Essays in Nonlinear Economic Dynamics
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Chapter 5

Nonlinear Mean Reversion in Stock Prices

5.1 Introduction

Does the stock market rationally reflect fundamental values? The stock price run-up of the late 90s revived the debate about the rationality of stock prices. In 2000 the Price-to-Dividends (PD) ratio for the S&P500 index reached a level of 85 against an historical average of approximately 25. The extreme behaviour compared to historical standards has been explained in different ways.

According to rational explanations, the rapid increase in stock prices reflects changes occurred in fundamental factors. They argue that the required rate of return has lowered significantly because of higher participation of investors to the stock market and changes occurred in consumers preferences. If investors discount future pay-offs at a lower rate, prices will increase. A similar result is obtained when the expected growth rate of dividends or earnings increases. These arguments were proposed by Heaton and Lucas (1999). However, they found that these explanations are not able to account for the large increase of the late 90s.

On the other hand, Campbell and Shiller (2001) argue that changes in fundamental factors are not large enough to explain changes in stock prices. In addition, historical evidence suggests that in periods followed by large collapses of stock prices the valuation ratios never reached such extraordinary levels. An alternative explanation is that prices experience large swings from fundamental valuations due to fads in investors expectations. Summers (1986) suggested that irrational fads create persistent deviations of prices from intrinsic valuations that are difficult to arbitrage away by rational investors. According to this approach, a combination of irrational expectations of some investors and limits to the arbitraging activities
of rational investors explains the deviations of stock prices from rational valuations. This view is also consistent with the empirical evidence of mean reversion and long-run predictability of stock prices. If the stock price reverts (in the long-run) back to its intrinsic value, a positive (negative) deviation predicts that prices will decrease (increase). Hence, the adjustment process creates a negative relation between the changes in prices and the deviation from the fundamentals that emerges at long horizons. Some theoretical models that try to capture this idea are DeLong et al. (1990a) and Brock and Hommes (1998).

In this chapter we investigate the role that fundamental factors played in the recent increase of stock prices. In particular, we use a dynamic version of the Present Value Model (PVM) that allows for time variation in the discount rate and the growth rate of dividends. The analysis of more than a century of the S&P500 index shows that the fundamental factors fail to explain the persistence of the deviations from intrinsic valuations, in particular in the late 90s. Shocks to the growth rate of dividends or to proxies for the discount rate, such as interest rates and returns volatility, die out very quickly compared to shocks to the stock prices. This indicates that the excessive persistence of the deviations from fundamentals could be caused by the overreaction of investors to fundamental news: they expect the effects of positive (negative) news about the fundamentals to be more persistent than it is rational. This evidence is consistent with the explanation of Summers (1986) that assumes that deviations follow a persistent AR(1) process. Recently, there has been a growing interest in modelling deviations of asset prices from intrinsic valuations using nonlinear models. A common result is that asset prices can be characterized as switching between two regimes: when deviations are small they follow a random walk process but when they are large they follow a stable AR process that contributes to the reversion of the price toward the fundamentals. Some studies along these lines are Gallagher and Taylor (2001) for stock prices and Kilian and Taylor (2003) and Taylor and Peel (2000) for exchange rates.

We investigate the issue of nonlinear mean reversion for yearly observations of the S&P500 index from 1871 until 2001. Estimation results for stock price data up to 1990 show that there is evidence for nonlinearity in the mean reversion process. In particular, when the price is close to the intrinsic value the deviations are very persistent and mean reversion is weak; however, when deviations are large the speed of adjustment increases and the price reverts back toward the fundamental value. The results suggest that in the mean reverting regime the half-life of a shock is approximately 3 years. When the 90s are included in the sample, there is strong evidence of nonlinearity in the transitory component. The estimation results indicate that the pattern of mean reversion has changed compared to the previous findings. Both close and far from the long-run equilibrium deviations are very persistent. So, there is no evidence that the speed of mean reversion becomes stronger for large deviations. We interpret these results as evidence that the extreme behaviour of prices in the 90s exacerbated
the persistence of the mean reversion process. Before the 90s, when a fad was driving the stock price away from the fundamentals, stabilizing forces were activated to weaken the persistence of the process. However, in the 90s the persistence became stronger and drove the PD ratio to unprecedented levels.

The chapter is organized as follows: section (5.2) introduces different notions of fundamental values used for empirical investigation. Section (5.3) describes the nonlinear model used for the deviations of stock prices from fundamentals. Section (5.4) discusses the estimation results and the evidence in support of the hypothesis of nonlinear mean reversion. Finally, section (5.5) concludes.

5.2 Fundamentals

A standard approach in asset valuation is to assume that the stock price satisfies

$$P_t = E_t \left[ \frac{1}{1 + r_{t+1}} (P_{t+1} + D_{t+1}) \right],$$

(5.1)

where $P_t$ is the price of the stock at the end of period $t$, $D_{t-1}$ is the stock dividend paid during period $(t+1)$ and $r_{t+1}$ is the required rate of return at time $(t+1)$. $E_t(\cdot)$ indicates the expectation conditional upon information available at time $t$. Solving Equation (5.1) forward for $T$ periods and applying the law of iterated expectations, we obtain

$$P_t = E_t \left[ \sum_{j=1}^{T} \left( \prod_{i=1}^{j} \frac{1}{1 + r_{t+i}} \right) D_{t+i} \right] + E_t \left[ \prod_{j=1}^{T} \frac{1}{1 + r_{t+i}} P_{t+T} \right].$$

(5.2)

The present value of holding the asset for $T$ periods is equal to the expected discounted value of its cash flows and the expected discounted value of the resale price. A typical assumption to rule out the occurrence of bubbles is

$$\lim_{T \to \infty} E_t \left[ \prod_{j=1}^{T} \frac{1}{1 + r_{t+i}} P_{t+T} \right] = 0,$$

(5.3)

called the transversality condition. This implies that by holding the asset in the infinite future, the price is equal to the expected discounted value of its future cash flows

$$P_t^* = E_t \left[ \sum_{j=1}^{\infty} \left( \prod_{i=1}^{j} \frac{1}{1 + r_{t+i}} \right) D_{t+i} \right].$$

(5.4)

where we indicate $P_t^*$ as the fundamental value. We define the growth rate of the dividend process $g_t$ as $D_{t+1} = (1 + g_{t+1}) D_t$, so that the fundamental value is given by

$$P_t^* = E_t \left[ \sum_{j=1}^{\infty} \left( \prod_{i=1}^{j} \frac{1 + g_{t+i}}{1 + r_{t+i}} \right) D_t \right].$$

(5.5)
The time variation of $g_t$ and $r_t$ and the nonlinearity in the pricing equation complicate the derivation of analytically tractable formulas. One approach to simplify the problem consists of assuming that the dividends growth rate and the required return are constant and equal to $g$ and $r$ (with $r > g$), respectively. Under these assumptions, Equation (5.5) implies that

$$P^*_t = mD_t. \quad (5.6)$$

where $m = (1 + g)/(r - g)$. The stock price at time $t$ is given by the cash flow times a multiple that depends on the ex-ante rate of return and the growth rate of dividends. This model is also known as the Gordon valuation formula and has recently been used by Heaton and Lucas (1999) to determine the rational valuation of stock prices and by Fama and French (2002) to evaluate the size of the risk premium. The model is very simple and makes some clear predictions about the behaviour of prices: prices will increase if $r$ is lowered, that is if investors discount at a lower rate future cash flows, or if $g$ increases, that is if dividends are expected to grow at a faster rate. Another implication of the model is that the Price-Dividend (PD) ratio should be constant over time.

However, the assumption that the dividend growth rate and the expected returns are constant seems unrealistic. It is possible to allow for time variation by following the approach of Poterba and Summers (1986). They approximate the pricing formula given in (5.5) by a first-order Taylor expansion around the mean of the required return, $r$, and the mean of the growth rate, $g$.

$$P^*_t \approx E_t \left[ \sum_{j=1}^{\infty} \left( \frac{1 + g}{1 + r} \right)^j + \frac{\partial P^*_t}{\partial r_{t+j}} \mid_r (r_{t+j} - r) + \frac{\partial P^*_t}{\partial g_{t+j}} \mid_r (g_{t+j} - g) \right] D_t \quad (5.7)$$

where the partial derivatives are given by

$$\frac{\partial P^*_t}{\partial r_{t+j}} \mid_r = \frac{D_t}{r - g} \cdot \beta^j. \quad (5.8)$$

$$\frac{\partial P^*_t}{\partial g_{t+j}} \mid_r = \frac{(1 + r)D_t}{(1 + g)(r - g)} \cdot \beta^j. \quad (5.9)$$

and $\beta = (1 + g)/(1 + r)$. Substituting the derivatives into Equation (5.7), we get

$$P^*_t = \left\{ \frac{1 + g}{r - g} - \frac{1}{(r - g)} E_t \left[ \sum_{j=1}^{\infty} \beta^j(r_{t+j} - r) \right] + \frac{1 + r}{(1 + g)(r - g)} E_t \left[ \sum_{j=1}^{\infty} \beta^j(g_{t+j} - g) \right] \right\} D_t. \quad (5.10)$$

In the pricing formula there are still the expectations of investors about future ex-ante returns and dividends growth rates. A typical assumption made in the literature is that the processes driving the required return and the dividend growth rate are AR(1), that is

$$E_t(r_{t+j} - r) = \rho^j(r_t - r) \quad (5.11)$$
\[ E_t(g_{t+j} - g) = \sigma^j (g_t - g). \]  
(5.12)

and the approximated pricing formula in Equation (5.10) becomes
\[ P^*_t = m_t D_t, \]  
(5.13)

where \( m_t \) is the time-varying multiplier given by
\[
m_t = \left\{ \frac{1 + g}{r - g} \left[ \frac{\rho(1 + g)}{(r - g)(1 + r - \rho(1 + g))} (r_t - r) + \frac{\sigma(1 + r)}{(r - g)(1 + r - \sigma(1 + g))} (g_t - g) \right] \right\}. \]  
(5.14)

This version of the fundamental value is known in the literature as the dynamic Gordon model because it expresses asset prices as a time-varying multiplier of the dividends. The multiplier in Equation (5.14) has a straightforward interpretation: if the required rate of return and the growth rate of dividends are constant and equal to their mean then it collapses to the static multiplier of Equation (5.6); however, time variations in the required rate of return and/or in the dividend growth rate change the level of the multiplier. The response of prices to changes in \( r_t \) and \( g_t \) is similar to the case of the static Gordon: if investors require at time \( t \) a return higher (lower) than the average \( r \), this will decrease (increase) the multiplier and consequently prices. On the other hand, if dividends grow at a higher (lower) rate at time \( t \), this will increase (decrease) the multiplier and will affect positively (negatively) stock prices. Equation (5.14) shows that the multiplier depends also on the AR coefficients in the expectations of the required return and the dividend growth rate. High \( \rho \) and \( \sigma \) imply that shocks to \( g_t \) and \( r_t \) will have a persistent effect on the multiplier and on prices. Analogously to the static case, the multiplier can be interpreted as the PD ratio: in this case the forcing variables, ex-ante returns and dividend growth rate, determine the dynamics of the ratio. The required rate of return is unobserved and many variables have been used as proxies. Campbell and Shiller (1989) used different notions of required returns: the risk-free interest rate plus a constant risk premium, the expected growth of real consumption times the coefficient of relative risk aversion plus a constant risk premium and another version in which the risk-free rate is constant and the risk premium is given by the conditional volatility of stock returns times the coefficient of relative risk aversion.

The extension to the dynamic Gordon model takes into account the possibility that time variation in interest rates, risk premia or growth rates could explain the large deviations of the PD ratio from its mean. The top plot in Figure (5.1) shows the PD ratio for yearly data from 1871 to 2001 of the S&P500 index\footnote{The dataset used is described in Shiller (1989). It consists of yearly observations of the price and dividends for the S&P500 Composite Stock Price Index from 1871 until 2001. We deflated the series by CPI index. The interest rate used is the return on four to six months commercial paper.}.
Figure 5.1: PD ratio

(a) Price-Dividend ratio for the S&P500 Composite Index from 1871 to 2001. The line indicates the average PD ratio of 25.78. (b) Log of the stock price and the static Gordon fundamental value. The multiplier is obtained by assuming $g = 0.018$ and $r = 0.057$.

It is clear that the static Gordon model is rejected by the large and persistent deviations of the ratio from its mean. It is also striking how the PD ratio increased during the 90s: while it has historically oscillated between approximately 10 to 35, after 1995 it exceeded this range to reach levels as high as 85. This is also apparent in the bottom plot of Figure (5.1) that shows the log of the real stock price and the log of the fundamental value.

It makes then sense to use the dynamic version of the Gordon formula in order to explain the large deviations by changes occurred in fundamentals. Figure (5.2) shows the time series properties of the dividend growth rate, the real riskless interest rate and the yearly volatility measured by the average squared monthly returns. The autocorrelation plots show that at yearly frequency only the interest rate has some significant linear dependence whereas both the growth rate of dividends and the volatility of the stock returns have no significant dependence. In addition, the autocorrelation in the riskless rate is quite small to be able to explain the large deviations of the stock price from the fundamental price. The last column of Figure (5.2) depicts the multiplier (equivalent to the PD ratio) in Equation (5.14) when the dividend growth rate or the required rate are allowed to vary. We follow the approach of Campbell and Shiller (1989) and use the risk-free rate (plus a constant risk premium) and the stock return volatility as proxies for ex-ante returns. In all cases, the multipliers do not have the persistence and variability displayed by the PD ratio in Figure (5.1).

The results of Campbell and Shiller (1989) and the evidence discussed here suggest that
Figure 5.2: Fundamental Factors

Plot of the time series (left), autocorrelation function (center) and multiplier as given in Equation (5.14) when the dynamic Gordon model includes time varying dividends growth rates, real interest rate and stock return volatility.

the fundamental factors should have high persistence to explain stock prices. Barsky and DeLong (1993) assume that prices are formed according to Equation (5.6) with the dividend growth rate following an ARIMA(0,1,1). This process contains a unit root and gives more persistence to the warranted fundamental value. However, there is no empirical evidence to support the assumption of a unit root in the dividend growth rate. Bansal and Lundblad (2002) provide evidence that at monthly frequency an ARMA(1,1) process has quite large AR and MA coefficients. However, Figure (5.2) suggests that for the yearly data analyzed here there is no evidence of statistical significance of an ARMA specification.

These results point to the fact that fundamental factors are not able to give a full account of the dynamics of stock prices. However, the failure of the rational valuation could be caused by misspecification of the fundamental process. This issue has been investigated by Donaldson and Kamstra (1996) in order to give a rational explanation for the bubble occurred in 1929. They used monthly data and simulated paths from Equation (5.5) assuming that the discount factor included a nonlinear component and ARCH innovations. In this way they allow for time variation in the fundamental factors without relying on the approximated pricing formula. They found that the stock price run-up and crash of 1929 was not caused by a bubble but it could be rationalized by considering nonlinear effects and heteroscedasticity in discount
factors. However, the results are based on the findings of significant linear structure in the monthly growth rate of dividends. As is clear from Figure (5.2), the yearly data do not show significant evidence of ARMA dependence. Hence, it is unlikely that their method performs successfully on the data analyzed here.

In the literature, there are two alternative interpretations to explain this failure: rational bubbles and irrational fads. The deviations are called rational bubbles when they are consistent with Equation (5.1), while irrational fads are not. In both cases, prices are decomposed into a permanent (or fundamental) and a transitory (or non-fundamental) component

\[ P_t = P^*_t + X_t, \]  

(5.15)

where \( P^*_t \) is as in Equation (5.5) or (5.10). To be consistent with a rational bubble model \( X_t \) has to satisfy the condition

\[ X_t = (1 + r)^{-1}E_t(X_{t+1}). \]  

(5.16)

where to simplify the notation we focus on the static Gordon model. This is a more general solution to Equation (5.2) because it does not satisfy the transversality condition. The characteristic of a bubble is that it grows indefinitely at rate \((1 + r) > 1\). A rational bubble always explodes, so that it cannot be an empirically relevant dynamic model for stock prices. Blanchard and Watson (1982) proposed a model in which the bubble switches between two states, one in which the bubble survives \((X_t > 0)\) with probability \(q\) and one in which it collapses \((X_t = 0)\) with probability \(1 - q\). Further refinement is the periodically bursting bubbles model proposed by Evans (1991). In this model, the bubble grows faster than the rate \((1 + r)\) if \(X_t\) is below a positive threshold while beyond it has a positive probability to burst. Many tests for the existence of bubbles in asset prices or exchange rates were proposed. For a survey see Flood and Hodrick (1990). The conclusion is that there is no evidence to support the existence of rational bubbles in asset prices.

An alternative explanation for the transitory component \(X_t\) is that it represents an irrational fad in investors sentiment that causes temporary deviations from fundamental valuations. This approach has been proposed by Summers (1986) and Poterba and Summers (1988). The assumption that \(X_t\) is a persistent stationary process is consistent with the evidence of mean reversion and long-term predictability in stock prices. Mean reversion was investigated mainly by using variance ratio tests. They showed that stock prices do not follow a random walk because the variance of returns over \(k\) periods is lower than \(k\) times the variance of one period return. In addition, long-horizon returns are negatively related to measures of deviations from the fundamentals, such as the PD ratio.

However, few attempts have been made to explicitly model the transitory component and investigate the possibility that it evolves in a nonlinear fashion. In the next section
we introduce a simple nonlinear model to explain the time variation in the mean reversion process.

5.3 Nonlinear Dynamics

A simple form of nonlinearity consists of assuming that a smooth nonlinear function regulates the switching between 2 linear regimes. This introduces time-variation in the coefficients of the linear AR model. The model is called STAR (Smooth Transition AR)\(^2\) and assumes that the deviations from the fundamentals scaled by the dividends, \(x_t\), evolves as

\[
x_t = \left\{ \phi_1' G_t(s_t, \gamma, c) + \phi_2' [1 - G_t(s_t, \gamma, c)] \right\} X_{t-1} + \epsilon_t
\]

(5.17)

where \(X_{t-1} = (1, x_{t-1}, ..., x_{t-p})'\) and the disturbance term \(\epsilon_t\) is i.i.d. with constant variance \(\sigma^2\). \(G_t(s_t, \gamma, c)\) is the function that regulates the transition from the first regime, with coefficient vector \(\phi_1\), to the second regime, where the dynamics evolves according to \(\phi_2\). \(s_t\) is the variable that determines the switch between regimes. In the application in the next section we use \(s_t = x_{t-d}\) for \(d \geq 1\). Two common choices of \(G_t(s_t, \gamma, c)\) are the logistic and the exponential function. The logistic version of the STAR model (called in the literature LSTAR) has transition function

\[
G_t(s_t, \gamma, c) = \left\{ 1 + \exp[-\gamma(s_t - c)] \right\}^{-1},
\]

(5.18)

where \(\gamma > 0\) determines the speed of transition and the threshold \(c\) determines the regime that is active. The logistic function varies smoothly from 0 to 1 as the transition variable, \(s_t\), becomes increasingly larger than the threshold \(c\). The other common choice for the transition function is the exponential, given by

\[
G_t(s_t, \gamma, c) = 1 - \exp[-\gamma(s_t - c)^2]
\]

(5.19)

and this version of the STAR model is called ESTAR. In this case the transition function smoothly approaches 1, the further \(s_t\) deviates (in either directions) from the threshold value \(c\).

These transition functions imply different dynamics for the process of mean reversion: the logistic implies an asymmetric adjustment if \(s_t\) is above or below the threshold \(c\). In contrast, the exponential implies a symmetric adjustment in both directions of the deviation. In other words, when using the logistic function we assume that negative and positive deviations revert back to the fundamental at different speeds, whereas using the exponential the speed

\(^2\)We largely simplify the discussion of STAR models according to the application at hand. For a more detailed discussion of this family of models see Teräsvirta (1994) and van Dijk et al. (2002).
of mean reversion is equal for negative and positive deviations. The choice of the transition function is a crucial issue for the interpretation of the results. We will test which type of transition seems to accommodate better the dynamics in the deviations of stock prices from the fundamental value.

The null hypothesis of linearity against STAR holds if either $H_0 : \phi_1 = \phi_2$ or $H_0 : \gamma = 0$. As discussed more extensively in Terasvirta (1994), under both null hypotheses the test statistics are affected by the presence of nuisance parameters that complicate the derivation of the asymptotic distribution. In order to overcome this identification problem, Luukkonen et al. (1988) proposed to approximate the transition function $G(t,s_1,\gamma,c)$ with a Taylor-expansion around $\gamma = 0$. This allows to derive an LM type statistic with a standard $\chi^2$ distribution. A 2nd order Taylor-series expansion of the exponential transition function around $\gamma = 0$, leads to the auxiliary regression

$$x_t = \beta_1 X_{t-1} + \beta_2 \tilde{X}_{t-1,1} + \beta_3 \tilde{X}_{t-1,2} + \beta_4 \tilde{X}_{t-1,3} + \epsilon_t$$

(5.20)

where $\tilde{X}_{t-1} = (x_{t-1},...,x_{t-\mu})'$ and the $\beta_j$ are reparametrizations of the vector of parameters $(\phi_1',\phi_2',\gamma,c)'$. The null hypothesis that $\gamma = 0$ against ESTAR corresponds to test that $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Similarly, a 3rd order expansion of the logistic function involves only the first four elements of the RHS of Equation (5.20) and the null of linearity can be tested as $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. The artificial regression in Equation (5.20) could also be used to guide the specification of the transition function. The reparametrizations of the expansion of the logistic function imply that the null holds if $\beta_1 = 0$ and $\beta_2 = 0$, whereas the expansion of the exponential function under the null involves only the second order term, that is, $\beta_2 = \beta_3 = 0$. We can design the following null hypotheses in order to test for evidence of STAR dynamics and the type of transition function that is more appropriate. The null hypotheses are

$$\text{(LM}_1) \quad H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

$$\text{(LM}_3) \quad H_0 : \beta_1 = \beta_2 = \beta_3 = 0|\beta_4 = 0$$

$$\text{(H}_0L) \quad H_0 : \beta_1 = \beta_3 = 0$$

$$\text{(H}_0E) \quad H_0 : \beta_2 = \beta_3 = 0$$

$\text{LM}_4$ and $\text{LM}_4$ are used as general tests of linearity against STAR dynamics. Instead, rejection of $\text{H}_0L$ suggests that a logistic transition should be preferred while rejection of $\text{H}_0E$ points to an exponential specification. The testing procedure is conditional on the lag $d$ used for the transition variable. By testing for different values of $d$, the tests are also useful in the selection of the optimal lag for the transition variable. In order to robustify inference in small samples we will use the $F$-version of the tests. A relevant issue in the implementation
of these models is the choice of $p$, the order of the AR regimes. We follow the approach of Teräsvirta (1994) by looking at the PACF and the order selected by AIC.

### 5.4 Estimation Results

We use the static Gordon valuation as our notion of fundamental value. From Figure (5.1) it is clear that the dynamic version of the Gordon model does not improve significantly with respect to the static version. We estimate the STAR model to the deviations of the price from the fundamental value scaled by the dividends, that is we define

$$x_t = \frac{X_t}{D_t} = \frac{P_t - P_t^*}{D_t}.$$  \hspace{1cm} (5.21)

Using the static Gordon model, $x_t$ is equivalent to the deviation of the PD ratio from the multiplier $m$. In what follows we analyze the time series both in the full sample and in the sub-sample from 1871 to 1990. This seems a natural choice because the late 90s might have changed dramatically the time series properties and the mean reversion dynamics of the deviations from the fundamentals.

Figure (5.3) shows the time series $x_t$ and the PACF. In the subperiod, the PACF suggests that there is dependence up to lag 3. This is also confirmed by the AIC selection criterion. However, in the full sample the dependence in the third lag is not significant and $p=2$ seems appropriate.

First, we tested for linearity of the time series of the transitory component against a STAR alternative. The p-values of the tests described in the previous section are given in Table (5.1).

In the sample up to 1990 the $LM_3$ and $LM_4$ tests reject at 5% significance level the null hypothesis of linearity. For both tests the rejection occurs in the 4th lag of the transition variable. The rejection for $LM_4$ is stronger than for the other test and this could support the choice of the exponential as transition function. More insights about the specification of the transition function come from the $H_{0,L}$ and $H_{0,E}$ tests. For both tests we reject the null of linearity: for $H_{0,L}$ the third and seventh lag have p-value 0.02 and for $H_{0,E}$ the lowest p-value is 0.04 in the fourth lag. Thus, contrary to the previous more general tests, the rejections favour the logistic specification.

The pattern of rejections changes dramatically when the sample is extended to include the last 10 years. Both $LM_4$ and $LM_4$ reject for $d$ up to lag 8. The test for logistic transition, $H_{0,L}$, rejects strongly from the second lag up to the fifth and also in the eight and tenth lag. Instead, $H_{0,E}$ rejects on the forth lag and in the seventh lag. These results might be explained by the run-up of the late nineties that attributes a higher weight to one tail of the distribution and gives more support to a logistic specification. As discussed in detail in
Figure 5.3: Deviations from the Fundamental

Time series of $x_t$, the deviation from the fundamentals scaled by the dividends. It can also be interpreted as the deviation of the PD ratio from its multiplier. The bottom plots are the PACF up to lag 10.

Teräsvirta (1994), LSTAR and ESTAR are to some extent substitutes. This might happen when an ESTAR model has most of the observations lying in one side of the threshold such that it can be reasonably approximated by an LSTAR specification.

The tests for linearity suggest that there is evidence to reject the null hypothesis both in the period 1871-1990 and in the full sample until 2001. The evidence about the specification of the transition function is mixed: there seems to be support for both specifications up to 1990 while the full sample favours more clearly the logistic function. Given the mixed evidence of the tests, we chose the best specification by estimating and selecting the models based on the AIC selection criteria for the period 1871-1990. The best model is an ESTAR specification with $d = 4$ as was also found by the tests. This result suggests that there is no evidence of asymmetry in the adjustment process of the stock price toward its long-run equilibrium. This confirms also the evidence of Kilian and Taylor (2003) and Taylor and Peel (2000) that estimated ESTAR models to exchange rates. However, the late 90s can be interpreted as evidence that positive deviations might have become more persistent than negative. This issue is relevant but it is still premature because it might be necessary to observe the evolutions of stock prices in the following years to conclude that there has been a break in the symmetry of the adjustment process.
Table 5.1: Linearity Tests

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<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.55</td>
</tr>
<tr>
<td>4</td>
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<td>0.01</td>
<td>0.08</td>
<td>0.04</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.04</td>
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<td>0.57</td>
<td>0.41</td>
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<td>0.36</td>
<td>0.92</td>
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<td>0.18</td>
<td>0.52</td>
</tr>
<tr>
<td>7</td>
<td>0.12</td>
<td>0.08</td>
<td>0.02</td>
<td>0.27</td>
<td>0.01</td>
<td>0.00</td>
<td>0.11</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.92</td>
<td>0.92</td>
<td>0.90</td>
<td>0.79</td>
<td>0.01</td>
<td>0.00</td>
<td>0.00</td>
<td>0.20</td>
</tr>
<tr>
<td>9</td>
<td>0.50</td>
<td>0.68</td>
<td>0.60</td>
<td>0.65</td>
<td>0.03</td>
<td>0.10</td>
<td>0.07</td>
<td>0.42</td>
</tr>
<tr>
<td>10</td>
<td>0.07</td>
<td>0.09</td>
<td>0.27</td>
<td>0.06</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
</tr>
</tbody>
</table>

The tests are as in Section (5.3) and are applied to $x_t$, the deviations of stock prices from the static Gordon fundamental value. The autoregressive order, $p$, was set to 3. The tests are implemented as F-tests. In bold p-values smaller than 5% significance level.

We use the same specification both in the estimation for the subsample and in the full sample. In this way we can interpret the changes that might occur in the estimation results. The estimation results are shown in Table (5.2). We performed a grid search for γ and c to initialize the NLLS estimation procedure. When a coefficient was not significant we dropped it from the regression and fitted the reduced model.

In the sample 1871-1990 the estimated coefficients are statistically significant and the residuals diagnostics proposed in Eitrheim and Teräsvirta (1996) do not show significant model misspecification. The mid-regime is characterized by an AR(3) with modulus 0.962, so that it is very persistent and close to having a unit root. The outer regime is a stationary process with dependence only in the first lag. The estimated model suggests that the dynamics of the deviations from the fundamentals is characterized by a very persistent process in the inner regime that drives the price away from the fundamental value; when the deviations get large the outer regime is activated and the process mean revert to the fundamental value.
Table 5.2: ESTAR Estimation

<table>
<thead>
<tr>
<th></th>
<th>1871-1990</th>
<th></th>
<th>1871-2001</th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$G(\cdot) = 0$</td>
<td>$G(\cdot) = 1$</td>
<td>$G(\cdot) = 0$</td>
<td>$G(\cdot) = 1$</td>
</tr>
<tr>
<td>$\phi_{0,t}$</td>
<td>-1.053</td>
<td>2.265</td>
<td>-1.152</td>
<td>3.033</td>
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<tr>
<td></td>
<td>[-2.475]</td>
<td>[1.968]</td>
<td>[-2.588]</td>
<td>[2.064]</td>
</tr>
<tr>
<td>$\phi_{1,t}$</td>
<td>0.712</td>
<td>0.794</td>
<td>0.738</td>
<td>0.989</td>
</tr>
<tr>
<td></td>
<td>[5.143]</td>
<td>[4.869]</td>
<td>[3.956]</td>
<td>[3.351]</td>
</tr>
<tr>
<td>$\phi_{2,t}$</td>
<td>-0.311</td>
<td></td>
<td>-0.308</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[-2.435]</td>
<td></td>
<td>[-1.977]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{3,t}$</td>
<td>0.531</td>
<td></td>
<td>0.52</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[4.758]</td>
<td></td>
<td>[3.137]</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.35</td>
<td></td>
<td>2.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[3.394]</td>
<td></td>
<td>[2.128]</td>
<td></td>
</tr>
<tr>
<td>$c$</td>
<td>2.87</td>
<td></td>
<td>-1.421</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[2.03]</td>
<td></td>
<td>[2.21]</td>
<td></td>
</tr>
</tbody>
</table>

1871-1990: $R^2 = 0.619$, AIC = 2.637, $\sigma = 3.645$, AR(1) = 0.34.
$\text{AR}(4) = 0.565$, $\text{LM}_{2} = 0.786$, $\text{LM}_{P} = 0.892$

1871-2001: $R^2 = 0.881$, AIC = 2.929, $\sigma = 4.228$, AR(1) = 0.11.
$\text{AR}(4) = 0.21$, $\text{LM}_{2} = 0.01$, $\text{LM}_{P} = 0.16$

Estimation results of Equation (5.17) with transition function given in Equation (5.19). The sample periods are 1871-1990 and 1871-2001; the t-values in parenthesis are obtained by Newey-West variance-covariance estimator. The adequacy tests are as in Eitrheim and Teräsvirta (1996): AR(p) is a test for residual serial independence, LM$_{2}$ test for no remaining nonlinearity and LM$_{P}$ for parameter constancy.
The outer regime has an AR(1) coefficient of 0.794. When this regime is completely active the half-life of a shock is approximately 3 years. It is interesting to analyze the behaviour of the transition function $G_t(x_{t-4}, \gamma, c)$ in Figure (5.4). The top plot shows the evolution over time of $G_t(\cdot)$. It is clear that there are wide variations in the transition function but the external regime is never completely active. This is due to the small estimated value of $\gamma$ that implies a very smooth transition also clear from the bottom plot in Figure (5.4). This description of the process seems consistent with the theoretical model of a fad. However, a new result is that the speed of mean reversion of the transitory component varies over time and depends on the magnitude of the deviation. This result captures the fact that investors beliefs might be very aggressive at the early stage of a fad while becoming increasingly concerned of the irrational mispricing when deviations become large. This confirms the evidence found in the exchange rate literature by Kilian and Taylor (2003) that show that a linear mean reverting process is not consistent with the findings of long-horizon predictability.

When the same specification is estimated to the sample 1871-2001 the $LM_{NL}$ adequacy test rejects the null hypothesis of no further nonlinearity in the data. This is to expect given the pattern of strong rejections in the linearity tests and the fact that the model was selected on the shorter sample. The tests for residuals autocorrelation do not reject...
at 10% significance level but they are much lower than in the shorter sample. Also the test for parameter constancy, $LM_{PC}$, has a much lower $p$-value. The results for the mid-regime confirm the previous findings. The coefficients of the AR(3) process have very similar magnitude and the modulus is 0.971. However, the estimated coefficients of the outer regime is 0.989 while before it was 0.794. The interpretation of this result is clear: to accommodate for the price behaviour after 1995 the model has to allow for more persistence in the outer regime. Figure (5.4) shows the transition function for the full sample estimation. It is quite similar to the plot in the shorter sample. However, the upper limit of 1 is reached in the last years of the sample, pointing to the fact that the increased persistence tries to accommodate the abrupt increase in stock prices. These results suggest that the outer regime

Figure 5.5: Transition Function: 1871-2001

![Transition function $G_t(x_{t-1}, \gamma, c)$ plotted in time and against $x_{t-1}$.](image)

has lost its stabilizing interpretation and it contributes also to exacerbate deviations from the fundamentals. The fact that we found in both regimes two processes very close to have a unit root suggest that the PD ratio is now more persistent than in the subsample period. In terms of the fads model, these results can be interpreted as evidence that the mean reversion is much slower than previously found and that the increased speed of adjustment when deviations are large is significantly weakened. This can also be interpreted as evidence that the findings of long-horizon predictability such as in Fama and French (1988) and Poterba and Summers...
(1988) will probably emerge at longer horizons than the typical 3-4 years.

5.5 Conclusion

It is a well documented fact that rational valuation models are not able to account for the dynamics of stock prices that are too volatile and take long swings away from intrinsic valuations. As we showed in this chapter, allowing for time variation in the discount rate and in the dividends growth rate does not improve significantly the explanatory power of the PVM presented in Section (5.2). The deviations of stock prices from the fundamental value are much more persistent than warranted by the factors that are assumed to determine the asset price dynamics.

An explanation proposed by Summers (1986) is that stock prices contain a temporary component associated with the sentiment of investors. When investors observe positive (negative) news about the fundamentals of an asset they expect the effect on the stock price to be more persistent than it is rational. This implies that shocks to stock prices are more persistent than warranted by shocks to fundamental factors as it is clear from Figure (5.2). In Summers (1986) and Poterba and Summers (1988) it is assumed that the transitory component follow a persistent AR process while the fundamental value evolves according to a random walk. This model implies that stock returns have small negative autocorrelations at short-horizons while they become large and negative at long-horizons. In other words, they display the same type of mean reversion that was found for various assets, such as stocks and exchange rates.

In this chapter we show that the assumption of a linear process for the deviations from the fundamentals is inappropriate. The transitory component is better explained by a nonlinear model that behaves like a unit root process when prices are close to the intrinsic value and follow a stable AR process when the deviations are large. This model implies that the speed at which stock prices revert towards the fundamentals is higher when deviations are large. This could be the result of the arbitraging activities of smart investors that act to correct the mispricing of a stock. When the stock price is farther away from the fundamentals, they will act more aggressively to correct the deviation that will cause the adjustment toward the mean.

However, this explanation seems not appropriate to explain the rapid stock price run-up of the late 90s. The stabilizing role of the outer regime has significantly lowered and there is weak evidence of mean reversion. After 1996, instead of experiencing fast adjustment toward the mean the stock price continued to deviate from the intrinsic value until 2000 when it started to correct downward. This fact is at odd with the previous interpretation of the role of rational arbitrageurs that cause the stock price to revert back. Probably, in the late 90s
the irrational expectations of a majority of investors about the persistence of stock prices prevailed on the stabilizing role of rational agents and drove the transitory component to unprecedented levels. A situation that could be probably associated with a bubble in stock prices.