Track Reconstruction and Point Source Searches with Antares

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Chapter 6
Point source searches

In this chapter, a new method, called the Likelihood Ratio (LR) method, to search for point-like sources of neutrinos will be described and the resulting sensitivity of the ANTARES detector for point sources will be presented. The method will be described in section 6.3 and the results are compared to the expected sensitivity\(^1\) of binned methods, which are discussed in section 6.2. Finally, in section 6.6, the sensitivity of ANTARES will be compared to a number of flux predictions and to results obtained by other experiments.

6.1 Introduction

The presence of a point source of neutrinos could be indicated by an excess of events from a certain direction in the sky. On the other hand, such an excess can be due to atmospheric neutrinos that, by coincidence, seem to come from the same point in the sky.

After a period of data taking, a set of reconstructed and selected events will have been collected. The reconstruction algorithm provides estimates for the zenith and azimuth angles $\theta$ and $\phi$. Furthermore, the time of detection $t$ is recorded. From the reconstructed neutrino direction and the detection time of the event, the corresponding celestial coordinates are calculated (standard software is used for this [78]). We use 'equatorial' coordinates: right ascension $\alpha^\text{ra}$ and declination $\delta$. As an example, the celestial coordinates of the events in a one-year sample of selected atmospheric neutrino events are shown in figure 6.1. Due to the geographical location of the detector, the part of the sky with declinations higher than 48° is always above the horizon and is thus never observed by ANTARES. In contrast, the part of the celestial sphere with declinations below $-48^\circ$, is always visible. The region in between is visible during some fraction of the day.

The sensitivity of the search depends on the assumptions that are made about the source. Throughout this chapter, two cases will be considered:

**Full sky search:** In a full sky search, a source of neutrinos is searched for anywhere in the observable sky.

**Fixed point search:** In this case, we try to determine whether there is a source of

\(^1\)The term 'sensitivity' is sometimes defined as the average upper limit an experiment can set [77]. Here the term is used, in general, as 'the capability to make a discovery'.
neutrinos at one of a number of predefined locations in the sky. The candidate source locations could, for example, correspond to known gamma ray sources, which may be expected to produce neutrinos.

Although it is more restrictive, a fixed point search is more sensitive than a full sky search, since the probability that fluctuations in the background produce an excess is smaller.

### 6.2 Binned methods

One way to conduct a full sky search for point sources is to divide the observable sky into square-like bins and to look for a bin which contains an excess of events. For a fixed point search, a similar method can be used, where one counts the number of events that fall within circular bins, which are centred on the coordinates of the candidate sources. In this section, the discovery potential of these methods will be estimated for a point source with an $E^{-2}$ energy spectrum. The objective of this exercise is to verify the validity and to quantify the improvement in performance of the more elaborated likelihood ratio method, which will be presented in section 6.3. A more detailed study of the performance of ANTARES using binned methods can be found in [79].

The size of the bins will be chosen such that the probability of discovering a point source at the $3(5)\sigma$ Confidence Level (CL) is maximal. This probability is related to the
number of background events in a bin, which is given by

\[ N_{\text{bg}} = R_{\text{bg}}(\delta) \Omega_{\text{bin}} \Delta T, \tag{6.1} \]

where \( \Omega_{\text{bin}} \) is the bin size, \( \Delta T \) is the observation time, \( R_{\text{bg}}(\delta) \) is the rate of background events per unit solid angle, which depends on the declination \( \delta \) of the bin and which is shown in figure 6.2. \( \Delta T \) is the observation time, which we take to be one year.

![Figure 6.2: Rate of selected atmospheric neutrino events as a function of declination.](image)

If no signal is present, the probability of observing at least \( N_c \) events is given by the sum of the Poisson probabilities:

\[ P(N \geq N_c|\langle N \rangle = N_{\text{bg}}) = \sum_{k=N_c}^{\infty} \frac{N_{\text{bg}}^k e^{-N_{\text{bg}}}}{k!}. \tag{6.2} \]

In order to discover a signal at, for example, 3(5)\( \sigma \) CL, the probability of observing \( N \) or more events in any of the bins must be less than \( 2.7 \times 10^{-3} (5.7 \times 10^{-7}) \) in the background-only case. This can, to good approximation, be translated into the following requirement on a single bin:

\[ P(N \geq N_c|\langle N \rangle = N_{\text{bg}}) \leq \frac{2.7 \cdot 10^{-3}}{N_{\text{bins}}}, \tag{6.3} \]

where \( N_{\text{bins}} \) is the total number of bins.

The value of \( N_{\text{bins}} \) depends on the type of search. In the case of a fixed point search, \( N_{\text{bins}} \) is simply the number of candidate sources under consideration. For a full sky search, the whole observable sky is filled with bins. In this case, the bin size is a function of the declination and the total number of bins is given by

\[ N_{\text{bins}} = 2\pi \int_{-90^\circ}^{45^\circ} \Omega_{\text{bin}}(\delta)^{-1} d\sin(\delta), \tag{6.4} \]
6.2. Binned methods

where the search region is defined by all declinations from $-90^\circ$ to $+45^\circ$, which corresponds to a total solid angle of about 10.7 sr.

The number of events needed in a bin for a $3\sigma$ discovery, $N_c$, may now be calculated as a function of the bin size for each value of the declination. As an example, the result for a full sky search is shown as the dashed line in figure 6.3 for a source at a declination of $-81^\circ$. This arbitrarily chosen value of the declination will be used throughout this chapter for various examples. It is clear that a small (large) bin size requires a small (large) number of events.

Figure 6.3: Dashed line: The number of events $N_c$ required in a bin at $\delta = -81^\circ$ for a $3\sigma$ discovery as a function of the bin size. Solid line: Mean of the total number of signal events $N_{\text{sig}}^c$ required from a point source to have a 50% probability of detection (at $3\sigma$ CL).

Figure 6.4: Average binning efficiency for a circular bin, which is centred on the true source coordinates, and for a square bin with the source at a random position within the bin.
In order to compare these results to the results from the LR method, we calculate the expectation value of the number of observed events in the bin \( \langle N \rangle_c \) so that there is a 50% probability that the actual number of observed events is at least \( N_c \). The relation between the two follows from Poisson statistics\(^2\).

The expectation value of the number of events in a bin needed for a discovery is the sum of the expectation value of the number of background events in the bin and the expectation value of the total number of signal events observed from a source \( N_{\text{sig}}^c \), multiplied by the binning efficiency \( \epsilon \):

\[
\langle N \rangle_c = N_{\text{bg}}(\Omega_{\text{bin}}) + \epsilon(\Omega_{\text{bin}})N_{\text{sig}}^c.
\]

The binning efficiency is defined as the probability that the reconstructed coordinates of an event lie within the bin if the coordinates of its astrophysical source are contained in it. This quantity has been calculated for a source with an \( E^{-2} \) spectrum and is shown in figure 6.4 for the case of a fixed point search, where a circular bin is centred on the coordinates of the source, and for a full sky search, where the source is located at a random position within a square-like bin. In the latter case, the binning efficiency is reduced, because the source may be located close to the edge of the bin.

Equation 6.5 is used to calculate the number of signal events a source must produce to yield a 50% chance of a discovery. For the full sky search, the result is shown in figure 6.3 as a function of the bin size. The discontinuities in the figure correspond to changes in the value of \( N_c \), which occur at values of \( \Omega_{\text{bin}} \) where the inequality in equation 6.3 is an equality. In between the discontinuities, the number of required events decreases with \( \Omega_{\text{bin}} \) due to the increase in binning efficiency.

The optimal bin size is determined from figure 6.3 as the value at which the expectation value of the number of required signal events is smallest (i.e. where the flux needed for a discovery is lowest); this occurs just before a step\(^3\). From the figure one sees that if the bins are chosen to be optimal for a 3\( \sigma \) discovery, the number of signal events needed is about 9.1 for a source at a declination of \(-81^\circ\). The optimal bin size is about \(0.95^\circ \times 0.95^\circ\) for this declination. At higher declination, where there is less background, the optimal bin sizes are somewhat larger. The corresponding binning efficiency is 60%. When optimised for a discovery at 5\( \sigma \) CL, the bin size is smaller: \(0.85^\circ \times 0.85^\circ\).

After the optimal bin size has been determined for each value of the declination, equation 6.4 is used to re-calculate the total number of bins. This procedure is repeated (a few times) until the result has converged. The result is a configuration of bins for which equation 6.3 is satisfied for each bin.

The final result is the discovery potential, which we have defined as the number of signal events needed for a 3\( \sigma \) discovery in 50% of the experiments. This quantity is calculated as a function of the declination and it will be shown in figures 6.13 (full sky search) and 6.17 (fixed point search), where it will be compared to the results obtained with the LR method.

\(^2\)\(\langle N \rangle_c \) is always smaller than \( N_c \); the relative difference is typically less than 10% for the values involved here (\( N_c \geq 4 \)).

\(^3\)At this point we make an assumption that is favourable to the binned method, since in reality, this 'fine-tuning' of the bin size seems not feasible.
6.3 Likelihood ratio search method

6.3.1 Motivation

The binned methods described in the previous sections are well known ways to search for point sources of neutrinos. The motivation to develop a new method is that such methods do not make optimal use of the following pieces of information:

**The events outside the bin:** In order to reduce the background, the size of the bin is chosen such that a fraction of the signal events falls outside the bin. The information contained in these events is lost.

**The distribution of the events within the bin:** Not only the number of events in a bin is important, but there is also information in the distribution of the events within the bin. For a fixed number of events, there are still configurations of events which 'look' more like they are the result of a point source than others.

**The energy of the events:** The reconstructed muon energies are, in general, distributed differently for signal and background events (see figure 5.9). This information can be used, even when the energy spectrum of the signal is not known a priori. Furthermore, the angular resolution varies with energy; high energy events should be close to the source coordinates in order to be compatible with the point source signal, whereas low energy events may have a larger deviation.

**The angular error estimate:** The accuracy of the reconstructed direction varies from event to event and is described by the error estimates provided by the reconstruction algorithm.

**The zenith angle of the events:** Typically, binned methods only use the celestial coordinates of the events. The expected number of background events can, however, be estimated most accurately from the zenith angle. Events from a range of azimuth angles contribute to the events at a given declination. Therefore information is lost when using only the celestial coordinates.

The likelihood ratio (LR) method, which will be introduced in the following sections, was developed with the objective to increase the sensitivity by making use of this information. It will be shown that the new method indeed leads to an increase in the sensitivity compared to binned methods.

6.3.2 Hypothesis testing

Searching for point sources involves testing the compatibility of the data with two hypotheses; traditionally the default one is called $H_0$ and the exceptional one is called $H_1$. In our case they are $H_0$: "Only atmospheric neutrinos are present." and $H_1$: "In addition to the atmospheric neutrinos, there exists a point source of neutrinos."

Testing the compatibility of the data with these two hypotheses is accomplished by computing the value of the so-called test statistic $\lambda(data)$, which can, in principle, be any function of the data. The idea is to choose the test statistic such that, if $H_0$ is true, it is
expected to have a different value from the case where $H_1$ is true. Thus, the value of $\lambda$ indicates whether the data is more compatible with $H_0$ or with $H_1$. The distributions of $\lambda$ for $H_0$ and $H_1$ can be computed (numerically if needed). Using this information, one can define a set of values that is unlikely to contain $\lambda$ if $H_1$ is true. This region is called the 'rejection region' $\omega$. If $\lambda$ lies within the rejection region, hypothesis $H_0$ is 'rejected' in favour of $H_1$.

It is possible that $\lambda$ is contained in the rejection region, even though $H_0$ is true. In this case $H_0$ will be wrongly rejected. In our case this means that a discovery is claimed, while in reality there is no source. The probability that this happens can be calculated and is related to the Confidence Level (CL) of the test:

$$1 - \text{CL} = P(\lambda \in \omega | H_0).$$

(6.6)

The CL is usually chosen beforehand and it defines the rejection region.

The probability to reject $H_0$ in favour of $H_1$ if $H_1$ is indeed the correct hypothesis is called the 'power' of the test:

$$\text{power} = P(\lambda \in \omega | H_1).$$

(6.7)

At a fixed level of significance, the power of the test corresponds to the sensitivity for discovering the signal. It depends on the level of separation of the distributions of the test statistic for $H_0$ and $H_1$, as is illustrated in figure 6.5. Searching for point sources with optimal sensitivity is therefore equivalent to choosing the best test statistic, which is the subject of the next section.

### 6.3.3 Likelihood ratio test statistic

There are many possibilities to choose the test statistic. In the case of a binned method, for example, the test statistic is 'the number of entries in the bin with the highest content'. In the method presented here, the test statistic is defined as the ratio of the probabilities of the data under the assumption that $H_1$ respectively $H_0$ is the correct model:

$$\lambda = \log \left[ \frac{P(\text{data}|H_1)}{P(\text{data}|H_0)} \right].$$

(6.8)

The corresponding test is called the Neyman-Pearson test, or likelihood ratio test [44, 80]. It has been shown to be the best possible test in case $H_0$ and $H_1$ are completely specified. Here, $H_0$ and $H_1$ have unknown parameters that need to be measured from the data. In such cases it is customary to use equation 6.8, with the additional requirement that the unknown parameters are chosen such that the probability of the data is maximised:

$$\lambda = \log \left[ \frac{P(\text{data}|H_1^{\text{max}})}{P(\text{data}|H_0^{\text{max}})} \right].$$

(6.9)

where $H_1^{\text{max}}$ is hypothesis $H_1$ with the (unknown) parameters chosen such that $P(\text{data}|H_1)$ is maximal. In other words, maximum likelihood (ML) estimates are used to determine the unknown parameters. The background-only hypothesis $H_0$ also has unknown parameters.
6.3. Likelihood ratio search method

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Figure 6.5: Illustration of hypothesis testing. The probability density functions of the test statistic for $H_0$ and $H_1$ are shown. The rejection region $\omega$ is the region to right of the vertical line. The filled regions are related to the confidence level and the power of the test.

For example, the normalisation of the atmospheric neutrino flux is uncertain. However, after a period of data taking, the ML estimators of the corresponding parameters can be estimated, and we can consider $H_0$ to be fully specified. In the remainder of this chapter, it is therefore assumed that $H_0$ does not contain unknown parameters.

In terms of the flux $\Phi(\theta, \phi, E, t)$, $H_0$ can be written as

$$H_0: \quad \Phi(\theta, \phi, E, t) = \Phi^{bg}(\theta, E), \quad (6.10)$$

where $\Phi^{bg}$ is the atmospheric neutrino flux, which is known to be independent of time and azimuth angle.

It is important that $H_1$ is precisely defined, since the power and interpretation of the search depend on what one is searching for. The most general hypothesis that we will consider for $H_1$ is: In addition to the known background, there is a point source, with a power law spectrum with (unknown) spectral index, and (unknown) normalisation at an (unknown) position in the sky. $H_1$ can be written as

$$H_1: \quad \Phi(\theta, \phi, E, t) = \Phi^{bg}(\theta, E) + \Phi^{sig}(\theta, \phi, E, t), \quad (6.11)$$

where $\Phi^{sig}$ is the neutrino flux from the point source, which depends on the time due to the rotation of the Earth. It can be expressed as

$$\frac{d\Phi^{sig}(\theta, \phi, E, t)}{dEd\Omega} = \varphi \cdot \left( \frac{E}{\text{GeV}} \right)^{-\gamma} \delta [\cos \theta - \cos \theta_0(t, \alpha^r, \delta)] \delta [\phi - \phi_0(t, \alpha^r, \delta)], \quad (6.12)$$

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where $\delta$ is the Dirac delta function and $(\phi_0, \theta_0)$ is the time dependent location of the source in the sky, which depends on the (fixed) celestial coordinates $\alpha^i, \delta$. The spectral index of the energy spectrum of the source is denoted by $\gamma$ and $\varphi$ is the flux 'normalisation'.

In the case of a full sky search, ML estimators have to be determined for the parameters $\varphi, \gamma, \alpha^i, \delta$, whereas for a fixed point search, only $\varphi$ and $\gamma$ are free.

### 6.3.4 Expressions for the likelihood

The crucial ingredient for calculating the test statistic is the probability of the data for a given hypothesis, i.e. for a given neutrino flux.

After the reconstruction and selection procedures, the data will consist of a number of uncorrelated events. The $i$th event is characterised by the detection time $t_i$ and the true values of the neutrino's zenith and azimuth angles and energy: $\theta_i, \phi_i$ and $E_i$. The observed quantities are the reconstructed angles $\hat{\theta}_i, \hat{\phi}_i$, the reconstructed muon energy $\hat{E}_i$, and the time of the event $t_i$. The time of the event can be determined with an accuracy of about a millisecond, which makes the uncertainty on the event time negligible ($t = \hat{t}$) since the induced angular error is below $10^{-5}$ degrees.

The probability of obtaining a set of events $(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i)$ for flux $\Phi$ is (see appendix A)

$$\sum_i \log P(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i | \Phi) = \sum_i \log \left[N(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i | \Phi)\right] - \langle N_{\text{tot}} \rangle(\Phi) + C, \quad (6.13)$$

where $N(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i | \Phi)$ denotes the expected density of events with the observed parameters $\hat{\theta}, \hat{\phi}, \hat{E}, t$ per unit$^5$ solid angle, energy and time due to the flux $\Phi$. $\langle N_{\text{tot}} \rangle(\Phi)$ is the expectation value of the total number of detected events produced by the flux $\Phi$. The constant $C$ does not depend on the flux and will be omitted in the remainder of this chapter, since it plays no role in the calculation of likelihood ratios or maximum likelihood estimates.

In general, $\langle N_{\text{tot}} \rangle$ and $N(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i | \Phi)$ can be calculated from the flux $\Phi$ and the knowledge of the detector behaviour. The total number of expected events is given by a convolution of the effective area for neutrinos, $A_{\nu}^{\text{eff}}$, with the differential flux:

$$\langle N_{\text{tot}} \rangle = \int A_{\nu}^{\text{eff}}(\theta, \phi, E) \frac{d\Phi(\theta, \phi, E, t)}{dE d\Omega} d\Omega dEdt, \quad (6.14)$$

where the integration is over the live-time of the detector, over all upward directions and over all energies that produce events in the detector for the flux under consideration. Similarly, $N(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i)$ is given by

$$N(\hat{\theta}, \hat{\phi}, \hat{E}, t) = \int P(\hat{\theta}, \hat{\phi}, \hat{E} | \theta, \phi, E) A_{\nu}^{\text{eff}}(\theta, \phi, E, t) \frac{d\Phi(\theta, \phi, E, t)}{dE d\Omega} d\Omega dE, \quad (6.15)$$

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$^4$We have explicitly made the term $(\frac{E}{\text{GeV}})^{-7}$ dimensionless, so that $\varphi$ always has the same units (e.g. GeV$^{-1}$ m$^{-2}$ s$^{-1}$), regardless of the value of $\gamma$.

$^5$Note that the units used to express $N$ do not play a role since the factors resulting from a change of units can be absorbed in the constant $C$. 

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where $P(\hat{\theta}, \phi, \hat{E}|\theta, \phi, E)$ is the probability density (per unit solid angle and per unit energy) of obtaining reconstructed values $\hat{\theta}, \phi$ and $\hat{E}$ when the true neutrino direction and energy are $\theta, \phi$ and $E$.

In general, the integral in equation 6.15 is difficult to evaluate and numerical integration is very time consuming. However, for the fluxes that are of interest here, some approximations can be made to simplify the expression.

### Likelihood for the background flux

Both the flux of atmospheric neutrinos and the neutrino effective area are known to be a smooth function of the zenith angle. In contrast, the function $P(\theta, \phi, \hat{E}|\theta, \phi, E)$ is sharply peaked around the true direction $(\theta, \phi)$: the angle between the reconstructed muon direction and the true neutrino direction is typically smaller than a few degrees, which is small compared to the scales on which the flux or the effective area change significantly. This justifies the following approximation:

$$P(\hat{\theta}, \phi, \hat{E}|\theta, \phi, E) \approx \delta(\phi - \phi)\delta(\cos \theta - \cos \hat{\theta})P(\hat{E}|E), \quad (6.16)$$

where $P(\hat{E}|E)$ is the probability density function for obtaining a reconstructed muon energy ($\hat{E}$) as a function of the true neutrino energy ($E$). Using this approximation, the density of expected events can be expressed as

$$N^{bg}(\hat{\theta}, \phi, \hat{E}) = \int P(\hat{E}|E)A^{\text{eff}}(\hat{\theta}, \hat{\phi}, E)d\Phi^{bg}(\hat{\theta}, E)dE. \quad (6.17)$$

The expected total number of background events, which contributes to $\langle N_{\text{tot}} \rangle$ in equation 6.13, can be calculated from equation 6.17 by integrating over $\hat{E}$ and the angles, and by multiplying with the live-time. However, this quantity gives a constant contribution to the likelihood and it can therefore be neglected.

In practice, it will be possible to determine $N^{bg}$ directly from the data, since, in first order, all events can be considered to be background. In this way, the influence of systematic uncertainties in the atmospheric neutrino flux and the effective area may be reduced.

### Likelihood for the signal flux

The differential flux from a point source with a power law energy spectrum with spectral index $\gamma$ is given by equation 6.12. Substitution into equation 6.15 yields

$$N^{\text{sig}}(\hat{\theta}, \phi, \hat{E}, t) = \varphi \int P(\hat{\theta}, \phi, \hat{E}|\theta, \phi, E)A^{\text{eff}}(\theta_0(t, \alpha^\text{ra}, \delta), \phi_0(t, \alpha^\text{ra}, \delta), E) \left(\frac{E}{\text{GeV}}\right)^{-\gamma}dE \quad (6.18)$$

The three-dimensional PDF $P(\hat{\theta}, \phi, \hat{E}|\theta, \phi, E)$ has been factorised. One factor is $P(\hat{E}|E)$, which was introduced in equation 6.16. The other is the probability density (per unit solid angle) for obtaining the reconstructed muon direction $(\hat{\theta}, \phi)$ for a true neutrino direction $(\theta, \phi)$. This PDF is known as the point spread function (PSF), since it describes the distribution of the reconstructed coordinates if the events come from the
same point in the sky. The PSF is parameterised as a function of the angle $\alpha_\nu$ between the directions of the neutrino and the reconstructed muon. The PDF of $\alpha_\nu$ is determined for each event separately from the true neutrino energy ($E$) and the estimate of the error on the muon direction $\hat{\alpha}_\mu$ (see section 4.7.2). The expression for $P(\hat{\theta}, \hat{\phi}, \hat{E}|\theta, \phi, E, \hat{\alpha}_\mu)$ thus becomes

$$P(\hat{\theta}, \hat{\phi}, \hat{E}|\theta, \phi, E) = P(\alpha_\nu|\hat{\alpha}_\mu, E)P(\hat{E}|E).$$

(6.19)

The factors $P(\alpha_\nu|\hat{\alpha}_\mu, E)$ and $P(\hat{E}|E)$ have been estimated from simulations as will be discussed in section 6.3.5. The expected density of events from the signal flux is then

$$N_{\text{sig}}(\theta, \phi, \hat{E}, t) = \varphi \int P(\alpha|\hat{\alpha}_\mu, E)A_\nu^{\text{eff}}(\theta_0(t, \alpha^{\text{ra}}, \delta), \phi_0(t, \alpha^{\text{ra}}, \delta), E)\left(\frac{E}{\text{GeV}}\right)^{-\gamma} dE.$$  

(6.20)

As expected, this expression contains four unknown parameters: the position of the source ($\alpha^{\text{ra}}, \delta$), the spectral index $\gamma$ and the flux normalisation $\varphi$.

The expectation value of the total number of signal events $\langle N_\nu \rangle$ gives the only relevant contribution to the term $\langle N_{\text{sig}} \rangle$ in equation 6.13, since the expected number of background events is constant (and can thus be absorbed in the constant $C$). It is calculated by

$$\langle N_\nu \rangle = \varphi \int A_\nu^{\text{eff}}(\theta_0(t, \alpha^{\text{ra}}, \delta), \phi_0(t, \alpha^{\text{ra}}, \delta), E)\left(\frac{E}{\text{GeV}}\right)^{-\gamma} dE dt.$$  

(6.21)

For speed, the integration over time of $A_\nu^{\text{eff}}(\theta_0(t, \alpha^{\text{ra}}, \delta))$ is performed by using a table of the time-averaged effective area as a function of the celestial coordinates, which is multiplied by the live-time.

In conclusion, the calculation of each of the terms discussed in this section involves only the (one-dimensional) integration over the energy, instead of the three-dimensional integration in equations 6.14 and 6.15. As a result, the computation is now fast enough to be used in a practical search method.

### 6.3.5 Ingredients of the likelihood calculation

Knowledge of the detector response is used in the search method to evaluate equations 6.17, 6.20 and 6.21. The information used consists of three parts: the PSF, the effective area and a table describing the behaviour of the estimate of the muon energy as a function of the neutrino energy.

The PSF has been parameterised because it is used directly for fitting the position of the source. The maximisation algorithm is expected to work better with smooth functions than with tabulated values. In contrast, the information on the behaviour of the energy reconstruction and the effective area are not used directly in the fit, but are convolved with the neutrino spectrum. Therefore, no parameterisations are needed for these two quantities and tabulated values are used.

**Point spread function**

The point spread function is related to the PDF of $\alpha_\nu$, which was shown in figure 5.6 for the case of an $E^{-2}$ spectrum. Here, this PDF is parameterised as a function of the
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Figure 6.6: Two examples of the distribution of the angle \( \alpha_\nu \) between the neutrino direction and the reconstructed muon direction for two arbitrarily chosen bins in the neutrino energy \( E \) and the error estimate \( \hat{\alpha}_\mu \) obtained from simulations. The line is a fit to the data using the functional form of equation 6.22. This parameterisation is used in the search method.

Neutrino energy \( E \) and the estimated error on the muon direction, \( \hat{\alpha}_\mu \). The PDF was parameterised for 12 × 8 bins\(^6\) in \( \log(E) \) and \( \log(\hat{\alpha}_\mu) \) respectively, using the following functional form:

\[
\frac{dP}{d\alpha_\nu} \propto \frac{\alpha_\nu}{1 + a\alpha_\nu^2 + b\alpha_\nu^4}.
\]

(6.22)

Two examples of the distribution of the angle \( \alpha_\nu \) and the fitted parameterisations are shown in figure 6.6.

The point spread function used in the search method (i.e. the term \( P(\alpha|\hat{\alpha}_\mu, E) \) in equation 6.20) is the probability density per unit solid angle. It is related to the PDF of \( \alpha_\nu \):

\[
P(\alpha|\hat{\alpha}_\mu, E) = \frac{dP}{d\Omega} = \frac{1}{2\pi \sin \alpha_\nu} \frac{dP}{d\alpha_\nu}.
\]

(6.23)

The form of equation 6.22 was chosen such that equation 6.23 is well behaved for \( \alpha_\nu = 0 \), which ensures that no singularities occur in the likelihood function.

**Neutrino effective area**

A plot of the neutrino effective area was already shown in figure 5.5. In the search method, a table of the neutrino effective area as a function of the neutrino energy and the zenith angle is used. The azimuth dependence of the neutrino effective area has been neglected throughout this analysis in order to decrease statistical bin-by-bin fluctuations.

\(^6\)In some bins, not enough events were simulated to fit the data. In that case, values of \( a \) and \( b \) (see equation 6.6) were copied from bins with the same \( \hat{\alpha}_\mu \), but higher \( E \).
PDF of the reconstructed muon energy

The probability density function of finding a muon with reconstructed energy $\hat{E}$ was determined from simulations as a function of the neutrino energy $E_\nu$. It is shown in figure 6.7.

![Figure 6.7: Probability density function of the reconstructed muon energy as a function of the true neutrino energy.](image)

6.3.6 Likelihood maximisation

To evaluate the test statistic, maximum likelihood estimates of the unknown parameters of the signal hypothesis are needed. This means that values for $\alpha^{ra}, \delta, \gamma$ and $\varphi$ must be found that maximise

$$\log P(\text{data}|\alpha^{ra}, \delta, \gamma, \varphi) = \sum \log [\mathcal{N}^{\text{sig}}(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i|\alpha^{ra}, \delta, \gamma, \varphi) + \mathcal{N}^{\text{bg}}(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i|\Phi^{\text{bg}})] - \langle N_s \rangle(\alpha^{ra}, \delta, \gamma, \varphi),\quad (6.24)$$

where $\langle N_s \rangle(\alpha^{ra}, \delta, \gamma, \varphi)$ is the expectation value of the number of events due to the signal flux and $\mathcal{N}^{\text{sig}}(\hat{\theta}_i, \hat{\phi}_i, \hat{E}_i, t_i|\alpha^{ra}, \delta, \gamma, \varphi)$ is given by equation 6.20.

For any given source position $(\alpha^{ra}, \delta)$, the factor $P(\alpha|\hat{\alpha}_\mu, E)$ in equation 6.20 will only be non-negligible for events with reconstructed celestial coordinates that are close to the source position. It is therefore sufficient to evaluate only one cluster of events at a time.

The clusters are selected by a clustering algorithm. A cluster can be built around each event by selecting all other events that are located within a cone with an opening angle of 1.25°. This opening angle is called the cluster size. It was chosen to be large compared to the typical angular resolution in order to ensure that all relevant events are selected. However, it can not be chosen too large, since the number of clusters increases very rapidly with the cluster size. The value of 1.25° was found to give an acceptable
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6.4.1 Distribution of the test statistic

The distribution of the test statistic for the background-only case has been obtained from a simulation of about 50000 one-year periods of data taking, containing, on average, 3650 selected atmospheric neutrino events each. These samples were generated by (over)sampling a set of \( \sim 66 \times 10^3 \) simulated events while randomly choosing the time of detection and azimuth angle, both of which enter into the calculation of the celestial coordinates. In each one-year sample, the likelihood ratio is determined for an average of 74 clusters of events. The highest value of the likelihood ratio is the test statistic \( \lambda \) obtained for the one-year experiment.

The distribution of \( \lambda \) for background-only experiments is shown in figure 6.8. A discovery is made if the test statistic exceeds a critical value \( \lambda_c \). The value of \( \lambda_c \) follows
Figure 6.8: Left: Distribution of the test statistic in case only background is present. An exponential function is fitted to the tail of the distribution to extrapolate to the 5σ level. Right: Cumulative distribution of the test statistic \( \int_{\lambda_c}^{\infty} \frac{dP}{d\lambda} d\lambda \). Indicated are the probabilities corresponding to the 3σ and 5σ CL.

from the distribution of \( \lambda \) in the case there is only background and from the desired confidence level (CL), since, by definition, \( 1 - \text{CL} \equiv P(\lambda_0 > \lambda_c | H_0) \). Values for \( \lambda_c \) have been determined from figure 6.8(right) for the confidence levels (CLs) of \( 1 - 2.7 \times 10^{-3} \) and \( 1 - 5.7 \times 10^{-7} \), which are also known as the 3σ and 5σ CLs. Due to the limited number of simulated experiments, it was not possible to determine the value of \( \lambda_c \) that corresponds to the 5σ CL by means of simulations. In order to estimate this value, an exponential function was fitted to the distribution in figure 6.8, which is used to extrapolate to the probability corresponding to the 5σ CL. This yields the following values for \( \lambda_c \) for 3σ and 5σ CLs: \( \lambda_c^{3\sigma} = 16.3 \) and \( \lambda_c^{5\sigma} = 26.1 \).

The distribution of the test statistic for a source at a declination of \( \delta = -81° \) is shown in figure 6.9 for \( N_s = 3, 6, 9 \) and 12 detected signal events. As expected, the value of \( \lambda \) is increased by the presence of the signal. Whereas \( N_s \) represents the actual number of signal events that are detected (i.e. pass the selection criteria), only the expectation value of \( N_s \), \( \langle N_s \rangle \), can be calculated from a given neutrino flux. \( N_s \) is distributed according to a Poisson distribution with mean \( \langle N_s \rangle \). The distribution of the test statistic for any value of \( \langle N_s \rangle \) can therefore be calculated by

\[
P(\lambda | \langle N_s \rangle) = \sum_{N_s=0}^{\infty} P(\lambda | N_s) \frac{\langle N_s \rangle^{N_s} e^{-\langle N_s \rangle}}{N_s!}.
\]

(6.26)

In practice the summation is truncated when the terms become negligible\(^7\).

\(^7\)The maximal value of \( N_s \) in the simulations is 14.
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Figure 6.9: Distribution of the test statistic for the background only case and for the case where a number of signal events are present. The point source signal has an $E^{-2}$ energy spectrum and is located at a declination of $-81^\circ$.

6.4.2 Examples

As an illustration of the LR method, three rare examples of clusters of events and the corresponding value of the test statistic are shown in figure 6.10. The cluster in figure 6.10.a consists of only four events, but since three of them are located very close to the fitted (and true) source position, the likelihood ratio has a value of 19.20, which means that the source would be discovered at $3\sigma$ CL. In contrast, when using the binned method discussed in section 6.2 at least 6 events are needed to discover a source at $3\sigma$ CL at this declination (see figure 6.3).

The cluster in figure 6.10.b is an example of a cluster that has a low value of the test statistic, 11.48, despite the relatively large number of events (i.e. 6). Typically, clusters with 6 signal events, have $\lambda \approx 15$, but this cluster is more background-like due to the large angular deviations and the low values of the reconstructed energy of the events.

Finally, the cluster in figure 6.10.c is an example of a background-only cluster that has a high value of the test statistic: $\lambda = 17.56$. The probability that such a cluster (or one with a even higher value of $\lambda$) appears in a one year background-only sample is about $10^{-3}$. Apart from the small angular separation between the events and the relatively high energies, the likelihood ratio of this cluster is large because it occurs at a declination of $15^\circ$, where the number of background events is relatively low. As a consequence the likelihood for the background hypothesis is low for this cluster, which enhances the value of the test statistic.
Figure 6.10: Examples of clusters of events and the corresponding test statistic. Filled circles represent signal events and open circles are background events. The $\log_{10}$ of the reconstructed energy and error estimate (in degrees) are indicated for each event, separated by a '/'. The + denotes the fitted source position, while ★ indicates the true source position. The concentric circles have radii of 0.5° and 1°, and are centred on the event that formed the seed of the cluster.

6.4.3 Fitted source position

For each cluster, the source coordinates are fitted to the data. As a result, we obtain an ML estimate of the source location. In figure 6.11 this fitted source position is compared to the true position of the source.

The resolution is defined as the median of the angular error on the source position, similar to the resolution of a single event (see section 5.2.3). As expected, the resolution on the source position improves as the number of detected events from the source, $N_s$, increases. The resolution could be expected to be approximated by $\alpha_{\gamma}^{-2}/\sqrt{N_s}$, where $\alpha_{\gamma}^{-2}$ is the median of the single-event resolution for an $E^{-2}$ neutrino source, which is 0.23°. The ML estimate of the source position is somewhat more accurate than this, as is shown in figure 6.11. In order to discover a source at $3\sigma(5\sigma)$ CL, about 6(9) events need to be detected. The position of the source can then be determined with an accuracy of about $0.08^\circ(0.07^\circ)$. 
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Figure 6.11: Left: Distribution of the error of the fit of the right ascension and the declination of the neutrino point source for a source producing 6 events in the detector. Right: Resolution of the determination of the position of the source as a function of $N_s$, i.e. the number of events produced by the source. The dashed line indicates the 'naive expectation' that results from scaling the single event resolution with $1/\sqrt{N_s}$.

6.4.4 Discovery potential

The probability of discovering a source (i.e. the power of the test) at the $3(5)\sigma$ level is given by the probability of $P(\lambda|\langle N_s \rangle)$ to exceed the value of $\lambda_c^{3\sigma}$ ($\lambda_c^{5\sigma}$), which was found in section 6.4.1. This probability is calculated using equation 6.26 and the result is shown in figure 6.12 as a function of $\langle N_s \rangle$ for a source at a declination of $-81^\circ$.

Figure 6.12: The probability for the discovery of a point source as a function of the expectation value of the number of observed signal events from the source (which is directly related to the flux of the source) at the $3\sigma$ and $5\sigma$ level after one year of data taking.
As a measure of the discovery potential, we take the number of events (or neutrino flux) needed for a 50% probability to discover the source. This quantity is shown in figure 6.13 and is compared to the discovery potential obtained with the binned method described in section 6.2. Compared to the binned method, the LR method needs roughly 40(35)% less events in order to make a $3(5)\sigma$ discovery after one year of data taking. Consequently, the LR method can discover a source at the $5\sigma$ level, while it is only a $3\sigma$ effect when using the binned method. Using equation 6.21, the average number of events needed for discovery can be directly translated into the normalisation of the required neutrino flux, which is shown in figure 6.14. In order to make a $5\sigma$ discovery, a neutrino flux is needed of roughly $1.4 \times 10^{-3} E^{-2} \text{GeV m}^{-2} \text{s}^{-1}$ for the lowest declinations.

![Figure 6.13: Discovery potential of the search after one year of data taking, in terms of the expectation value of the number of signal events required to yield a probability of 50% for discovery of the source in a full sky search as a function of the declination of the source. Also shown is the result obtained with the binned method.](image)

**Discovery potential after two years of data taking**

The discovery potential was also determined for a two-year data taking period. The results are shown in figure 6.15. Since there are more background events in the two-year sample, between roughly 5% and 20% more events are needed, depending on the declination. As a result, the flux that can be discovered is a factor 1.7 to 1.9 lower than the flux that can be discovered after one year.

**6.4.5 Exclusion limit**

If a small value of the test statistic is observed, no discovery can be claimed, but the existence of an intense source of neutrinos can be excluded. This results in a declination dependent upper limit on the flux.

If, after a period of data taking, a value $\lambda_0$ is obtained, then, all signal hypotheses that would yield a value greater than $\lambda_0$ in at least 90(99)% of the cases can be excluded.
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Figure 6.14: Neutrino flux from a point source needed to yield a probability of 50% of discovery of the source in a one-year full sky search as a function of the declination.

Figure 6.15: Left: Number of events needed for a 3(5)σ discovery of a point source, after two years of data taking. Right: Flux needed for discovery of a point source after two years of data taking.

at 90(99)% CL. As a characteristic value for \( \lambda_0 \), we take the median of the distribution of the test statistic in case there is only background. Using a similar calculation as for the discovery potential, the corresponding flux limit is found in terms of the expectation value of the number of signal events \( \langle N_s \rangle \). The result is shown in figure 6.16. Since \( \lambda_0 \) is the median of the distribution, the result should be interpreted as follows: *If there is no point source of neutrinos, there is a 50% probability that we will be able to set the limit...*
shown in figure 6.16 or a stronger one.

Figure 6.16: Left: Median exclusion limit for an $E^{-2}$ source anywhere in the sky in terms of the number of events produced by that source. The probability of being able to set this limit or a stronger one is 50% in the case that there is no signal. Right: Median exclusion limit on the $(E^{-2})$ flux from a source anywhere in the sky.

6.5 Fixed point searches

In this section, the LR method will be used to 'search' for a source at a number of predefined locations in the sky. This restriction increases the sensitivity for a source at the specified location by reducing the 'trial factor' that stems from searching at many different locations in the sky. While it is unlikely that, in reality, the search will be restricted to only one single point, the upper limit on the flux that is obtained from such a search is an important quantity, as will be explained in section 6.5.3.

6.5.1 Distribution of the test statistic

For a single-point search, the distribution of the test statistic $\lambda$ for the background-only case is shown in figure 6.17. In many cases, the fitted flux is zero, resulting in $\lambda = 0$. In these cases, the fluctuations in the background are less signal-like than the average background cluster. The tail of the distribution is approximately proportional to $e^{-s\lambda}$, with $s \approx 1$. The fitted values of $s$ vary between 0.92 and 0.99, depending on the declination. The exponential distribution may be intuitively understood: for the background-only case, the probability to obtain a given likelihood ratio is inversely proportional to the likelihood ratio. Under the assumption that the ML estimates of the free parameters in $H_1$ are normally distributed around the true values, it can be shown [80] that $\lambda$ is distributed
6.5. Fixed point searches

Figure 6.17: Left: Distribution of the test statistic for a fixed point search for simulated background-only experiments. The line is an exponential function that was fitted to the data. Right: Cumulative distribution, indicating the probability to obtain a value of the test statistic that is higher than the ordinate. Indicated are the probabilities corresponding to the 3σ and 5σ confidence levels and the line corresponding with a probability of 50%. The line corresponds to the fit in the left figure.

according to a $\chi^2(r)$ distribution, where $r$ is the number of unknown parameters in $H_1$. This assumption in not strictly valid here, since the true value ($\varphi = 0$) is at the boundary of the allowed region ($\varphi > 0$), which results in the peak at $\lambda = 0$. However, when considering only the cases with positive $\varphi$, the distribution is reasonably well approximated by a $\chi^2(2)$ distribution, i.e. $dP/d\lambda \propto \chi^2(2) = e^{-\lambda}$.

If multiple candidate sources are being considered, the test statistic is taken to be the likelihood ratio of the most signal-like candidate (i.e. the one with the highest likelihood ratio). The distribution of the test statistic is shown in figure 6.18 for the cases where 10 and 100 candidate sources are being considered. The sources were assumed to have random positions, uniformly distributed in $\sin(\delta)$. The PDF of $\lambda$ scales linearly with the number of candidates for large values of $\lambda$. This is expected as long as the candidate sources are separated by angular distances much larger than the resolution of the detector.

The distribution of the test statistic when a signal is present is shown in figure 6.19 for several values of the number of selected signal events. Again, the presence of one or more signal events results in an increase in the test statistic.

6.5.2 Discovery potential

The probability of discovering the source at the 3(5)σ CL has been calculated from the distributions of $\lambda$ in the same manner as described in section 6.4.4. The number of signal events needed for a 50% discovery probability is shown in figure 6.21 for the cases where
1,10 and 100 source candidates are considered.

The number of signal events required for a discovery with the binned method is also shown in figure 6.17. The LR method requires typically about 15% less events to discover a source. While this is a modest improvement, it means, for example, that 100 candidate sources can be considered instead of 10 while retaining the same sensitivity for a $5\sigma$ discovery.

The neutrino flux needed for a discovery is shown in figure 6.17(right). It can be compared to the flux needed for a discovery in a full sky search (figure 6.14). If the search is restricted to 10 sources, for example, the flux needed for a discovery is roughly half the

Figure 6.18: Distribution of the test statistic in a fixed point search for background-only when 1,10 and 100 candidate neutrino sources are being considered. The candidate sources are uniformly distributed in the sky. The lines are exponential fits to the distributions.

Figure 6.19: Distributions of the test statistic in a fixed point search for background-only and one candidate source (filled histogram), and for the case where a number of signal events (1 to 6, from left to right) are present in addition to the background. The declination of the source is $-81^\circ$.
6.6. Discussion

Point source searches

![Figure 6.20: Probability to discover the source in a one-year fixed point search with one candidate source as a function of the expectation value of the number of detected events for a declination of $-81^\circ$.](image)

flux that would be required in a full sky search.

6.5.3 Upper limits

It is unlikely that a point source search will be restricted to only a single point in the sky. However, the value of the test statistic obtained for a particular point in the sky can also be used to set an upper limit on the flux from that particular direction. The value of the limit depends on the observed value of the test statistic $\lambda_0$, which is distributed as shown in figure 6.17 under the assumption that no neutrino source is present. The mean value of $\lambda$ varies between 0.5 ($\delta = -90^\circ$) and 0.2 ($\delta = 40^\circ$). The resulting mean number of events from a point source that can be excluded is shown in figure 6.22 for 90% and 99% CL.

A flux limit can be determined in this way for each point in the sky individually. However, the presence of a point source at any location is not ruled out by these limits, in contrast to the limits discussed in section 6.4.5.

6.6 Discussion

The LR method that has been presented in this chapter is a valuable tool for detecting point sources with ANTARES. Compared to the more conventional binned method, the discovery potential is increased by up to 40%. This means that sources can be discovered at the $5\sigma$ CL that can only be detected at about the $3\sigma$ CL with the binned methods. Below some more aspects of the method are discussed.

- A disadvantage of the LR method is its complexity. Both the principle and the implementation of the method are more complicated than for binned methods. Furthermore, the LR method relies on knowledge of the detector response, which is implemented via parameterisations and tables of the point spread function, the neutrino effective area and the response of the energy estimator. Inaccuracies in any of these will result in a degraded sensitivity of the search method. However,
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Figure 6.21: Left: Expectation value of the number of detected events from an $E^{-2}$ neutrino point source needed to yield a 50% probability of discovering the neutrino point source as a function of declination. The three curves correspond to a search in which 1, 10 and 100 (from bottom to top) candidate sources are considered. Also shown are the results from the binned method. Right: Neutrino flux needed to give a 50% discovery probability as a function of the source declination for a search with 1, 10 and 100 (from bottom to top) source candidates.

Figure 6.22: Left: Mean upper limit that can be set on the mean number of signal events $\langle N_s \rangle$ from an $E^{-2}$ neutrino point source as a function of declination. Right: Mean upper limit that can be set on the neutrino flux from an $E^{-2}$ point source as a function of declination.
the probabilistic statements made will remain valid; i.e. the method does not require perfect knowledge of the detector response in order to accurately determine the significance of a cluster of events. Moreover, binned methods require knowledge of the detector response as well. Knowledge of the angular resolution, for example, is used to optimise the bin size. The influence of systematic uncertainties in the angular resolution, acceptance (effective area) and energy estimation on the sensitivity should be studied in the future for both methods.

- The LR method was developed to be an optimal way to search for point sources: at any confidence level, the discovery potential (power) should be optimal. The derivation in the first part of this chapter explicitly states the approximations made that could lead, in principle, to degradation of the method compared to the full maximum likelihood ratio test.

- The LR method does not require any optimisation. No special cuts are required on the input sample and there are no bin sizes to be optimised. The disadvantage of such optimisations is that one needs to make a choice what to optimise for. A binned method optimised for maximum discovery potential at e.g. 3σ CL will, in general, not be optimal at other confidence levels nor for setting an upper limit. Note that, in the comparison with the binned methods, the bin size was always chosen to be optimal for the stated CL.

- By simply fixing or releasing parameters in the fitting routine that finds the most likely signal hypothesis, one can change the scope of the search-method. For example the search could be restricted to look only for sources with a particular spectrum or to look only for sources at specific points in the sky. Each of these searches will have its own interpretation and discovery potential.

6.6.1 Comparison with models and experiments

Searches for point sources of neutrinos have been conducted by various experiments. No sources have been discovered, and upper limits on the flux have been published by the MACRO [81] and AMANDA-II [82] experiments. These are 'point-by-point' limits of the type discussed in section 6.5.3. The established limits are shown in figure 6.23 and can be compared to the limit that ANTARES expects to set in one year. The MACRO results are for a live-time of 6.3 years. The AMANDA-II limit is obtained from 197 days of data taken in the year 2000. It will be further improved as more data is analysed. Also indicated is the limit that is expected to be set after one year of taking data with the IceCube detector, which is being built on the South Pole [84, 85] and is planned to be completed by 2011. In the northern hemisphere, ANTARES will improve on the present limits after one year of data taking. The flux needed for a 5σ discovery is also shown in the figure. The limits set by MACRO do not exclude the possibility that ANTARES will discover a source at 5σ CL. For the part of the sky which is visible to both ANTARES and AMANDA, the latter experiment already excludes this possibility, although one should

8A search was also performed by the Super-Kamiokande [83] collaboration, but only limits on the neutrino induced muon flux have been published.
bear in mind that the AMANDA limit in the figure represents the average limit and that there are source candidates for which the limit is higher.

It should be stressed that any inefficiencies due to dead time of the detector or high levels of optical background have been neglected in the calculation for ANTARES. Depending on the circumstances, the sensitivity presented here may only be reached after several years of operating the detector, especially since the optical background rate is often much higher than 60 kHz, which is the value used in the simulations (see section 2.6).

A number of flux models were already shown in figure 1.3. Most of the fluxes in this figure are too low to be detected by ANTARES, even after several years of data taking. This confirms the general notion that neutrino telescopes require an effective area of the order of a km$^2$. Nevertheless, the AGN core model \cite{14} predicts fluxes in the sensitivity range of ANTARES. Hence, the most luminous AGNs could be detected if a significant fraction of the non-thermal energy emitted by these objects is due to hadronic interactions.

In addition, figure 6.23 shows neutrino flux predictions for a number of Galactic microquasars taken from \cite{26}. The intense fluxes predicted imply that microquasars may be the first objects that will be detectable by neutrino telescopes. Many microquasars show strong temporal variability. The fluxes are given for the active state and may only be applicable for a fraction of the time. Furthermore, oscillations are not taken into account, so the expected flux of $\nu_e$'s is actually a factor two lower. If these predictions are correct, ANTARES will start to be sensitive to the most intense microquasars within a year of data taking.

Finally, a model for the neutrino flux due to GRBs is shown in the figure. In this model, the neutrinos are due to accelerated protons, which interact first in the remnants of the progenitor star (in this case mainly the hydrogen envelope) and later in internal shocks in the GRB jet. GRBs could be detected \cite{87} using external information (provided by satellites) by selecting events that are closely correlated to the GRB in direction and time. The search will then be virtually background free. The results presented here are thus conservative estimates for the discovery potential of ANTARES for GRBs.

$^9$In \cite{86}, the diffuse flux is given. We have integrated this over $2\pi$ sr, i.e. the instantaneous field of view of the detector.
Figure 6.23: Upper limits (90% CL) on the neutrino flux as a function of declination. Established limits are shown from MACRO (limit on a selected number of sources [81]) and AMANDA-II (average limit) [82]. The expected average limits after one year of data taking are shown for ANTARES and IceCube [84]. The flux needed for a 5σ discovery at ANTARES after one year of data taking is also shown.