The pion Form Factor from Lattice QCD

van der Heide, J.

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Summary

In nature one usually distinguishes four elementary forces, the electromagnetic, weak, and strong interaction, and gravity. These forces dominate physics on a certain scale. The most important force in our daily life for instance, is the electromagnetic interaction between atoms and molecules. On the most fundamental level, inside e.g. protons, on the other hand, physics is dominated by the strong force. This is the domain of (anti-)quarks and gluons. These elementary building blocks cannot be observed directly; they can only be detected in pairs or triplets, called hadrons. This is called confinement.

The properties of these composite particles can be calculated from the theory describing the strong force, Quantum Chromodynamics (QCD). Because of the distinctive features of the strong force, confinement and the large value of the coupling constant, analytical methods are not applicable to perform the calculations. Therefore, one turns to numerical methods. They consist of describing our continuous world in terms of a discrete space-time grid or lattice. In this way the computations one has to perform become finite and the use of computers becomes possible, but is also implies that one can only describe physics in a finite box with finite resolution. In order to still simulate a system which resembles the real world as much as possible, the lattice must be both large and fine. This demand makes the calculations very time consuming and one has to turn to supercomputers in which multiple processors compute simultaneously, often for several months. Using such methods, masses and decay times of hadrons have been calculated with considerable success directly from the underlying theory. In this thesis, we have investigated how photons (light) interact with the lightest hadron, called the pion. From this calculation, we can extract the form factor, which in turn provides us information on the internal structure of the pion from first principles. The methods and results of this study will now be summarised in somewhat more detail.

Over the years, Quantum Chromodynamics has been established as the correct theory of the strong interaction through impressive quantitative agreement between experiment and theory. Although many results have been obtained in the perturbative sector, relatively few concern physics on the scale of a hadron. Since perturbative methods fail in this regime, other techniques have to be employed. In this thesis, we use lattice methods to investigate the structure of hadrons, a typical non-perturbative phenomenon. The basis of this approach consists of the discretisation of space-time. This discretisation acts as a regulator, rendering all observables finite. The number of degrees of freedom is limited, and therefore numerical methods can be used to calculate the integral one encounters in the path integral formalism. The immense computational
effort involved requires parallel processing and the use of super computers.

In this thesis we have focused on the structure of the pion, with the emphasis on the electromagnetic form factor and the charge radius. These quantities directly reflect the spatial extent of non-perturbative phenomena, in particular the intriguing confinement of QCD. It is therefore crucial to obtain it from first principles, without any model assumptions. Lattice QCD is of course not free of approximations, but the main advantage over other non-perturbative approaches is that it, in principle, does not use ad hoc assumptions. We have worked in the quenched approximation, using a $24^3 \times 32$ lattice with coupling constant $\beta = 6.0$ $(a \approx 0.105 \text{ fm})$ for our $T = 0$ investigation. We did our simulations for 5 different values of the (valence) quark mass. At finite temperature, we used a lattice of size $32^3 \times 8$, but kept the other parameters the same.

In comparison with earlier work on the pion form factor we have employed improvement techniques to reduce the discretisation errors of our calculation. We have used only non-perturbatively determined improvement and renormalisation constants to ensure that the remaining errors are of $\mathcal{O}(a^2)$. One of the results of this thesis is the investigation of the effect of improvement of the current operator on the form factor. It was found that the results based on the improved current are significantly smaller than those of the commonly used conserved current. The difference with the renormalised local or continuum current was found to be very small. Using another value for the improvement constant $c_V$ from the literature, our results indicate that the $\mathcal{O}(a^2)$ effects can still be as large as 10 %.

The earlier calculations of the form factor were done using pion masses larger than about 1 GeV. We have used masses ranging from 1 GeV down to 360 MeV, only 2.5 times the physical mass. It was found that our results for the form factor were decreasing with mass, coming very close to the experimental values for the lighter pion. Moreover, the theoretical calculations were seen to produce the same type of $Q^2$ behaviour. This showed us that a highly non-perturbative feature can be obtained from lattice QCD. With our calculations, we have thus shown from first principles that interacting point-like quarks produce a particle which has finite dimensions and internal structure.

It was also confirmed that a VMD inspired monopole form described our data very well when used as a parametrisation. Using this monopole form as a fit ansatz, we obtained the charge radius of the pion from the low $Q^2$-behaviour of the form factor without any further assumptions. It was found that the charge radius thus obtained showed a considerable mass dependence, in contrast to the radius based on the less reliable Bethe-Salpeter approach. We extrapolated our results to the chiral limit and obtained a physical value for the radius. Choosing the most reliable extrapolation, we found $\langle r^2 \rangle = 0.36(2) \text{ fm}^2$. This value lies below the experimental value of $\langle r^2 \rangle = 0.439(8) \text{ fm}^2$, but only amounts to a 10 % deviation of the RMS.

The question remains, how we can proceed with lattice methods to bridge the (small) gap with experiment. A simple further lowering of the quark mass is not the path to follow. First it becomes increasingly more difficult to invert the fermion matrix. For physical values for the mass this is even impossible. Furthermore, in the improved
scheme we have chosen, the presence of so-called exceptional configurations increases with decreasing mass, which make the analysis cumbersome. An extrapolation to the physical limit thus remains necessary. A more promising extension of the present study would be the inclusion of dynamical (sea) quarks. Effects of these fermions were estimated to be small for the pion in case of density-density correlators. Nevertheless, further research is necessary in this area. Since Wilson quarks have no chiral symmetry, another improvement would be to use chiral symmetric actions. Investigations in this area are presently pursued by several collaborations.

In the second part of this thesis, we focused on the temperature dependence of the above mentioned quantities. This was done in view of relativistic heavy ion collisions. In these experiments, nuclei are collided into each other to create a hot and dense medium. When the energy density of this fireball is large enough, the anticipated quark-gluon plasma will be formed. In such a plasma, hadrons no longer exist. Instead, the deconfined (anti)quarks and gluons form a collectively interacting system in which the particles move almost freely. In order to describe the processes and phenomena occurring during and right after the creation of the fireball, we need to know how static properties of hadrons modify under changes in the environment. Using again lattice methods, we calculated the pion form factor at finite temperature for the first time. Since it is expected that changes occur very rapidly only in the vicinity of the phase transition, we have done our calculations at 0.93 $T_c$.

The parametrisation of the pion-photon matrix element in terms of form factor(s) is more involved at finite temperature. Not only do we have more form factors, but they also depend on more scalar variables. Since the temporal extent was decreased to increase the temperature, the extraction of observables from temporal correlation functions becomes impossible, or at best rather difficult. We therefore had to switch to spatial correlators. This introduced extra dependencies into the form factors due to the different structure of the dispersion relation. We choose specific kinematics (same restrictions as at $T = 0$) to reduce the number of form factors and thus to facilitate the extraction of the main form factor. In order to investigate the dependence of the form factor on the momentum of the pion in the heat bath, we chose three different final momenta in our calculation. The same analysis techniques as for the zero temperature data were used.

A dependence of the form factor on the pion three-momenta was observed for all pion masses, although the 'breaking' of Lorentz symmetry due to the presence of a heat bath, was more notable for lower pion masses. The form factor results for the two lowest three-momenta could not be distinguished. Thus it seems that the dependence only becomes an important effect at larger pion momentum. This is reflected in the radii, which we obtained again from the low $Q^2$-behaviour of the form factor. It was found that the VMD inspired monopole parametrisation still provides a proper description of our data. The radius for the combined lower momenta data is significantly larger than for the highest pion momenta considered. For the radius of a pion moving through a static heat bath, we find $\langle r^2 \rangle = 0.39(2)$ for the lower momenta and $\langle r^2 \rangle = 0.28(3)$ for the higher momentum.
Summary

The data are not directly comparable to the zero temperature results because of various reasons. First, although temporal and spatial correlators yield the same results at zero temperature, this does not need to be so at finite temperature due to possible modifications in the dispersion relation. Secondly, the specific choice of reference frame amounts to the breaking of Lorentz symmetry, introducing more dependencies on kinetic variables. Probably the most natural comparison would be with the radius of a pion at rest in the heat bath. Unfortunately, with our methods it is impossible to obtain the radius of a pion with zero momentum. A possibility would be to calculate the form factor for various pion momenta and perform an extrapolation. With our data and statistics, we are not able to do this. However, the data we do have indicate that the radius of a pion at rest is somewhat larger than the radius of the pion with the smaller momentum. This means that a pion at rest at $T = 0.93 T_c$ is also larger than a free pion at $T = 0$ as calculated on the lattice.

Comparison of the form factor at $T = 0.93 T_c$ with the zero temperature results yields the general conclusion that the changes are rather small. The error bars overlap, but the differences are systematic and therefore significant. Overall, we thus find that close to the critical temperature, the changes in the properties of hadrons are small, indicating a very rapid phase transition.