Optical trapping and manipulation of atoms near surfaces
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An atom interferometer using spontaneous decay

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H.B. van Linden van den Heuvell,
submitted for publication.

In this chapter we investigate the question of whether Michelson type interferometry is possible where the role of the beam splitter is played by a spontaneous Raman transition. This question arises from a detailed inspection of trajectories of atoms bouncing inelastically from an EW mirror. Each final velocity can be reached via two possible paths, with the Raman transition occurring either during the ingoing or the outgoing part of the trajectory. At first sight, one might expect that the spontaneous character of the Raman transfer would destroy the coherence and thus the interference. We investigated this problem by numerically solving the Schrödinger equation and found that interference fringes in velocity space nevertheless are expected, even when random photon recoils are taken into account.
8.1 Introduction

Throughout this thesis we have considered processes where an atom undergoes spontaneous Raman pumping between two internal atomic states that interact differently with the applied evanescent fields. This process has been used as a mechanism to efficiently load low-dimensional traps. In earlier experiments we studied the behavior of atoms that reflect from an evanescent-wave mirror while undergoing such a spontaneous Raman transition during their reflection [46]. When the repulsive potential experienced by the final state is lower, the atoms lose kinetic energy during this process and hence bounce inelastically from the potential.

The final velocity of an atom depends on the position where it made the Raman transfer. Looking at the trajectories in detail we see that two trajectories can end up at the same final velocity. An atom can be transferred to the second state on the ingoing or the outgoing part of its trajectory, as is shown in Fig. 8.1(a). Cognet et al. [131] have performed similar experiments where the transfer to the second internal state was by a stimulated Raman transfer. They observed interference patterns, in the form of Stuckelberg oscillations in the diffraction efficiency of atoms from polarization gradients in an evanescent-wave mirror.

This leaves the intriguing question whether interference is possible when the transfer is by a spontaneous Raman transition. At first sight one would answer this question negatively, since the spontaneously emitted photon could in principle be detected, and thus would provide "which way" information. Furthermore, a spontaneous process is generally considered incoherent and not deterministic due to the random direction in which the photon is emitted and the resulting random recoil to the atom. Intuitively, both effects would destroy the interference.

On the other hand there are several indications that spontaneous Raman transitions are not necessarily incoherent processes. Loudon [132] derives that the linewidth of the spontaneous emission of an atom that is optically driven by a weak field is equal to the linewidth of the driving field and not to the linewidth of the transition that is driven. Eichmann et al. [133] report interference of the spontaneous emission of two well localized trapped ions. The experiments of Cline et al. [134] show that the phase of the atomic wave function is preserved over the spontaneous process. They trapped $^{85}$Rb atoms in a far off-resonance dipole trap with a detuning that is much larger than the fine-structure splitting between the $D_1$ and $D_2$ line. The observed trap life time could only be explained by coherently adding the spontaneous Raman scattering amplitudes of the $D_1$ and $D_2$ lines. The posed question will be answered in this chapter by solving the Schrödinger equation.

This chapter is structured as follows. We first present a semi-classical picture, that we use to make qualitative predictions about the behavior of the interference effects, if present. The question whether interference is possible will be answered by solving the Schrödinger equation in two different ways. The first method will employ stationary analytical solutions of the time-independent Schrödinger equation, but is limited to monochromatic wave functions. The second method will propagate a wave packet by numerically solving the time-dependent Schrödinger equation. The last section will focus on the implications and requirements of a future experiment.
8.2 Semi-classical description

We start the description of inelastic bouncing in a (semi) classical way. In the first part we derive the classical velocity distribution of the inelastically bouncing atoms. After that we give a semi-classical picture of the phase difference between the two possible trajectories. Based on this picture we derive how the possible interference patterns change on variation of the available experimental parameters.

8.2.1 Primary rainbow caustic

We consider atoms bouncing inelastically from an evanescent-wave (EW) mirror, defined by a repulsive potential \( V_1 \exp(-2\kappa z) \). Here \( \kappa^{-1} \) is the decay length of the EW field, as defined by Eq. (2.19). An atom in state \( |1\rangle \), with incident velocity \( -v_i \) \((v_i > 0)\) moves, in a classical description, on a trajectory

\[
z(t) = z_0 + \frac{1}{\kappa} \ln(\cosh(v_i \kappa t)),
\]

with \( z_0 \) the position of the turning point of the atom, given by

\[
z_0 = \frac{1}{2\kappa} \ln \left( \frac{2V_1}{mv_i^2} \right).
\]

The atom's trajectory through phase space is given by

\[
v_i^2(z) = v_i^2 - \frac{2V_1}{m} e^{-2\kappa z}.
\]

While moving in the repulsive potential, the atom can make an optical Raman transition to state \( |2\rangle \). The repulsive potential \( V_2 \exp(-2\kappa z) \) experienced by this latter state is significantly lower. The ratio \( V_2/V_1 \) depends on the detuning of the EW with respect to the \( D_1 \) line. For convenience, we define \( \beta = V_2/V_1 \), thus \( \beta < 1 \). The transfer leads to a loss of potential energy of the atom so that it will leave

Figure 8.1: (a) An atom that is transferred from state \( |1\rangle \) to state \( |2\rangle \) can be transferred on the ingoing or the outgoing part of the trajectory. These two paths possibly interfere. (b) Caustic in the final velocity distribution of an inelastic EW mirror for initial velocity \( v_i = 1 \) and ratio between the potentials \( \beta = 0.1 \).
the potential with a velocity $v_t \leq v_i$. The trajectory through phase space after this transfer is given by

$$v_t^2(z) = v_i^2 - \frac{2\beta V_i}{m} e^{-2kz}. \quad (8.4)$$

The velocity $v_t$ with which an atom in state $|2\rangle$ leaves the potential depends on its velocity $v_t$ at the moment of the transfer. By combining Eqs. (8.3) and (8.4) it follows that the final velocity $v_f$ depends on the transfer velocity $v_t$ as

$$v_f(v_t) = \sqrt{v_t^2 + 2\beta (v_t^2 - v_i^2)}. \quad (8.5)$$

The spontaneous Raman transfer leads to a caustic in the velocity distribution of atoms that leave the potential in state $|2\rangle$. There is a large population of the outgoing velocity class around $\sqrt{\beta}v_t$, which corresponds to atoms that are transferred in the turning point, with $v_t = 0$. A transfer in the motional turning point is most likely since atoms spend a long time there (classically zero velocity) and the scattering rate is highest, because the light intensity is highest. There is a tail to larger velocities, up to a maximum of $v_{in}$, which corresponds to atoms that are pumped over on their way in or their way out of the potential.

Classically, the caustic, the distribution of final velocities, is proportional to $\partial v_f / \partial v_t$. This distribution is shown in Fig. 8.1(b). In this description depletion of the initial state $|1\rangle$ is not taken into account, so the results are only valid for a small transfer rate.

Like many processes that occur near the edge of a classically allowed domain, this caustic maps on the well known description of the rainbow, as was already noticed earlier, and several analogies between the two phenomena are possible [46]. Furthermore, we have successfully observed and characterized this caustic distribution. The minimum final velocity $\sqrt{\beta}v_t$ is called the rainbow velocity in analogy with the rainbow angle for an optical rainbow caustic.

### 8.2.2 Supernumerary rainbows

There is, however, an intriguing question. According to Eq. (8.5) both atoms that are transferred to state $|2\rangle$ at velocities $\pm v_t$, i.e. on their way in and out of the potential, end up on the same trajectory through phase space and will thus leave the evanescent field with the same final velocity $v_f$. This is also shown in Fig. 8.1(a). This raises the question whether there could be interference between the part of the atomic wave function transferred on the ingoing trajectory and the part, describing the same atom, transferred on the outgoing trajectory. This interference would manifest itself in velocity space. Following the rainbow analogy, one could call these interference patterns supernumerary rainbows. In our case the question about interference is highly non-trivial, because a spontaneous Raman process is involved that may destroy the coherence between the paths.

We will give a semi-classical analysis of this problem and a method to determine the phase difference between the two paths. Furthermore two mechanisms will be discussed that could destroy the interference. The first is related to the atom's initial
8.2 Semi-classical description

phase-space distribution, which is an argument that is generally applicable to atom interferometers. The second is more specific for our case and involves the effect of the spontaneous recoil.

**Phase difference between trajectories**  A schematic representation of two trajectories through phase space of atoms that enter the potential in state $|1\rangle$ and leave the potential in state $|2\rangle$, is shown in Fig. 8.2(a). The phase difference $\Delta \varphi$ between the two trajectories is given by

$$\Delta \varphi = \frac{m}{\hbar} \int_{-v_f}^{v_i} (z_1(v) - z_2(v)) \, dv,$$

where $z_1(v)$ and $z_2(v)$ correspond to the inverse of Eqs. (8.3) and (8.4) respectively. The transfer velocity $v_t$ can be derived from Eq. (8.5). This phase difference in Eq. (8.6) is proportional to the area between the two curves, indicated in gray in Fig. 8.2(a). From the evaluation of the integral for various parameters we learn that the fringe period decreases for increasing initial velocities $v_i$, for increasing final velocities $v_f$, for smaller $\beta$ and for larger decay length (smaller $\kappa$).

**General argument for matter-wave interferometers**  The center-of-mass motion and position of an atom can be described by a wave packet which is subject to Heisenberg’s uncertainty relation $\Delta z \Delta v_z \geq \hbar/2m$. The distribution of uncertainty between position $z$ and momentum $m v_z$ is determined by the experimental preparation procedure of the atoms.

For a wave packet that initially has a large spread in momentum, and a well defined position, it is not possible to unambiguously determine the phase difference between the two possible paths, since the wave packet is spread out over several classical trajectories through phase space. This is indicated by the dashed curves in Fig. 8.2(a). With such a wave packet it is possible to determine whether the atom transferred to state $|2\rangle$ on its ingoing or outgoing path, by observing the timing of the spontaneously emitted photon. The spatial compactness of the wave packet enables this measurement. Therefore, it is not expected that wave packets with this shape show interference.

On the other hand, a minimum uncertainty wave packet with a narrow initial momentum spread, and therefore some extension in the spatial dimension, will more closely follow a classical trajectory through phase space, as defined by Eqs. (8.3) and (8.4). This is indicated by the dotted curve in Fig. 8.2(a). Due to this, the phase difference between the two paths, equivalent to the gray area of phase space enclosed by the two paths, is well defined. It comes as no surprise that the two points in phase space where a transition to the final trajectory is possible are covered simultaneously by the wave packet. This is the reason that no “which way” information can be obtained by observing the spontaneously emitted photon.

Thus the initial trade-off between position and momentum uncertainty in a bandwidth limited wave packet determines whether interference can be a priori excluded.
Figure 8.2: (a) Phase space trajectories of atoms being repelled by an evanescent potential. Atoms initially in state $|1\rangle$ can be transferred to state $|2\rangle$ and continue on a different path through phase space. The transfer to such a trajectory can occur either on the ingoing branch of the trajectory, at point A, or on the outgoing branch, at point A'. The accumulated phase difference between these two paths, indicated by the enclosed gray area, may give rise to interference effects. A wave packet with a large initial velocity spread, indicated by the dashed curve, can not overlap both transfer points at the same time. Therefore "which way" information can be obtained by observing the timing of the spontaneously emitted photon. We do not expect that such a wave packet will lead to interference. A wave packet with a small initial velocity spread, as indicated by the dotted curve, can overlap both transfer points simultaneously. No "which way" information can be obtained from the emitted photon and we expect that interference is possible in this case. (b) When the transfer is a spontaneous Raman process, the momentum kick due to the stochastic nature of the spontaneous emission process has to be taken into account. For a given $z$ component of the recoil this gives rise to extra phase factors, indicated by the dark gray areas for a small final velocity and by the light gray areas for a higher final velocity.

or not. This requirement is necessary but not sufficient, since in the actual realization the incoherent nature of the transfer between states must yet be taken into account.

**Specific argument for interferometers with spontaneous decay** The random direction of the spontaneously emitted photon can be taken into account in the motion of the atom by a random momentum jump. This makes the atom propagate on a different trajectory through phase space than it would have without the random recoil. Since we treat our atomic bounce as a one-dimensional problem, only the $z$ component of the recoil is of importance to us. The momentum changes are indicated by horizontal arrows in the phase-space diagram of Fig. 8.2(b).

For a single atom, or a collection of distinguishable atoms, the size and sign of the $z$ component of the spontaneous recoil could be measured by detecting in which direction the photon was emitted. Due to this possibility there will be a set
of interference patterns, each with a well defined recoil direction. By disregarding the information present in the scattered photons, we probe the incoherent sum of all these interference patterns [135].

The transfer moments that contribute to a particular final velocity of one of these interference patterns are not symmetrical around the moment of closest approach anymore, as indicated by the points A and A' in Fig. 8.2(b). The phase difference between the two paths is different with respect to the recoilless case, and it depends on the direction of the recoil. It is indicated by the dark gray areas in Fig. 8.2(b). In order for the interference to be experimentally observable the difference between the interference patterns with a certain recoil direction should not be too large. This means that the phase difference between recoil components in the ±z directions should be less than π. For larger final velocities these phase corrections get larger as is apparent from the larger area around the points B and B' in Fig. 8.2(b). So we expect the visibility of the interference to decrease for larger final velocities.

8.3 Time-independent approach

The question whether interference is visible will be answered by considering analytical stationary solutions of the Schrödinger equation for particles with a total energy \( p_0^2/2m \). We show that the interference survives the incoherent nature of the spontaneous Raman transfer to state |2\rangle for various experimental parameters. Furthermore the qualitative predictions of section 8.2 are confirmed.

8.3.1 Analytical stationary solutions

The time-independent Schrödinger equation

\[
-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2} \psi_1(z) + V_1 e^{-2\kappa z} \psi_1(z) = \frac{p_0^2}{2m} \psi_1(z),
\]

(8.7)
describes stationary states with a total energy \( p_0^2/2m \) on the potential \( V_1 \exp(-2\kappa z) \). It thus describes particles with momenta ±\( p_0 \) in the asymptotic limit of large \( z \). This is one of the few examples where the eigenfunctions of the Schrödinger equation are analytically known. The solutions are given by [136]

\[
\psi_1(z) = \sqrt{\frac{4}{\pi \kappa}} \sinh \left( \frac{\pi p_0}{\hbar \kappa} \right) K_{i\nu_0/k} \left( \frac{\sqrt{2mV_1}}{\hbar \kappa} e^{-\kappa z} \right),
\]

(8.8)

where \( K_\alpha(\beta) \) is the Bessel-K function of order \( \alpha \) evaluated for argument \( \beta \) [137].

Figure 8.3 shows \(|\psi_1(z)|^2 \) for an evanescent-wave potential with \( \kappa = k_0/8 \), with \( k_0 \) the wave vector of a photon, and for several values of the initial momentum \( p_0 \). To separate them, for better visibility, they are shown at an offset of their own energy \( p_0^2/2m \). It is clearly visible that far from the surface, where the exponential potential vanishes, the wave function closely resembles that of a free particle. Also obvious is that the higher energy solutions move deeper into the evanescent wave potential. Note that in the classically forbidden region the wave function vanishes very rapidly, more rapidly than exponential.
Figure 8.3: Examples of density distribution $|\psi_1(z)|^2$ with $\psi_1(z)$ a stationary solution of Schrödinger’s equation with an exponentially decaying potential as given by Eq. (8.8). Plots are shown for initial momenta of $p_0 = \hbar k_0$, $2\hbar k_0$, $2.5\hbar k_0$, and $3\hbar k_0$, and are shown at an offset of their own energy $p_0^2/2m$. The decay length of the evanescent wave is $\kappa^{-1} = k_0/8$.

### 8.3.2 Spontaneous Raman transfer

An atom with initial momentum $p_0$ that is initially in state $|1\rangle$ can make a Raman transition to state $|2\rangle$ by scattering photons from the optical evanescent potential. In this new state the interaction of the atoms with the evanescent potential is changed and they are now repelled by a lower potential of height $V_2$. The atom can make a transition to several of the eigenfunctions $\psi_{2,p}(z)$ with final momentum $p$ in the potential $V_2 \exp(-2\kappa z)$.

The final wave function in momentum space is given by

$$\phi_k(p) \sim \int_0^\infty \psi_1(z) e^{-\kappa z - ikz} \psi_{2,p}(z) dz,$$

where the recoil due to the spontaneous Raman transition is taken into account by projecting the initial wave function with an extra momentum factor $\psi_1(z) \exp(-ikz)$ onto the eigenfunction $\psi_{2,p}(z)$. Here $\hbar k$ is the momentum component of the recoil in the $z$ direction. The coupling between these two states is determined by the Rabi frequency, which is proportional to the electric field amplitude, and is thus given by $\exp(-\kappa z)$ and not by $\exp(-2\kappa z)$ which is proportional to the evanescent-wave intensity. As already discussed in section 8.2.2, there will be interference patterns $|\phi_k(p)|^2$ for every value $\hbar k$ of the recoil. A measurement constitutes of the sum of all these possible interference patterns. In this derivation we will assume an isotropic distribution of the recoil momentum $\hbar k$. This is not straightforward, since the distribution depends on the polarization of the spontaneously emitted photon. We will come back to this point in section 8.5.3. This leads to

$$|\phi(p)|^2 = \int_{-k_0}^{+k_0} |\phi_k(p)|^2 dk.$$
8.3 Time-independent approach

8.3.3 Results and discussion

Fig. 8.4 shows the behavior of the momentum distribution for various values of the $z$ component of the photon recoil $hk$. It is calculated using Eq. (8.9), with an initial momentum $p_0 = 2\hbar k_0$, a potential steepness $\kappa = k_0/8$, and potential reduction $\beta = 0.2$. Some points are immediately apparent from this graph. First of all, as expected, the main part of the momentum distribution extends between the initial momentum $p_0$ and the rainbow momentum $\sqrt{\beta}p_0$, both indicated by vertical dotted lines. The distributions “peak” near the rainbow velocity, resembling the caustic distribution. Secondly, for every recoil direction there are supernumerary rainbows visible. Although the interference washes out for larger values of the recoil, the remaining interference fringes are present at more or less the same final momenta. They only shift slightly to lower final momenta for larger values of the recoil. This already indicates that the spontaneous recoil does not completely wash out the interference. The behavior of the interference is independent of the direction of the recoil. This makes sense if one realizes that a photon emitted on the ingoing part of the trajectory has the same effect on the momentum distribution as a photon that is emitted in the opposite direction on the outgoing part of the trajectory.

Fig. 8.5 shows the results of calculations of Eq. (8.10) for different experimental parameters. The dashed lines are calculations of $|\phi_0(p)|^2$, without a spontaneous recoil, so the expected interference patterns for a coherent splitting process. The solid lines are calculations of $|\phi(p)|^2$ in which the effect of the recoil has been incorporated. Indeed the summation over the spontaneous recoil does not destroy the interference pattern, even for these extremely low initial momenta. The small parts of the distribution $|\phi_0(p)|^2$ at momenta smaller than the rainbow momentum and larger than the incident momentum are evanescent matter waves that extend into the classically forbidden regions.

Several of our predictions that were made for the general case of a coherent interferometer are noticeable in these graphs. Indeed the fringe spacing decreases
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Figure 8.5: Interference patterns calculated for different parameters. Solid lines: $|\phi(p)|^2$, with the effect of a spontaneous recoil, and dashed lines: $|\phi_0(p)|^2$, without the effect of a spontaneous recoil. The dotted lines indicate the initial momentum and the rainbow momentum.

for larger initial and final momenta, for longer decay lengths $\kappa^{-1}$ of the evanescent field, and for smaller values of $\beta$. Furthermore, as predicted for the case of an incoherent interferometer, the visibility of the graphs in which the effect of the recoil has been taken into account decreases for larger final momenta.

### 8.4 Time-dependent approach

In the previous section we have shown that the incoherent nature of the spontaneous Raman transfer does not prevent us to observe interference. In this section we will show that the interference phenomena will also be visible for a wave packet with
8.4 Time-dependent approach

Figure 8.6: Evolution of (a) the density distribution $|\psi_1(z, t)|^2$ and (b) the real part of the wave packet $\psi_1(z, t)$, calculated for an evanescent wave with $\kappa = \kappa_0/8$, and a wave packet with initial height $z_0 = 75/k_0$, initial width $\sigma_z = 7/k_0$, and initial momentum $p_0 = 2\hbar k_0$.

a finite momentum spread. In the analysis we will closely follow the Monte-Carlo wave-function approach [138, 139].

8.4.1 Transfer-free evolution

We consider the evolution of a diffraction limited wave packet in state $|1\rangle$

$$\psi_1(z, t = 0) = \sqrt{\frac{1}{(2\pi)^{1/2}\sigma_z}}e^{ik_0z}e^{-\frac{(z-z_0)^2}{4\sigma_z^2}}$$  \hspace{1cm} (8.11)

with initial height $z_0$, initial width $\sigma_z$, and initial momentum $p_0 = \hbar k_z$. It is normalized such that $\int |\psi(z, 0)|^2dz = 1$ and $\int (z - z_0)^2 |\psi(z, 0)|^2dz = \sigma_z^2$. The evolution of the wave packet is calculated by numerically solving the time-dependent Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\psi_1(z, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial z^2}\psi_1(z, t) + V_1e^{-2\kappa z}\psi_1(z, t),$$  \hspace{1cm} (8.12)

when it reflects from the evanescent-wave potential with a potential height $V_1$ at $z = 0$ using the Quantum kernel [141] package in Mathematica [140]. This results in a wave packet $\psi_1(z, t)$ at time $t$.

An example of such an evolution is shown in Fig. 8.6, where the density distribution and the real part of the wave function are plotted versus position $z$ and time $t$. This graph is calculated for an evanescent wave with $\kappa = \kappa_0/8$, and a wave packet with initial height $z_0 = 75/k_0$, initial width $\sigma_z = 7/k_0$, and initial momentum $p_0 = 2\hbar k_0$. The spatial width leads to a width of the momentum distribution of $\sigma_p = \hbar k_0/7$. 

In these graphs several characteristics of the wave packet are visible. In Fig. 8.6(b) the difference between the phase velocity and the group velocity is apparent. Furthermore the dispersion of the wave packet is observable: its initial width is smaller than its final width. Also the slight tilt of the interference fringes during the reflection is due to dispersion. Atoms that arrive later in their turning point are the slower atoms that move less deep into the evanescent potential.

### 8.4.2 Spontaneous Raman transfer

At a time $\tau$ a transition to state $|2\rangle$ occurs, and the evolution abruptly continues on a potential that is a factor $3$ lower. This corresponds to a Raman transition of an atom to another internal state that has a weaker interaction with the evanescent wave.

Immediately after the transfer the wave function in state $|2\rangle$ is described by

$$
\psi_{\tau,k}(z,\tau) = N\psi_1(z,\tau)e^{-\kappa z}e^{-ikz}.
$$

where $N$ denotes a normalization factor. The first exponent represents the mode function of the absorbed evanescent photon, and the second exponent the mode function of the spontaneously emitted photon. The evolution of this wave function can now be continued up to a time $t_{\text{end}}$, leading to a wave function $\psi_{\tau,k}(z,t_{\text{end}})$. When $t_{\text{end}}$ is large enough the entire wave packet effectively propagates in free space, so that the momentum distribution remains constant. The Fourier transform

$$
\phi_{\tau,k}(p) \sim \mathcal{F}(\psi_{\tau,k}(z,t_{\text{end}}))
$$

of the wave packet at this time is the wave function in momentum space that endured a momentum kick $\hbar k$ at its transfer time $\tau$.

Every atom of the sample that is transferred to state $|2\rangle$ is transferred at a random transfer time $\tau$ and undergoes a random recoil kick $\hbar k$ due to the spontaneously emitted photon. The transfer rate $\Gamma(\tau)$ at a certain time $\tau$ is given by

$$
\Gamma(\tau) \sim \int_0^\infty \psi_1^*(z,\tau)V_1(z)\psi_1(z,\tau)dz.
$$

We again assume an isotropic distribution of the recoil momentum $\hbar k$. We have to add contributions with different $\tau$ and $k$ in an incoherent way. In contrast to the original description of the Monte-Carlo wave-function method we do not evaluate the momentum distribution for a discrete number of random transfer moments $\tau$ and recoil momenta $\hbar k$, but integrate numerically over these values. For the momentum distribution as a function of the wave vector of the spontaneously emitted photon we get

$$
|\phi_k(p)|^2 \sim \int_0^{t_{\text{end}}} \Gamma(\tau)|\phi_{\tau,k}(p)|^2d\tau.
$$

A subsequent integration over this wave vector yields

$$
|\phi(p)|^2 = \int_{-k_0}^{k_0} |\phi_k(p)|^2dk
$$

for the momentum distribution of a sample of atoms.
8.4.3 Results and discussion

Fig. 8.7 shows examples of the density and momentum distributions as a function of the transfer time $\tau$ in the absence of recoil, thus for $k = 0$. The data for a certain value of time $\tau$ represents the final wave function if the evanescent field had been switched on for a very short time at time $\tau$. The plots are calculated for
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Figure 8.8: Interference patterns calculated for different parameters. Solid lines: $|\phi(p)|^2$, with the effect of a spontaneous recoil, and dashed lines: $|\phi_0(p)|^2$, without the effect of a spontaneous recoil. The dotted lines indicate the initial momentum and the rainbow momentum.

an evanescent wave with a decay length $\kappa^{-1} = 8/k_0$, a ratio of the two potentials $\beta = 0.2$ and a wave packet with initial height $z_0 = 75/k_0$, initial width $\sigma_z = 7/k_0$ and initial momentum $p_0 = 2\hbar k_0$. The momentum spread due to the finite size of the wave packet is $\sigma_p = \hbar k_0/7$. Both the density distribution $|\psi_{r,0}(z, t_{end})|^2$ and the real part of the wave function $\psi_{r,0}(z, t_{end})$, and the momentum distribution $|\phi_{r,0}(p)|^2$ and the real part of the wave function in momentum space $\phi_{r,0}(p)$ are shown.

Figs. 8.7(a) shows a line with high density that we interpret as atoms that are transferred in their turning point. It is extended in time due to the finite width of the wave packet and it is slanted because atoms that are transferred later have less time to fly away from the potential. We interpret the less intense area at larger distances and larger transfer times as atoms that are transferred on the outgoing part of their trajectory and thus leave the potential with a large velocity. In Figs. 8.7(c, d) is visible that atoms which are transferred either very early or very late, indeed leave the potential with unchanged momentum. Fig. 8.7(c) also shows that around the motional turning point, for $\tau$ approximately between 30 $m/\hbar k_0^2$ and 45 $m/\hbar k_0^2$, a large population near the rainbow momentum $\sqrt{3}p_0$ is created. It is furthermore visible that the supernumerary rainbow contributions that are created at different transfer times $\tau$ have the same final momentum. The phase fluctuations in momentum space, that are visible in Fig. 8.7(d), are of no importance since the various $\tau$ contributions add incoherently.

Fig. 8.8 shows graphs of $|\phi(p)|^2$ for some parameters. Fig. 8.8(a) must be
compared with Fig. 8.5(a), Figs. 8.8(b)-(d) must be compared with Fig. 8.5(c). It is obvious that even for a wave packet with a finite momentum spread the interference effects are still present. The parameters for Figs. 8.8(b)-(d) are equal except for the initial width \( \sigma_z \) of the wave packet and thus the momentum spread \( \sigma_p = \hbar/\sigma_z \). It is clear that the interference fringes are more apparent for a wave packet with a smaller momentum spread.

### 8.5 Experimental considerations

This section discusses the implications of the results of the calculations that are presented in this chapter for an actual experiment. Several aspects are considered. Firstly the implication of the low initial velocity that is used in the calculations. Secondly proper physical levels of \(^{87}\text{Rb}\) atoms are chosen to fulfill the role of states |1\> and |2\>. Subsequently we consider the implications of a real probability distribution of directions in which the spontaneously emitted photon is emitted. This distribution has been taken isotropic throughout this chapter. Finally we consider a method to detect the resulting final momentum distribution.

#### 8.5.1 Computational limitations - experimental implications

The calculations presented in this chapter are for unrealistically low initial velocities \( v_i \). This is because both calculation procedures turned out to be limited by computational resources. The calculation time becomes inconveniently long to evaluate the procedure for experimentally realistic incident velocities of \( \sim 30\ \text{cm/s} \) \([46]\), which corresponds to \( \sim 50v_{\text{rec}} \), with \( v_{\text{rec}} \) the recoil velocity of the atom due to the scattering of a photon.

For the time-independent approach the evaluation in Mathematica \([140]\) of the Bessel-K functions of high imaginary order becomes very slow, even though these functions are analytically known. For the time-dependent approach the number of sampling points, necessary for the numerical evaluation, becomes too large due to the highly oscillatory character of the incident wave packet. However, we expect even better signals for realistic values for the initial velocity \( v_i \), so the calculation represents a worst case.

#### 8.5.2 Rb levels and optical pumping

So far we have considered levels |1\> and |2\> without discussing which physical level they correspond to. In reality we usually deal with multi-level atoms, that moreover include sub-structure. Each of these (sub-)levels has a different interaction with the evanescent field. If more (sub-)levels contribute to the signal the predicted interference can be washed out.

For \(^{87}\text{Rb}\) atoms a convenient choice for state |1\> would be the \(|Fm\rangle = |1, 0\rangle\) ground state and the \(|Fm\rangle = |2, \pm 1\rangle\) ground states for the |2\> state. The evanescent field needs to be linearly polarized and blue detuned with respect to the \( F = 1 \rightarrow \)
$F' = 2$ transition of either the $D_1$ or $D_2$ line. For this choice of parameters, atoms in state $|1\rangle$ can only make transitions to state $|2\rangle$. Transitions via the $F' = 0$ excited state can not decay to state $|2\rangle$, and hence do not contribute to the signal. Transitions via the $|F'm'\rangle = |1.0\rangle$ or $F' = 3$ excited stated are forbidden due to selection rules. Only a transition over the $|F'm'\rangle = |2.0\rangle$ excited state contributes to the transition from state $|1\rangle$ to state $|2\rangle$. This excited state can decay to either of the $|2.\pm 1\rangle$ ground states. Since these states interact identically with the evanescent field, their interference patterns will overlap.

We performed experiments to observe the interference effects that are described in this chapter. Our attempts were hampered by the fact that the incident atoms were not optically pumped [90]. If interference was present it was obscured by the superposition of several of these interference patterns. Moreover the velocity spread of the sample of atoms might have been too large to observe interference.

### 8.5.3 Distribution of the spontaneously emitted photons

The distribution of wave vectors of the spontaneously emitted photon has been assumed isotropic throughout this chapter. In reality the polarization of the photon is either linear or circular, each having a corresponding wave-vector distribution.

The intensity distribution of a linearly polarized photon is given by $I(\theta) = \frac{3}{8\pi} \sin^2 \theta$, with $\theta$ the angle with the $z$ axis. This leads to a distribution of the $z$ component of the wave vector of $\frac{3}{8}(1 - (k/k_0)^2)$. For a $\sigma^\pm$ circularly polarized photon the intensity distribution is given by $I(\theta) = \frac{3}{16\pi} (1 + \cos^2 \theta)$, which leads to a distribution of the $z$ component of the wave vector of $\frac{3}{8}(1 + (k/k_0)^2)$. For the intensity distributions, see e.g. [47].

From these distributions it is clear that a linearly polarized spontaneously emitted photon is advantageous in order to maximize the visibility of the interference pattern, since the components for small $|k|$ contribute more than the components for larger $|k|$. However, as discussed in the previous sub-section, for $^{87}$Rb atoms the spontaneously emitted photon needs to be either $\sigma^+$ or $\sigma^-$ polarized. This will lead to a decrease of the interference visibility as compared to the graphs calculated in the previous two sections. However, it will still be possible to observe interference.

### 8.5.4 Detection

In an actual experiment the momentum distribution eventually maps on the position distribution since the decay length of the evanescent wave $\kappa^{-1}$ is short with respect to the height of the motional turning point due to gravity. One way to perform this experiment is similar to the inelastic bouncing experiments [46]. The velocity distribution can be determined from e.g. an absorption image of the sample of atoms at a sufficiently long time after the reflection.

Another possibility is to perform a beam experiment with an atomic beam incident at a grazing angle with respect to the dielectric surface that is used to create the evanescent wave. The velocity component of the atoms parallel to the surface will not be altered, while the velocity component perpendicular to the surface will
show the interference as discussed in this chapter. A spatially resolving detector
down the path of the atomic beam will show the interference patterns. This setup
has the advantage of being a continuous experiment, which will enhance the signal
to noise ratio.

8.6 Discussion and conclusions

By solving the Schrödinger equation numerically we have been able to answer the
question “Is it possible to have an atom interferometer on basis of beam splitters
that involve spontaneous emission?” with an unambiguous yes.

The intuitive objections to whether this is possible have been refuted. The semi-
classical arguments that were presented in section 8.2 have been confirmed by the
full quantum-mechanical calculations. “Which way” information due to the possi-
bility of detecting the spontaneously emitted photon can be prevented by choosing
a sufficiently narrow velocity uncertainty. A wave packet can cover both transfer
points in phase space simultaneously if its velocity is defined accurately enough. The
incoherent nature of a spontaneous emission process due to the random recoil direc-
tion of the atom is visible in all the calculated interference curves, but does not lead
to a complete scrambling of the interference. For larger final velocities, for which the
transfer points are separated more and the acquired random phase is consequently
larger, the visibility of the interference fringes indeed decreases. Furthermore the
fringe period indeed qualitatively shows the behavior that was predicted on the basis
of the semi-classical calculations.
An atom interferometer using spontaneous decay