substitution reflects the properties of the cost function. Hicks (1970) finds that the elasticity of factor demand \( \lambda \) (or derived demand) is related to the elasticity of substitution and introduces a new concept: the partial elasticity of complementarity (\( c \)), which is related to the inverse of the factor demand in a linear homogenous production function, i.e \( c = \frac{1}{\sigma} \). The partial elasticity of complementarity measures the effect of a change in the quantity of one factor on the price of another factor, holding marginal cost and quantities of other factors constant. Note that the level of output is not constant. In other words, the partial elasticity of complementarity measures the degree by which the joint contribution of inputs increases production.

The perfect inverse relationship between the two elasticities is based on the duality theory of production and cost functions. Satô and Koizumi (1973) formally demonstrate form of the duality relation between elasticities of substitution and complementarity for the \( n \) factor case under the assumption of constant returns to scale\(^\text{12}\).

In the partial elasticity of substitution concept, two factors, \( i \) and \( j \), are

\[ p\text{-substitutes if } \sigma_{ij} > 0 \quad \text{(A.3)} \]
\[ p\text{-complements if } \sigma_{ij} < 0 \quad \text{(A.4)} \]

\(^{12}\)Syrquin and Hollender (1982) extend this concept for a production function which is not characterised by constant returns to scale.
In the partial elasticity of complementarity, the two factors, \( i \) and \( j \), are

\[
q\text{-substitutes if } c_{ij} < 0 \quad \text{(A.5)}
\]
\[
q\text{-complements if } c_{ij} > 0 \quad \text{(A.6)}
\]

When there are only two production factors, the two factors must be \( q \)-complements (complements in the production of a variable output) and \( p \)-substitutes (substitutes in production of the fixed output). However, a similar relationship does not hold for the multi-factor case, i.e., \( c \neq \frac{1}{\sigma} \), unlike the two-factor case in which \( c = \frac{1}{\sigma} \). If there are \( n \) inputs, there must be at least \( n - 1 \) \( p \)-substitutes and \( n - 1 \) \( q \)-complements from \( n(n - 1)/2 \) partial elasticities.

The formal derivation of the elasticities of substitution and complementarity is as follows (Hicks 1970; Sato and Koizumi 1973; Hamermesh 1993)

\[
Y = f(L_1, \ldots, L_n), \text{ with } f_i > 0, f_{ii} < 0 \quad \text{(A.7)}
\]

where \( Y \) is output and \( L_i \) are factors of production and \( f_i, (f_{ii}) \) denotes the first (second) derivative of the production function.

The associated cost function is

\[
C = g(w_1, \ldots, w_n, Y) \text{ with } g_i > 0 \quad \text{(A.8)}
\]

where \( w_i \) are input prices.

The profit maximisation condition is met when, using the production and cost functions

\[
f_i - \lambda w_i = 0, \quad i = 1, \ldots, n \quad \text{(A.9)}
\]
\[
L_i - \mu g_i = 0, \quad i = 1, \ldots, n \quad \text{(A.10)}
\]

where \( \lambda \) and \( \mu \) are Lagrangian multipliers.

The partial elasticity of substitution is the percentage effect of a change in \( \frac{w_i}{w_j} \) on \( \frac{L_i}{L_j} \) holding output and other input prices constant, given by
\[ \sigma_{ij} = \frac{Y}{L_i L_j} \frac{F_{ij}}{|F|} \text{ where } |F| = \begin{bmatrix} 0 & f_1 & \cdots & f_n \\ f_1 & f_{11} & \cdots & f_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ f_N & f_{N1} & \cdots & f_{NN} \end{bmatrix} \] (A.11)

the bordered Hessian determinant of the equilibrium conditions (A.7) and (A.9), and \( F_{ij} \) is the cofactor of \( f_{ij} \) in \( F \).

An alternative definition of the partial elasticity of substitution based on the cost function is

\[ \sigma_{ij} = \frac{C g_{ij}}{g_i g_j} \] (A.12)

The partial elasticity of complementarity show the percentage effect on \( \frac{w_i}{w_j} \) of a change in the input ratio \( \frac{L_i}{L_j} \) holding marginal cost and other input quantities constant. It is defined, using the production function

\[ c_{ij} = \frac{Y f_{ij}}{f_i f_j} \] (A.13)

where \( f_i = \frac{\partial f}{\partial L_i}, f_j = \frac{\partial f}{\partial w_i} \) and \( f_{ij} = \frac{\partial^2 f}{\partial L_i \partial L_j} \).

The partial elasticity of complementarity can also be defined from the cost function, in a similar way

\[ c_{ij} = \frac{C G_{ij}}{w_i w_j |G|} \] (A.14)

where \(| G |\) is the determinant of the bordered Hessian matrix that results from totally differentiating (A.8) and (A.10), and \( G_{ij} \) is the cofactor of \( g_{ij} \) in that matrix.

In summary, the elasticity of substitution registers the cross-elasticity of derived demand, holding output level constant when price of input is exogenous, while the elasticity of complementarity does the same thing, but holding marginal cost constant when quantity is changed.