Essays in Nonlinear Economic Dynamics
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Chapter 4

Exchange Rate Dynamics and Unpredictability

4.1 Introduction

The work of Meese and Rogoff (1983) showed that structural exchange rate models were not able to beat the simple random walk in out-of-sample prediction. Cheung et al. (2002) repeated a similar exercise adding the last 20 years of data and confirmed the earlier results of Meese and Rogoff. A possible reason for this failure is that fundamental variables used in the structural models play only a minor role in the determination of the exchange rate. This explanation was suggested in survey studies of exchange rate traders such as Frankel and Froot (1987), Allen and Taylor (1992), Lui and Mole (1998) and, more recently, Cheung and Chinn (2001). They find that traders perceive the effects of macroeconomic variables on exchange rates as a long-term phenomenon. For shorter horizons, their expectations are more influenced by non-fundamental factors, such as over-reaction to news, trading based on technical analysis and bandwagon effects. Hence, even though the structural models could explain the long-term dynamics of the exchange rate, at shorter horizons other (non-fundamental) factors play a relevant role in driving the exchange rate.

This evidence suggests that a successful model of the exchange rate should combine both fundamental and non-fundamental factors. An early model along these lines is Frankel and Froot (1990). The model departs from rational expectations by assuming agents are one of two types: fundamentalists or chartists. Fundamentalists form expectations about the exchange rate using fundamental factors, such as macroeconomic variables. Chartists, on the other hand, only use information about the past history of the exchange rate itself and extrapolate to form their expectations. The interaction of these types of traders is able to generate patterns with large deviations from the fundamentals. However, in the long term
the stabilizing role of the fundamentalists' expectations drives the exchange rate back to its long-run equilibrium. Recently, Kilian and Taylor (2003) found evidence in favour of a similar model. They suggest that the interaction between fundamentalists and chartists traders could explain the findings of nonlinear mean reversion in exchange rates, that is, the long-term tendency of the exchange rate to adjust toward its fundamental value in a nonlinear fashion. The model implies that changes in the real exchange rate are unpredictable when it is close to the long-run equilibrium but become predictable when the exchange rate deviates consistently from its fundamental value. Other papers finding evidence of threshold dynamics are Obstfeld and Taylor (1997) and Taylor et al. (2001).

In addition to the difficulty to explain the short-run dynamics by structural models, time series methods are also unable to make significant improvements in out-of-sample prediction. Diebold and Nason (1990) used nonparametric regression techniques on weekly exchange rates and failed to achieve any improvement. However, for long-horizon returns Mark (1995) found that there is significant evidence of predictability. These mixed results are consistent with the previous discussion of the mean reverting behaviour of real exchange rates. At short-horizons they change in an unpredictable way but, in the long term, fundamental factors become relevant and determine the adjustment of the nominal exchange rate toward the fundamental value.

In this chapter, we propose an empirical model of exchange rate determination along the tradition of the chartist-fundamentalist approach. As in earlier models, we assume that traders have heterogeneous expectations of the exchange rate: a group of traders (fundamentalists) considers fundamental factors while another group (chartists) considers non-fundamental factors, such as past returns. We introduce a threshold type mechanism to model the time variation in the extrapolation rate of the chartists. We assume that they switch between two regimes according to the absolute change in the exchange rate being smaller or larger than a constant value. Further, chartists extrapolate more aggressively when the exchange rate increasingly deviates from the fundamentals. Estimation results suggest that there is evidence in support of the model for most of the currencies. In addition, the model is able to achieve statistically significant improvements in out-of-sample prediction for some of the currencies. This is a novel result given the evidence in the literature of short-horizon unpredictability.

The favourable in- and out-of-sample results suggest that we should detect predictability even at short-horizons. In order to understand the apparent contradiction between our findings and the results discussed previously, we follow an approach similar to Kilian and Taylor (2003). We simulate the proposed exchange rate model and apply the tests that are typically used in investigating short- and long-horizon predictability to these simulated series. In particular, we use the nonparametric prediction approach of Diebold and Nason (1990) and
the long-run regression of Mark (1995). We find that the test for short-term predictability has extremely low power in detecting the nonlinearity in the model at the available sample size. On the other hand, the long-horizon test reproduces the typical finding of increasing predictability when the horizon is longer. We interpret these results as suggesting that the failure to predict exchange rates changes out-of-sample could be explained by the existence of weak form of nonlinearities and/or small samples.

The outline of the chapter is as follows: In Section (4.2) we describe the model and in Section (4.3) we show the estimation results for monthly exchange rates. In Section (4.4) we test whether the proposed structural model can actually explain the empirical puzzles previously mentioned. Finally, Section (4.5) concludes.

4.2 The Model

We develop a model of exchange rate determination in which investors have heterogeneous expectations. We assume the economy is populated by two types of traders. One group of traders, the fundamentalists, expects the exchange rate to converge to its long-run fundamental value. We assume their expectations are given by

\[ E_t^f(s_{t+1}) = s_t + \phi_f(f_t - s_t), \]

where \( s_t \) is the log of the exchange rate at time \( t \), \( f_t \) is the log of the fundamental value and \( \phi_f \) is a parameter. We can rewrite this equation in terms of expected change in the log exchange rate as

\[ E_t^f(r_{t+1}) = \phi_f \text{dev}_t, \]

where \( r_{t+1} = s_{t+1} - s_t \) is the one-period return and \( \text{dev}_t = f_t - s_t \) indicates the deviation of the fundamental value from the exchange rate. The fundamentalists expect tomorrow’s price to include an adjustment component that corrects the mispricing of the exchange rate with respect to the fundamental value. In terms of returns they expect the next period return to be proportional to the mispricing. A mean reversion behaviour of prices to the fundamentals implies that the coefficients \( \phi_f \) should typically be positive and less than 1. If the exchange rate at time \( t \) is above (below) its long-run equilibrium they expect a depreciation (appreciation) of \( s_t \).

The second type of traders, the chartists, attribute a relevant role to the information extracted from the exchange rate itself. For short horizons they believe that information contained in macroeconomic variables does not contribute to the prediction of exchange rates. However, we assume that the extrapolation rate depends on the magnitude of the deviation from the fundamentals. We model the expectations of this group as
$E_t^c(s_{t+1}) = s_t + \delta_t(s_t - s_{t-1}), \quad (4.2)$

where $\delta_t$ is a time-varying coefficient that captures the sentiment, or confidence, chartists have in the continuation or reversal of past returns. In terms of returns the expectation is

$E_t^c(\delta_{t+1}) = \delta_tr_t.$

A positive $f_t$ is associated with bandwagon expectations in the sense that traders expect a trend to persist. If an appreciation (depreciation) of the exchange rate is observed they also expect a change in the same direction in the next period. On the other hand, if $\delta_t$ is negative they expect a reversal of the exchange rate in the following period. We interpret $\delta_t$ as indicating the sentiment that chartists have in the continuation of a trend. We assume that

$$\delta_t = \begin{cases} \delta_1|\text{dev}_t| & \text{if } |r_t| \geq c \\ \delta_2|\text{dev}_t| & \text{otherwise}, \end{cases} \quad (4.3)$$

where the sentiment, $\delta_t$, depends on the absolute deviation from the fundamentals and switches between two regimes, depending on the absolute return being bigger or smaller than a constant threshold value $c$. A long-standing discussion is the role of chartists’ expectations in stabilizing or destabilizing the market. A negative $\delta_t$ is stabilizing in the sense that changes in the exchange rate are expected to reverse, whereas a positive $\delta_t$ is destabilizing in the sense that positive changes tend to persist and create trends in $s_t$. If the parameters $\delta_1$ and $\delta_2$ in Equation (4.3) have negative (positive) signs then the chartists have a stabilizing (destabilizing) role. However, allowing $\delta_t$ to vary over time captures the fact that chartists might switch between destabilizing and stabilizing expectations. Opposite signs of the coefficients across regimes represents a situation in which they have stabilizing expectations in one regime and destabilizing ones in the other. This is an interesting hypothesis to test since it implies that chartists might contribute to correct or exacerbate deviations from the long-run equilibrium. In addition, the $\delta_t$ captures the fact that chartists extrapolate more aggressively when the exchange rate deviates more from the fundamentals: if $\delta_1$ or $\delta_2$ are positive they become more confident about the continuation of the trend; on the other hand, if the parameters are negative they expect a stronger correction in the direction of the fundamentals.

We assume that investors have myopic mean-variance demand functions given by

$$d_t^f = \alpha^f[E_t^f(s_{t+1}) - s_t] \quad (4.4)$$

$$d_t^c = \alpha^c[E_t^c(s_{t+1}) - s_t], \quad (4.5)$$
where the \( \alpha \)'s are reaction coefficients and depend on the risk aversion coefficient. Finally, we assume that a risk-neutral market-maker aggregates the demands of the traders and adjusts the price according to the following rule

\[
s_{t+1} = s_t + \alpha^m \left[ d_t^p + d_t^c \right],
\]

(4.6)

where \( \alpha^m \) indicates the reaction coefficient of the market-maker.

### 4.3 Empirical Evidence

We analyze monthly exchange rates from the beginning of 1974 to the end of 1998. The currencies we consider are the German mark (DM), Japanese yen (JY), Canadian dollar (CD), French franc (FF) and the British pound (BP) against the US dollar. As the fundamental value we assume the PPP (Purchasing Power Parity) given by

\[
f_t = \pi_t - \pi_t^*,
\]

where \( \pi_t \) and \( \pi_t^* \) indicate the log of the CPI index in the US and the foreign country, respectively. Figure (4.1) shows the time series and the linear properties of the exchange rates. In the first column of graphs it is immediately clear that there are large and persistent deviations of the nominal exchange rate from the PPP value for all currencies. The third column also highlights that nominal returns do not show any linear autocorrelation structure for lags up to 15. Besides, the squared returns do not show significant linear dependence either.

The model described in the previous section has a very simple structure that can be easily estimated. Setting all reaction coefficients equal to 1, \( \alpha^L = \alpha^C = \alpha^m = 1 \), it implies the following model for returns

\[
r_{t+1} = \delta_1 |dev_{t,r} \cdot r_{t-p} I_t \cdot |r_{t+1} > c| + \delta_2 |dev_{t,r} \cdot r_{t-p} I_t \cdot |r_{t+1} < c| + \phi_f dev_{t-s} + \epsilon_{t+1}.
\]

(4.7)

where \( I_{(A)} \) denotes the indicator function that assumes the value 1 if A is true and zero otherwise, and \( \epsilon_t \) is an i.i.d. observational noise term. In Equation (4.7) we generalize the model described in Section (4.2) to have lags \( (p,q,r \) and \( s) \) in the expectations different from 0. We will search for the lags that best fit the data. Equation (4.7) is estimated by OLS because, conditional on the threshold \( c \), the model is linear. We will perform a grid search for the optimal value of \( c \).

The estimation results for the sample period of 1974 to the end of 1994 are in Table (4.1). For each currency we present the estimated parameters and the HCCE t-values, the \( F \)-test for linearity, the Root Mean Square Prediction Error (RMSPE) obtained by out-of-sample one-step ahead predictions for the last 48 observations (from the beginning of 1995 to the end of 1998) and the \( DM \) test proposed by Diebold and Mariano (1995) to compare predictive
Figure 4.1: Exchange Rates

Plots for the major currencies against the US dollar for the sample period 1974-1998. The first column shows the nominal exchange rate and the PPP fundamental value, the second column shows the returns, the last two column are the ACF of the returns and the squared returns, respectively.
accuracy\textsuperscript{1}. Table (4.1) shows the estimation results when we assume that $\delta_1 = \delta_2 = \delta_0$ are equal across regimes, i.e. we estimated the following regression

$$r_{t+1} = \delta_0|\text{dev}_{t-1}|r_{t-1-p} + \phi_f\text{dev}_{t-s} + \epsilon_{t+1},$$  

(4.8)

where $\delta_0$ is a parameter. We indicate Equation (4.8) as the linear model. $F^{lin}$ is the statistic to test for linearity proposed by Hansen (1996) and Hansen (1997) and test whether the threshold mechanism is significant. We used 1000 simulations to calculate the p-value of the test statistic.

As shown in Table (4.1), for the linear model the lagged deviation from the fundamental price is significant at the 5\% level for all the currencies, but the lagged value of the returns is not significant. Instead, when the nonlinear model is considered most of the coefficients are significant at the 5\% level. The only exception is the CD, for which the evidence of a threshold dynamics is weak. This is also confirmed by the linearity test, which strongly rejects the null hypothesis of linearity for DM, FF and BP, rejects at the 10\% level for the JY and does not reject for the CD.

For most of the currencies the dependence of the return on the lagged deviation occurs at lag 12 (the 7\textsuperscript{th} is also significant for CD). The coefficient $\phi_f$ varies between 0.018 for the JY and 0.037 for the BP, implying a slow adjustment towards the fundamentals. An interpretation of the results in terms of the model in Section (4.2) is that fundamentalists base their expectations about next month's exchange rate on the level of today's exchange rate with a typical 3\% monthly adjustment of the deviation from the long-run equilibrium that occurred one year ago. This result is consistent with the survey analysis of Frankel and Froot (1987) and Allen and Taylor (1992), from which it emerged that investors use information about fundamentals when forming long-term expectations.

The significant nonlinear dependence on past returns occurs on the first lag, with the exception of the BP, where the third lag is also significant. This confirms the survey findings previously cited, in which 90\% of the respondents attributed a relevant role to technical analysis in the formation of expectations for short horizons. For all currencies, the sign of the estimated coefficients of $\delta_t$ are negative in the outer regime (for absolute returns higher than $c$) and positive in the inner regime (absolute returns smaller than $c$). This is evidence in support of the mixed influence of chartists' expectations on prices: when the exchange rate appreciation (depreciation) is smaller than the threshold the chartists expect it to persist; on the other hand, when the observed change is larger than $c$ they expect a reversal of the change. Thus, there is evidence supporting the hypothesis that chartists act as a destabilizing force when absolute returns are small, but contribute to the stabilization of the exchange rate.

\textsuperscript{1}We implement the $DM$ test with the correction proposed by Harvey \textit{et al.} (1997).
Table 4.1: Estimation Results

<table>
<thead>
<tr>
<th></th>
<th>$\delta_0$</th>
<th>$\phi_f$</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\sigma_f$</th>
<th>$F^{lin}$</th>
<th>RMSPE</th>
<th>DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>DM</td>
<td>-0.047</td>
<td>-1.09</td>
<td>4.19</td>
<td>0.032</td>
<td>0.028</td>
<td>29.44</td>
<td>0.963</td>
<td>-0.89</td>
</tr>
<tr>
<td>$dev_{t-11}$</td>
<td>0.027</td>
<td>(1.98)</td>
<td>0.028</td>
<td>(2.31)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>JY</td>
<td>0.38</td>
<td>-0.65</td>
<td>1.14</td>
<td>0.029</td>
<td>0.018</td>
<td>7.43</td>
<td>0.961</td>
<td>-1.33</td>
</tr>
<tr>
<td>$dev_{t-11}$</td>
<td>0.017</td>
<td>(1.81)</td>
<td>0.018</td>
<td>(2.35)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td>-0.73</td>
<td>-1.68</td>
<td>2.28</td>
<td>0.015</td>
<td>0.07</td>
<td>3.89</td>
<td>1.04</td>
<td>0.38</td>
</tr>
<tr>
<td>$dev_{t-6}$</td>
<td>-0.07</td>
<td>(-2.6)</td>
<td>-0.07</td>
<td>(-2.66)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dev_{t-11}$</td>
<td>0.09</td>
<td>(3.4)</td>
<td>0.09</td>
<td>(3.6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FF</td>
<td>-0.37</td>
<td>-0.84</td>
<td>3.89</td>
<td>0.029</td>
<td>0.031</td>
<td>17.42</td>
<td>0.94</td>
<td>-2.18</td>
</tr>
<tr>
<td>$dev_{t-11}$</td>
<td>0.029</td>
<td>(1.95)</td>
<td>0.031</td>
<td>(2.31)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BP</td>
<td>0.50</td>
<td>3.78</td>
<td>0.025</td>
<td>0.037</td>
<td>0.032</td>
<td>15.55</td>
<td>1.10</td>
<td>1.41</td>
</tr>
<tr>
<td>$r_{t-2}$</td>
<td>-0.34</td>
<td>(-2.04)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$dev_{t-11}$</td>
<td>0.032</td>
<td>(2.11)</td>
<td>0.037</td>
<td>(2.41)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimation results for the major currencies indicated in the first column *vis-a-vis* the U.S. dollar. The linear model is as in Equation (4.8) and the nonlinear model is as in Equation (4.7). The lags used in the regressors are indicated in the second column. In the switching mechanism we selected $|dev_t|$ for all the currencies and $|r_t|$ except for JY where the switching variable is $|r_{t-1}|$. The values in parenthesis are the HCCE t-values. $F^{lin}$ indicates the F-test for linearity and in parentheses the simulated p-value; RMSPE indicates the root mean square prediction error for the last 48 observations; $\mathcal{DM}$ indicates the statistic for the test of equality of the model forecast compared with the no-change forecast and the one-sided p-values in parentheses (a negative value of the statistics indicate that the structural model improves over the random walk model). Asymptotically it is standard normally distributed. We perform a one-sided test.
when they observe large movements in the exchange rate. In addition, the larger the deviation is from the fundamentals the more aggressive they become in stabilizing (destabilizing). The estimated value of the threshold varies from 1.5% for the CD to a maximum of 3.2% for the DM.

To evaluate the role of this mechanism more effectively we plotted the estimated dynamics of $\delta_t$ in Figure (4.2) for the case of the DM. In the top plot the coefficient $\delta_t$ is plotted against time; the bottom plot depicts the exchange rate and the PPP fundamental value. It is clear that $\delta_t$ is an indicator of chartist sentiment. In periods of large deviations they become nervous about the trading signals they extrapolate from the data. This fact is particularly clear in the period from 1980 to 1985, when a persistent appreciation of the USD occurred with respect to the DM that was not supported by an increase in the long-run equilibrium of the PPP. This period is associated with large variability in chartist sentiment, which reacted more nervously and aggressively to changes in prices as the deviation became larger. It is also interesting to note that in 1985 the extrapolation was so aggressive that chartists expected the next month’s change to be even higher than the last observed change. We also observe that $\delta_t$ becomes negative in some cases. This confirms that they might have a stabilizing role when deviations are associated with large changes in the exchange rate (with respect to the threshold).

The in-sample results are quite promising and point towards the significant explanatory power of the model. The out-of-sample prediction was carried out by re-estimating the model for each of the last 48 months of the sample and making a 1-step ahead forecast. The $\mathcal{DM}$ statistics reported show significant evidence of predictability of the model for FF and JY at the 10% significance level. In the case of DM, JY and FF, the $\mathcal{DM}$ statistic has a negative value, meaning that the structural model has more accurate forecasts than the random walk model. Hence, for 2 of the 6 currencies analyzed the in-sample evidence is confirmed even by the out-of-sample test. For DM and BP the out-of-sample results are not significant but the null of linearity can be safely rejected. For the CD there is evidence that the mean reversion to the fundamentals is stronger than for the other currencies.

4.4 Evidence of Predictability?

In Section (4.3) we concluded that the proposed model fits significantly well in-sample but that there is less robust evidence of significant out-of-sample predictive power. This result was already pointed out by Meese and Rogoff (1983) for linear models. More recently, Diebold and Nason (1990) used nonparametric techniques to investigate nonlinear predictability for weekly exchange rates in a time series framework. They found significant in-sample improvements for a wide range of currencies, but out-of-sample the nonparametric method did not
Figure 4.2: Chartists Extrapolation Coefficient

The evolution of $\delta_t$ for the DM exchange rate from 1974.1 to 1994.12 (top plot) and the log exchange rate and PPP fundamental value (bottom plot).

improve significantly over the random walk. They pointed to 3 possible reasons for the lack of out-of-sample improvements: (1) there might be dependence that occurs in even moments that cannot be exploited to improve predictions of the conditional mean, (2) evidence for in-sample nonlinearities could be spuriously caused by outliers and structural shifts and (3) weak nonlinearities are present in the conditional mean but it is difficult to exploit them in prediction at the typically available sample sizes. Further investigation of possible nonlinearities in the data was carried out by Meese and Rose (1991). They regressed nonparametrically the change in the exchange rate on the economic variables, such as interest rates, money growth rates and inflation rates. No significant improvement was found.\(^2\)

On the other hand, the unpredictability of exchange rate returns for short horizons has been challenged by significant evidence of predictability in long-horizon returns. Mark (1995) found evidence that the deviation of the exchange rate from the fundamental value has significant explanatory power for multi-period returns. In addition, a typical finding is that the explanatory power increases with the horizons at which the returns are calculated.

\(^2\)However, out-of-sample predictability was found by Lisi and Medio (1997) using nonparametric techniques for monthly exchange rates returns.
An heuristic analysis of Figure (4.1) suggests that the returns do not seem to be affected by the presence of outliers or heteroscedasticity. Thus, as Diebold and Nason (1990) pointed out, it could be that the interaction of weak nonlinearities and small samples is responsible for the lack of out-of-sample predictability for the exchange rates. The structural model proposed here could be used as a laboratory to investigate this issue. The model incorporates a nonlinearity, due to the switching mechanism in the chartists’ expectations, which should be captured by nonparametric time series methods. But it also has a linear adjustment process to the long-run equilibrium, due to the fundamentalists’ expectations that are thought to be responsible for the long-horizon predictability. It seems natural, therefore, to generate time series from the structural model of the typical sample size available in empirical research and to apply the techniques of Diebold and Nason (1990) and Mark (1995) to investigate the issue of predictability in exchange rates. In this way, we test the ability of these tools to detect dependence and nonlinearity if the true model is the one proposed in Section (4.2).

We simulated 1000 series from the structural model by setting the parameters at the estimated values in Section (4.3) for the different currencies. We assume that the fundamental process follows a driftless random walk process with the variance of the innovations calibrated at the estimated variances of the observed fundamentals.

### 4.4.1 Short-Horizon unpredictability

We use the locally weighted regression (LWR) framework of Cleveland and Devlin (1988). Previous applications of LWR to exchange rates are Diebold and Nason (1990) and Meese and Rose (1991). We assume returns are generated by the following process

\[
r_{t+1} = m(X_t) + \epsilon_t, \tag{4.9}
\]

where \( X_t \) is a vector of lagged values of \( r_t \) and \( \epsilon_t \) is an i.i.d. disturbance term. The LWR method estimates the conditional mean function at the point \( x \), \( m(x) \), by minimizing the following quantity

\[
\sum_{t=1}^{n-1} \{ r_{t+1} - \alpha - \beta(X_t - x) \}^2 K \left( \frac{\| X_t - x \|}{d_k(x)} \right), \tag{4.10}
\]

where \( \hat{m}(x) = \hat{\alpha} \). \( K(\cdot) \) is the tricube kernel defined as

\[
K(u) = \begin{cases} 
(1 - u^3)^3 & \text{for } 0 \leq u < 1 \\
0 & \text{otherwise},
\end{cases}
\]

\( \| \cdot \| \) indicates the euclidean distance and

\[
d_k(x) = \begin{cases} 
\| X_{x|k} - x \| & \text{for } 0 < h \leq 1 \\
\| X_{x|u} - x \| h^{1/\rho} & \text{for } h > 1.
\end{cases}
\]
\(X_{x(k)}\) denotes the \(k\)-th nearest neighbor of \(x\), and \(k\) is the integer part of \(hn\). The bandwidth \(h\) can be interpreted as the parameter that regulates the smoothness of the local linear fit.

Given the evidence in the previous section that nonlinearities occur mainly in the first lag, we set \(X_t = r_{t-1}\). We evaluate the prediction accuracy of the nonparametric regression with the \(D_M\) test statistic and the no-change forecast as an alternative. A negative value of the test statistic means that the nonparametric method has a lower forecasting error than the alternative. Asymptotically, the test statistic is standard normally distributed.

In Table (4.2) we present the results of the nonparametric prediction of the last 4 years of observations for the currencies analyzed in Section (4.3) and for the bandwidth \(h\) that varies from 0.1 to 1. For all the currencies we cannot reject the null hypothesis that the predictions of the LWR method are significantly more accurate than the no-change forecast. For DM and JY, the RMSPE is as low as 0.92 but the improvement is not statistically significant. For DM the best bandwidth is 0.8, which has a \(D_M\) statistic of -1.05, and does slightly better than the model estimated in the previous section, which had a statistic of -0.89. For the JY, the situation is reversed and the structural model improves slightly over the pure time series approach. Both prediction methods are close to a significance level of 10%. For both CD and BP, the conditional model does not improve over the no-change forecast and the structural model does significantly worse than the nonparametric regression. A striking result is obtained for the FF: the time series approach reaches a lower RMSPE of 0.98, which is not significant. Instead, the predictions of the structural model are significant at the 5% significance level. This might be due to the fact that the structural model also accounts for information about the dynamics in the fundamental value in addition to the nonlinear autoregressive mechanism implied by the expectations of the chartists.

These results show that for monthly data there is weak evidence of the predictive ability of nonparametric regression to beat the random walk model. This confirms the results of Diebold and Nason (1990), who used weekly exchange rates.

In Table (4.3) we show the results of the LWR prediction method to time series generated from the structural model at the estimated parameter values. In the table we show the power of the \(D_M\) test, i.e. the frequency of rejection of the null hypothesis of equal prediction accuracy of LWR with respect to the no-change forecast. We simulate 1000 time series of length 300, of which the last 48 observations were predicted.

The striking result in Table (4.3) is that the power is extremely low across bandwidths and currencies. The highest power is reached for the DM: when testing at the 5% significance level the \(D_M\) test rejects 13% of the times and 23% when testing at the 10% significance level. This is not surprising because DM is the currency that showed the highest evidence against the null of linearity in Table (4.1). The other currencies show very low power and in
Table 4.2: Short-Horizon Predictability Test

<table>
<thead>
<tr>
<th>$h$</th>
<th>$\text{DM}$</th>
<th>$\text{JY}$</th>
<th>$\text{CD}$</th>
<th>$\text{FF}$</th>
<th>$\text{BP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{RMSPE}$</td>
<td>$\text{DM}$</td>
<td>$\text{RMSPE}$</td>
<td>$\text{DM}$</td>
<td>$\text{RMSPE}$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.17</td>
<td>1.25</td>
<td>0.96</td>
<td>-0.36</td>
<td>1.07</td>
</tr>
<tr>
<td>0.2</td>
<td>0.98</td>
<td>-0.23</td>
<td>0.93</td>
<td>-0.76</td>
<td>0.99</td>
</tr>
<tr>
<td>0.3</td>
<td>0.95</td>
<td>-0.59</td>
<td>0.93</td>
<td>-0.81</td>
<td>0.98</td>
</tr>
<tr>
<td>0.4</td>
<td>0.94</td>
<td>-0.72</td>
<td>0.92</td>
<td>-0.86</td>
<td>0.97</td>
</tr>
<tr>
<td>0.5</td>
<td>0.93</td>
<td>-0.85</td>
<td>0.92</td>
<td>-0.90</td>
<td>0.96</td>
</tr>
<tr>
<td>0.6</td>
<td>0.93</td>
<td>-0.93</td>
<td>0.92</td>
<td>-0.94</td>
<td>0.96</td>
</tr>
<tr>
<td>0.7</td>
<td>0.92</td>
<td>-0.99</td>
<td>0.92</td>
<td>-0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>0.8</td>
<td>0.93</td>
<td>-1.05</td>
<td>0.92</td>
<td>-1.02</td>
<td>0.96</td>
</tr>
<tr>
<td>0.9</td>
<td>0.94</td>
<td>-0.99</td>
<td>0.91</td>
<td>-1.11</td>
<td>0.96</td>
</tr>
<tr>
<td>1.0</td>
<td>0.98</td>
<td>-0.41</td>
<td>0.89</td>
<td>-1.27</td>
<td>0.98</td>
</tr>
</tbody>
</table>


some cases even lower than the significance level\(^3\).

The conclusion is that the nonlinearity in the structural model is hardly detectable by the nonparametric methods used in the exchange rate literature. This might be consistent with the third explanation offered by Diebold and Nason (1990). The combination of weak nonlinearities and small samples might cause the power of the test to be so low as to make short-term predictability statistically undetectable.

### 4.4.2 Long-Horizon predictability

As previously mentioned, Mark (1995) showed that multi-period returns are consistently more predictable than short period returns. To investigate this property he proposed the following regression

$$r_{t+k} = \alpha_k + \beta_k \text{dev}_t + \epsilon_{t+k}.$$  \hspace{1cm} (4.11)

\(^3\)The fact that the power is lower than the significance level could be because of the finite-sample distortion of the test.
Table 4.3: Power of the Short-Horizon Predictability Test

<table>
<thead>
<tr>
<th>$h$</th>
<th>DM</th>
<th>JY</th>
<th>CD</th>
<th>FF</th>
<th>BP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5%</td>
<td>10%</td>
<td>5%</td>
<td>10%</td>
<td>5%</td>
</tr>
<tr>
<td>0.1</td>
<td>0.06</td>
<td>0.10</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>0.2</td>
<td>0.10</td>
<td>0.16</td>
<td>0.02</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>0.3</td>
<td>0.11</td>
<td>0.19</td>
<td>0.02</td>
<td>0.07</td>
<td>0.02</td>
</tr>
<tr>
<td>0.4</td>
<td>0.12</td>
<td>0.21</td>
<td>0.03</td>
<td>0.08</td>
<td>0.03</td>
</tr>
<tr>
<td>0.5</td>
<td>0.13</td>
<td>0.21</td>
<td>0.03</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>0.6</td>
<td>0.13</td>
<td>0.22</td>
<td>0.03</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>0.7</td>
<td>0.12</td>
<td>0.23</td>
<td>0.03</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>0.8</td>
<td>0.12</td>
<td>0.23</td>
<td>0.03</td>
<td>0.09</td>
<td>0.03</td>
</tr>
<tr>
<td>0.9</td>
<td>0.10</td>
<td>0.21</td>
<td>0.04</td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td>1.0</td>
<td>0.01</td>
<td>0.06</td>
<td>0.02</td>
<td>0.06</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Power of the out-of-sample predictability test against the alternative of the structural model. The simulated time series are of length 300 of which the last 48 are predicted. The test is the DM test for prediction accuracy. The entries indicate the power of the test, i.e., the frequency of rejection of the null hypothesis of equal accuracy of the LW and no-change predictions when the time series are simulated from the structural model at the estimated parameter values of the different currencies.

where the $k$-period return is given by $r_{t+k} = s_{t+k} - s_t$ and $dev_t$ is the deviation of the fundamental value from the exchange rate. If the coefficient $\beta_k$ is significantly different from zero it implies that the observed deviation has explanatory power for the returns over the following $k$ periods. In addition, the economically interesting alternative is $\beta_k > 0$: if the exchange rate is lower (higher) than the fundamental value ($dev_t > 0$) then in the next $k$ periods the exchange rate will appreciate (depreciate). In other words, the exchange rate will mean revert to the long-run equilibrium in the next $k$ periods. The empirical findings in Mark (1995) are that by increasing $k$ the $\beta_k$ increases in magnitude and becomes significantly positive; also the $R^2$ of the regression consistently increases.

In Table (4.4) we show the results for the monthly returns from 1974 to 1998 (we use the full sample in this case). It is clear that by increasing the horizon for most of the currencies.
### Table 4.4: Long-Horizon Predictability Test

<table>
<thead>
<tr>
<th>$k$</th>
<th>DM</th>
<th>JY</th>
<th>CD</th>
<th>FF</th>
<th>BP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\beta_k$</td>
<td>$R^2$</td>
<td>$\beta_k$</td>
<td>$R^2$</td>
<td>$\beta_k$</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.07†</td>
<td>0.05†</td>
<td>0.02</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>8</td>
<td>0.15†</td>
<td>0.11†</td>
<td>0.05</td>
<td>0.04</td>
<td>0.16</td>
</tr>
<tr>
<td>12</td>
<td>0.25†</td>
<td>0.18†</td>
<td>0.09</td>
<td>0.10</td>
<td>0.26†</td>
</tr>
<tr>
<td>16</td>
<td>0.37†</td>
<td>0.25†</td>
<td>0.14</td>
<td>0.16†</td>
<td>0.39†</td>
</tr>
<tr>
<td>20</td>
<td>0.48†</td>
<td>0.32†</td>
<td>0.18</td>
<td>0.23†</td>
<td>0.51†</td>
</tr>
<tr>
<td>24</td>
<td>0.58†</td>
<td>0.39†</td>
<td>0.22</td>
<td>0.32†</td>
<td>0.62†</td>
</tr>
</tbody>
</table>

Estimation results of the long-horizon regression for the full sample of monthly observations. † indicates significance at the 5% level and ‡ at the 10% level of the asymptotic test $H_0: \beta_k = 0$ against the alternative that $\beta_k > 0$. t-values corrected for HCCE with $2(k - 1)$ lags.

$\beta_k$ increases in magnitude and becomes significantly positive. Also the $R^2$ of the regression increases and becomes larger.

We performed a one-sided test of the null hypothesis that $\beta_k = 0$ against the alternative that $\beta_k > 0$ on time series simulated from the structural model. In Table (4.5) we show the frequencies of rejection of the null hypothesis at the 5 and 10% significance levels.

It is clear that the rejections increase with $k$ for all the currencies, indicating that the structural model has the feature of long-horizon predictability observed in empirical research. However, the rejections are too frequent for the short forecast horizons. In Table (4.4) only JY and BP have a $\beta_k$ significant for $k = 1$, whereas this is not the case for the other currencies.

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1. Recently, the statistical validity of this type of regression has been questioned. An underlying assumption in Equation (4.11) is that the exchange rate and the fundamental value are cointegrated so that $derv_t$ is a stationary process. However, there is weak evidence for the cointegration of $s_t$ and $f_t$ for different notions of fundamental value. The implication is that the deviation from the fundamentals, $derv_t$, is non-stationary and the statistical theory to make inference on Equation (4.11) breaks down. Here we ignore this issue. For further details see Berber and van Dijk (1998) and Berkowitz and Giorgianni (2001).
4.5 Conclusion

In this chapter we proposed a simple model of the dynamics of exchange rates inspired by the chartist-fundamentalist approach. We found that combining dependence on fundamental variables and on lagged values of returns is supported by the data. The fundamentalists expect the exchange rate to adjust toward the long-run equilibrium with a typical correction of approximately 3% monthly on the deviation occurred one year ago. In addition to the stabilizing influence of this group, also chartists contribute to correct the mispricing of the exchange rate: we found that when chartists observe an absolute change beyond a threshold (of approximately 2-3%), they expect a reversal in the next period. On the other hand, for absolute returns smaller than the threshold they believe changes will persist. There is also evidence that chartists become more aggressive in extrapolating trends in exchange rates when the deviation from the long-run equilibrium is larger.

The in-sample significance of the model is also supported by evidence of out-of-sample predictability for two of the six currencies analysed. This result might be explained by the fact that the model combines fundamental and non-fundamental factors instead of considering them separately.

However, an important issue is how to rationalize the findings of nonlinearity and predictability with the results of Diebold and Nason (1990). We show that applying nonparametric techniques to predict out-of-sample time series simulated from the structural model has very low power in detecting the dependence in the data. We interpret this as evidence that the existence of weak nonlinearities and small samples are responsible for the unpredictability at short-horizons of exchange rates.
Finally, after Meese and Rogoff (1983) the out-of-sample performance of models has been the benchmark to judge the robustness of the findings of in-sample dependence in exchange rates. However, little attention has been paid to the issue of the statistical properties of the tests used in evaluating the accuracy of out-of-sample predictions and the role of nonlinearities. A priority for future research is to develop inference methods that are robust in small and moderate samples.