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Chapter 4

The Influence of Europe’s Single Market on Monetary Policy

Summary

This chapter studies the implications of Europe’s single market. Small costs of international trade in goods and services may cause a large home bias in consumer spending and can explain seemingly excessive short-run exchange rate volatility. The European single market (declining costs of international trade) will reduce the home bias in consumption. As a result, monetary policy becomes less effective in terms of stabilising consumption, but better able to influence the general price level.

4.1 Introduction

Markets for goods and services in the European Union are becoming more and more integrated. This is the result of a variety of developments. In particular, border controls have been abolished, markets are being liberalised, public procurement has been opened to foreign firms and technical standards are being harmonised. Moreover, the internet makes it easier and cheaper to order goods and services from abroad. Head and Mayer (2000) provide empirical evidence that the bias towards buying domestically-produced products has been on a downward trend since the 1970s.

In spite of the trend towards more integration, markets for the same products continue to be segmented. This can be illustrated by the home bias in consumer spending. The aforementioned study by Head and Mayer finds that, in the period 1993-95, Europeans purchased twelve times more from domestic producers than from equally-distant (in a geographic sense) foreign producers.
The costs of cross-border transactions are still larger than for transactions concluded between parties within the same country. These extra costs may arise from information costs, or from other implicit trade barriers. The importance of information costs was first highlighted by Stigler (1961) and has been explored further by Salop (1977), Bester and Petrakis (1995) and others. Examples are the costs of finding out the quality of foreign goods and foreign counterparties, costs related to differences in distribution channels, costs of cross-border advertisements, etc. Prices are more transparent to buyers from the same country: consumers have better access to domestic price information, since newspapers, magazines and tv programmes are distributed largely within national borders. Other implicit trade barriers include different product standards and national regulations which require firms to comply with specific rules in order to enter another national market. In Europe, several steps have been taken to reduce the costs involved in cross-border transactions, but many obstacles remain.¹

This chapter studies the consequences of the remaining barriers to cross-border competition. This may shed some light on the implications of Europe's single market for the home bias in consumption and for the effectiveness of the ECB's monetary policy. The analysis is conducted in the context of the Obstfeld and Rogoff (1995) framework, which allows for imperfect competition and nominal rigidities (in casu: short-run wage stickiness). I explicitly incorporate ‘transportation costs’ for trade between two countries into the model. ‘Transportation costs’ is a catch-all term: it can be interpreted as any kind of barrier to trade. Hence, the terms transportation costs and trade costs will be used interchangeably in this paper. The introduction of costs for international trade implies that the model deviates from the standard Obstfeld and Rogoff (1995) model. First, the law of one price need not hold. Second, trade costs lead to an endogenous home bias in consumer spending.

I obtain the following results. First, small trade costs may lead to a substantial home bias in consumer spending. I will show numerically that the large decline in the home bias for European countries since the late 1970s found in the literature is consistent with a relatively small reduction in trade costs. The policy implication is the existence of a window of opportunities for the completion of the European Union's Single Market.

Second, the large short-run exchange rate volatility that we observe in practice can be explained by the presence of international trade costs in the goods market. The intuition is as follows. In the presence of wage stickiness and a constant mark-up (as in the current paper), the short-run relative price level can only adjust if the exchange

¹See OECD (1997) for examples of regulatory reforms co-ordinated across borders, which have reduced the costs of international transactions. See ECB (2001, Table 2) for an overview of remaining obstacles to trade in the internal market.
rate changes. Positive costs of international trade lead to a home bias in spending, so only a limited share of goods is affected by exchange-rate movements. As a result, relatively large short-run exchange rate movements may be required to attain short-run money market equilibrium. I will show numerically that when the costs associated with international trade are substantial, short-run exchange rate movements in response to monetary shocks can become extremely large. By reducing the costs involved in cross-border transactions, the Single Market initiative is likely to contribute to exchange rate stability between the euro area and the other EU member states.

Third, the Single Market initiative may have important consequences for the transmission of ECB monetary policy. More specifically, in an environment of declining costs of international trade, monetary policy becomes less powerful in terms of affecting consumption, but more effective in terms of influencing the general price level. The intuition is as follows. Positive costs of international trade lead to a home bias in consumer spending. This implies that a monetary expansion will lead to a larger surplus on the short-run current account, so that a larger accumulation of net foreign assets takes place, facilitating a larger long-run consumption differential. Intertemporal consumption smoothing ensures that the short-run consumption differential is also larger. Moreover, a smaller share of the resulting consumption increase ‘leaks’ to other countries, re-inforcing the increase in the terms of trade, thus enhancing further the transmission from money to consumption. It follows that a decline in the costs of international trade reduces the effectiveness of monetary policy with respect to consumption. The effectiveness of monetary policy with respect to the general price level is the flip-side of its effectiveness with respect to consumption (this follows from the long-run equilibrium conditions in the money market). Therefore, a decline in the costs of international trade enhances the effectiveness of monetary policy with respect to the general price level.

This chapter is related to several earlier papers in the literature. One is Warnock (1999), who incorporates a home bias in Obstfeld and Rogoff’s model. He imposes an exogenous home bias in preferences: consumers derive more utility from consuming domestically-produced goods than from foreign-produced goods.² By contrast, there is no home bias in preferences in this chapter: the home bias is endogenous and is caused by the interplay of costs of international trade and imperfect competition (a finite price-elasticity of demand). A second paper in this field is Hau (2000), who introduces a home bias in spending by adding non-tradable goods to the model. By contrast, there are only tradable goods in the model in this chapter.

²See Neven, Norman and Thissé (1991) for an earlier, and more extensive modeling of consumer attitude towards foreign products.
Another paper which is related to the current chapter is Obstfeld and Rogoff (2000). They focus on the cost of international trade and its possible implications for several empirical puzzles in international macroeconomics, including the home bias in consumer spending. Their paper studies a number of special cases for a small country, without developing and solving a fully-fledged model, as this paper does. Their two-goods model leads to a remarkable discontinuity in prices. This is caused by their assumption that the home country is a small open economy, which may either import or export the good that it is endowed with. Since the world price of the home good is supposed to be given, the presence of trade costs implies that the home price of this good is below the price on the world market when the home country is a net exporter, whereas it is above the world market price when the home country is a net importer. As a result, there is a discrete jump in prices at the point where the home country changes from a net importer to a net exporter. My results, derived in the context of a model with two large countries and a continuum of goods, do not suffer from this peculiarity.

The remainder of this chapter is organised as follows. Section 4.2 presents the basic model and derives the equilibrium conditions assuming optimal behaviour by households and firms. In section 4.3, a steady state solution and a log-linearised version of the model are presented. Section 4.4 analyses the dynamics. Section 4.5 discusses money shocks. Section 4.6 concludes this chapter.

4.2 The model

4.2.1 Market structure and preferences

The world consists of two countries, Home and Foreign, which are completely symmetric. Both countries are inhabited by a continuum of consumer-producers. Producers in the home country are indexed by $z \in [0, \frac{1}{2}]$, producers in the foreign country are indexed by $z \in (\frac{1}{2}, 1]$. Each of them produces a single differentiated good.

Household preferences are defined over an intertemporal utility function which includes a consumption index, real money balances and work effort:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} [\log C_s + \chi \log (\frac{M_s}{P_s}) - \frac{\kappa}{2} L_s^2],$$

where $U$ is the lifetime utility of a representative household in the home country, $C$ is real composite consumption index, $M$ is the amount of nominal money balances held by the representative household,$^3$ $P$ is a consumption-based price deflator, $L$ is the amount

$^3$The money-in-the-utility function approach can be rationalised by arguing that real money balances
of labour used in production, \( \beta \) is the discount factor, \( \chi \) is a parameter which captures the benefit from holding real money balances and \( \kappa \) captures the disutility from work effort. Finally, \( s \) and \( t \) denote the moment in time. Time subscripts will be suppressed whenever possible.\(^4\)

All goods are tradable and sold in the world market. The elasticity of substitution between goods (whether produced at Home or in Foreign) is \( \theta \). The Home consumption index of the composite good is defined by

\[
C = \left[ \int_0^1 [c(z)]^{\frac{\theta-1}{\theta}} \, dz \right]^\frac{\theta}{\theta-1}. \tag{4.2}
\]

and the Foreign consumption index of the composite good is defined by

\[
C^* = \left[ \int_0^1 [c^*(z)]^{\frac{\theta-1}{\theta}} \, dz \right]^\frac{\theta}{\theta-1}. \tag{4.3}
\]

The parameter \( \theta \) is assumed to be larger than one.\(^5\) In the remainder of this paper, Foreign variables are denoted with an asterisk (*) and a different indexation of producers \( (z = f \text{ for Foreign and } z = h \text{ for Home}) \). Apart from that, the mathematical expressions for Foreign variables are identical to those found for the Home country, unless explicitly stated otherwise.

The consumption-based price index (defined as the minimum nominal amount of money needed to purchase one unit of composite real consumption, \( C \)) is:

\[
P = \left[ \int_0^1 [p(z)]^{1-\theta} \, dz \right]^{\frac{1}{1-\theta}}, \tag{4.4}
\]

where \( p(z) \) is the money price of good \( z \).

As mentioned before, this paper allows for cross-border trade costs. The presence of cross-border trade costs implies the possibility that the same good sells for a different price in both countries, i.e. the law of one price may not hold. The exact relationships between prices for individual goods will follow from profit maximisation by firms.

There is a single financial asset, an internationally traded riskless real bond denominated in the composite consumption good. There is no capital in the model, hence firms' profits consist entirely of monopoly rents. Firms are entirely domestically owned and allow agents to save time in conducting transactions [Warnock (1999)].

\(^4\)Note that work effort enters the utility function with a negative sign, reflecting the household's preference for leisure.

\(^5\)Since the marginal revenue of an additional unit of output is negative when the elasticity of demand is less than one, \( \theta > 1 \) is required for a positive level of output in equilibrium.
profits are immediately handed back to the owners (households). Labour is immobile between countries, so that labour income remains in the own economy. The period budget constraint (in nominal terms) for a representative household of Home can then be written as

\[ P_t F_t + M_t = P_t (1 + r_{t-1}) F_{t-1} + M_{t-1} + W_t L_t + \Pi_t - P_t C_t - P_t T_t, \]  

(4.5)

where \( F_t \) is the stock of bonds held by the representative household on date \( t \). \( r_{t-1} \) is the real interest rate on bonds between \( t - 1 \) and \( t \). \( W_t \) is the nominal wage rate, \( \Pi_t \) is its firm’s profits and \( T_t \) is a lump-sum tax or transfer. Interest rates and taxes are denominated in terms of the composite good. The constraint (4.5) states that the households holdings of cash and bonds must be equal to the previous period’s cash and bond holdings, increased by the income from its labour and the ownership of its firm and bonds, and reduced by spending on consumption and tax payments.

I assume that the government budget is balanced at all times. Moreover, I assume that there is no government spending. All seigniorage revenues are redistributed in the form of transfers

\[ 0 = T_t + \frac{M_t - M_{t-1}}{P_t}. \]  

(4.6)

Further, the nominal interest rate \( i_t \) is given by

\[ 1 + i_t = (1 + r_t) \frac{P_{t+1}}{P_t}. \]  

(4.7)

### 4.2.2 Maximisation of household utility

The household’s maximisation problem is not directly affected by international trade costs. The household’s maximisation problem can be separated into an intratemporal and an intertemporal problem. I solve the intratemporal problem first.

Individual Home consumption of good \( z \) is given by [see Obstfeld and Rogoff (1996, p. 664) for a derivation]:

\[ c(z) = \left( \frac{p(z)}{P} \right)^{-\theta} C. \]  

(4.8)

The demand for good \( z \) is decreasing in its relative price, with a price elasticity of \( \theta \).

Having solved the household’s intra-temporal maximisation problem, I turn to the intertemporal optimisation problem. The representative household maximises life-time
utility (4.1), subject to the period budget constraint (4.5) which must be satisfied in every single period. The first-order conditions are (see Appendix A for the derivation):

\begin{align}
C_{t+1} &= \beta(1 + r_t)C_t, \\
\frac{M_t}{P_t} &= \chi C_t \left(1 + \frac{i_t}{i_t}\right), \\
L_t^s &= \frac{1}{\kappa P_t} \frac{1}{C_t}.
\end{align}

Equation (4.9) is the standard Euler equation. It indicates how the total consumption of goods is spread over time. This equation follows from the condition that the marginal utility from consuming an additional unit of the composite consumption good in period \(t\) be equal to the discounted marginal utility from saving the resources for one more period (earning a real interest rate of \(r_t\)) and consuming them in period \(t + 1\). The money demand equation (4.10) follows from the condition that the marginal utility derived from consumption in period \(t\) be equal to the marginal utility derived from holding cash balances during that period and spending the resulting cash balances on consumption in period \(t + 1\). The labour supply equation (4.11) is the result of the condition that the marginal utility from consuming the additional labour income be equal to the marginal disutility of the required work effort.

### 4.2.3 Maximisation of firm profits

Goods markets are imperfectly competitive. This follows from the fact that the substitutability between different varieties of the single differentiated good in this model is finite (\(\theta < \infty\)). Imperfect competition is an important ingredient of the model. First, it permits the explicit analysis of pricing decisions. Second, equilibrium prices set above marginal cost rationalise demand-determined output in the short run, since firms gain from a marginal expansion of the output level. Third, imperfect competition implies that equilibrium output falls below the social optimum. See, for example, Lane (2001).

The representative home-country firm \(z\) is a monopolist in the production of a good \(z\). The firm uses only one input: labour. Labour is homogeneous. The production process exhibits constant returns to scale

\[ y(z) = \alpha l(z), \]

where \(\alpha\) is labour productivity and \(y(z)\) is output of good \(z\).

The costs of international trade take the form of a 'melting loss': only a fraction
1 − τ of all exported goods reaches its destination.\textsuperscript{6,7} I assume 0 ≤ τ < 1, i.e. a strictly positive fraction of exports 1 − τ reaches its destination and a non-negative fraction τ fails to reach its destination. The goods markets clearing condition, which I will use to solve the firms’ optimisation problem below, takes into account this ‘melting loss’:\textsuperscript{8}

\[ y(z) = c(z) + \frac{1}{1-\tau}c^*(z). \quad (4.13) \]

The labour market is assumed to be competitive.\textsuperscript{9} I assume that labour is immobile internationally, but fully mobile within national borders. It follows directly that, in equilibrium, there will be a single wage rate \( W (W^*) \) in each country. I assume that firms can price-discriminate between countries.

Firm profits are

\[ \Pi(z) = p(z)c(z) + Xp^*(z)c^*(z) - Wl(z), \quad (4.14) \]

where \( X \) is the nominal exchange rate, i.e. the price of one unit of Foreign currency expressed in the Home currency. The two currencies are in a floating exchange rate regime.

The representative Home firm \( z \) chooses \( p(z) \) and \( p^*(z) \) in order to maximise profits (4.14), subject to the demand function (4.8) and its Foreign counterpart, the production function (4.12) and clearing of goods markets (4.13). I assume that the firm takes Home and Foreign aggregate demand \( C \) and \( C^* \) as given. International trade costs enter the profit function implicitly: observe that ‘melted’ exports cause production costs, but do

\[ \text{\textsuperscript{6}This is a standard way of modelling in the literature, also known as ‘iceberg transportation costs’. See, for instance, Obstfeld and Rogoff (2000).} \]

\[ \text{\textsuperscript{7}Under alternative specifications (e.g. firms pay an import tariff to the government of the foreign country, or exporting firms need to hire additional workers for ensuring compliance with foreign product rules), I obtain very similar results. However, these specifications require adjustments to the expressions for the household budget constraint and the current account equation, which make the resulting solution more complicated.} \]

\[ \text{\textsuperscript{8}The melting loss for Home-produced good \( z \) is a fraction \( \tau \) of Home exports of good \( z \), i.e.: } mL(z) = \tau[y(z) - c(z)]. \text{Foreign imports are a fraction } 1 - \tau \text{ of Home exports: } c^*(z) = (1 - \tau)[y(z) - c(z)]. \text{Therefore, the melting loss for the Home-produced good must be a fraction } \frac{1}{1-\tau} \text{ of Foreign imports: } mL(z) = \frac{1}{1-\tau}c^*(z). \text{The goods market clearing condition for Home-produced good } z \text{ is } y(z) = c(z) + c^*(z) + mL(z), \text{which can be simplified to equation (4.13) in the main text.} \]

\[ \text{\textsuperscript{9}The assumption of competitive labour markets implies that firms and workers take the wage rate as given when making decisions on goods production (labour demand) and labour supply respectively. This assumption simplifies the computations, but is inessential for the purpose of this paper.} \]
The model

not generate revenues. The first-order conditions are (see Appendix B for derivation):

\[ p(z) = \left( \frac{\theta}{\theta - 1} \right) \frac{W^*}{\alpha}, \]  

\[ p^*(z) = \frac{1}{X(1 - \tau)} \left( \frac{\theta}{\theta - 1} \right) \frac{W}{\alpha}. \]  

These equations represent the optimal pricing rules for the representative Home firm (i.e., for \( z \in [0, \frac{1}{2}] \)) under monopolistic competition. The prices in equations (4.15)-(4.16) are expressed in local currency, i.e., \( p^*(z) \) is in foreign currency. Firms set the Home price equal to unit labour costs plus a mark-up of \( \frac{1}{\theta - 1} \). They set the Foreign price equal to the foreign currency equivalent of the Home product price multiplied by \( \frac{1}{1 - \tau} \). As the goods market moves towards full competition \((\theta \to \infty)\) profit margins tend to zero.

Similarly, the optimal pricing rules for the Foreign firm (i.e., for \( z \in (\frac{1}{2}, 1] \)) are

\[ p^*(z) = \left( \frac{\theta}{\theta - 1} \right) \frac{W^*}{\alpha}, \]  

\[ p(z) = \frac{X}{X(1 - \tau)} \left( \frac{\theta}{\theta - 1} \right) \frac{W^*}{\alpha}. \]  

It follows directly from the optimal pricing rules (4.15)-(4.18) that

\[ Xp^*(z) = \frac{1}{1 - \tau} p(z), \text{ for } z \in [0, \frac{1}{2}], \]  

\[ p(z) = \frac{1}{1 - \tau} Xp^*(z), \text{ for } z \in (\frac{1}{2}, 1]. \]  

It is optimal for firms to price-discriminate between markets. Optimal price setting implies that firms fully pass on the costs of international trade to consumers abroad. Thus, the law of one price does not hold.\(^{11}\)

---

\(^{10}\)When melting losses occur, one needs to distinguish carefully between firm output and firm income. However, it turns out that, under optimal price setting, both are equal in the model in this paper. Output \( y(z) \) must satisfy the production function (4.12) and the goods market clearing condition (4.13). Income \( \tilde{y}(z) \) is defined by \( p(z) \tilde{y}(z) = WI(z) + II(z) = p(z)c(z) + Xp^*(z)c^*(z) \). Using equation (4.19), which follows directly from optimal price-setting, I find \( p(z) \tilde{y}(z) = p(z)c(z) + \frac{1}{1 - \tau} p(z)c^*(z) \). Dividing both sides by \( p(z) \) and using equation (4.13) yields \( \tilde{y}(z) = y(z) \). Hence, I will be able to use the terms output and income interchangeably. Strictly speaking, the former includes melted exports whereas the latter does not, but (as will be seen further on in this subsection) optimal price setting guarantees that the higher price received for exports precisely offsets the negative impact of the melting loss on income.

\(^{11}\)The firm income per unit produced for the foreign market is the same as per unit produced for the domestic market. In terms of firm revenue, the price charged abroad, which is a fraction \( \tau \) higher than
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The labour demand function follows directly from the production function (4.12):

\[ l^d(z) = \frac{1}{\alpha} y(z). \]  

(4.21)

I will assume the labour market to clear in equilibrium. The labour market clearing condition for the home country is:

\[ L^* = \int_0^1 l^d(z)dz. \]  

(4.22)

with \( L^* \) as given by equation (4.11) and \( l^d(z) \) given by equation (4.21).

4.2.4 The home bias

Combining the consumer demand equation (4.8) and its Foreign counterpart with the price relationships (4.19)-(4.20) and assuming symmetry between producers in the same country yields the expressions for Home consumption of Home-produced and Foreign-produced goods

\[ c(h) = \left( \frac{p(h)}{P} \right)^{-\theta} C, \]

\[ c(f) = \left( \frac{XP^*(f)}{P(1-\tau)} \right)^{-\theta} C. \]

In equilibrium, under international symmetry:\textsuperscript{12}

\[ \frac{c(h)}{c(f)} = (1-\tau)^{-\theta} > 1. \]  

(4.23)

Thus, transportation costs and the price-elasticity of demand together determine the distribution of consumer spending over domestically-produced and imported goods. Let \( \delta \) be the share of income spent on domestically-produced goods:\textsuperscript{13}

\[ \delta = \frac{c(h)}{y(h)} = \frac{c(h)}{c(h) + \frac{1}{1-\tau}c(f)} = \frac{1}{1 + (1-\tau)^{\theta-1}}, \]  

(4.24)

the price charged in the domestic market, precisely offsets the melting loss, which amounts to a fraction \( \tau \) of the volume of exports. Only in the special case that these costs tend to zero (\( \tau \to 0 \)), it becomes optimal to sell goods for the same price in both countries and the law of one price is restored, as in Obstfeld and Rogoff (1995).

\textsuperscript{12}Equation (4.23) is derived using the two previous equations, equation (4.20) and international symmetry. Under international symmetry, the prices of Home-produced and Foreign-produced goods on the Home market are identical, after correcting for trade costs, i.e. \( p(h)/p(f) = (1-\tau) \).

\textsuperscript{13}It does not matter whether \( \delta \) is defined in nominal or real terms. Note that Home income equals Home output: \( Wl(h) + \Pi(h) = p(h)c(h) + Xp^*(h)c^*(h) = p(h)y(h) \). Thus, the share of nominal income spent on (the nominal value of) domestically-produced goods is \( \delta = p(h)c(h)/[p(h)y(h)] = c(h)/y(h) \).
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where I have used equations (4.13), (4.23) and international symmetry.\(^{14}\)

I define the home bias \(\beta\) as the share of income spent on domestically-produced goods in excess of the ‘neutral’ level, where the neutral level of spending on domestically-produced goods is the share of the Home country in world output. In the special case of two countries of equal size, the neutral spending ratio is equal to 1/2. Thus:

\[
\beta = \delta - \frac{1}{2}.
\]

When markets are fully integrated (\(\tau = 0\)), both countries consume identical goods baskets (\(\delta = \frac{1}{2}\), so that \(\beta = 0\)). When markets are less than fully integrated (\(\tau > 0\)), there is a bias towards spending on domestically-produced goods (\(\delta > \frac{1}{2}\), so that \(\beta > 0\)), since firms charge a higher price to foreign customers and since consumers are sensitive to relative prices. Theoretically, when firms would charge a lower price abroad than in their home market, the home bias could actually be negative.

When the price-elasticity of demand is very low (\(\theta \to 1\)), there is no home bias (\(\lim_{\theta \to 1} \beta = 0\)). The consumers’ ‘love for variety’ is strong enough to overcome even high costs to international trade. By contrast, when the price-elasticity of demand is very high (\(\theta \to \infty\)), positive transportation costs (\(\tau > 0\)) imply that firms are unable to compete in foreign markets. Consumers are very sensitive to prices, so they are unwilling to pay more for the variety that foreign goods bring.\(^{15}\) They will spend all money on domestically-produced goods (the home bias attains its maximum value: \(\lim_{\theta \to \infty} \beta = \frac{1}{2}\)) and no international trade will take place. Even in less extreme cases, small transportation costs can cause a large home bias, as we will see below.

Table 1 shows the sensitivity of the home bias with respect to the price-elasticity of demand (\(\theta\)) and the costs of international trade (\(\tau\)). The literature provides some (implicit) estimates for the price-elasticity of demand. Obstfeld and Rogoff (2000) cite a number of papers that have estimated product mark-ups. According to their survey, most authors find results that correspond to a price-elasticity of demand (\(\theta\)) in a range between 3.5 and 6,\(^{16}\) but some find a price-elasticity as high as 20 for OECD countries.

\(^{14}\)Alternatively, one could look at the share of domestically-produced goods in consumption, defined as \(\gamma = c(h)/[c(h) + c(f)] = 1/[1 + (1 - \tau)\theta]\). The difference between the share of domestically-produced goods in consumption (\(\gamma\)) and the share of domestically-produced goods in income (or equivalently: output) (\(\delta\)) is caused by the melting loss. The difference between the two will be quantitatively relevant only when \(\tau\) is sufficiently large and \(\theta\) is not too large. I have chosen to focus on \(\delta\) in the main text, since this variable will help to simplify the notation in the loglinearization of the model later on in this chapter.

\(^{15}\)When the degree of substitutability (\(\theta\)) increases, the product variety is lower. Therefore, the additional variety that foreign goods bring is also lower.

\(^{16}\)The markup equals \(1/(\theta - 1)\), so that \(\theta = 3.5\) is associated with a markup of 40%, \(\theta = 6\) corresponds
The literature also provides a basis to quantify trade costs (τ). Smith and Venables (1988) estimate the tariff-equivalent of intra-EC trade barriers to be between 23 and 44%. Verwaal and Cnossen (2000) conclude that the compliance costs of new VAT and statistical requirements alone have a lower bound of 5% of the value of intra-EU trade. Rose and Van Wincoop (2001) find that the tariff-equivalent of not using the same currency is 26%. These findings in the literature explain the choice of the parameters in Table 1.

### Table 1: Transportation costs and home bias

<table>
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<th>home bias (β)</th>
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<tr>
<td>5%</td>
<td>23%</td>
<td>27%</td>
<td>2.8</td>
</tr>
<tr>
<td>20%</td>
<td>49%</td>
<td>1%</td>
<td>86.7</td>
</tr>
<tr>
<td>50%</td>
<td>50%</td>
<td>0%</td>
<td>&gt;200</td>
</tr>
</tbody>
</table>

to a markup of 20% and θ = 20 is associated with a markup of 5%. 
4.2. The model

Table 1 illustrates that the higher the price-elasticity of demand and the higher the costs of international trade, the larger the home bias. Small trade costs can cause a large home bias. For the average price-elasticity reported in the literature \((\theta = 5)\), a 20% cost is sufficient to lead to a substantial home bias \((\beta = 21\%)\). In the case of \(\theta = 20\), trade costs of just over five per cent are sufficient to generate a similar home bias.

The quantitative results in Table 1 are relevant only for large economic areas.\(^\text{17}\) But even for regions like the euro area and the United States, the quantitative results from a model with two countries of equal size can only give a very rough indication.

Several recent papers focus on the ratio of consumer spending on home-produced and imported goods, as defined by equation (4.23): the ‘spending ratio’. The spending ratios reported in the empirical literature vary greatly. McCallum (1995) finds that, after controlling for population size and distance, Canadian consumers are 22 times more likely to buy from a Canadian producer than from a US producer.\(^\text{18}\) At the other end of the spectrum, Wei (1996) reports a spending ratio of 2.5 for OECD countries, after controlling for economic size, distance between countries, geographic location and a possible linguistic tie. Wolf (1997) examines whether the home bias is also present at the sub-national level (i.e. whether it applies to trade between regions). He concludes affirmatively, but also finds that the international home bias is larger (by a factor five) than the intra-national home bias. A recent study by Head and Mayer (2000) puts the spending ratio for European countries at 12, down from 21 in the late 1970s.

The home bias \((\beta)\) has a one-to-one relationship with the share of foreign trade in national output \((1 - \delta)\). The latter variable is observable from a country's national accounts. For the euro area and the United States, the trade share amounts to 10-15% of GDP.

What reduction in the costs of international trade is required to explain the substan-

\(^\text{17}\)In this model, both countries account for half of world output, so that in the absence of trade costs (i.e. when \(\tau = 0\)), the share of trade in national income is 50%. Small open economies, such as the Netherlands and Belgium, account for only 1-2% of world output. Therefore, in the absence of trade costs, the share of trade in national income should be 98-99% for these countries. Thus, the home bias caused by the costs of international trade needs to be measured against another (higher) baseline value for small countries.

\(^\text{18}\)The finding of segmented national markets is confirmed by Engel and Rogers (1996) who study price dispersion across the US-Canadian border, rather than the volume of trade. They find that the distance between cities explains a significant amount of the variation in the prices of similar goods in different cities. But the variation of the price is much higher for two cities located in different countries than for two equidistant cities in the same country, even after taking into account such factors as the influence of the nominal exchange rate on the calculation of cross-border prices, the role of sticky prices and the relative homogeneity of labour markets within countries.
tial decline in the home bias for European countries between the late 1970s and the late 1990s reported by Head and Mayer? If $\theta = 5$ (the average price-elasticity reported in the literature) is believed to be realistic (it is probably even lower for luxury goods), then Head and Mayer's empirical finding that the spending ratio for European countries has declined from twenty-one to twelve implies that the cost of international trade has declined from 45% to 40% of the value of the traded products. However, if we believe $\theta = 20$ to be more realistic (as may be the case for commodities, like wheat), the implied decline in the cost of international trade is from 14% to 12%. This suggests that a relatively small decline (of two to five percentage points) in trade costs would be sufficient to explain the observed reduction in the home bias over a twenty-year period. But, again, one should be careful not to overstate the validity of the exact numbers for real-world countries.

Two results emerge from the considerations above. First, for realistic values of the price-elasticity parameter, small trade costs lead to a substantial home bias in international trade. Thus, small trade costs are sufficient to explain the empirical finding that only a relatively small part of income is spent on foreign-produced goods. Second, the substantial decline in the home bias for European countries since the late 1970s can be explained by a relatively small reduction in trade costs. The policy implication is the existence of a window of opportunities for the completion of the Single Market.

4.3 Loglinearising the model

The model does not yield simple closed-form solutions for general paths of exogenous variables, due to monopoly pricing and the endogeneity of output. Therefore, in order to study the dynamics, the model will be linearised around a symmetric steady state. The first step in this direction is deriving the solution for the initial symmetric steady state.

4.3.1 A symmetric steady state

In a steady state, all exogenous variables are constant. Steady-state values will be represented by overbars. It follows directly from (4.9) that real interest rate equality holds across countries in the steady state. The steady state world real interest rate $\bar{r}$ is\(^{19}\)

$$\bar{r} = \frac{1 - \beta}{\beta}.$$  \hspace{1cm} (4.25)

\(^{19}\)Equation (4.25) is equivalent to $\beta = \frac{1}{1-\bar{r}}$, showing that $\beta$ is the usual discount factor.
4.3. Loglinearising the model

From the household budget constraint (4.5), it follows that steady state consumption is (see Appendix C for derivation):

\[ C = rF + \frac{P^p}{P} Y, \]

(4.26)

where \( P^p \) is the output deflator and \( P \) is the consumption-based price index of goods in the Home country. The output deflator is the aggregate of price of domestically-produced goods. This measure is unaffected by trade costs. The consumption-based price index is affected not only by the output deflator \( P^p \), but also by import prices [which are related to the Foreign output deflator as in equation (4.20)].

In steady state, the current account must be balanced, but this is not necessarily the case for the trade account.\(^{20}\) Equation (4.26) shows that the home country can run a trade deficit in steady state \((P^C > P^pY)\),\(^{21}\) but only if it owns interest-bearing net foreign assets \((F > 0)\).\(^{22}\)

We now have a system of 20 independent equations in 21 endogenous variables (see Appendix D). Given that the model contains more endogenous variables than independent equations, there would be a multiplicity of solutions for the steady state.

Following Obstfeld and Rogoff (1995), we impose the starting condition of zero net foreign assets:

\[ F_0 = 0. \]

\(^{20}\)Equation (4.26) is the current account equation. It is derived by adding up the private budget constraint (which follows from substituting the expression for firm profits into the household’s budget constraint) and the government’s (balanced) budget constraint. Transportation costs do not affect the expression for the household budget constraint (4.5) and the government budget constraint (4.6), so this must also be true for the current account equation (4.26) (= the national budget constraint). However, transportation costs do affect the value of several variables in these constraints. Transportation costs also affect the change in the current account position \((dF/C_0)\) in response to exogenous shocks (see formula (4.50) below).

\(^{21}\)The share of melted exports is not included in the trade balance, since the melted goods are not sold to foreign residents and therefore do not generate income to domestic producers. Under optimal price setting, firm income is not affected by the size of the melting loss, since this is fully recovered by charging a higher price for the same product abroad. Therefore, as shown in footnote 10, firm income equals firm output and both can be represented by a single variable \(Y\).

\(^{22}\)Optimal pricing by firms implies that export prices are equal to domestic producer prices multiplied by a correction factor for trading costs \((\frac{1}{1+\tau} P^p)\) and that import prices (Foreign export prices) are equal to Foreign producer prices expressed in the Home currency multiplied by a correction factor for trading costs \((\frac{1}{1+\tau} X(P^p)^*)\) [see equations (4.19)-(4.20)]. The terms of trade equal export prices divided by import prices: \( P^p/X(P^p)^* \). The terms of trade are favourable if export prices exceed import prices.

The costs of international trade do not affect the terms of trade, because of the assumption that these costs are equal for both countries.
which closes the model in the sense that the solution is now uniquely determined.

Combining (4.11), (4.12), (4.15) yields

\[ Y_0 = \alpha \left( \frac{\theta - 1}{\theta \kappa} \right)^{\frac{1}{2}}, \]  

\[ \bar{C}_0 = \alpha \left( \frac{\theta - 1}{\theta \kappa} \right)^{\frac{1}{2}} \left\{ \frac{1}{2} [1 + (1 - \tau)^{\theta - 1}] \right\}^{\frac{1}{\theta - 1}}. \]

The expression for output corresponds to the result reported by Obstfeld and Rogoff (1995). Steady state output increases as the economy moves towards more competition \((\theta \uparrow)\). In the absence of transportation costs \((\tau = 0)\), output and consumption are equal in the steady state \((Y_0 = \bar{C}_0)\). However, in general, steady state consumption will be smaller than (or equal to) output, due to the presence of costs of international trade.\(^{23}\)

The shape of the money demand equation is unaffected by trade costs:

\[ \frac{M_0}{P_0} = \left( \frac{\chi}{1 - \beta} \right) \bar{C}_0. \]  

Using equation (4.26) and the starting condition of zero net foreign assets, it follows immediately that:

\[ \frac{\bar{C}_0}{Y_0} = \frac{\bar{P}_0^P}{\bar{P}_0} = \left\{ \frac{1}{2} [1 + (1 - \tau)^{\theta - 1}] \right\}^{\frac{1}{\theta - 1}} \leq 1. \]

Due to the presence of transportation costs, the consumption-based price index will be higher than the production-based price index in equilibrium: \(\bar{P}_0 > \bar{P}_0^P\). An increase in the costs of international trade affects the consumption-based price index \((\bar{P}_0)\) via two channels.\(^{24}\) First, it has an upward impact on the price of imports (as costs are passed on to the consumer). Second, an increase in trade costs enhances the home bias, thus reducing the import weight in total consumption. Whereas higher priced imports push up the consumption-based price index, the increase in the home bias has a moderating

\(^{23}\)Note that steady state output is not affected by the transportation costs. This is explained by the fact that higher transportation costs lead to lower Foreign import demand, but also lead to higher iceberg transportation losses, which require shipments in excess of Foreign import demand. The lower Foreign import demand is precisely offset by the higher excess shipments required. In terms of firm profits, lower Foreign import demand and higher export prices exactly offset each other. Thus, transportation costs are borne entirely by consumers. This feature greatly simplifies the algebra.

\(^{24}\)The level of the output deflator in the initial steady state \((\bar{P}_0^P)\) is not affected (directly or indirectly) by the level of transportation costs. Therefore, the ratio \(\bar{P}_0^P/\bar{P}_0\) is affected by transportation costs only via their impact on the consumption-based price index \(\bar{P}_0\).
impact. The first effect dominates. Thus, an increase in the costs of international trade unambiguously leads to an increase in the general price level. The implication is that the completion of the European Single Market (i.e. a reduction in trade costs) would lead to a permanent decline in the general price level (and a temporarily lower inflation rate).

It is directly clear from equations (4.1), (4.12), (4.27), (4.28) and (4.29) that a reduction in trade costs is welfare-enhancing in a steady state equilibrium: the equilibrium levels of consumption and real money balances increase, whereas the equilibrium level of work effort is unaffected.

### 4.3.2 Loglinearisation

To allow for asymmetries between the two countries, it is helpful to log-linearise the model around the initial steady state. Define $\tilde{Z}_t = dZ_t/Z_0$, i.e. variables with a hat denote percentage changes from the initial steady state. It will be assumed that the costs of international trade ($\tau$) are constant.

The labour demand function (4.21) is the easiest one to linearise. Taking logs, taking total differentials and evaluating at the initial steady state yields that changes in labour demand are proportional to changes in output

$$\tilde{L} = \tilde{Y}. \tag{4.30}$$

Similarly, it follows from the optimum pricing rule (4.15) that changes in the output price deflator are proportional to changes in the nominal wage

$$\tilde{P}^p = \tilde{W}. \tag{4.31}$$

The linearised form of the first-order conditions of the households’ intertemporal optimisation problem [(4.9), (4.10), (4.11)] is

$$\tilde{C}_{t+1} = (1-\beta)\tilde{r}_t + \tilde{C}_t, \tag{4.32}$$

$$\tilde{M}_t - \tilde{P}_t = \tilde{C}_t - \beta[\tilde{r}_t + \tilde{P}_{t+1} - \tilde{P}_t], \tag{4.33}$$

$$\tilde{L} = \tilde{W} - \tilde{P} - \tilde{C}. \tag{4.34}$$

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25 The second effect explains the fact that for $\tau \to 1$ (i.e. the product almost entirely 'melts' during transportation), the consumption-based price index does not go to infinity, but asymptotically approaches $P_0 = (2)^{\tau^{-1}} P^p_0$.

26 Neven, Norman and Thissé (1991) obtain a similar result in a different model.

27 It is quite natural to model the completion of the internal market as a reduction in the cost of intra-EU trade. See, for instance, Smith and Venables (1988).
The consumer price index is affected not only by the price of domestically-produced goods, but also by the price of imported goods. From equations (4.4) and (4.20), it follows that

\[ P = \left\{ \int_0^{\frac{1}{2}} p(z)^{1-\theta} dz + \int_{\frac{1}{2}}^1 \left[ \frac{X}{1-\tau} p^*(z) \right]^{1-\theta} dz \right\}^{\frac{1}{1-\theta}} = \]

\[ = \left\{ \frac{1}{2} (PP)^{1-\theta} + \frac{1}{2} \left[ \frac{X}{1-\tau} (PP)^* \right]^{1-\theta} \right\}^{\frac{1}{1-\theta}}. \]

where the second equality follows from symmetry among producers in the same country.

Linearisation of these equations yields the expressions for small percentage deviations of consumer prices from the initial steady state

\[ \hat{P} = \delta \hat{P}^P + (1 - \delta)(\hat{P}^P)^* + \hat{X}, \quad (4.35) \]

with \( \delta \) as defined in equation (4.24). The corresponding equation for the Foreign country is

\[ \hat{P}^* = \delta (\hat{P}^P)^* + (1 - \delta)[\hat{P}^P - \hat{X}], \quad (4.36) \]

An increase in the Home output price deflator leads to a less than proportional increase in the Home and Foreign consumer price indices (CPIs). Due to the presence of the (endogenous) home bias, an increase in the output price deflator has a larger impact on the Home price level than on the Foreign price level. The effect of exchange-rate movements on the CPIs depends on the trade costs: the larger the trade costs, the larger the home bias in spending, thus the smaller the effect of a given change in the exchange rate on the CPIs.

In the absence of trade costs \( (\tau = 0 \Rightarrow \delta = \frac{1}{2}) \), subtracting (4.36) from (4.35) yields the familiar purchasing power parity (PPP) relationship.

Linearising the expression for demand for individual goods (4.8), substituting the market clearing condition (4.13) and aggregating over all goods yields

\[ \hat{Y} = \delta \hat{C} + (1 - \delta)\hat{C}^* - \theta \left\{ \hat{P}^P - \delta \hat{P} - (1 - \delta)[\hat{P}^* + \hat{X}] \right\}. \quad (4.37) \]

Note that trade costs enter the equations (4.35), (4.36) and (4.37) of the log-linear model via \( \delta \) (which positively depends on trade costs and on the price-elasticity of demand).

## 4.4 Dynamics

This section studies the model's dynamics. Importantly, I assume that wages cannot adjust in the short run, but are fully flexible in the long-run. I will first discuss the
long-run steady state equilibrium, then the short-run equilibrium and finally the effect of international wealth transfers, which play an important role in the model's dynamics. This will provide some insight, before I turn to solving the model explicitly for money shocks in section 4.5.

4.4.1 Comparing steady states

The difference between national consumer price indices depends on the national output deflators and the exchange rate (see Appendix F for derivation):

\[ \frac{\Delta c^d}{P} = (2\delta - 1)(\frac{\Delta P}{P})^d + 2(1 - \delta)\frac{\Delta X}{X}. \] (4.38)

In the basic Obstfeld and Rogoff model, consumer spending is equally divided over Home and Foreign output. The somewhat unrealistic result of this assumption is that the difference between consumer price indices and the difference between national output deflators are uncorrelated in their model (as can be seen by setting \( \delta = \frac{1}{2} \) in the equation above). Here, the difference between consumer price indices and the difference between national output deflators are positively correlated \((2\delta - 1 > 0)\), as one would expect. The exchange rate has a positive, but less than proportionate \((0 < 2(1 - \delta) < 1)\) impact on the CPI difference.

The difference between national outputs is as follows:

\[ \frac{\Delta Y^d}{Y} = -4\delta(1 - \delta)\theta(\frac{\Delta P}{P})^d + 4\delta(1 - \delta)\theta\frac{\Delta X}{X} + (2\delta - 1)\frac{\Delta C}{C}. \] (4.39)

This equation leads to the following observations. The first term on the right-hand side indicates that relative output is declining in its own relative price: if the relative price of Home output increases, lower demand for the Home product will induce lower Home

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28 Recall that the initial steady state which is symmetric internationally. Long-run steady state changes, however, can be asymmetric.

29 For all OECD countries for which data are available, the difference between the change in the national GDP deflator and the change in the US GDP deflator is positively correlated to the difference between the change in the national consumption deflator and the change in the US consumption deflator over the 1961-2000 period. The (unweighted) average correlation coefficient is .84. This result appears robust for the choice of the benchmark country (here: the United States).

30 Note that the exchange rate is more than just a scaling factor in this model. The profit margin is constant: see pricing formula (4.15). Therefore, short-run wage rigidities directly translate into short-run output price rigidities. This implies there is a role for the exchange rate in facilitating the achievement of an equilibrium.
output. The existence of positive trade costs has a moderating impact on the own-price effect $|\delta > \frac{1}{2}$ implies $4\delta(1 - \delta)\theta < \theta$. The reason is that positive trade costs imply a home bias in consumer spending ($\tau > 0 \Rightarrow \delta > \frac{1}{2}$), so that the general price level is more strongly correlated with the price of domestic output. This limits the real price increase of Home output, which is the relevant price for consumer demand [see equation (4.8)]. The second term on the right-hand side shows that an appreciation of the Foreign currency ($\tilde{X} > 0$) will stimulate Home output, by reducing the price of Home output for Foreign consumers and raising the price of Foreign output for Home consumers. However, positive trade costs reduce the impact of exchange rate movements again: $\delta > \frac{1}{2}$ implies $4\delta(1 - \delta)\theta < \theta$. The intuition is that positive trade costs lead to a home bias in consumer spending, which means that a smaller share of consumption involves cross-border transactions (i.e. the transactions affected by exchange rate movements). The third term on the right-hand side in equation (4.39) is new. This term would not arise when consumer spending were equally divided over home and foreign output (i.e. when $\delta = \frac{1}{2}$). The presence of a home bias implies that most of an increase in consumer spending falls on the domestic economy. Therefore, a relative increase in Home consumer spending induces a larger positive impact on Home than on Foreign output. In the basic Obstfeld and Rogoff model, the consumption differential and the output differential are uncorrelated. Here, I obtain the more plausible result that there is a positive correlation between both ($2\delta - 1 > 0$).

4.4.2 Short-run equilibrium

Given a constant mark-up and constant labour productivity, the assumption of short-run nominal wage stickiness directly implies short-run stickiness of output prices [see equation (4.15)]. However, the consumption price index also incorporates imported goods. Therefore, it will change whenever the exchange rate changes (see Appendix

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31In the absence of trading costs ($\tau = 0 \Rightarrow \delta = \frac{1}{2}$), equation (4.39) reduces to $\hat{Y} = -\theta(\hat{P}^d) + \tilde{P} X$, as in Obstfeld-Rogoff (1995). Thus, an increase in the relative price of Home output will lead to a more than proportional decline of Home output (the coefficient is $\theta > 1$).

32More than half of the products sold in the Home market are produced in the Home country. Therefore, the weight of Home products in the Home CPI is more than one half. Note that, using equation (4.38), equation (4.39) can be rewritten as: $\hat{Y} = -\theta(\hat{P}^d) + 2(1 - \delta)\theta X + (2\delta - 1)\hat{P}^d + (2\delta - 1)\tilde{C}^d$.

33For all OECD countries for which data are available, the difference between national GDP growth and US GDP growth is positively correlated to the difference between national consumption growth and US consumption growth over the 1961-2000 period. The (unweighted) average correlation coefficient is .70. This result appears robust for the choice of the benchmark country (here: the United States).
4.4. Dynamics

F for the full short-run model):

\[ \hat{P}^d = 2(1 - \delta) \hat{X} \leq \hat{X}, \quad (4.40) \]

where the inequality follows directly from \( \delta \geq \frac{1}{2} \). Thus, the presence of trade costs implies that the exchange rate has a less than proportionate impact on the international CPI difference.\textsuperscript{34}

Short-run aggregate output is:

\[ \hat{Y}^d = 4\delta (1 - \delta) \theta \hat{X} + (2\delta - 1) \hat{C}^d. \quad (4.41) \]

For \( \delta = \frac{1}{2} \), this equation reduces to \( \hat{Y}^d = \theta \hat{X} \). In the more general form, the impact of the exchange rate on the output differential is reduced by the home bias (since the home bias in consumption reduces the relative importance of imported goods). This seems more realistic than the finding from the basic Obstfeld and Rogoff model that exchange rate movements have a strong impact on output differences. In the current model, the effect is still somewhat strong, but less so than in the basic model. Moreover, the home bias introduces a positive feedback of the consumption differential on the output differential \( (2\delta - 1 > 0) \), which adds another element of realism to the model.

4.4.3 International wealth transfers

International wealth transfers play an essential role in the model’s dynamics. In particular, a temporary shock may affect the long-run equilibrium via its impact on the international distribution of wealth. International wealth transfers are endogenously determined in the short run. I will discuss the impact of wealth transfers on the long-run equilibrium here, before solving the model explicitly for shocks to the money supply and the ensuing wealth shifts in the next section.

The model’s symmetry admits a simple solution approach. I first solve for differences between Home and Foreign variables and then for world aggregates. This approach also provides a better insight in the underlying intuition.

From the linearised versions of equation (4.26) and its Foreign counterpart:

\[ \hat{C}^d = 2 \hat{Y}^d \hat{F} + \hat{Y}^d (\hat{P}^d) - \hat{P}^d. \quad (4.42) \]

\textsuperscript{34}I obtain a similar result in chapter 5 of this thesis. However, a crucial difference is that the less than proportionate impact of the exchange rate on the general price level follows from the presence of non-tradable goods in the model in chapter 5, whereas it follows from the existence of positive trade costs in the current chapter.
If output were exogenous (i.e. when labour input were fixed), a wealth transfer equal to 1% of steady state consumption to the Home country would lead to a steady-state international consumption differential of $2\overline{r}$%: Home country residents would raise consumption by $\overline{r}$% and Foreign residents would reduce consumption by $\overline{r}$%, where $\overline{r}$ is the interest rate received on the wealth transfer. However, output is endogenous in this model: the net wealth transfer leads Home residents to work less and enjoy more leisure. Foreign country residents do the opposite, leading to the following (negative) international output differential:

$$\dddot{Y}^d = -\frac{\overline{r}dF}{C_0}.$$  \hspace{1cm} (4.43)

The endogeneity of output causes the consumption differential between the Home and Foreign country to be smaller than $2\overline{r}dF/C_0$.

The home country long-run terms of trade, given by

$$\dddot{TOT} = (\dddot{P}^d)^d - \dddot{X} = \frac{\delta}{(1-\delta)(2(\theta - 1)\delta + 1)} \frac{\overline{r}dF}{C_0},$$  \hspace{1cm} (4.44)

improve when the Home country receives a transfer. This improvement is driven by the labour-leisure decision. The Home country residents' decision to work less reduces Home output and therefore has an upward effect on the terms of trade. In the absence of trade costs ($\tau = 0$ and hence $\delta = \frac{1}{2}$), a wealth transfer of 1 percent of $C_0$ would lead to a change in the terms of trade by $\overline{r}/\theta\%$. For positive trade costs ($\tau > 0$ and hence $\delta > \frac{1}{2}$), the terms of trade improve by more than that. The intuition is that the additional Home demand caused by a wealth transfer falls mainly on Home-produced goods, thus re-inforcing the price increase of Home goods relative to the price of Foreign goods.

The impact of the terms of trade on consumption is reduced by the presence of positive trade costs:\footnote{Recall that the terms of trade are given by $P_N/P_M$. Using the optimal pricing formulas, the terms of trade can be rewritten as $P_N/P_M = P^d/[X(P^d)^+]$. The terms of trade are favourable if export prices exceed import prices. Thus, an increase of domestic output prices leads to an improvement in the terms of trade. See footnote 22.}

$$\overline{(P^d)^d} - \dddot{P}^d = \frac{2\delta}{2(\theta - 1)\delta + 1} \frac{\overline{r}dF}{C_0} = 2(1 - \delta)\dddot{TOT} \leq \dddot{TOT},$$  \hspace{1cm} (4.45)

where the inequality follows from $\tau \geq 0$ (which implies $\delta \geq \frac{1}{2}$). Intuitively, trade costs imply that only a relatively small share of output is traded, so that terms of trade changes
have a relatively small impact on overall consumption. However, this reduction in the terms of trade impact on consumption is dominated by the magnifying impact of trade costs on the size of the long-run terms of trade change [equation (4.44)].

From equations (4.42), (4.43) and (4.45), the consumption differential is

\[
\frac{\tilde{C}^d}{C^d} = \frac{2\theta \delta + 1}{2(\theta - 1)\delta + 1} \frac{\tilde{F}dF}{C_0} \geq \frac{\theta + 1}{\theta} \frac{\tilde{F}dF}{C_0},
\]

where, again, the inequality follows from \( \tau \geq 0 \) (which implies \( \delta \geq \frac{1}{2} \)). Thus, for positive trade costs, the consumption differential is larger than the value reported by Obstfeld and Rogoff (1995). The reason is that the home bias induces the additional consumption caused by a wealth transfer to be mainly spent on Home goods. Thus, any additional Home consumption benefits Foreign producers only to a limited extent and therefore does not generate income abroad to the same extent as when the additional consumption were equally divided over Home and Foreign goods.

## 4.5 Money shocks

This section focuses on monetary shocks, because the role of nominal rigidities is best illustrated in the case of monetary shocks and it is this kind of disturbance that flexible-price models are least well-equipped to handle. See Lane (2001). I will take the case of an unanticipated permanent increase in the Home money supply, i.e. \( \tilde{M} = \tilde{M}^d \).

### 4.5.1 Long-run impact

One implication for monetary policy follows directly from the previous subsection. As will be seen below, an expansion of the Home money supply induces a short-run depreciation of the Home currency and a short-run current account surplus for the Home country, which is balanced by a net transfer of financial assets from Foreign to Home. The accumulation of net foreign assets implies that, in the long run, Home residents can afford to consume more and work less. The long-run terms of trade improve (this is required to achieve long-run equilibrium on the current account), which reinforces the possibility for Home residents to consume more. Thus, equation (4.46) implies that a money supply shock, via the wealth effect, has an upward effect on the long-run consumption differential. In other words, money can have real effects in the long-run via the

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\(^{37}\)This can be seen by noting that \((\tilde{P}^d)^d - \tilde{P}^d\) is increasing in \(\delta\) (and thus increasing in \(\tau\)).
International trade costs imply that monetary policy becomes more effective with respect to the international consumption differential. The intuition is as follows. First, positive costs of international trade imply that a money impulse leads to a larger accumulation of net foreign assets. Trade costs cause a home bias in consumption, making the general price level less sensitive to the exchange rate [see equation (4.38)]. Therefore, a larger short-run exchange rate depreciation is required to reach equilibrium in the money market. The larger short-run exchange rate depreciation induces a larger current account surplus, i.e. more net foreign assets are accumulated. The additional interest income on net foreign assets facilitates a larger long-run consumption differential. Secondly, the home bias implies that the additional Home consumption caused by the wealth transfer is mainly spent on Home goods (i.e. there is a smaller ‘import leak’). This widens the consumption differential further. Thus, the presence of trade costs reinforces the transmission from money to consumption via the accumulation of net foreign assets and via the terms of trade.

Next, we turn to the international price differential. Combining the steady-state version of equation (4.33) with equation (4.46) yields:

$$\hat{P}^d = \hat{M}^d - \frac{2\delta + 1}{2(\theta - 1)} \frac{\tau dF}{\delta + 1} \leq \frac{\hat{M}^d}{\theta} - \frac{\theta + 1}{\theta} \frac{\tau dF}{\delta + 1},$$

(4.47)

where, again, the inequality follows from $\tau \geq 0$ (which implies $\delta \geq \frac{1}{2}$). The wealth redistribution effect which causes money to affect long-run real consumption differentials, also implies that the long-run price differential changes less than proportionately to a permanent money shock: $\hat{P}^d < \hat{M}^d$.

The presence of positive trade costs reduces the influence of monetary policy on the international price differential. The intuition is as follows. A monetary impulse affects

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38 The long-run effect of money on consumption should not be overstated. It is in the order of magnitude of the real interest rate, which is explained by the fact that it derives from the yield on net foreign assets accumulated in the short run.

39 The larger short-run exchange rate depreciation induced by a higher value of $\delta$ leads to a larger output differential, even though a higher $\delta$ also reduces the impact of the exchange rate on output. The short-run current account balance can be written as: $dF/\hat{C}_0 = \hat{Y}^d - \hat{M}^d$, showing that a larger output differential implies a larger short-run surplus on the current account.

40 Technically, the first effect is that the value of $dF/\hat{C}_0$ in equation (4.46) is increasing in $\delta$ [as will be shown in equation (4.50) below], whereas the second effect is that the coefficient of $dF/\hat{C}_0$ in equation (4.46) is increasing in $\delta$.

41 Technically, this is the flip-side of the higher effectiveness of money with respect to consumption. Long-term equilibrium in the money market requires that a permanent increase in the money supply is
prices via wages and via the exchange rate.\footnote{From equations (4.31) and (4.38), the international price differential is a weighted average of the international wage differential and the change in the exchange rate: $\nabla^d = a\nabla W + (1 - a)\nabla X$, where $a = 26 - 1$.} Trade costs imply that more net foreign assets are accumulated after a Home money expansion (see above). The larger accumulation of net foreign assets implies a larger net demand for Foreign-produced products. The larger net demand for Foreign products means that a larger improvement in the Home terms of trade is required, which is achieved via a smaller long-run depreciation of the Home currency [see equation (4.44)]. Thus, trade costs reduce the impact of a Home money shock on the exchange rate. Therefore, trade costs also reduce the effectiveness of a money shock with respect to the international price differential.

We have seen that positive costs of international trade enhance the effectiveness of monetary policy with respect to consumption and reduces the effectiveness of monetary policy with respect to the general price level. It follows that in an environment of declining costs of international trade, monetary policy becomes less effective with respect to consumption, but more effective with respect to the general price level.

Trade costs do not affect the long-run value of world aggregates (see Appendix F).\footnote{This does not imply that trade costs only have distributive effects in this model. Recall that in this section, I consider small changes in variables. From equation (4.28), trade costs do have a negative impact on the equilibrium value of consumption.} As with zero trade costs, money is neutral at the world level in the long run. World money shocks will translate one-for-one into price increases at the world level.

### 4.5.2 Short-run impact

Next, turn to the short run. Recall that I consider the impact of an unanticipated permanent shock to the Home money supply.

In the short run, wages cannot adjust to money shocks. As a result, monetary policy is able to affect output and consumption. As shown in Appendix F, trade costs do not affect the short-run change in world aggregates.

The reduced-form solutions for international differences are:

- fully reflected in an increase in the nominal value of consumption. In other words, equilibrium requires that a money shock is distributed over real consumption and prices. Trade costs make monetary policy more effective with respect to international consumption (see above). Therefore, positive costs of international trade must reduce the effectiveness of monetary policy with respect to the general price level.
It is easy to show that $\hat{Y}^d > \hat{C}^d$. The intuition is that output becomes demand-determined when wages are rigid. Under monopolistic competition, prices are set above the marginal cost of production. Therefore, at the margin, it is profitable for firms to accommodate additional demand by producing more output.\textsuperscript{44} As we shall see below, a monetary expansion in the home country causes a depreciation of the home currency ($\hat{X} > 0$). This induces net foreign demand for Home goods ($\hat{Y}^d > \hat{C}^d$). Both $\hat{Y}^d$ and $\hat{C}^d$ are increasing in $\delta$, i.e. the output differential and consumption differential increase in trade costs. The intuition is as follows. First, we have already seen that the long-run consumption differential is increasing in trade costs. Intertemporal consumption smoothing implies that the short-run consumption differential must also be increasing in trade costs. Secondly, a larger short-run exchange rate depreciation is required to achieve money market equilibrium. The larger exchange-rate depreciation leads to a larger net foreign demand for Home goods, i.e. the net foreign demand for Home goods is also increasing in trade costs. If the short-run consumption differential and the net foreign demand for Home goods are both increasing in trade costs, this must also be the case for the output differential.

The short-run current account (which equals the change in net foreign assets) has the following reduced form (see Appendix F):

$$\frac{d\hat{F}}{\hat{C}_0} = \frac{2\delta\theta - 1}{2 + \bar{r}(2\delta\theta + 1)} \hat{M}^d. \quad (4.50)$$

An expansion of the Home money supply leads to a short-run surplus on the Home current account.\textsuperscript{45} The larger the trade costs ($\tau$), the larger $\delta$, the larger the short-run current account surplus [the coefficient of $\hat{M}^d$ on the right-hand side of equation (4.50) is increasing in $\delta$]. The intuition is that the larger the home bias in consumption, the larger the short-run exchange rate depreciation required to reach equilibrium in the money market (as the price differential become less responsive to the exchange rate).

\textsuperscript{44}As stated, this is only true at the margin, i.e. for small increases in demand. See Blanchard and Kiyotaki (1987).

\textsuperscript{45}In the special case $\delta = \frac{1}{2}$ (no trading costs, hence no home bias), the expression reduces to $\frac{d\hat{F}}{\hat{C}_0} = \frac{\theta - 1}{2 + \bar{r}(1 + \theta)} \hat{M}^d$, as found by Obstfeld-Rogoff (1995).
The larger exchange rate depreciation induces a larger current account surplus. Observe that the current account effect is larger, despite the fact that trade flows are depressed by the presence of trade costs.

### 4.5.3 Exchange rate dynamics

The exchange rate plays a substantial role in this model. The reason is that the assumptions of short-run nominal wage rigidity and a constant mark-up directly imply that short-run relative prices ($\hat{P}^d$) can only change due to exchange-rate movements. As pointed out before, the impact of a given exchange rate movement on the price differential is smaller, the larger the home bias.

The long-run solution for the exchange rate is:

$$\hat{X} = \frac{\hat{X}^d}{M} = \frac{1}{1 + \frac{\delta}{(1 - \delta)[2(\theta - 1)\delta + 1]}} \frac{\bar{\tau}_d\bar{F}}{C_0} < \frac{\hat{X}^d}{M} - \frac{1 + \theta \bar{\tau}_d\bar{F}}{C_0},$$

where the expression on the right-hand side of the inequality is the solution in the absence of trade costs ($\tau = 0 \Rightarrow \delta = \frac{1}{2}$) and the inequality itself follows from the presence of positive trade costs ($\tau > 0 \Rightarrow \delta > \frac{1}{2}$). Thus, the home bias in consumption reduces the impact of money shocks on the long-run value of the exchange rate. The reduced-form solution is

$$\hat{X} = \frac{1}{2 + \frac{\tau}{(2\delta - 1)}} \left\{ \frac{2(1 + \tau)}{(1 - \delta)[2\delta(\theta - 1) + 1]} \right\} \frac{\hat{X}^d}{M}. \tag{4.51}$$

It is straightforward to show that the Home currency will show a long-run depreciation in response to a money shock, unless $\delta$ is sufficiently large, that is unless trade costs are above a certain level (see below).

The short-run solution for the exchange rate is

$$\hat{X} = \frac{1}{2 + \frac{\tau}{(2\delta + 1)}} \left\{ \frac{1}{1 - \delta} + \frac{\bar{\tau}(2\delta + 1)}{2\delta(\theta - 1) + 1} \right\} \frac{\hat{X}^d}{M}. \tag{4.52}$$

It is easy to show that exchange rate overshooting occurs in this model (i.e. $\hat{X} < \hat{X}$; see Appendix H). Theoretical explanations for the empirical observation that exchange rate movements are much more volatile than goods prices go back to Dornbusch (1976). He presents an IS-LM model with sticky prices and rational expectations. The intuition behind his exchange rate overshooting result is that, under short-run price stickiness, an expansion of the money supply must be followed by a decline in interest rates in order to restore equilibrium in the money market. The lower level of Home interest rates implies that, under rational expectations, agents must expect an appreciation of the
domestic currency. However, long-run equilibrium requires a net depreciation. This can only happen if the exchange rate overshoots in the short run. The basic Obstfeld and Rogoff (1995) model with tradables only has no exchange rate overshooting, but several extensions of this model do. As in Dornbusch’s paper, the overshooting result in the New Open Macro-economics literature depends on the assumption that goods prices (or wages) adjust slowly relative to financial asset prices. In Hau (2000) and chapter 5 of this thesis, overshooting is caused by the presence of non-tradables. Given wage stickiness and a constant mark-up, the price of Home and Foreign non-tradables cannot adjust in the short run. Thus, the adjustment of Home and Foreign general price levels depends on exchange rate changes and money market equilibrium requires a relatively large short-run adjustment of the nominal exchange rate. In these papers, exchange rate volatility is inversely related to openness (the share of tradables). The present model contains only tradable goods, as in Warnock (1999). There, a bias in consumer preferences causes a home bias in consumer spending. Here, a home bias in consumer spending is caused by the presence of trade costs. The intuition behind the overshooting in this chapter and Warnock (1999) is similar to Hau (2000) and chapter 5: output prices cannot adjust in the short run, so the relative price level can only change due to short-run exchange rate changes. A larger home bias in spending (caused by either asymmetric preferences or trade costs) implies that a relatively small share of goods are affected by exchange rate movements. Thus, a larger short-run exchange rate movement is required to attain short-run money market equilibrium.

When the costs associated with international trade are substantial, short-run exchange rate movements in response to monetary shocks can become extremely large. Thus, the presence of international trade costs can explain large short-run exchange rate volatility. In fact, when trade costs are sufficiently large ($\delta \rightarrow 1$), the short-run depreciation of the Home currency in response to a Home money shock becomes so large ($\widehat{X} \gg 0$) that a long-run appreciation of the Home currency ($\widehat{X} < 0$) is required in order to achieve general equilibrium. Thus, the impact of money on the long-run exchange rate is ambiguous in general. In Table 2 illustrates these characteristics numerically.

\[ More precisely, given $\theta$ and $\delta$, there is a unique admissible value of $\tau$ for which $\frac{d\widehat{X}}{d\widehat{M}} = 0$. For lower trading costs, the Home currency shows a long-run depreciation in response to an increase in the Home money supply ($\frac{d\widehat{X}}{d\widehat{M}} > 0$). For higher trading costs, the Home currency shows a long-run appreciation in response to an increase in the Home money supply ($\frac{d\widehat{X}}{d\widehat{M}} < 0$). See Appendix G for an informal proof. \]
4.5. Money shocks

Table 2: Exchange rate response to a 1% Home money shock, under positive costs to international trade

<table>
<thead>
<tr>
<th>Trade cost</th>
<th>Short-run XR response</th>
<th>Long-run XR response</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$\hat{X}$</td>
<td>$\hat{X}$</td>
</tr>
<tr>
<td>$\theta = 3$ :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.00%</td>
<td>0.96%</td>
</tr>
<tr>
<td>20%</td>
<td>1.22%</td>
<td>0.95%</td>
</tr>
<tr>
<td>50%</td>
<td>2.32%</td>
<td>0.90%</td>
</tr>
<tr>
<td>70%</td>
<td>5.54%</td>
<td>0.79%</td>
</tr>
<tr>
<td>$\theta = 5$ :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.0%</td>
<td>0.9%</td>
</tr>
<tr>
<td>20%</td>
<td>1.6%</td>
<td>0.9%</td>
</tr>
<tr>
<td>50%</td>
<td>7.4%</td>
<td>0.7%</td>
</tr>
<tr>
<td>70%</td>
<td>53.5%</td>
<td>-0.7%</td>
</tr>
<tr>
<td>$\theta = 10$ :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.1%</td>
<td>0.8%</td>
</tr>
<tr>
<td>20%</td>
<td>3.3%</td>
<td>0.7%</td>
</tr>
<tr>
<td>50%</td>
<td>&gt;100%</td>
<td>-5.0%</td>
</tr>
<tr>
<td>$\theta = 20$ :</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>1.3%</td>
<td>0.7%</td>
</tr>
<tr>
<td>20%</td>
<td>21.9%</td>
<td>-0.0%</td>
</tr>
<tr>
<td>50%</td>
<td>&gt;100%</td>
<td>&lt; -100%</td>
</tr>
</tbody>
</table>

In all the results in Table 2, I have assumed $\bar{r} = 3\%$ and $dM^d = 1\%$. As can be seen in the Table, the exchange rate response to an increase in the Home money supply is quite sensitive to the elasticity of substitution ($\theta$) and the costs of international trade ($\tau$). Let us take the costs of international trade to be 20% of the value of traded goods. Then, if we regard $\theta = 5$ as plausible, a one percent increase in the Home money supply will lead to a 0.9% increase in the long-run exchange rate. Short-run exchange rate overshooting will occur, but only to a limited degree: the short run exchange rate will
increase by 1.6%. If we regard $\theta = 20$ to be more plausible, a one percent increase in the Home money supply will have a negative, but almost negligible impact on the long-run value of the exchange rate. However, the short-run exchange rate will increase by over 20%. Table 2 also shows that a long-run depreciation of the Home currency ($\widetilde{X} > 0$) is far more plausible than a long-run appreciation ($\widetilde{X} < 0$).

When the costs associated with international trade are substantial, short-run exchange rate movements in response to monetary shocks can become extremely large. Thus, the presence of costs to international trade and imperfect competition can explain quite high short-run exchange rate volatility in response to money shocks.

### 4.6 Conclusion

This chapter has studied the consequences of the remaining barriers to cross-border competition. This shed some light on the implications of Europe’s single market for the home bias in consumption and for the effectiveness of the ECB’s monetary policy. The analysis is conducted in the context of the Obstfeld and Rogoff (1995) framework, which allows for imperfect competition and nominal rigidities. I explicitly incorporate ‘transportation costs’ for trade between two countries into the model. The introduction of costs for international trade implies that the model deviates from the standard Obstfeld and Rogoff (1995) model. This has several implications. First, the law of one price need not hold. Second, trade costs lead to an endogenous home bias in consumer spending.

This approach leads to the following results. First, for realistic values of the price-elasticity parameter, small trade costs lead to a substantial home bias in consumption. This result is in line with the more general finding in the literature that even small transaction costs can have significant economic effects. The policy implication is that a small reduction in trade costs, such as in the context of the Single Market programme, may have substantial effects.

Second, the presence of international trade costs in the goods market can explain large short-run exchange rate volatility. The intuition is as follows. As a result of short-run rigidities, the short-run relative price level can only adjust when the exchange rate changes. Costs of international trade lead to a home bias in spending, which implies that only a relatively small share of goods is affected by the exchange rate. As a result, large short-run exchange rate movements may be required to attain short-run money market equilibrium.

Third, the Single Market initiative may have important consequences for the transmission of ECB monetary policy. More specifically, it causes monetary policy to become
less powerful in terms of stabilising consumption, but more effective in terms of influencing the general price level. The intuition behind this result is that the presence of positive costs of international trade lead to a home bias in spending, which reinforces the transmission from money to consumption via the accumulation of net foreign assets and the terms of trade. Therefore, a decline in the costs of international trade reduces the effectiveness of monetary policy with respect to consumption. The effectiveness of monetary policy with respect to the general price level is the flip-side of its effectiveness with respect to consumption (this follows from the long-run money market equilibrium conditions). Therefore, a decline in the costs of international trade implies that monetary policy becomes more effective with respect to the general price level.
Appendices

A First-order conditions for households

The representative household maximises life-time utility (4.1), subject to the period budget constraint (4.5). The Lagrangian function is:

\[
L_t = \sum_{s=t}^{\infty} \beta^{s-t} \left[ \log C_s + \chi \log \left( \frac{M_s}{P_s} \right) - \frac{\kappa}{2} L_s^2 \right] + \\
+ \lambda_s [P_s F_s + M_s - P_s (1 + r_{s-1}) F_{s-1} - M_{s-1} - W_s L_s - \Pi_s + \\
+ P_s C_s + P_s T_s].
\]

The first-order conditions are

\[
\frac{\partial L_t}{\partial C_s} = \beta^{s-t} \frac{1}{C_s} + \lambda_s P_s = 0, \\
\frac{\partial L_t}{\partial M_s} = \beta^{s-t} \frac{\chi}{M_s} + \lambda_s - \lambda_{s+1} = 0, \\
\frac{\partial L_t}{\partial L_s} = -\beta^{s-t} \kappa L_s - \lambda_s W_s = 0, \\
\frac{\partial L_t}{\partial F_s} = \lambda_s P_s - \lambda_{s+1} P_{s+1} (1 + r_s) = 0.
\]

Combining these conditions, eliminating \( \lambda \) and using equations (4.4) and (4.7) yields equations (4.9)-(4.11) in the main text.

The transversality condition

\[
\lim_{T \to \infty} R_{t,t+T} \left( F_{t+T} + \frac{M_{t+T}}{P_{t+T}} \right) = 0,
\]

with

\[
R_{t,t+T} = \left( \prod_{s=t}^{T} (1 + r_{s-1})^{s-t} \right)^{-1}
\]

guarantees that the marginal present value of an additional unit of bond and money holdings at the end of the decision horizon is zero.\(^{47}\)

B First-order conditions for firms

The representative firm sets the prices of its products so as to maximise profits

$$\max_{\{p(z), p^*(z)\}} \Pi(z) = p(z)c(z) + Xp^*(z)c^*(z) - Wl(z).$$

Substituting the goods market clearing condition (4.13) and the production function (4.12) into the profit function yields

$$\Pi(z) = [p(z) - \frac{W}{\alpha}]c(z) + [Xp^*(z) - \frac{W}{\alpha(1 - \tau)}]c^*(z)$$

It is assumed that the firm takes aggregate demand as given. Thus:

$$\frac{\partial C}{\partial p(z)} = \frac{\partial C^*}{\partial p^*(z)} = 0.$$ 

It is also assumed that Home prices do not affect Foreign demand and vice versa (that is: unless the price difference creates room for arbitrage, but it will turn out that it is optimal for firms to set local prices so that there is no room for arbitrage [assuming that all other agents face trade costs which are at least as large as those faced by firms]). Thus:

$$\frac{dc(z)}{dp^*(z)} = \frac{dc^*(z)}{dp(z)} = 0.$$ 

Then, the first-order conditions are

$$\frac{d\Pi(z)}{dp(z)} = [p(z) - \frac{W}{\alpha}] \frac{dc(z)}{dp(z)} + c(z) = 0,$$

$$\frac{d\Pi(z)}{dp^*(z)} = [Xp^*(z) - \frac{W}{\alpha(1 - \tau)}] \frac{dc^*(z)}{dp^*(z)} + c^*(z) = 0,$$

It follows from the goods demand function (4.8) and its foreign counterpart that

$$\frac{dc(z)}{dp(z)} = -\theta \frac{c(z)}{p(z)},$$

$$\frac{dc^*(z)}{dp^*(z)} = -\theta \frac{c^*(z)}{p^*(z)},$$

so that the first-order conditions for the representative Home firm can be simplified to the conditions (4.15)-(4.16) in the main text.
C Steady-state consumption

Equation (4.26) can be derived as follows. In the steady-state the individual household’s budget constraint (4.5) reduces to:

\[
\overline{PF} + \overline{M} = \overline{P}(1 + \tau)\overline{F} + \overline{M} + \overline{WL} + \overline{\Pi} - \overline{FC} - \overline{PT}.
\]

Eliminate common terms, divide both sides by \(\overline{P}\) and note that the government budget must be balanced in steady state. Then:

\[
\overline{C} - \left(\frac{\overline{W}}{\overline{P}}\overline{L} + \frac{\overline{\Pi}}{\overline{P}}\right) = \tau \overline{F},
\]

i.e. domestic spending minus domestic income equals interest earned on net foreign assets. By definition

\[
\overline{\Pi} = \int_0^1 \pi(z)dz = \int_0^1 \left[p(z)c(z) + Xp^*(z)c^*(z) - WL(z)\right]dz =
\]

\[
= \int_0^1 \left[p(z)c(z) + \frac{\overline{p}(z)}{\overline{X}(1 - \tau)}c^*(z) - WL(z)\right]dz =
\]

\[
= \int_0^1 \left[p(z)c(z) + \frac{1}{1 - \tau}c^*(z)\right] - WL(z)\right]dz =
\]

\[
= \int_0^1 \left[p(z)\gamma(z) - WL(z)\right]dz =
\]

\[
= \frac{\overline{PP}}{\overline{P}}Y - WL,
\]

where the final equality follows from symmetry among Home country producers.

The goods price paid by Home country consumers is a weighted average of the price received by Home country producers and the price of imported goods.

Substituting the definition of profits into the equation above implies

\[
\overline{C} = \tau \overline{F} + \frac{\overline{PP}}{\overline{P}}\overline{Y}.
\]

The corresponding condition for the Foreign country is

\[
\overline{C}^* = \tau^* \overline{F}^* + \frac{(\overline{PP})^*}{\overline{P}^*}\overline{Y}^*.
\]

Using world real interest equality (4.25) and the fact that world net foreign assets must be zero (i.e. \(\overline{F} + \overline{F}^* = 0\)), the latter equation can be rewritten as

\[
\overline{C}^* = -\tau \overline{F} + \frac{(\overline{PP})^*}{\overline{P}^*}\overline{Y}^*.
\]
4.6. Appendices

D Determinacy of the initial steady state

The following ten equations hold at the level of individual countries: (4.4), (4.7), (4.9), (4.10), (4.11), (4.13), (4.14), (4.15), (4.21), (4.26). In the steady state, using the assumption of symmetry among producers in each country, the equations for the Home country are:

\[
\bar{P} = \left[ \frac{1}{2} \left[ P^p \right]^{1-\theta} + \frac{1}{2} \left[ \frac{\bar{X}}{1 - \tau} (P^p)^* \right]^{1-\theta} \right]^{\frac{1}{1-\theta}}, \\
\bar{i} = \bar{r}, \\
\bar{r} = \frac{1 - \beta}{\beta}, \\
\frac{M}{\bar{P}} = \chi \bar{C} \left( \frac{1 + \bar{i}}{\bar{i}} \right), \\
\bar{L} = \frac{1}{\kappa} \frac{1}{\bar{P}} \frac{1}{\bar{C}}, \\
\bar{Y} = \frac{1}{2} \left( \frac{\bar{P}^p}{\bar{P}} \right)^{-\theta} \bar{C} + \frac{1}{2} (1 - \tau)^{\theta-1} \left( \frac{\bar{P}^p}{\bar{X} \bar{P}^*} \right)^{-\theta} \bar{C}^*, \\
\bar{\Pi} = \bar{P}^p \bar{Y} - \bar{W} \bar{L}, \\
\bar{P}^p = \left( \frac{\theta}{\theta - 1} \right) \frac{\bar{W}}{\alpha}, \\
\bar{L} = \frac{1}{\alpha} \frac{\bar{Y}}{\bar{i}}, \\
\bar{C} = \bar{r} \bar{F} + \frac{\bar{P}^p}{\bar{P}} \bar{Y}.
\]

Therefore, we have a total of (2x10 = 20) equations. The above system of equations contains 21 endogenous variables: ten for the Home country: \( \bar{Y}, \bar{C}, \bar{P}, \bar{P}^p, \bar{W}, \bar{L}, \bar{\Pi}, \bar{F}, \bar{r}, \bar{i} \), ten for the Foreign country: \( \bar{Y}^*, \bar{C}^*, \bar{P}^*, (\bar{P}^p)^*, \bar{W}^*, \bar{L}^*, \bar{\Pi}^*, \bar{F}^*, \bar{r}^*, \bar{i}^* \) and one variable that relates to both countries: \( \bar{X} \).

As in Obstfeld and Rogoff (1995), the model is closed by imposing the starting condition of zero net foreign assets:

\( \bar{F}_0 = 0. \)

E Determinacy of the loglinearised model

In this Appendix, I will first consider the long-run. Thereafter, I will consider the equations for the short-run and the equations which relate both periods to each other.
In the long-run, the level of the net foreign assets $\bar{F}$ is predetermined. There are seven equations for the Home country: (4.30), (4.31), (4.33), (4.34), (4.35), (4.37) and the linearised version of the current account equilibrium condition (4.26). Similarly, there are seven corresponding equations for the Foreign country. Denoting long-run steady state changes by hatted overbars, the equations for the Home country are:

$$\hat{P}^p = \hat{W},$$
$$\hat{Y} = \delta \left\{ -\theta [\hat{P}^p - \hat{P}] + \hat{C} \right\} + (1-\delta) \left\{ -\theta [\hat{P}^p - \hat{X} - \hat{P}^*] + \hat{C}^* \right\} =$$
$$= \delta \hat{C} + (1-\delta)\hat{C}^* - \theta \left\{ \hat{P}^p - \delta \hat{P} - (1-\delta)[\hat{P}^* + \hat{X}] \right\},$$
$$\hat{L} = \hat{W} - \hat{P} - \hat{C},$$
$$\hat{\bar{M}} - \hat{P} = \hat{C},$$
$$\hat{\bar{P}} = \delta \hat{P}^p + (1-\delta)[\hat{X} + (\hat{P}^p)^*],$$
$$\hat{\bar{C}} = \frac{\delta \bar{F}}{\bar{C}} + \hat{P}^p - \hat{P} + \hat{Y}.$$

The above system consists of 14 equations. One of the equations also follows directly from combining other equations in the system (this is seen most easily by writing the system in terms of world aggregates and country differences). Thus, there are 13 independent equations.

In the long run, there are 13 endogenous variables: six for the Home country: $\hat{Y}, \hat{C}, \hat{P}, \hat{P}^p, \hat{W}, \hat{L}$, six for the Foreign country: $\hat{Y}^*, \hat{C}^*, \hat{P}^*, (\hat{P}^p)^*, \hat{W}^*, \hat{L}^*$ and one that relates to both countries: $\hat{X}$. Thus, the subsystem of long-run equations is exactly determined.

Next, turn to the short run. The short-run nominal wage rate is fixed. There are four equations for Home: (4.30), (4.31), (4.35), (4.37) and four corresponding equations for Foreign. Denoting short-run changes by hatted variables (no overbars), the short-run equations for Home are

$$\hat{P}^p = 0,$$
$$\hat{L} = \hat{Y},$$
$$\hat{P} = \delta \hat{P}^p + (1-\delta)[\hat{X} + (\hat{P}^p)^*] =$$
$$= (1-\delta)\hat{X},$$
$$\hat{Y} = \delta \left\{ -\theta [\hat{P}^p - \hat{P}] + \hat{C} \right\} + (1-\delta) \left\{ -\theta [\hat{P}^p - \hat{X} - \hat{P}^*] + \hat{C}^* \right\} =$$
$$= \delta \hat{C} + (1-\delta)\hat{C}^* + 2\theta \delta (1-\delta)\hat{X}.$$

Three equations for each country relate the short run to the long run: (4.32), (4.33) and
the linearised version of the current account definition \( dF = (P^P/P)Y - C \):

\[
\begin{align*}
\hat{C} - \hat{C} &= (1 - \beta)\hat{r}, \\
\hat{M} - \hat{P} &= \hat{C} - \beta\hat{r} - \frac{\beta}{1 - \beta}(\hat{P} - \hat{P}), \\
\frac{d\hat{F}}{\hat{C}_0} &= \hat{Y} - \hat{C} - \hat{P}.
\end{align*}
\]

The above system consists of \((2 \times 4 + 2 \times 3 = 14)\) equations. One of the equations can be derived from combining other equations in the system (this is seen most easily by writing the system in terms of world aggregates and country differences). Thus, again, there are 13 independent equations. There are 13 endogenous variables: five for the Home country: \( \hat{Y}, \hat{C}, \hat{P}, \hat{P}^p, \hat{L} \), five for the Foreign country: \( \hat{Y}*, \hat{C}*, \hat{P}*, (\hat{P}^p)*, \hat{L}* \) and three variables that relate to both countries: \( \hat{X}, \hat{r} \) and \( d\hat{F} \). Thus, the subsystem of short-run equations is exactly determined.

**F Solution of the loglinearised model**

**F.1 Country differences and world aggregates**

It is straightforward to rewrite the equations of the loglinearised model in terms of differences between Home and Foreign variables and in terms of world aggregates. Define: \( x^d = x - x^* \) and \( x^w = \frac{1}{2}(x + x^*) \), for any variable \( x \).

The long-run equations can be rewritten as six independent equations in world aggregates:

\[
\begin{align*}
\hat{Y}^w &= \hat{L}^w, \\
(\hat{P}^P)^w &= \hat{W}^w, \\
\hat{M}^w - \hat{P}^w &= \hat{C}^w, \\
\hat{L}^w &= \hat{W}^w - \hat{P}^w - \hat{C}^w, \\
\hat{P}^w &= (\hat{P}^P)^w, \\
\hat{Y}^w &= \hat{C}^w,
\end{align*}
\]

and seven independent equations in country differences:
Note that \( d\overline{F} \) is predetermined (in the short run, as will be seen below): the short-run trade balance determines the transfer of net foreign assets \( (d\overline{F}) \) which, combined with the requirement that the long-run current account must be balanced, determines the long-run trade balance. Thus, there are 13 independent equations in 13 endogenous variables: \( \hat{Y}^w, \hat{C}^w, \hat{P}^w, (\hat{P}^p)^w, W, L, \hat{Y}, \hat{C}, \hat{P}, (\hat{P}^p)^d, \hat{M}, L, \hat{X} \).

Next, turn to the short run and recall that nominal wages are fixed in the short run. The short-run equations (and the equations relating the short and long run) can be rewritten as six independent equations in world aggregates:

\[
\begin{align*}
\hat{Y}^w &= \hat{L}^w, \\
(\hat{P}^p)^w &= 0, \\
(\hat{P})^w &= 0, \quad \hat{Y}^w = \hat{C}^w, \\
\hat{C}^w - \hat{\overline{C}}^w &= (1 - \beta) \hat{\overline{r}}, \\
\hat{M}^w - \hat{\overline{P}}^w &= \hat{C}^w - \beta \hat{\overline{r}} - \frac{\beta}{1 - \beta} (\hat{P}^w - \hat{\overline{P}}^w),
\end{align*}
\]

and seven independent equations in terms of country differences:

\[
\begin{align*}
\hat{Y}^d &= \hat{L}^d, \\
(\hat{P}^p)^d &= 0, \quad \hat{P}^d = 2(1 - \delta) \hat{X}.
\end{align*}
\]
4.6. Appendices

\[ \hat{Y}^d = 4\delta(1 - \delta)\theta \hat{X} + (2\delta - 1)\hat{C}^d, \]
\[ \hat{C}^d - \hat{D}^d = 0, \]
\[ \hat{M}^d - \hat{P}^d = \hat{C}^d - \frac{\beta}{1 - \beta} (\hat{P}^d - \hat{P}^d), \]
\[ 2 \frac{dF}{C_0} = \hat{Y}^d - \hat{C}^d + (\hat{P}^d)^d - \hat{P}^d. \]

Note that \( \hat{P}^d \) and \( \hat{C}^d \) are determined in the long run (see above). Thus, there are 13 independent equations in 13 endogenous variables: \( \hat{Y}^w, \hat{C}^w, \hat{P}^w, (\hat{P}^p)^w, \hat{L}^w, \hat{Y}^d, \hat{C}^d, \hat{P}^d, \]
\( (\hat{P}^p)^d, \hat{L}^d, \hat{X}, dF, \hat{r}. \)

**F.2 Long-run and short-run solution for shocks**

Now solve the model for a permanent money shock in the Home country \( (\hat{M} = \hat{M}) \). The solution of the model is given below. The results are discussed in the main text.

I first solve the long-run part of the model. This yields the following long-run solution for world aggregates:

\[ \hat{Y}^w = \hat{C}^w = \hat{L}^w = 0, \]
\[ \hat{P}^w = (\hat{P}^p)^w = \hat{W} = \hat{M}. \]

The long-run solution for country differences is:

\[ \hat{Y}^d = \hat{C}^d = \frac{-\hat{r}dF}{C_0}, \]
\[ \hat{C}^d = \frac{2\delta + 1}{2(\theta - 1)\delta + 1} \frac{\hat{r}dF}{C_0}, \]
\[ (\hat{P}^p)^d = \frac{\hat{W} = \hat{M}}{\hat{C}^d}, \]
\[ \hat{P}^d = \frac{\hat{C}^d}{\hat{M}} \frac{2\delta + 1}{2(\theta - 1)\delta + 1} \frac{\hat{r}dF}{C_0}. \]

The long-run solution for the exchange rate and the terms of trade is:

\[ \hat{X}^d = \frac{\hat{C}^d + \delta}{1 - \delta} \frac{\hat{r}dF}{C_0}, \]
\[ (\hat{P}^p)^d - \hat{X}^d = \frac{\hat{C}^d}{1 - \delta} \frac{\hat{r}dF}{C_0}. \]
Next, solve the short-run part of the model. Recall that nominal wages are fixed in the short run. This yields the following short-run solution for world aggregates:

$$\hat{Y}_w = \hat{C}_w = \hat{L}_w,$$
$$\hat{P}_w = (\hat{P}^p)_w = 0,$$
$$\hat{C}_w = (1 - \beta)\hat{M}_w + \beta\hat{C}_w + \beta\hat{P}_w.$$

The short-run solution for country differences is:

$$\hat{Y}_d = \hat{L}_d = 2\delta (1 - \beta)\hat{M}_d + (-2\delta + 2\delta \beta + 2\delta - 1)\hat{C}_d + 2\delta \beta \hat{P}_d,$$
$$\hat{C}_d = \hat{C}_d,$$
$$(\hat{P}^p)_d = 0,$$
$$\hat{P}_d = (1 - \beta)\hat{M}_d - (1 - \beta)\hat{C}_d + \beta\hat{P}_d.$$

The short-run change in the interest rate and the short-run current account balance are:

$$\hat{\rho} = -\hat{M}_w + \hat{C}_w - \frac{\beta}{1 - \beta} \hat{P}_w.$$

$$\frac{d\hat{F}}{C_0} = \frac{1}{2} \left[ (2\delta - 1)(1 - \beta)\hat{M}_d + (-2\delta + 2\delta \beta + 2\delta - 1 - \beta)\hat{C}_d + (2\delta - 1)\beta \hat{P}_d \right].$$

The short-run change in the exchange rate is:

$$\hat{X} = \frac{1}{2(1 - \delta)} \left\{ (1 - \beta)\hat{M}_d - (1 - \beta)\hat{C}_d + \beta\hat{P}_d \right\}.$$

The short-run terms of trade are equal to minus the exchange rate:

$$(\hat{P}^p_\rho)_d = -\hat{X} = -\hat{X}.$$

**F.3 Reduced-form solution for main variables**

The long-run and the short-run equations can be combined, in order to obtain the closed form solution of the model. Recall that I consider the special case of a permanent money shock, i.e. $\tilde{M} = \tilde{M}$. The long-run variables that appear in the short-run solution ($\hat{C}$ and $\hat{P}$) are combined with the short-run variable that appears in the long-run solution ($d\hat{F}/C_0$). Also, $\beta$ has been eliminated from the equations by recalling that $\tilde{\rho} = \frac{1 - \beta}{\beta}$. 

This yields:

\[
d\bar{C} = \frac{2\delta \theta - 1}{2 + \bar{r}(1 + 2\delta \theta)} \bar{M}^d,
\]

\[
\hat{C}^d = \frac{\bar{r}(2\delta \theta - 1)(2\delta \theta + 1)}{[2\delta(\theta - 1) + 1][2 + \bar{r}(1 + 2\delta \theta)]} \bar{M}^d,
\]

\[
\hat{P}^d = \left[1 + \frac{\bar{r}(1 - \delta)(2\delta \theta + 1)}{2\delta(\theta - 1) + 1}\right] \frac{2}{2 + \bar{r}(1 + 2\delta \theta)} \bar{M}^d,
\]

\[
\hat{C}^w = 0,
\]

\[
\hat{P}^w = \bar{M}^w.
\]

Substituting the expressions for \(\hat{C}^w\) and \(\hat{P}^w\) into the earlier equation for \(\hat{r}\) yields a simplified expression for the short-run change in the world interest rate:

\[
\hat{r} = -(1 + \frac{1}{\bar{r}})\bar{M}^w.
\]

Substituting the results for \(\frac{d\bar{C}}{dC_0}, \hat{C}^w, \hat{C}^d, \hat{P}^w, \hat{P}^d\) into the equations that were obtained earlier for output, consumption and the general price level yields reduced-form solutions for those variables. The long-run solution for world aggregates was already obtained above:

\[
\hat{Y}^w = \hat{C}^w = \hat{L}^w = 0,
\]

\[
\hat{P}^w = (\hat{P}^d)^w = \hat{W}^w = \hat{M}^w.
\]

The long-run solution for country differences is:

\[
\hat{Y}^d = \hat{L}^d = -\frac{\bar{r}(2\delta \theta - 1)}{2 + \bar{r}(1 + 2\delta \theta)} \bar{M}^d,
\]

\[
\hat{C}^d = \frac{\bar{r}(2\delta \theta - 1)(2\delta \theta + 1)}{[2\delta(\theta - 1) + 1][2 + \bar{r}(1 + 2\delta \theta)]} \bar{M}^d,
\]

\[
\hat{P}^d = \left[1 + \frac{\bar{r}(1 - \delta)(2\delta \theta + 1)}{2\delta(\theta - 1) + 1}\right] \frac{2}{2 + \bar{r}(1 + 2\delta \theta)} \bar{M}^d.
\]

The signs of the long-run response coefficients can be determined:

\[
\frac{d\hat{Y}^w}{d\bar{M}} > 0, \quad \frac{d\hat{C}^w}{d\bar{M}} = 0, \quad \frac{d\hat{P}^w}{d\bar{M}} > 0,
\]

\[
\frac{d\hat{Y}^d}{d\bar{M}} < 0, \quad \frac{d\hat{C}^d}{d\bar{M}} > 0, \quad \frac{d\hat{P}^d}{d\bar{M}} > 0.
\]
The long-run solution for the exchange rate and the terms of trade are:

\[ \hat{X} = \frac{1}{2 + \bar{r}(2\delta\theta + 1)} \left\{ 2(1 + \bar{r}) - \frac{\bar{r}\delta(2\delta\theta - 1)}{(1 - \delta)[2\delta(\theta - 1) + 1]} \right\} \hat{M}^d, \]

\[ \hat{\text{TOT}} = \hat{P}^p - \hat{P}^p \hat{X} = \frac{\bar{r}\delta(2\delta\theta - 1)}{(1 - \delta)[2\delta(\theta - 1) + 1][2 + \bar{r}(1 + 2\delta\theta)]]} \hat{M}^d. \]

Next, turn to the short-run solution. The short-run reduced-form solution for world aggregates is:

\[ \hat{Y}^w = \hat{C}^w = \hat{L}^w = \hat{M}^w. \]

\[ \hat{P}^w = (\hat{P}^P)^w = 0. \]

The short-run solution for country differences is:

\[ \hat{Y}^d = \left[ 1 + \frac{2(2\delta\theta - 1)}{2 + \bar{r}(1 + 2\delta\theta)} \right] \hat{M}^d, \]

\[ \hat{C}^d = \frac{\bar{r}(2\delta\theta - 1)(2\delta\theta + 1)}{[2\delta(\theta - 1) + 1][2 + \bar{r}(1 + 2\delta\theta)]} \hat{M}^d, \]

\[ \hat{P}^d = \left[ 1 + \frac{\bar{r}(1 - \delta)(2\delta\theta + 1)}{2\delta(\theta - 1) + 1 + 1} \right] \frac{2 + \bar{r}(1 + 2\delta\theta)}{2 + \bar{r}(1 + 2\delta\theta)} \hat{M}^d. \]

The signs of the short-run response coefficients can easily be determined

\[ \frac{d\hat{Y}^w}{d\hat{M}^w} > 0, \quad \frac{d\hat{C}^w}{d\hat{M}^w} > 0, \quad \frac{d\hat{P}^w}{d\hat{M}^w} = 0, \]

\[ \frac{d\hat{Y}^d}{d\hat{M}^d} > 0, \quad \frac{d\hat{C}^d}{d\hat{M}^d} > 0, \quad \frac{d\hat{P}^d}{d\hat{M}^d} > 0. \]

The short-run solution for the exchange rate and the terms of trade are:

\[ \hat{X} = \frac{1}{2 + \bar{r}(2\delta\theta + 1)} \left\{ \frac{1}{1 - \delta} + \frac{\bar{r}(2\delta\theta + 1)}{2\delta(\theta - 1) + 1} \right\} \hat{M}^d, \]

\[ \hat{\text{TOT}} = \hat{P}^p - (\hat{P}^P)\hat{X} = -\hat{X}. \]

G Exchange rate dynamics

**Lemma:** Given values for \( \theta \) and \( \bar{r} \), there is a unique admissible value of \( \tau \) (say \( \tau^* \)) for which \( d\hat{X}/d\hat{M}^d = 0 \). For trade costs lower than \( \tau^* \), the Home currency shows a long-run depreciation in response to an increase in the Home money supply \( (d\hat{X}/d\hat{M}^d > 0) \). For trade costs higher than \( \tau^* \), the Home currency shows a long-run appreciation in response to an increase in the Home money supply \( (d\hat{X}/d\hat{M}^d < 0) \).
Proof: The reduced-form solution of the long-run exchange rate is:

\[ \hat{X} = \frac{1}{2 + \bar{\tau}(1 + 2\delta \theta)} \left( 2 + \bar{\tau} \left[ 2 - \frac{\delta(2\delta \theta - 1)}{(1 - \delta)[2(\theta - 1)\delta + 1]} \right] \right) \hat{M} \]

Setting the response coefficient equal to zero yields two possible solutions:

\[ \delta = \frac{4(\theta - 1)(1 + \bar{\tau}) - 2 - \bar{\tau} \pm \sqrt{\text{root}}}{4[2(\theta - 1)(1 + \bar{\tau}) + \bar{\tau} \theta]}, \]

where

\[ \text{root} = (16\theta^2 + 8\theta - 7)\bar{\tau}^2 + (32\theta^2 - 8\theta - 4)\bar{\tau} + 16\theta^2 - 16\theta + 4. \]

One of these solutions involves a value \( \delta < \frac{1}{2} \), which would imply negative trade costs:

\[ \delta = \frac{4(\theta - 1)(1 + \bar{\tau}) - 2 - \bar{\tau} - \sqrt{\text{root}}}{4[2(\theta - 1)(1 + \bar{\tau}) + \bar{\tau} \theta]} < \frac{4(\theta - 1)(1 + \bar{\tau})}{4[2(\theta - 1)(1 + \bar{\tau})]} = \frac{1}{2}. \]

Therefore, only one admissible solution for \( \delta \) remains:

\[ \delta = \frac{4(\theta - 1)(1 + \bar{\tau}) - 2 - \bar{\tau} + \sqrt{\text{root}}}{4[2(\theta - 1)(1 + \bar{\tau}) + \bar{\tau} \theta]}. \]

Next, note that \( d\hat{X}/d\hat{M} < 0 \) for \( \delta = \frac{1}{2} \), whereas \( d\hat{X}/d\hat{M} < 0 \) for \( \delta \to 1 \). Therefore, there must be a unique value of \( \delta \) (\( 0 < \delta < \frac{1}{2} \)) for which \( d\hat{X}/d\hat{M} = 0 \). Next, recall that for given \( \theta \), there is a one-to-one positive relationship between \( \delta \) and \( \tau \). The lemma follows immediately.

H Exchange-rate overshooting

The long-run and short-run money demand equations are:

\[ \hat{M}^d - \hat{P}^d = \hat{C}^d, \]
\[ M^d - \bar{P}^d = \hat{C}^d - \frac{\beta}{1 - \beta} (\hat{P}^d - \bar{P}^d). \]

Subtract and note that \( \hat{C}^d = \hat{C}^d \) (Appendix F) and that, in the case of permanent money shocks, \( \hat{M} = M^d \). Then:

\[ \hat{P}^d - \bar{P}^d = -\frac{\beta}{1 - \beta} (\hat{P}^d - \bar{P}^d). \]
It follows directly that the long-run price differential must equal the short-run price differential:

$$\frac{P^*}{P} = \hat{P}^d.$$

Recall that

$$\hat{P}^d = (2\delta - 1)(\hat{P}^p)^d + 2(1 - \delta)\hat{X},$$

$$\hat{P}d = 2(1 - \delta)\hat{X}.$$

It follows immediately that

$$\hat{X} = \hat{X} + \frac{2\delta - 1}{2(1 - \delta)}(\hat{P}^p)^d.$$

In the special case $\delta = \frac{1}{2}$, as in Obstfeld and Rogoff (1995), no exchange-rate overshooting will take place ($\hat{X} = \hat{X}$). However, since the output price differential responds positively to a Home money shock $[d(\hat{P}^p)^d/d\hat{M} > 0]$, it follows that in the case of positive trade costs ($\delta > \frac{1}{2}$) exchange rate overshooting ($\hat{X} < \hat{X}$) is required to ensure that the above equilibrium condition $\frac{P^*}{P} = \hat{P}^d$ is satisfied.