Measurement of the W boson mass and width with the L3 detector

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Chapter 5

W mass and width measurement

In this chapter the determinations of $M_W$ and $\Gamma_W$ are discussed. The mass $M_W$ and width $\Gamma_W$ are extracted from the distribution of the reconstructed invariant masses of the selected events. This is called the direct reconstruction method. The parameters $M_W$ and $\Gamma_W$ are estimated with a maximum likelihood fit.

The fitting procedure is applied for the semi-leptonic and the fully hadronic channels for the years 1998, 1999 and 2000 at the different centre-of-mass energies. Finally, the sources of systematics are determined and the combination of the measurements is presented.

5.1 Likelihood function

In the direct reconstruction method the mass $M_W$ and width $\Gamma_W$ are determined with a maximum likelihood fit method. This parameter estimation method uses the invariant mass distribution after the kinematic fit with the equal mass constraint.

The probability of getting a measurement $M_{\text{inv}}$ within a small interval $[M_{\text{inv}}, M_{\text{inv}} + dM_{\text{inv}}]$ is given by the probability density

$$dP = \omega dM_{\text{inv}},$$

with

$$\omega(M_{\text{inv}}, M_W, \Gamma_W) = \frac{1}{f \cdot \sigma_{WW}^{\text{acc}}(M_W, \Gamma_W) + \sigma_{bg}^{\text{acc}}}
\times \left\{ f \cdot \frac{d\sigma_{WW}^{\text{acc}}}{dM_{\text{inv}}}(M_{\text{inv}}, M_W, \Gamma_W) + \frac{d\sigma_{bg}^{\text{acc}}}{dM_{\text{inv}}}(M_{\text{inv}}) \right\}.$$  

The parameter $\omega(M_{\text{inv}}, M_W, \Gamma_W)$ is the normalised differential cross-section $d\sigma/d\Omega$ at the measured invariant mass $M_{\text{inv}}$ in the presence of background. The accepted signal $W$-pair production differential cross-section is denoted by $\frac{d\sigma_{WW}^{\text{acc}}}{dM_{\text{inv}}}(M_{\text{inv}}, M_W, \Gamma_W)$, while $\frac{d\sigma_{bg}^{\text{acc}}}{dM_{\text{inv}}}(M_{\text{inv}})$ is the accepted background differential cross-section. The factor $\left( \frac{1}{f \cdot \sigma_{WW}^{\text{acc}}(M_W, \Gamma_W) + \sigma_{bg}^{\text{acc}}} \right)$ in equation 5.3, normalises the probability distribution which is essential for the maximum likelihood method. $\sigma_{WW}^{\text{acc}}(M_W, \Gamma_W)$ and $\sigma_{bg}^{\text{acc}}$ are the total accepted cross-sections of signal and background. The factor $f$ satisfies

$$f \cdot \sigma_{WW}^{\text{acc}}(M_W, \Gamma_W) + \sigma_{bg}^{\text{acc}} = \frac{N_{\text{data}}}{L}.$$  

(5.4)
5.2. Box method

Hence \( f \) normalises the cross-section to the measured \( W \)-pair cross-section. The fluctuations in the number of data are assigned to the \( WW \) signal.

The likelihood of getting a certain set of measurements \( M_{\text{inv}}, i = 1, N_{\text{data}}, \) i.e. the likelihood of an observed mass distribution in the data is the product of the individual event probabilities \( \omega(M_{\text{inv}}^i, M_W, \Gamma_W) \) for each selected event \( i \),

\[
L(M_W, \Gamma_W) = \prod_{i=1}^{N_{\text{data}}} \omega(M_{\text{inv}}^i, M_W, \Gamma_W). \tag{5.5}
\]

The likelihood in equation (5.5) should be maximised in order to determine the mass.

In practice, it is convenient to consider the negative logarithm of the likelihood function

\[
-\log(L) = - \sum_{i=1}^{N_{\text{data}}} \log \omega(M_{\text{inv}}^i, M_W, \Gamma_W). \tag{5.6}
\]

The product of the individual probabilities is transformed into the sum of their logarithms which is numerically more stable. Because of the fact that the MINUIT \[60\] package is used, which is optimised for minimising functions, the log-likelihood is multiplied by \(-1\). As a consequence equation (5.6) should be minimised.

In a one-parameter fit only the \( W \) boson mass \( M_W \) is extracted. In the one-parameter fit the \( \Delta \) is treated as a free parameter while \( \Gamma_W \) is fixed to the Standard Model relation

\[
\Gamma_W = \frac{3G_F M_W^3}{2\sqrt{2}\pi} \left(1 + \frac{2\alpha_s(M_W^2)}{\frac{3\pi}{2}}\right). \tag{5.7}
\]

In the two-parameter fit the mass as well as the width of the \( W \) boson are simultaneously determined. Both the width and mass, \( \Gamma_W \) and \( M_W \), are treated as free and independent parameters in the two-parameter fit.

5.2 Box method

The likelihood cannot be calculated analytically due to the selection criteria and smearing effects, which should be taken into account. Therefore a Monte Carlo method [61] is used to calculate the cross-section and differential cross-sections in the log-likelihood function (5.6). In this method the events from a Monte Carlo simulation are used to calculate the cross-sections. Since the Monte Carlo simulation includes as an integral part the acceptance and smearing, these effects are automatically taken into account in the calculations.

For the calculation of the cross section and differential cross section the so-called box method is used.

The differential cross-section can be calculated from the number of Monte Carlo events \( N_{\text{box}}^i \) in an interval around a data point corresponding with a mass measurement \( M_{\text{inv}}^i \). This method may also be used in a multi-dimensional space where in general the interval is called a box. If the size of the chosen interval is \( V_{\text{box}}^i \), the differential cross-section is approximated by

\[
\frac{d\sigma^{\text{acc}}}{dM_{\text{inv}}} (M_{\text{inv}}^i) \approx \frac{\Delta\sigma^{\text{acc}}}{\Delta M_{\text{inv}}} (M_{\text{inv}}^i) = \frac{1}{\mathcal{L}_{\text{MC}}} \cdot \frac{1}{V_{\text{box}}^i} \sum_{j=1}^{N_{\text{box}}^i} 1. \tag{5.8}
\]
The Monte Carlo luminosity is determined from the total number of Monte Carlo events and the total process cross-section without cuts:

\[ \mathcal{L}_{MC} = \frac{N_{MC}}{\sigma^{tot}}. \]  

(5.9)

Only in the limit where the number of events in the box is infinite and the size of the box is infinitely small in equation (5.8), the exact value is obtained. Figure 5.1 shows the principle of the box method.

The box size is chosen to be as small as possible to approximate the exact value while the number of events in the box (box occupancy) has to be large enough to reduce the statistical error of the calculation in the box. Earlier studies [41] have shown that an optimal performance is reached if the box size is 0.5 GeV. A supplementary constraint on the maximum number of Monte Carlo events per box of 1000, fulfills the requirement of even smaller box sizes than 0.5 GeV, while keeping sufficient events in the box. Another constraint on the minimum number of events per box of 100 prevents the increase of the statistical error for certain data points.

The results of these conditions are graphically represented in figure 5.2. The number of events in a box is between 100 and 1000. In the region where the distribution \(M_{inv}\) is close to its maximum the maximal box occupancy of 1000 events is reached while the box volume decreases to about 0.1 GeV. The differential cross-section, which is the ratio between the box occupancy and the box volume normalised to \(\mathcal{L}_{MC}\), has its peak in this region. Going away from this region the box volume increases, whereas the number of events per box remains 1000, until the size of the box is 0.5 GeV. At this point the box size remains constant at 0.5 GeV and the box occupancy decreases from 1000 to 100 events. At the tails the Monte Carlo events are less dense occupied. The box volume must increase very rapidly to fulfill the requirement of a minimum of 100 events.

The Monte Carlo prediction for the cross-section is given by:

\[ \sigma_{WW}^{acc} = \frac{1}{\mathcal{L}_{MC}} \sum_{j=1}^{N_{MC}} \frac{1}{N_{acc}}. \]  

(5.10)

where \(N_{acc}^{MC}\) is the total number of selected Monte Carlo events.

### 5.3 Monte Carlo re-weighting method

The cross-section and differential cross-section as determined by equations (5.10) and (5.8) are only given for the Monte Carlo generated mass \(M_W\) and width \(\Gamma_W\). To compare the reconstructed mass spectrum of the data with the Monte Carlo mass spectrum, other Monte Carlo simulations with different generated \(M_W\) and \(\Gamma_W\) are necessary to find the parameters which minimise the log-likelihood. In principle, an infinite set of Monte Carlo samples is needed with all possible values for \(M_W\) and \(\Gamma_W\). Creating several samples with a sufficient number of events would not be feasible due to the large amount of computing time. A solution which circumvents the creation of several Monte Carlo event samples, thus reducing the amount of computing time, is the re-weighting method.
5.3. Monte Carlo re-weighting method

For every data point $i$ with invariant mass $M_{\text{inv}}^i$, an interval is chosen, the box volume $V_{\text{box}}^i$. From the number of Monte Carlo events in this interval the differential cross-section can be approximated.

Figure 5.1: For every data point $i$ with invariant mass $M_{\text{inv}}^i$, an interval is chosen, the box volume $V_{\text{box}}^i$. From the number of Monte Carlo events in this interval the differential cross-section can be approximated.
Figure 5.2: Illustration of the box occupancy ($N_{box}$) and box volume ($V_{box}$) for the box algorithm with the required constraints of a fixed box size of 0.5 GeV unless the number of events is outside the range of 100 and 1000 events. The normalised ratio between the box occupancy and box volume is the differential cross-section.
5.4. Jet pairing

In the re-weighting method each Monte Carlo event is assigned a weight

\[ w_i(M_W^{fit}, \Gamma_W^{fit}) = \frac{|M_{ffff}^{fit}(M_W^{fit}, \Gamma_W^{fit}, p_1, p_2, p_3, p_4)|^2}{|M_{ffff}^{MC}(M_W^{MC}, \Gamma_W^{MC}, p_1, p_2, p_3, p_4)|^2} \]  

(5.11)

which is the ratio of the squared matrix element \(|M|^2\) for the process \(e^+e^- \rightarrow WW \rightarrow fff\) for the fit parameters \(M_W^{fit}\) and \(\Gamma_W^{fit}\) to the squared matrix element for a generated mass and width, \(M_W^{MC}\) and \(\Gamma_W^{MC}\). The four-momenta of the final state fermions, \(p_1, p_2, p_3, p_4\), are the generator level values.

The Monte Carlo generator EXCALIBUR [44,45] is used to calculate the four fermion final state matrix elements. In EXCALIBUR the photon radiation is not included. However, the Monte Carlo events generated by KANDY [12] do include ISR and FSR photons. The fermion system should be transformed to a new system without radiation.

ISR reduces the centre-of-mass energy \(\sqrt{s}\) and is corrected for by calculating a new effective centre-of-mass energy \(\sqrt{s'}\). The four-momenta of the final state fermions are boosted to the summed ISR photon momentum. The FSR photon momentum is added to the fermion which emitted this photon.

Every event of the Monte Carlo sample can be re-weighted according to the generator level information to obtain a new Monte Carlo with mass \(M_W^{fit}\) and width \(\Gamma_W^{fit}\). The differential cross-section and cross-section are now determined from the weighted Monte Carlo sample as,

\[ \frac{d\sigma_{WW}}{dM_{inv}}(M_W^{inv}) = \frac{1}{L_{MC}} \cdot \frac{1}{V_{box}} \cdot \sum_{j \in B_i} w_j(M_W^{fit}, \Gamma_W^{fit}) \]  

(5.12)

\[ \sigma_{WW} = \frac{1}{L_{MC}} \cdot \sum_{i=1}^{N_{MC}} w_i(M_W^{fit}, \Gamma_W^{fit}), \]  

(5.13)

where \(B_i\) is the set of Monte Carlo events for a data point \(i\) in a box. All the events have a weight of 1 in equations (5.8) and (5.10), which is now modified in equations (5.12) and (5.13) by re-weighting the events.

Figure 5.3 shows the invariant mass spectrum of a Monte Carlo sample of 80.5 GeV which is re-weighted to 80.0 GeV and 81.0 GeV. The shift of the peak of the invariant mass spectrum, with respect to the original Monte Carlo to a lower and higher mass respectively, for the re-weighted spectrum, can be clearly observed.

5.4 Jet pairing

Chapter 4 describes the kinematic fit to improve the resolution of the measurement, while the previous sections explain the method to extract the mass from the data.

Before proceeding to the test of the fit method and to the actual fit of the data, an additional remark should be made about the four jet channel.

The \(qqqq\) channel has four jets. Since the equal mass constraint is imposed in the kinematic fit, the jets should be paired together before the 5C-fit is applied. Three different pairings can be made. To determine the mass for this channel it is necessary to find the pairs which come from
5.4. Jet pairing

Figure 5.3: The invariant mass spectrum of a Monte Carlo sample with a generated mass of $M_w = 80.5$ GeV (solid line) is re-weighted to represent a Monte Carlo invariant mass spectrum with generated mass $M_w = 80.0$ GeV (dashed line) and $M_w = 81.0$ GeV (dotted line).

The methods typically have a pairing efficiency of about 74% for the lower energy points which drastically decreases to 65% for increasing centre-of-mass energies.

The best performance is obtained with a neural network based approach [64]. The neural network is trained at all centre-of-mass energies. The following eight input variables are used in the neural network:

- Difference of invariant masses: $\Delta M = |M(\text{pair1}) - M(\text{pair2})|$;
- Sum of invariant masses: $\Sigma M = M(\text{pair1}) + M(\text{pair2})$;
- Sum of di-jet angles: $\Sigma \alpha = \alpha(\text{pair1}) + \alpha(\text{pair2})$.
  The di-jet angle, $\alpha(\text{pair})$, is the angle between two jets;
- Minimum di-jet angle: $\min \alpha = \min(\alpha(\text{pair1}), \alpha(\text{pair2}))$;
5.4. Jet pairing

- Difference of pair energies: $\Delta E = |E(\text{pair1}) - E(\text{pair2})|$;
- Minimum energy difference: $\min \Delta E = \min(\Delta E(\text{pair1}), \Delta E(\text{pair2}))$;
- Matrix element: $|M(M_W, \text{pair1}, \text{pair2})|^2$.
  The matrix element is calculated with EXCALIBUR, in which $M_W = 80.50\text{ GeV}$;
- Difference of pair charges: $\Delta Q = |Q(\text{pair1}) - Q(\text{pair2})|$.

Not only the efficiency is higher with about 77% compared to other methods, but it is also almost unchanged at all centre-of-mass energies.

In figure 5.4 the correct pairs show a peak at higher values of the neural network output, while the wrongly paired events peak at lower values of the neural network output. A small peak is seen for the correct pairs at the lower neural network output values. This comes from badly reconstructed events or events for which the separation between two pairings is difficult to establish.

Figure 5.4: Neural network output of the pairing algorithm at $\sqrt{s} = 207\text{ GeV}$ in fraction of events. The wrong pairing is peaked towards zero. The correctly paired events show a peak at higher values of the neural network output.

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The pairs are arranged according to their neural network output. The pair with the highest neural network output of an event is classified as the first pairing and consequently the second highest output as the second pairing and the lowest neural network output as the third pairing. Figure 5.5 shows the mass distribution of the Monte Carlo sample at $\sqrt{s} = 207$ GeV for the three pairings. The correct pairing and wrong pairing are shown on the total sample. One can conclude from this that the efficiency of the pairing decreases and the information on the mass reduces conform the classification of the pairing. Hence, I will only use the first pairing for the $qqqq$ events.

![Graphs showing W mass distribution for different pairings](image)

Figure 5.5: The first pairing has the highest pairing efficiency and shows the W mass distribution correctly, in contrast to the second pairing. The second pairing has a lower pairing efficiency and does not exhibit the mass peak in the total sample. The third pairing is even worse and does also not contain any useful information on the mass.
5.5 Testing the fit method

Sections 5.1, 5.2 and 5.3 describe the procedure to extract the W mass from the data samples. To ensure that the implementation is correct and the fit program determines the correct mass and width of the W boson with the right statistical error, two tests are performed.

In the first test the bias and linearity are examined. The second test, which is described in section 5.5.2, verifies whether the estimated statistical error of the fit is correctly determined.

5.5.1 The linearity test

To ensure that the fit program does determine the measured parameters, $M_W$ and $\Gamma_W$, correctly and to show it has no bias, large Monte Carlo samples with different underlying generated parameters, five for the mass fit and another five for the width fit, are used to fit a reference Monte Carlo sample. Approximately 10,000 events are generated per baseline Monte Carlo sample.

The reference sample has been generated at $\sqrt{s} = 189$ GeV with $M_W = 80.50$ GeV and $\Gamma_W = 2.11$ GeV. This sample is treated as data. The baseline Monte Carlos are produced at five different generated mass values ranging from 80.00 GeV to 81.00 GeV with steps of 0.25 GeV with a corresponding $\Gamma_W$ calculated by the Standard Model relation in equation (1.38). Similarly, different Monte Carlo samples are produced with generated $\Gamma_W$ varying from 1.51 GeV to 2.71 GeV with steps of 0.30 GeV at the fixed mass value of 80.50 GeV.

Figure 5.6 graphically represents the results when the reference sample has been fitted with five different baseline Monte Carlo samples. The left hand-side of figure 5.6 shows the effect on the mass measurement, while the right-hand side illustrates the effect on the width. In the upper part of these figures the generated values are plotted on the abscissa, while along the ordinate the fit values are set. The diagonal line gives the ideal positions of the points. The lower part of the figures shows the difference between the fitted and generated values.

A straight line has been fitted through the difference of fitted and generated value points, i.e.

$$f(M_W) = b_M + s_M(M_W - 80.50),$$

$$f(\Gamma_W) = b_T + s_T(\Gamma_W - 2.11).$$

(5.14)

(5.15)

Parameters $b_M$ and $b_T$ are the biases of the fits at the reference points. Their slopes are parametrised by $s_M$ and $s_T$ respectively. The straight line fits give $b_M = 12 \pm 13$ MeV, $s_M = -0.039 \pm 0.038$ for the mass and $b_T = 19 \pm 30$ MeV, $s_T = 0.036 \pm 0.078$ for the width. One can observe in the figures that the fitted $M_W$ and $\Gamma_W$ obtained from the mass and width extraction method are well reproduced within the statistical error.

5.5.2 Statistical error test

In the fit procedure the statistical uncertainty of the mass and width measurement are estimated. These errors are calculated from the log-likelihood curve as the difference between the determined mass at the minimum of the curve and the mass at the point where the log-likelihood is 0.5 higher than at its minimum. The statistical errors depend on the number of data events as well as on the invariant mass distribution.
5.5. Testing the fit method

To confirm that the estimation provided by the fit program is done properly, a Monte Carlo test is performed. By taking events randomly out of one baseline Monte Carlo sample, many small samples can be constructed. These are handled in the same way as data in the mass and width fit program, by fitting them with the parent Monte Carlo. The mass and error are extracted for each small sample and plotted. The root-mean-square (RMS) of the extracted mass distribution should agree with the average of the estimated error distribution.

The size of the samples is chosen such that it agrees with the measured cross-section. To prevent any bias in the estimation, the randomly chosen events are excluded from the baseline Monte Carlo. Approximately 1,000 of such samples are constructed and fitted.

Figure 5.7 shows the histograms for the fitted mass values and the error estimation for the $q\bar{q}e\nu$ channel at 189 GeV with Monte Carlo samples generated at a mass of 80.50 GeV and a width of 2.11 GeV. The average value of the fitted mass distribution is 80.498 $\pm$ 0.012 GeV. Within the error the generated mass value is very well reproduced. The RMS of the mass distribution of 174 $\pm$ 9 MeV is compared with the average value of 169 $\pm$ 16 MeV of the error estimation distribution. The error on the average fit error comes from the spread of the distribution.

Figure 5.8 shows the plots for the width calculations. The average of the distribution in the left plot is 2.084 $\pm$ 0.013 GeV. For the width fit distribution the RMS yields 429 $\pm$ 10 MeV and the average of the error 430 $\pm$ 48 MeV.

Comparing the results the RMS and the average of the error shows these values to be compatible within their statistical error for both the mass and width.
5.6 Mass and width fit results

The W mass and width are determined in the data samples at each energy point with the un-binned log-likelihood fit by means of the box and reweighting method as described in sections 5.1, 5.2 and 5.3.
Table 5.1 and table 5.2 contain the results of the analyses for the $qqev$ and $qq\mu\nu$ channels respectively. Both the one- and two-parameter fits are shown. The number of events is the number after rejecting events with a fit probability lower than 5%, to remove background events and badly reconstructed events. Also the range where the fit is applied is restricted to the invariant mass above 65 GeV. Events which are far from the peak are not sensitive to the mass. The Monte Carlo sampling density for these events is very low, which causes the box-size to be more than 1 GeV. The kinematical edge defines the end point of the range in which the fit is performed.

There are too few data at 208 GeV for the two-parameter fit to converge.

Table 5.1: The mass and width of the W boson extracted from the $qqev$ final state at $\sqrt{s} = 189 - 208$ GeV. The one-parameter fit and two-parameter fit results are presented. The number of events is the number of events used in the fit.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>#data events</th>
<th>one-parameter fit</th>
<th>two-parameter fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>189</td>
<td>269</td>
<td>80.06 ± 0.18</td>
<td>80.05 ± 0.19</td>
</tr>
<tr>
<td>192</td>
<td>58</td>
<td>80.64 ± 0.48</td>
<td>80.52 ± 0.54</td>
</tr>
<tr>
<td>196</td>
<td>121</td>
<td>80.50 ± 0.31</td>
<td>80.47 ± 0.33</td>
</tr>
<tr>
<td>200</td>
<td>117</td>
<td>80.32 ± 0.29</td>
<td>80.31 ± 0.30</td>
</tr>
<tr>
<td>202</td>
<td>46</td>
<td>80.24 ± 0.50</td>
<td>80.25 ± 0.51</td>
</tr>
<tr>
<td>205</td>
<td>125</td>
<td>80.31 ± 0.29</td>
<td>80.24 ± 0.34</td>
</tr>
<tr>
<td>207</td>
<td>178</td>
<td>80.40 ± 0.22</td>
<td>80.43 ± 0.20</td>
</tr>
<tr>
<td>208</td>
<td>6</td>
<td>79.17 ± 0.79</td>
<td>-</td>
</tr>
</tbody>
</table>

The results of the fit for the $qq\tau\nu$ channel are shown in table 5.3. Only the hadronically decaying part is used in the reconstruction. The jet-jet part of the event is rescaled with a factor so that the sum of the energy of the hadronic jets is equal to half the centre-of-mass energy. Events with an invariant mass higher than 65 GeV only are considered in the fit.

Events with a fit probability lower than 5% are also rejected in the $qqqq$ case, as in the $qqev$ and $qq\mu\nu$ channels. The accepted range of the invariant mass is from 65 GeV upwards. The pairing with the highest probability is used in the fit in table 5.4.

Figure 5.9 shows the log-likelihood function for the $qqqq$ data at 208 GeV, which shows a proper parabolic curve, which confirms the fact that the error is symmetric. The minimum of this curve is assigned as the measured mass in this data sample.

Combining all the energy points for the measurements of each channel gives the results in table 5.5. These results are obtained by adding the log-likelihood curves for all energy points of a specific channel. The result of such a combination of log-likelihood curves is presented in Figure 5.10. Figure 5.11 shows the combined data and reweighted Monte Carlo fit.
5.6. Mass and width fit results

Table 5.2: The mass and width of the W boson extracted from the \(q\bar{q}\mu\nu\) final state at \(\sqrt{s} = 189 - 208\) GeV. The one-parameter fit and two-parameter fit results are presented. The number of events is the number of events used for the fit.

<table>
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</thead>
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<tr>
<td></td>
<td></td>
<td>(M_W) [GeV]</td>
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</tr>
<tr>
<td>189</td>
<td>269</td>
<td>80.13 ± 0.21</td>
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</tr>
<tr>
<td>192</td>
<td>52</td>
<td>80.58 ± 0.52</td>
<td>80.61 ± 0.44</td>
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<tr>
<td>196</td>
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<td>80.15 ± 0.41</td>
</tr>
<tr>
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<td>80.37 ± 0.86</td>
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<td>67</td>
<td>80.40 ± 0.54</td>
<td>80.34 ± 0.63</td>
</tr>
<tr>
<td>205</td>
<td>113</td>
<td>80.24 ± 0.47</td>
<td>80.23 ± 0.48</td>
</tr>
<tr>
<td>207</td>
<td>207</td>
<td>80.13 ± 0.32</td>
<td>80.12 ± 0.32</td>
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<tr>
<td>208</td>
<td>20</td>
<td>79.50 ± 0.79</td>
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</table>

Table 5.3: The mass and width of the W boson extracted from the \(q\bar{q}\tau\nu\) final state at \(\sqrt{s} = 189 - 208\) GeV. The one-parameter fit and two-parameter fit results are presented. The number of events is the number of events used in the fit.

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</tr>
<tr>
<td>189</td>
<td>430</td>
<td>80.12 ± 0.28</td>
<td>80.12 ± 0.29</td>
</tr>
<tr>
<td>192</td>
<td>57</td>
<td>80.36 ± 0.84</td>
<td>80.37 ± 0.92</td>
</tr>
<tr>
<td>196</td>
<td>222</td>
<td>80.69 ± 0.49</td>
<td>80.75 ± 0.52</td>
</tr>
<tr>
<td>200</td>
<td>181</td>
<td>80.45 ± 0.51</td>
<td>80.44 ± 0.52</td>
</tr>
<tr>
<td>202</td>
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<td>79.82 ± 0.74</td>
<td>79.71 ± 1.07</td>
</tr>
<tr>
<td>205</td>
<td>164</td>
<td>81.60 ± 0.56</td>
<td>81.66 ± 0.63</td>
</tr>
<tr>
<td>207</td>
<td>287</td>
<td>80.34 ± 0.45</td>
<td>80.35 ± 0.48</td>
</tr>
<tr>
<td>208</td>
<td>17</td>
<td>79.68 ± 1.04</td>
<td>-</td>
</tr>
</tbody>
</table>

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Figure 5.9: The log-likelihood curve for the $qqqq$ data at 208 GeV. The curve is shifted along the y-axis, by subtracting the minimum value of the log-likelihood, $-\Delta \ln(L)$. The curve is parabolic around the minimum. This attests that the errors are symmetric.
Table 5.4: The mass and width of the W boson extracted from the $qqqq$ final state at $\sqrt{s} = 189 - 208$ GeV. The one-parameter fit and two-parameter fit are presented. The number of events is the number of events used for the fit.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ [GeV]</th>
<th>#data events</th>
<th>one-parameter fit</th>
<th>two-parameter fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>189</td>
<td>794</td>
<td>80.51 ± 0.13</td>
<td>80.51 ± 0.13</td>
</tr>
<tr>
<td>192</td>
<td>146</td>
<td>80.12 ± 0.30</td>
<td>80.12 ± 0.33</td>
</tr>
<tr>
<td>196</td>
<td>368</td>
<td>80.30 ± 0.19</td>
<td>80.29 ± 0.19</td>
</tr>
<tr>
<td>200</td>
<td>422</td>
<td>80.17 ± 0.17</td>
<td>80.16 ± 0.18</td>
</tr>
<tr>
<td>202</td>
<td>162</td>
<td>80.49 ± 0.24</td>
<td>80.48 ± 0.21</td>
</tr>
<tr>
<td>205</td>
<td>360</td>
<td>80.15 ± 0.18</td>
<td>80.15 ± 0.18</td>
</tr>
<tr>
<td>207</td>
<td>599</td>
<td>80.48 ± 0.15</td>
<td>80.46 ± 0.15</td>
</tr>
<tr>
<td>208</td>
<td>33</td>
<td>79.68 ± 0.45</td>
<td>79.57 ± 0.40</td>
</tr>
</tbody>
</table>

Table 5.5: The mass and width of the W boson extracted for the combination of the energy points $\sqrt{s} = 189 - 208$ GeV for each channel. The one-parameter fit and two-parameter fit results are presented.

<table>
<thead>
<tr>
<th>channel</th>
<th>#data events</th>
<th>one-parameter fit</th>
<th>two-parameter fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$qq\nu$</td>
<td>920</td>
<td>80.27 ± 0.10</td>
<td>80.24 ± 0.11</td>
</tr>
<tr>
<td>$qq\mu\nu$</td>
<td>980</td>
<td>80.21 ± 0.13</td>
<td>80.21 ± 0.13</td>
</tr>
<tr>
<td>$qq\tau\nu$</td>
<td>1474</td>
<td>80.41 ± 0.17</td>
<td>80.36 ± 0.17</td>
</tr>
<tr>
<td>$qqqg$</td>
<td>2884</td>
<td>80.34 ± 0.07</td>
<td>80.35 ± 0.06</td>
</tr>
</tbody>
</table>
Figure 5.10: The log-likelihood distribution with the minimum subtracted, $-\Delta \ln(L)$, for the \textit{qqeu} channel. All separate log-likelihood curves at the different energy points are added together to extract the combined result. The curve is parabolic around the minimum.
Figure 5.11: Distribution of the invariant mass for data and the reweighted Monte Carlo.