The pion Form Factor from Lattice QCD

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Introduction

One thing the history of science has taught us, is that the smallest elements can always be broken down into even smaller components, although for a very long time, it was assumed that the atom was indivisible. After the discovery of the nuclei, it was soon realised that they too showed signs of a substructure, indicating the existence of smaller objects, the nucleons. All these building blocks are observed and studied individually as free particles in an experiment. The nucleons were later also found to be built from yet smaller constituents. But at this level, the trend has been broken: while we know that nucleons are built from quarks and gluons, they can only be detected indirectly, and they have never been observed and studied as individual particles. How hard one tries, the very structure of the laws of nature has prevented us from isolating a single quark or gluon and measure it. This is due to the confining nature of the interaction between the quarks, which is mediated by the exchange of gluons. If one tries to split e.g. a meson (a quark-antiquark bound state), the invested energy is converted into additional quark-antiquark pairs and we end up with multiple composite particles.

As of now, quarks and leptons are regarded as the elementary building blocks of matter. To describe all the interactions between these elementary particles, the Standard Model (SM) has been developed over the years. In nature one usually distinguishes four fundamental interactions: electro-magnetism, the weak and the strong interaction and gravity. Only the first three are described in the SM. In order to incorporate these three very differently behaving forces, the model actually is a collection of tightly interwoven theories, each with very distinguishable characteristics. Despite their obvious differences, they share one important common feature: they are all described through local gauge theories. Gauge theories are based on symmetries observed in nature and the (necessary) invariance of physics under a transformation by an element belonging to that symmetry group. The description of gauge theories can be found in text books such as [1–3] and is beyond the scope of this work.

This thesis deals with the properties of hadrons, the bound states of quarks and/or anti-quarks, which can satisfactorily be described through interactions governed by the strong force alone. We restrict ourselves to the theory of this interaction, described in the SM by Quantum Chromodynamics (QCD).

Over the years, QCD has established itself as the correct microscopic theory, mainly by impressive agreement between theory and experiment in the small coupling sector, where perturbative methods are valid. At the scale of the size of a hadron, however, non-perturbative methods have to be applied. Comparatively few results were obtained in this sector of QCD. It is therefore an obvious challenge to derive the internal structure
of a hadron from first principles, entirely within QCD [4]. Global features of hadrons, like charge, spin and isospin, represent no challenge and are trivially included in most models. However, specific features that are actually testing details of our theoretical understanding of the internal structure of hadrons are observables like form factors or polarisabilities.

In this thesis we will focus on the structure of a pion, the lightest hadron, using lattice QCD (LQCD) as the non-perturbative method. In the first part we will discuss a free pion. At first glance, the pion, which consists of a quark and an anti-quark, looks like a manageable two-body system. There have been many descriptions of the pion based on effective low energy models or QCD inspired approaches. One feature all these models share is that confinement, the most striking feature of QCD, is - in one way or another - put in by hand. This is of course an unwanted step when one sets out to calculate the pion form factor or its charge radius, which concern the form and size of QCD confinement. Here one obviously wants to proceed from first principles, from the fundamental QCD Lagrangian itself.

The pion form factor reflects the influence of the interacting quarks on the coupling of the photon to the pion. If the four-momentum transfer squared is zero, the photon 'sees' no substructure and the form factor is equal to the total charge of the meson. For non-zero four-momentum transfer squared the form factor changes, reflecting the substructure probed by the photon. In case of a time-like photon, the form factor will show resonances, most notably from the $\rho$- and $\omega$-meson. For space-like photons, the form factor shows only the tail of the $\rho$ resonance.

As an example, we consider elastic electron scattering from a pion. The differential cross section of this process can be written as the purely electromagnetic scattering of a point particle times the form factor squared

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{point}} |F(q)|^2. \quad (0.1)$$

The electromagnetic part is calculated using perturbation theory in the (small) electromagnetic coupling constant. The form factor itself can be extracted from the three-point function for the pion-photon interaction

$$\Gamma_\mu = \langle \pi(p_f)|j_\mu(0)|\pi(p_i)\rangle, \quad (0.2)$$

which we will evaluate using lattice techniques. In the non-relativistic limit, the form factor is defined as the Fourier transform of the normalised charge distribution $\rho(x)$. This connection will later be used to obtain the electromagnetic radius of the pion.

Two groups have already calculated the pion charge form factor on the lattice. The pioneering work was done by Martinelli et al. [5], followed by more detailed calculations of Draper et al. [6–10]. Working with heavy pions of the order of 1 GeV, one of the findings of the latter work was that the pion form factor could be described reasonably well by a monopole form, as suggested by vector meson dominance. In our work, we will extend these calculations to much lower mass. Furthermore, we will employ
an improvement program which ensures that discretisation errors, inherent in lattice techniques, are reduced significantly.

In the mid seventies, it was realised that the theory governing the strong force, QCD, exhibits a phase transition for higher densities and/or temperatures [11–13]. In this regime, the strong interaction is not confining anymore and the quarks and gluons are not restricted inside hadrons. This state of matter is commonly known as the quark gluon plasma. It has not yet been confirmed experimentally beyond all doubt, although strong indications of its existence have been observed at CERN and RHIC. It is believed to have existed in the very early stages of the universe, about $10^{-6}$ sec after the Big Bang and presumable exists in the core of cold and very dense neutron stars.

As an aside, it is interesting to note that in the mid sixties, before the formulation of QCD (thus before the postulation of quarks and gluons), it was already realised that in the hadronic phase the temperature is bounded [14]. The highest temperature was estimated to be $\approx 160$ MeV, not very different from the temperature of the phase transition ($\approx 150$ MeV).

To create and observe the quark gluon plasma in a laboratory, one needs powerful particle accelerators and huge detectors. Nuclei with a large number of nucleons, e.g. gold ion, are accelerated close to the speed of light and collided into each other. The number of nucleons needs to be large, in order to produce a large, dense and/or hot collectively interacting system for which the laws of thermodynamics are valid. An introduction to the expected properties and signals of the quark gluon plasma can be found in [15,16].

If the energy per nucleon in the collision is less than about 10 GeV, the nuclei will break up and create a region of high baryon density in the centre of mass (stopping scenario). On the other hand, for energies on the order of 100 GeV and higher, the colliding nuclei will penetrate through each other and leave a region of high energy density, but essentially zero baryon density in the centre of mass (transparent scenario). If in either case the energy density is large enough, the hot QGP will be formed after an initial thermalisation time of the order of $10^{-23}$ seconds. Depending on the temperature, the quarks in this deconfined phase may still form bound states due to residual interactions.

The system expands because of the thermodynamic pressure and cools down. When the energy density reaches a certain critical value, confinement reoccurs and the quarks and gluons of the plasma start to form hadrons again (hadronisation). The system further expands as a dense hadron gas, in which the particles still interact heavily with each other. This gas will eventually freeze out when the distance between the hadrons is too large for the strong force to have a significant influence.

Since the quark gluon plasma itself can not be measured directly, its existence can only be inferred from the information carried by the particles created at the different stages of the collision that reach the detector. The analysis of the various direct and indirect probes of the QGP is a highly non-trivial task. For example, the production rates of detected particles might be influenced by changes in the properties of the
hadrons due to the surrounding, hot medium.

The question thus arises whether and how the properties of these hadrons change in this environment. In the second part of this thesis, we will investigate these temperature changes of hadron properties by calculating correlation functions and the form factor of the pion embedded in a hot medium. This medium has zero baryon density and a temperature of $0.93 \, T_c$\textsuperscript{1}. Several lattice studies have already investigated the screening masses and wave functions of hadrons at finite temperature. It was found that these remain unchanged up to $0.93 \, T_c$\textsuperscript{17} in comparison with $T = 0$ results. The spectral functions however, indicate a shift in the mass and a broadening of the width of both the lowest pseudo scalar and the vector meson. Our study of a form factor concerns space like photons and is the first investigation of the pion form factor at finite temperature using lattice techniques. Under the clean circumstances we choose, it addresses the question whether and how a hadron changes its structure as it approaches the temperature where hadrons cease to exist as bound quark-anti-quark systems.

This thesis is organised as follows. In Chapter 1, the theory of the strong interaction will be outlined, followed by the lattice implementation in Chapter 2. The simulation techniques are discussed in Chapter 3. In Chapter 4 the analysis techniques and the results for the free two-point function are presented; in Chapter 5 the same is done for the free three-point function. The results at finite temperature are collected in Chapter 6. Conclusions are drawn in the last chapters separately.

\textsuperscript{1}The temperature at which the phase transition occurs is referred to as the critical temperature, $T_c$.\label{fn:critical_temp}