Rule-based constraint propagation: theory and applications
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Chapter 11

Final Remarks

11.1 Summary

One ideal of constraint programming is to be declarative: modelling the problem should be independent of the solution algorithm. The rule-based approach to constraint propagation which we examined in this thesis is entirely within this line; rules express constraint propagation declaratively.

Propagation by rules. In the three chapters following the introductory part, we dealt with generic issues concerning rule-based propagation. We developed theoretically well-founded techniques and implemented them as concrete tools.

We started with showing how a generic fixpoint computation algorithm can be instantiated to the improved scheduling algorithm R for constraint propagation rules. The central observation is this: the information that the condition of a rule succeeds entails more than just that it is now correct to apply the rule body. It also may state something about other rules: truth of their conditions, or relevance of their body. Such information can be used to accelerate the fixpoint computation, in particular by shrinking the set of participating rules. Most usefully, these connections can be pre-computed. Moreover, since fixpoint computations for the purpose of constraint propagation are called in successive rounds, rules removed in one round need not be reconsidered in subsequent rounds. The set of propagation rules shrinks as search and propagation progresses.

The removal of a rule during a fixpoint computation in the way just described indicates a 'local redundancy'. Rather similarly, a rule can also be (globally) redundant. This is the case if the common fixpoints of a rule set do not change if the rule is removed. Surprisingly, the relation between redundancy and what we call 'local redundancy' here is not straightforward: a redundant rule is not always 'locally redundant', nor does the inverse statement hold true.

Empirical examinations of the two techniques, i.e., using the R rule scheduler that dynamically reduces the set of involved functions, and removing redundant
rules from a rule set as a pre-process, showed that both can be very useful. They also appear to be orthogonal as far as improving the efficiency is concerned. Both techniques are thus of relevance for efficient rule-based constraint propagation.

We also dealt with the origin of constraint propagation rules. In contrast to existing automatic rule generation methods, we considered incremental rule generation, based on modifying and combining rules instead of relying on the constraint definition. For a concrete rule type, the membership rules, we examined various cases of incremental rule generation, with a focus on the associated propagation.

**Applications.** The usefulness of these techniques was evaluated in concrete applications. Our study of the automatic test pattern generation problem led to three models with different multi-valued logics. These logics naturally capture structural properties of the problem. In turn, the constraints arising from these models were directly applicable to the rule-based propagation techniques we developed.

Our approach to modal satisfiability checking was similar in the sense that we also applied a non-Boolean logic to express structural and heuristic information. Constraint propagation could in part be dealt with by the techniques mentioned above such as the R algorithm and redundancy removal during pre-processing. However, several complex constraints required manually designed and implemented propagation rules. The resulting constraint-based modal-SAT solver exhibited a performance competitive with an approach relying on an advanced, purely propositional solver.

Array constraints are not directly amenable to rule-based propagation as discussed above. However, the problem of their propagation can be tackled with a rule-based design approach. For two local consistencies, we derived concrete rules from generic rule templates embodying the desired propagation, and turned them subsequently into propagation algorithms.

In the area of qualitative spatial representation and reasoning, information is naturally organised in arrays. We discussed an alternative to the standard constraint-based model of qualitative space, and we argued for its advantages, which eminently lie in the ease of modelling several spatial aspects. Such models use simple constraints to which our rule-based propagation techniques are directly applicable.

Our approach to constraint-based qualitative simulation builds on the alternative model for (static) qualitative spatial reasoning, and in this way shows its flexibility. We used temporal logic formulas to represent knowledge about change, which yielded compact formalisations. We provided translations of such formulas into constraints, which allows to link them by propagation.
11.2 Outlook

We gather some future research issues involving rule-based propagation.

One question concerning propagation rules for simple constraints is how succinct this approach generally is. Consider membership rules, for example. While it is in principle possible to generate all GAC-enforcing membership rules for any given constraint, clearly their number will often be large. An intuitive argument for this is that the same atomic piece of information, a non-solution, occurs multiple times in different rules. It would be useful to better understand how the number of GAC-enforcing membership rules, with and without redundancy removal, depends on the number of solutions or also the structure of the constraint.

An alternative, interesting road for rule-based propagation is indicated in Sections 7.4.2 (p. 115) and 8.2.1, where we manually derived rules for complex, 'global' constraints from intensional definitions. It would be instructive to formalise in detail what derivation steps were taken, in order to ultimately automate these rule derivations. Such an approach could be based on a constraint definition in the shape of a logical formula, from which logical implications are derived that in turn are viewed as constraint propagation rules.

A further question regards the issue of control. In the R scheduler, we left open in which order rules are chosen and applied. The rule selection strategy clearly influences the length of the computation, however.

Finally, we must raise the issue of how efficient, in terms of complexity, propagation algorithms based on constraint propagation rules can in principle be. Instead of the localised view on a problem taken by each rule individually, a global view may afford more useful structuring of the information [Maher, 2002a]. This point comes to mind, for example, when one regards 'global' constraints whose propagation algorithms rely on graph-theoretic methods and of which the underlying propagation rules are not directly obvious.

Such observations may suggest a necessary departure from the concept of a constraint propagation rule in the pure sense of Def. 2.2.1. It may not necessarily require us to leave the rule-based paradigm, however; for example, see [Ganzinger and McAllester, 2002] where classical algorithms with optimal complexity are described in rule form.

To conclude this dissertation: in our view the rule-based paradigm provides a useful and elegant approach to constraint propagation, and, while much remains to be explored, we think it has great potential.