Chapter 3

Physics and detector simulation

In this chapter the tools that are used to simulate the neutrino interactions and the response of the detector are described.

High energy muons can travel up to tens of kilometres in rock or water before they are stopped. Muons from charged current neutrino-nucleon interactions occurring far from the detector can thus still be detected. In order to evaluate the acceptance of the detector, neutrino interactions must therefore be generated in a large volume surrounding the detector (see figure 3.1). On the other hand, the emission of Cherenkov light and the production of secondary particle showers need to be simulated only when the particles are close to the detector. The simulation of Cherenkov light and the development of hadronic or electro-magnetic showers is therefore restricted to a smaller volume, called the 'can'. This can is a cylindrical volume that encompasses the detector with a margin of a few times the attenuation length of light, so that Cherenkov light from particles outside the can does not need to be simulated.

Generating the neutrino interactions and propagating the muons to the can is the purpose of the event generator GENHEN. Aspects of the event generation are discussed in section 3.2. The simulation of the Cherenkov light and the detector response is performed by two detector simulation packages, KM3 and GEASIM, which are described in section 3.3. In section 3.1 it is discussed how the generated events can be weighted with a prediction for a neutrino flux in order to calculate an expected event rate.

3.1 Monte Carlo scheme and event weighting

In this section the event weighting method used in ANTARES is explained by calculating the rate of detectable muon events for an assumed neutrino flux.

The total rate of detected events originating from neutrino interactions occurring in a geometric volume $V$ is given by the following integral:

$$R = \int P_{\bar{\nu}}(E, \vec{d}) \sigma(E) \rho(x) N_A \frac{d\Phi(E, \vec{d})}{dE d\Omega} P^{\text{det}}(\vec{x}, \vec{d}, E) d\vec{x} d\vec{d} d\Omega dE,$$

(3.1)

where the following quantities have been introduced:

$\vec{x}$: the position of the neutrino interaction
3.1. Monte Carlo scheme and event weighting

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Figure 3.1: Overview of the simulation scheme: neutrino interactions are generated in a large (tens of kilometres) volume. The resulting muons are propagated to the can; only inside the so-called can, the Cherenkov light and the detector response are simulated.

\( \vec{d} \): the direction of the neutrino

\( \frac{d\Phi(E, \vec{d})}{dE d\Omega} \): the differential neutrino flux, i.e. the number of neutrinos per unit energy \( E \), solid angle \( \Omega \), area and time. This is the flux of neutrinos before they enter the Earth.

\( \rho(x)N_A \): the number of target nucleons per unit volume, given by the density \( \rho \) times Avogadro’s number \( N_A \),

\( \sigma(E) \): the total charged current neutrino-nucleon cross-section (see section 3.2.1),

\( P_{\oplus}(E, \vec{d}) \): the probability of a neutrino of energy \( E \) to traverse the Earth without undergoing an interaction (see section 3.2.2) and

\( P^{\text{det}}(\vec{x}, \vec{d}, E) \): the probability to detect and reconstruct the event.

The detection probability \( P^{\text{det}}(\vec{x}, \vec{d}, E) \) depends on the following:

- the energy and direction of the muon produced at the interaction vertex, which are determined by the kinematics of the neutrino interaction (see section 3.2.1)
- the probability that the muon reaches the detector and its direction and energy when it does so (see section 3.2.3)
- the characteristics of the detectable light produced by the muon and the response of the detector (see section 3.3)
reconstruction and selection algorithms (see chapters 4 and 5).

A detailed evaluation of $P_{\text{det}}(\vec{x}, \vec{d}, E)$ is only feasible by means of a simulation. We will therefore evaluate integrals like equation 3.1 by means of Monte Carlo integration (see e.g. [44]), which means that the integral is approximated by evaluating the integrand at a number of randomly chosen points in the overall phase space. An integral over a function $f(x, y)$ can, for example, be approximated by

$$\int f(x, y) \, dx \, dy \approx \frac{(x_2 - x_1)(y_2 - y_1)}{N_{\text{gen}}} \sum_{i=1}^{N_{\text{gen}}} f(x_i, y_i),$$

where the $x_i$ and $y_i$ are random numbers, uniformly distributed in the intervals $[x_1, x_2]$ and $[y_1, y_2]$ respectively. In the context of simulations, each set of these random numbers is called a (simulated) 'event'. This two-dimensional example generalises in a straightforward way to every dimension.

To calculate the integral of equation 3.1, random variables $\vec{d}$, $\vec{x}$, and $E_i$ are generated for the neutrino direction, the position of the interaction and the energy of the neutrino respectively. In addition, $P_{\text{det}}$ is determined for each event; i.e. for each event, the neutrino interaction, muon propagation, detector response and reconstruction are simulated\(^1\). The value of $P_{\text{det}}$ is either 1 or 0 depending on whether the event is detected, reconstructed and selected in the final analysis or not.

The direction of the neutrinos is generated uniformly in the cosine of the zenith angle $\theta$ in the range $[\theta_{\text{min}}, \theta_{\text{max}}]$ and in the azimuth angle in the range $[0, 2\pi]$. In contrast, the energy of the interacting neutrinos is not generated uniformly in $E$ but according to a simple power law spectrum: $\frac{dN_{\text{gen}}}{dE} \propto E^{-\alpha}$, where $\alpha$ typically has a value of 1.4. This is done in order to ensure that roughly equal numbers of events are simulated for each energy decade. In order to apply equation 3.2 to the integral of equation 3.1, the latter has been rewritten:

$$R = \int \sigma(E) \rho(\vec{x}) N_A \frac{d\Phi(E, \vec{d})}{dE d\Omega} E^\alpha \left( \frac{1}{1 - \alpha} \right) E^{1 - \alpha} d\vec{x} d\cos(\theta) d\phi$$

Now, the integral can be evaluated by Monte Carlo integration,

$$R = \frac{\Delta}{N_{\text{gen}}(1 - \alpha)} \sum_{i=1}^{N_{\text{gen}}} \sigma(E_i) \rho(\vec{x}_i) N_A \frac{d\Phi(E_i, \vec{d}_i)}{dE d\Omega} P_i^{\text{det}} E_i^\alpha,$$

where the value for $E_i$ is drawn from a distribution in which $E_i^{1 - \alpha}$ is uniformly distributed (i.e. $\frac{dN}{dE} \propto E^{-\alpha}$). $\Delta$ is known as the phase space of the generation:

$$\Delta = 2\pi V \times (\cos \theta_{\text{max}} - \cos \theta_{\text{min}}) \times (E_{\text{max}}^{1 - \alpha} - E_{\text{min}}^{1 - \alpha}).$$

To (re)calculate the event rate for any given neutrino flux it suffices to keep track of the 'generation weight' of each event, which is defined as

$$w_i = \Delta \frac{1}{1 - \alpha} E_i^\alpha \sigma(E_i) \rho N_A P_i^{\text{det}} \sigma(E_i, \vec{d}_i).$$

\(^1\)In this simulation many more random numbers are generated (e.g. the kinematic variables of the neutrino interaction). Since these variables are not used explicitly in the event weights, they are omitted here.
The rate of detected events resulting from a flux $\Phi$ can then be calculated as follows:

$$R = \frac{1}{N} \sum_{i=1}^{N} w_i \frac{d\Phi(E_i, \vec{\theta}_i)}{dE d\Omega}.$$  \hspace{1cm} (3.7)

In the special case where all neutrinos originate from a point-like astrophysical source, the direction of the neutrinos is uniquely determined, but varies with time $t$ due to the rotation of the Earth. In this case the integral in equation 3.1 reduces to an integral over $\vec{x}$ and $E$. Equation 3.4 then becomes

$$R(t) = \frac{\Delta^*}{N(1 - \alpha)} \sum_{i=1}^{N_{\text{gen}}} P_{\nu}(E_i, \vec{\theta}_i) \sigma(E_i) \rho(\vec{x}_i) N_{\text{det}} \frac{d\Phi(E_i, t_i)}{dE} E_i^\alpha,$$  \hspace{1cm} (3.8)

where $\frac{d\Phi(E,t)}{dE}$ is the differential flux from the point source, and

$$\Delta^* = V(E_{\text{max}}^{1-\alpha} - E_{\text{min}}^{1-\alpha}).$$  \hspace{1cm} (3.9)

A specialised 'point source mode' has been included in the event generation software to simulate events from point sources.

### 3.2 Event generation

Neutrino induced events are generated using the GENHEN package, which is described in detail in [45]. The purpose of this program is to generate all particles that could generate detectable light. A large number of neutrino interactions (typically a few times $10^{10}$) is generated in a cylinder surrounding the detector. This cylinder is typically 25 km in radius and height in order to ensure that all interactions that could lead to a muon in the detector are simulated. This size is determined from the maximal muon range (see section 3.2.3) that is associated with the highest neutrino energy that is generated (typically $E_{\nu}^{\text{max}} = 10^7$ GeV). The GENHEN program simulates the neutrino interaction and the propagation of the muon to the can. Cuts are made on the muon energy and direction in order to avoid full simulation of events with a negligible probability of producing a muon on the can.

Interactions of neutrinos and anti-neutrinos are generated separately and are weighted with the corresponding fluxes. For models of cosmic neutrinos, the two fluxes are usually assumed to be equal.

#### 3.2.1 Neutrino interactions

In general, two types of neutrino interactions are generated:

**Quasi elastic and resonant scattering.** Interactions such as $\nu_\mu + n \rightarrow \mu^- + p$ (quasi-elastic) and $\nu_\mu + p \rightarrow \mu^- + \Delta^{++}$ (resonant) give a small contribution to the total cross-section (about 10% for $E \gtrsim 10$ GeV and less than 1% for $E > 500$GeV). They are simulated with the RESQUE package [46].
Charged current deep inelastic scattering. Charged Current (CC) Deep Inelastic Scattering (DIS) events are generated with the LEPTO [47] package, which provides sampling of the first order double differential charged current cross-section to obtain the characteristics of the outgoing muon and the struck quark. The subsequent hadronisation is modelled by the Lund string model, which is implemented in LEPTO via calls to PYTHIA 5.7 and JETSET 7.4 [48].

Since the DIS process dominates the observed event rate, the corresponding cross-sections are discussed below.

Deep inelastic scattering cross-section

In the simulation, it is assumed that the target consists of an equal amount of protons \( p \) and neutrons \( n \). The combination \( N = \frac{1}{2}(n + p) \) is referred to as an isoscalar nucleon.

The leading order double differential cross-section of the DIS charged current process \( \nu + N \rightarrow \mu^- + X \) can be expressed as

\[
\frac{d^2\sigma}{dxdy} = \frac{2G_F^2ME}{\pi} \left( \frac{M_W}{Q^2 + M_W^2} \right)^2 [xq(x, Q^2) + (1 - y)^2x\bar{q}(x, Q^2)],
\]

(3.10)

where \( x, y \) and \( Q^2 \) are the kinematic variables characterising the kinematics of the process (see e.g. [49]): \( y \) is the inelasticity \( y = (E - E_\mu)/E \), the scaling variable \( x \) is defined as \( x = Q^2/2M(E - E_\mu) \) and \( -Q^2 \) is the invariant square of the momentum transferred between the neutrino and the outgoing muon, and \( x = Q^2/2M(E - E_\mu) \). Furthermore, \( G_F \) is the Fermi coupling constant, \( M \) is the mass of the target nucleon, \( E \) is the energy of the incident neutrino and \( M_W \) is the mass of the \( W \) boson. The distribution function \( q(x, Q^2) \) contains contributions from the down \( (d) \), strange \( (s) \) and bottom \( (b) \) quarks in the nucleon \( N \), i.e.

\[
q = \frac{1}{2}(d_p + s_p + b_p + d_n + s_n + b_n),
\]

(3.11)

where e.g. \( s_p \) denotes the density of strange quarks \( (s) \) in the proton \( (p) \). In the same way, the anti-quark distribution \( q(x, Q^2) \) can be expressed as

\[
\bar{q} = \frac{1}{2} (\bar{u}_p + \bar{c}_p + \bar{u}_n + \bar{c}_n),
\]

(3.12)

where \( c \) refers to the charm quark and where the contribution from top quarks has been neglected.

Under the assumption of isospin symmetry, the quark densities in the neutron are related to those in the proton: \( u_n = d_p, \bar{u}_n = \bar{d}_p \). It is furthermore assumed that the sea quark distributions in the neutron and proton are equal. As a result, \( q \) and \( \bar{q} \) can be expressed entirely in terms of quark density functions in the proton:

\[
q = \frac{1}{2}(d_p + u_p + 2s_p + 2b_p)
\]

(3.13)

\[
\bar{q} = \frac{1}{2}(d_p + \bar{u}_p + 2c_p).
\]

(3.14)
3.2. Event generation

The cross-section for anti-neutrino scattering is obtained from equation 3.10 after interchanging the above quark and anti-quark distributions $q$ and $\bar{q}$.

Parameterisations of the quark distributions are extracted from fits to data from experiments. Here, the most recent fit obtained by the CTEQ collaboration, called CTEQ6D\textsuperscript{2} [50] is used. Figure 3.2 shows the cross-section for neutrinos obtained by integration of (3.10) with the CTEQ6D parameterisations of the quark density function and with the older versions CTEQ5D and CTEQ3D. The latter is in agreement (to the level of $\sim 1\%$) with published values [51], which have been obtained using CTEQ3D and which are also shown in figure 3.2. This figure also shows the cross-section obtained from the LEPTO package using the CTEQ6 parameterisations. This cross-section, which is actually used in the simulation, is smaller than the cross-section obtained from direct integration of equation 3.10. At low energies the difference may be due to a cut on the invariant mass of the hadronic final state which is implemented in LEPTO. At high energies the discrepancy is due to the fact that the cross-section used by LEPTO does not take into account scattering off $b$ quarks. It is not trivial to modify LEPTO to include the contribution from $b$ quarks, since for a large part of the kinematic phase space the reaction is suppressed due to the large mass of the top quark. This effect is taken into account in the direct integration of equation 3.10 by using the 'slow-rescaling' prescription [52], as was done in [51]. The LEPTO package was thus not modified, with an underestimation of the cross-section (up to $10\%$ at $E = 10^8$ GeV) as a result.

At neutrino energies $E \gtrsim 10^8$ GeV there is a significant contribution to the cross-section from scattering off quarks at very small values of $x$ ($x \lesssim 10^{-6}$). In this regime, the quark densities are poorly constrained by measurements and there are dramatic differences between the different versions of the CTEQ routines. This results in a large discrepancy between the CTEQ3 and CTEQ5 results: at $E = 10^{12}$ GeV, the cross-section differs by about an order of magnitude. This difference could be interpreted as a kind of systematic uncertainty on the cross-section due to the unknown behaviour of the quark densities at low $x$. However, it is expected that the behaviour of the CTEQ3 and CTEQ6 parameterisations is more correct than that of CTEQ5, since in the latter, the quark densities essentially vanish in the low-$x$ regime. As can be seen in the figure, the LEPTO package fails to integrate the cross-section for energies above $10^9$ GeV.

The CTEQ6 parameterisations of the quark densities have error estimates associated with them which allows for a calculation of the uncertainty of the neutrino-nucleon cross-section stemming from the uncertainty of the quark density functions. This uncertainty turns out to be of the order of a few percent in the energy range $10 - 10^8$ GeV. This is shown in figure 3.2(right).

Composition of the interaction medium

As mentioned above, the interactions are generated for a target nucleus consisting of an equal amount of protons and neutrons. This is a valid assumption if the interaction occurs in the rock ('standard rock' [53] has atomic mass $A = 22$, atomic number $Z = 11$ and density $\rho = 2.65 \text{ g cm}^{-3}$). However, for water ($A = 18$, $Z = 10$, $\rho = 1.04 \text{ g cm}^{-3}$) this is

\textsuperscript{2}The 6 is a 'version number' and the D refers to the factorisation scheme that was used while performing the fits: in this case the so called DIS-scheme.
3.2. Event generation

Figure 3.2: Left: Cross-sections of the charged current DIS process $\nu_\mu + N \rightarrow \mu^- + X$ obtained by integration of equation 3.10 using the CTEQ3, CTEQ5 and CTEQ6 parameterisations of the quark density distributions. Also shown are points taken from [51] which were calculated with CTEQ3. The cross-section calculated by the LEPTO package using CTEQ6 is also shown. Right: The same results normalised to the CTEQ6 result. The uncertainty on the CTEQ6 result due to uncertainty of the quark density functions is indicated by the filled area.

not the case. The average cross-section for scattering on a water nucleon can be written as

$$\sigma_{\text{H}_2\text{O}} = \sigma_N + \frac{1}{18}(\sigma_p - \sigma_n). \quad (3.15)$$

In the low energy DIS regime, $\frac{\sigma_p}{\sigma_n} \approx \frac{1}{2}(2)$ for (anti-)neutrinos and the error is of the order of $\frac{1}{18} \times \frac{2}{3} \approx 4\%$. At high energies the error is smaller, since the difference between $\sigma_n$ and $\sigma_p$ decreases as the contribution from sea quarks becomes more important. Hence, the non-isoscalarity of the interaction medium can be neglected.

3.2.2 Neutrino absorption in the Earth

Neutrinos with energies above a TeV have a non-negligible probability to undergo a charged current interaction in the Earth before reaching the vicinity of the detector. The Earth is therefore opaque to very high energy neutrinos.

The amount of matter $\Sigma$ that the neutrino encounters while traversing the Earth was taken from [51]; it is shown in figure 3.3 as a function of the zenith angle $\theta$. The column density seen by neutrinos with $\theta > 145^\circ$ is enhanced due to the increased density of the Earth’s core. The probability that the neutrino survives its journey through the Earth, $P_\oplus$, is given by

$$P_\oplus(E, \theta) = e^{-\Sigma(\theta)N_A\sigma(E)} \quad (3.16)$$
3.2. Event generation

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and is shown in figure 3.3(right) as a function of the energy and zenith angle of the neutrino. As was mentioned in section 3.1, this probability is taken into account in the calculation of the expected event rate.

Neutral Current (NC) interactions of neutrinos in the Earth will result in a decrease in energy and a deviation from the original direction. At the relevant energies, the NC cross-section is about a factor 2 smaller than the CC cross-section. A rough estimate of the fraction of neutrinos undergoing a NC interaction but not a CC one, yields 15% at most. This effect has been neglected.

3.2.3 Muon propagation

While travelling from the interaction point to the detector, the muon loses energy. Furthermore its direction is affected by multiple Coulomb scattering. The propagation of the muon through rock and sea water must therefore be simulated. For this, the MUSIC [54] package was used, which provides both simulation of the energy loss and of multiple scattering\(^3\). In the following, the physics processes included in the MUSIC code and the relevant results are described.

\(^3\)There exist other muon propagation codes, like the PROPMU package, which includes multiple scattering, but has inaccuracies in the treatment of the energy loss. This package was used only for the propagation of atmospheric muons from sea level to the detector (see section 3.5.2) because it executes more quickly than MUSIC.
Energy loss

The muon can lose energy via the following processes:

1. **ionisation**: Atoms in the medium are ionised. The energy transfer to the electrons is usually modest, but occasionally the electrons obtain a non-negligible fraction of the muon energy. Such electrons are called 'knock-on electrons' (or δ-rays).

2. **bremsstrahlung**: In the nuclear electric field, the muon radiates off a photon.

3. **pair production**: An $e^+e^-$ pair is produced.

4. **photonuclear interactions**: A virtual photon is exchanged with a nucleus.

The average energy loss per unit length as implemented in the MUSIC software is shown in figure 3.4 for each of these processes. Below about a TeV ionisation is the dominant energy loss mechanism and the energy loss is roughly constant at about 0.3 GeV/m. At high energies, pair production and bremsstrahlung are the dominating processes. For these processes, the energy loss is roughly proportional to the energy of the muon. Distributions of the muon range (i.e. the distance the muon can travel before it is stopped) are shown in figure 3.5 for various values of the initial muon energy.

Angular deviation

Deviations of the direction of the muon are predominantly caused by multiple Coulomb scattering off atomic nuclei. However, the processes responsible for the energy loss described above can also deflect the muon. Both effects are taken into account in MUSIC. As is shown in figure 3.6, for the majority of the events, the scattering angle between the neutrino and the initial muon is about an order of magnitude larger than the angle caused by multiple scattering of the muon itself. In conclusion, multiple scattering gives a small contribution to the angular resolution, but is nevertheless simulated.

3.3 Detector simulation

In this section the simulation of the detector response to particles is described. As explained before, this simulation is only done for particles inside the can volume. The production of Cherenkov light is simulated for the muon itself and secondary particles that may be produced. The propagation of Cherenkov light is also simulated. The two programs used are called KM3 and GEASIM.

The GEASIM package uses GEANT3 [55] to perform full tracking of all particles through the detector volume. For each particle, the arrival time of the Cherenkov light incident on the OMs is calculated analytically. The number of Cherenkov photons is calculated taking into account the attenuation of the light. A drawback of GEASIM is that scattering of the Cherenkov light is not simulated. Furthermore, the full particle tracking results in long execution times especially for the simulation of high energy muons, which produce large secondary showers.
3.3. Detector simulation

Figure 3.4: Average energy loss per water equivalent metre for muons in rock and sea water as a function of the muon energy. For the energy loss in water, the contributions of the different processes are shown separately. The data for this figure were taken from the MUSIC code.

The KM3 program uses a modified version of the MUSIC package to propagate the muon through the detector. This propagation is done in one metre steps. If the energy loss of the muon over this distance exceeds a critical value (0.3 GeV), an Electro-Magnetic (EM) shower is initiated at a random position on the path. In order to provide a fast simulation of the shower development and light scattering, the photons are sampled from tables containing the average photon fields produced by the muon or by EM showers. These tables are obtained beforehand from a full simulation (using GEANT3) of a large number of muons and EM showers. In this simulation, the Cherenkov photons are individually tracked through the water, including the effects of light scattering. A drawback of the KM3 package is that there are no facilities to simulate hadronic showers.

In order to obtain a realistic estimate of the angular resolution of the detector, it is important that light scattering is included for the simulation. Therefore, KM3 was used for the simulation of the light produced by the muon and by secondary particles. The hadrons produced in the neutrino interaction have to be simulated in the (relatively rare) case that the interaction takes place near the instrumented volume (i.e. in the can). In this case, the hadronic shower is simulated using GEASIM.
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Figure 3.5: Distribution of the muon range in rock and water for different muon energies expressed in water equivalent km.

Figure 3.6: Angle between the neutrino and the produced muon (interaction) and the angle between the muon at the interaction vertex and the muon entering the can, which is due to multiple scattering.
The simulations of the optical modules, the PMT response and the front-end electronics are identical in both packages. They are described in section 3.3.2. The following section describes the simulation of the generation of the Cherenkov light and its propagation through the sea water as is implemented in KM3.

### 3.3.1 Cherenkov light

The number of Cherenkov photons as a function of the wavelength is given by equation 2.2 and the angle with respect to the muon track, $\theta_C$, is given by equation 2.1. These expressions are used in the simulation of the light produced by the muon itself and the charged particles in the hadronic or electro-magnetic showers. The simulation of the light emitted by charged particles is fairly simple. The propagation of the light to the OM is more complicated and is discussed in the next sections.

**Absorption and scattering**

The influence of absorption and scattering of light is taken into account by a model that has been tuned to data acquired during measurements at the ANTARES site [56]. In these measurements a short (10 ns) light pulse was used to illuminate an OM which was located at a distance of either 24 or 44 metres from the light source. The absorption and scattering parameters have been determined, respectively, from the relative intensities and the arrival time of the light.

The absorption length could be determined with good accuracy: the errors are typically smaller than 1 m. However, in various measurements taken at different times of the year, different values were obtained. The interpretation is that the medium exhibits genuine variations. Figure 3.7(left) shows the measurements and the model of the wavelength dependence of the absorption length that is used in the simulation. The shape of this distribution was taken from [57], while the normalisation was adjusted to match the measurements. At short distances from the muon track, the average absorption length can be computed by weighting this distribution with the $\lambda^{-2}$ spectrum of the Cherenkov light, which yields an absorption length of 22 m. At larger distances from the muon the absorption length will effectively increase, up to 55 m, since photons with short absorption lengths have already been absorbed.

Scattering of light can be described by a scattering length (i.e. the mean distance between two scatterings) and a distribution of the angle between the initial and final direction of the photon at each scattering. For the angular distributions, two cases were considered:

- **Scattering off molecules.** This is described by Rayleigh scattering, which has a known distribution of the scattering angle.

- **Scattering off sedimentary particles (and perhaps microscopic organisms).** For this type of scattering, the distribution of the scattering angle has been measured.

The corresponding distributions of the scattering angle are shown in figure 3.8. Both are taken from [58]. The data were fitted using a combination of the two cases: each scattering
has a probability $\eta$ of being a Rayleigh scattering and a probability $1-\eta$ of being a 'particle' type scattering. The values for $\eta$ and the scattering length were fitted simultaneously to the data. Since the particle scattering is strongly peaked in the forward direction, it has relatively little influence on the time distribution of the arriving light. As a result, the contribution of particle scattering could not be accurately determined. The most important effect of the scattering for the reconstruction of muon tracks is the influence of the time delay of the light. This can be expected to be well reproduced. The reason for this is that the scattering parameters have been extracted from the measurements of exactly this quantity.

**Light velocity and dispersion**

The relevant velocity for the propagation of the Cherenkov light is the group velocity $v_g$, as was pointed out only relatively recently [59]. The group velocity of light is defined as

$$v_g = \frac{d\omega}{dk} = -\lambda^2 \frac{d\nu}{d\lambda}$$  \hspace{1cm} (3.17)

where $\omega = 2\pi \nu$ and $k = 2\pi / \lambda$ ($\lambda$ and $\nu$ are respectively the wavelength and frequency of the light). Using the standard dispersion relation, this can be expressed in terms of the phase velocity $v_p \equiv (c/n)$

$$v_g = v_p \cdot \left(1 + \frac{\lambda}{n} \frac{dn}{d\lambda}\right),$$  \hspace{1cm} (3.18)

where $n$ is the index of refraction of the medium.

An empirical model to describe the behaviour of the refractive index in sea water was adopted from [60], with additional correction for the pressure of $1.4 \times 10^{-5}$ bar$^{-1}$ [61].
3.3. Detector simulation

The phase and group velocities from this model are shown in figure 3.9. The velocity has also been measured at the ANTARES site [56]. The model agrees well with the measured value at \( \lambda = 460 \text{ nm} \), but disagrees at \( \lambda = 370 \text{ nm} \). The origin of this discrepancy is not understood.

The wavelength dependence of the speed of light induces a spread in the arrival times of the Cherenkov photons on the PMT. This effect is called dispersion. In the simulation programs (both KM3 and GEASIM), the speed of light is taken to be constant (as the group velocity at 460 nm), but dispersion is taken into account afterwards by sampling tables of the relative delays induced by dispersion. These tables have been obtained in a dedicated study [45], which used the model from [60] with the pressure correction. The distribution of the relative delays as a function of the distance travelled by the light is shown in figure 3.9(right).

It may be noted that the somewhat weaker (compared to the model) wavelength dependence of the group velocity that is suggested by the data would result in less dispersion than the model used in the simulation.

3.3.2 Simulation of the hardware

The response of the optical module to the Cherenkov photons is simulated by taking into account the quantum efficiency of the PMT, the transparency of the glass sphere and the optical gel and the effective area of the photocathode. All these effects are a function of the wavelength. The efficiency of the OM is also dependent on its orientation with respect to the direction of the photons. Figure 3.10(left) shows the quantum efficiency for both the bare PMT and the PMT enclosed in the glass sphere. Figure 3.10(right) shows the angular dependence of the efficiency of the total optical module\(^4\).

\(^4\)By convention, an angle of incidence of 180° means the photon hits the PMT head-on, while 0° means it hits the (insensitive) rear of the OM.
3.4 Light detection

Using the detector simulation discussed in the previous section, some important characteristics of the detected light can be evaluated.

The first quantity of interest is the number of photons that is detected by an OM. This is shown in figure 3.11 as a function of the distance travelled by the photon for different values of the muon energy \( E_\mu \). As expected, the number of detected photons increases approximately linearly with the muon energy in the energy range above 1 TeV. At \( E_\mu \approx 1 \) TeV, the contribution of the light from the muon accounts for about 50% of the total light.

The front-end ARS chip integrates the analogue signal from the PMT over a typical time window of 25 ns. This is simulated by summing the number of detected photons in that window. After the integration, the ARS cannot take data for about 250 ns. A second ARS, connected to the same PMT, digitises signals arriving afterwards. The time resolution for single photo-electron signals is 1.3 ns (see section 2.5.3) and decreases for higher amplitudes. To simulate this the hit times are smeared using a Gaussian function with a width \( \sigma = 1.3 \) ns/\( \sqrt{N_e} \), where \( N_e \) is the number of simultaneously detected photons.

The amplitude measurement is simulated by smearing the integrated number of photons with an empirical function. This function results in a (roughly Gaussian) smearing of about 30%. The dynamic range of the charge integration corresponds with a signal of about 20 photo-electrons. This effect is not included in the simulation.

Figure 3.9: Left: Phase velocity of light according to the model of Quan and Fry and the group velocity derived from it. Also indicated are direct measurements at the ANTARES site. The statistical uncertainty on the measurements is smaller than the size of the dots. Right: Distribution of the delay in arrival time due to dispersion for hits from dispersed light (with respect to hits travelling at the group velocity at 460 nm) as a function of the distance travelled by the light. The lines indicate the central value and the RMS.
3.4. Light detection

Figure 3.10: Left: The quantum efficiency as a function of the wavelength for the bare PMT and for the PMT enclosed in the glass sphere. Right: The dependence of the acceptance on the angle of incidence of the photon on the optical module.

production. The number of observed photons can be approximated by

$$N(b) = N_{1m} \frac{1}{b} e^{-b/\lambda_{eff}},$$

(3.19)

where $b$ is the photon path length and $\lambda_{eff}$ is the effective absorption length. A reasonable agreement with the distribution due to the muon alone is found for a value of $\lambda_{eff} = 38$ m. The number of detected photons due to the muon alone 1 metre away from the muon track $N_{1m}$ is about 100. Equation 3.19 is plotted in figure 3.11 and is in good agreement with the full simulation.

The time information obtained from the PMT signals is of crucial importance for the reconstruction of muons. The arrival time of the photons is expressed relative to the expected arrival time $t^{th}$ that can be calculated from the parameters of the muon track, as will be explained in section 4.1. The distribution of the resulting time residuals $r = t - t^{th}$ is shown in figure 3.12 for a muon with an energy of 1 TeV. The contributions from the muon itself and the secondary electrons are shown separately for both scattered and unscattered photons. Photons originating from the muon and arriving at the OM without scattering carry the most precise timing information: their arrival time is only perturbed by dispersion and the TTS of the PMT and hence the residual distribution is sharply peaked at $r = 0$. Photons that originate from secondary electrons or that have scattered, are often delayed with respect to this time. However, also for these photons the distribution peaks at $r \approx 0$, which means that such photons can still be used in the reconstruction process. The distribution for background photons corresponds to a background rate of 60 kHz (see section 3.5.1).
3.4. Light detection

Figure 3.11: Number of photons detected by an optimally oriented OM as a function of the photon path length for different muon energies. The contribution from Cherenkov light produced by the muon alone is shown separately. The result of equation 3.19 is also shown (muon analytic).

Figure 3.12: Distribution the time residuals for photon arrival times relative to the direct muon signal. Contributions are shown for scattered and unscattered photons originating from the muon itself and from secondary electrons and positrons. All hits occurring within a distance of 100 m from the track are included in this figure.

3.4.1 Effect of the front-end electronics

As explained in section 3.3.2 the PMT signal is integrated over a period of 25 ns. Due to the integration, arrival times of photons separated by less than 25 ns can not be observed. Instead, these signals are combined to give hits with larger amplitudes. The time of the resulting hit corresponds to the time of the first photon.

The effect of the electronics on the hit times is illustrated in figure 3.13(left), which shows the hit time residuals. Due to the charge integration, the region between the main...
peak and 25 ns is depleted. An increase in the amount of detected photons can be seen at residuals just above 25 ns. Note also, that for large amplitude hits, the main peak is shifted slightly to the left. This is because the time of the hit corresponds to the time of the first photon, which arrives, on average, earlier if the number of photons is large.

The number of hits of a given amplitude as a function of the photon path length is shown in figure 3.13(right). When the OM is close to the track, the average number of photo-electrons is significantly larger than one (see figure 3.11). Hence the hits will predominantly consist of large amplitude hits. At distances larger than 15 m, most hits are due to single photo-electrons.

![Figure 3.13: Left: Distribution of the hit times relative to the expected arrival time for three ranges in the hit amplitudes. For comparison, the original distribution of the arrival times without simulation of the electronics is reproduced from figure 3.12 (top curve, labeled 'all photons'). Right: Average number of hits detected as a function of the photon path length for different hit amplitudes.](image)

### 3.5 Background

#### 3.5.1 Background photons

In addition to the light from the muon and its secondaries, background light due to β-decay of $^{40}K$ and bioluminescence will be detected (see section 2.6). This background is simulated by adding hits to the simulated physics events. The time window for the generation of background hits ranges from 1500 ns before the time of the first signal hit to 1500 ns after the time of the last signal hit. The assumed background rate is 60 kHz per PMT. As was shown in section 2.6, this value corresponds to the background rate measured at the ANTARES site, but only during periods of low optical activity. For the periods with increased bioluminescence, the performance of the detector will be degraded.
compared to the simulations presented here. Quantification of the impact of the elevated background rates should be the subject of future studies.

Besides complicating the reconstruction of genuine events, random background hits can also form patterns that mimic the signal produced by muons. Such pure background events could be reconstructed as up-going muons. The full impact of this background is currently under study and is not included in this analysis. Due to its combinatoric nature, this type of background can be reduced by selecting events with a large number of hits compatible with the muon track. Hence it is expected that this background will predominantly influence physics studies involving low energy (≤ 100 GeV) muons.

### 3.5.2 Atmospheric muons

Energetic down-going muons produced by interactions of cosmic rays with the Earth’s atmosphere can reach the detector. Even though PMTs face downwards, many of these muons produce detectable light. The vast majority of these events can be rejected by selecting only up-going tracks as neutrino candidates. The remaining background stems from down-going muons that are wrongly reconstructed as up-going. Particularly dangerous, in this respect, are bundles of muons that are produced in the same air shower. The atmospheric muon background is simulated using the HEMAS package [62], which performs a full simulation of the atmospheric shower, starting from the interaction of the primary nucleus and resulting in muons at sea level. The muons are then propagated to the detector using the PROPMU package [53].

There are several limitations in the simulation of atmospheric muons. The high rate of atmospheric muons, implies that, with the limited computing power and storage space available, only several hours of this background could be fully simulated. Also, the zenith angle of the primary particles is limited to the range between 0 and 60° by the HEMAS package. Moreover, this simulation was performed with a somewhat different detector geometry (14 strings of 30 floors with 12 m floor spacing). A small sample using the new detector geometry was used to verify that no dramatic changes occur due to the difference in detector geometry. In particular, the distributions of the variables used in the event selection (see chapter 5) were checked to be compatible. New simulations, using the correct detector geometry and the more widely used CORSIKA package [63] for the shower simulation, are currently in production, but were not completed in time to be incorporated in this analysis.

### 3.5.3 Atmospheric neutrinos

Neutrinos that are produced in air showers initiated by cosmic rays, form a direct background to cosmic neutrinos. They can only be distinguished by their energy spectrum, which is known to be steep for atmospheric neutrinos but may be much flatter for cosmic neutrinos.

A distinction is made between neutrinos produced by the decay of pions and kaons, which dominate the flux at energies below 10^5 GeV and neutrinos produced by the decay of charmed mesons, which may dominate above that energy. For historical reasons, the flux of the former is called the conventional flux, while the latter are referred to as 'prompt'
neutrinos.

The conventional spectrum of atmospheric neutrinos has been calculated by various groups, e.g. FLUKA[64], Bartol [40] and HKKM[65]. The calculations rely on measurements of the energy spectrum and composition of cosmic rays and on models of the hadronic interactions producing the charged pions and kaons that decay into neutrinos. Differences in these assumptions lead to differences of the order of 20% in the predicted neutrino fluxes, which can be taken as a measure of the systematic uncertainty of the atmospheric neutrino flux.

Calculations of the prompt neutrino flux depend on models of the production cross section of charmed mesons in proton-nucleon collisions. This cross-section is poorly constrained by experiment. A review of prompt neutrino production is given in [66], where fluxes are presented for different models of charmed particle production: Recombination Quark Parton Model (RQPM), perturbative QCD (pQCD), and Quark Gluon String Model (QGSM). For each model, the ingredients of the calculation (e.g. the parameterisation used to represent the flux of primary cosmic rays) are varied, yielding a range of fluxes. In figure 3.14 the range of allowed prompt neutrino fluxes is shown for each of the charm production models. This illustrates that, in contrast to the models for the conventional neutrino flux, the uncertainty on the prompt neutrino flux is very large. The models of the prompt neutrino flux differ by two orders of magnitude.

Since atmospheric neutrinos form a background for the physics analysis, we have chosen to use the models that give the highest flux for both the conventional and prompt neutrino flux, namely the 'Bartol' flux [40], combined with the maximal prediction for the RQPM model for the prompt neutrinos.

![Figure 3.14: Models of atmospheric neutrino fluxes. Left: three models for the conventional atmospheric neutrino flux. Right: The Bartol model and three models for prompt neutrino fluxes taken from [66]. The results have been integrated over all directions.](image)

The dependence of the atmospheric neutrino flux on the zenith angle is shown in fig-
Figure 3.15. At energies below 100 TeV, the neutrino flux is enhanced near the horizon, because horizontal CRs interact high in the atmosphere, where the density is low. Therefore, the resulting mesons have more chance to decay before interacting [7]. At energies where the prompt neutrino flux dominates, there is basically no angular dependence, since the charmed mesons promptly decay before interacting (hence the name prompt neutrinos).

![Figure 3.15: The dependence of the atmospheric neutrino flux on the zenith angle of the neutrino for different values of the neutrino energy. The model used is from Bartol [40] + (maximal) RPQM [66].](image)

In this thesis, only muon neutrinos are taken into account. Atmospheric electron neutrinos that are reconstructed as up-going muons could contribute to the background. However, the flux of atmospheric electron neutrinos is heavily suppressed at high energies (e.g., at 1 TeV, $\Phi_{\nu_e}/\Phi_{\nu_\mu} \approx 20$). The reason is that the muons that produce the electron neutrinos (via decay) lose a large fraction of their energy in the atmosphere before decaying.

Furthermore, the effects of neutrino oscillations have been neglected. This can be expected to be a good approximation at high energies: above energies of a few hundred GeV, the oscillation length is already larger than the diameter of the Earth. The effect of oscillations of atmospheric neutrinos on the number of selected events will be quantified in section 5.2.4.