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Chapter 6

Mean Reversion, Bubbles and Beliefs Heterogeneity in Stock Prices

6.1 Introduction

In the late 90s, commonly used valuation ratios (such as the Price-to-Dividends (PD) and Price-to-Earnings (PE)) were higher than the levels reached during periods that anticipated stock market crashes. The euphoria for the stock market was driven by the surge of interest for the internet sector. In the late 90s companies in this sector had negative earnings and still reached extreme valuations. Detailed empirical evidence is given in Ofek and Richardson (2002). They estimated that in 1999 the internet sector had a PE ratio of 605. This should be compared with a ratio of 45 for the aggregate stock market and an historical peak (before the 90s) of around 25. This implies that investors had very optimistic expectations about the earnings growth of the sector and were willing to pay extreme prices to hold stocks.

A rational explanation of the high stock market valuations is that the fundamental factors that determine stock prices were all very positive. The state of the economy was good, companies profitability was increasing, technological factors (such as internet) boosted the productivity of the economy and the inflation was low and under control. All factors justified a higher level of stock prices. However, the valuations of the late 90s were extreme and to many they appeared only partially explained by fundamental factors. An alternative explanation is that investors had irrational expectations. They bought stocks for non-fundamental reasons, such as optimism about the growth of the economy or just for speculative purposes.

Evidence supporting an irrational component in investors expectations was e.g. given by Shiller (1989). He did a questionnaire survey of individual and institutional investors after
the one-day drop of 22.6% in October 1987. One of the findings is that investors did not trade on negative fundamental news about the economy but reacted to the price change itself. In addition, in the months before the crash many investors were buying stocks although they perceived the market to be overvalued. The survey indicates that 93% of the institutional investors that were net buyers prior to the crash were aware of the overvaluation. They also declared to expect the overvaluation to be corrected in the following 3 to 6 months. On the other hand, during the day of the crash they were very anxious because the overvaluation was likely to explain the drop that was occurring in stock prices. Further evidence is provided by Bauge (2000). She used survey data on small investors to investigate their expectations and asset holdings. She found that small investors increase their stocks holdings when they become optimistic about the stock market and decrease them when they are pessimistic. In addition, they increase their exposure to stocks after market run-ups and decrease it when they experience drops in stock prices. This supports the hypothesis that investors react to non-fundamental news, such as the information they extrapolate from the stock prices.

A traditional "efficient markets" argument against the existence and survival of irrational traders is that rational investors would arbitrage away the mispricing. In other words, by taking large positions opposite to the mispricing they would drive the stock price back to its fair valuation and cause losses to irrational traders. However, Shleifer and Vishny (1997) suggested that rational investors may have "limits to arbitrage", that is, they are constrained in their arbitraging activity. If this happens, the overvaluation (undervaluation) might survive. They justify the limits to arbitrage by an agency relation between investors and arbitrageurs (or fund manager). The arbitrageurs manage investor's funds using a strategy that profits from the correction of mispricing situations. However, the stock price may not correct toward the rational valuation at the typically short horizons at which investors evaluate the performance of their fund managers. In this case, they perform poorly compared to the outlook of the stock market and persuade investors to withdraw funds. This would weaken the capital available for their arbitraging activity.

This discussion suggests that some of the basic hypotheses supporting the rationality of stock prices might fail to hold. Some investors act in a non-rational way. They extrapolate information from the price series and demand stocks even when they realize the overvaluation. On the other hand, rational investors might be limited in their ability to arbitrage away the deviations from the fundamental value. The combination of these elements could explain the stock price run-up in the late 90s.

In this paper, we estimate a model proposed earlier by Brock and Hommes (1998). The model incorporates both of the discussed features. Investors are assumed to have heterogeneous beliefs about future payoffs of the risky asset. They form their expectations in a bounded rational fashion, based on past prices and dividends. In particular, investors have
different opinions about the degree of persistence of the mispricing. As discussed above, investors might buy overvalued stocks because they expect prices to adjust to the fundamental value only at long-horizons. At the same time, other investors might expect the mispricing to have reached unsustainable levels and move against it. The second key element of the model is the evolutionary selection of the beliefs based on the performance. If a belief performed relatively well in the recent past, it attracts more investors and funds. On the other hand, the beliefs that performed poorly experience withdraws from disappointed investors. The beliefs selection mechanism allows for time variation in the aggregate sentiment of the market. The variability of investors sentiment found empirical support in Shiller (1999). Based on the answers to a survey, he constructed indexes for the expectations of a bubble and for the confidence of investors. The indexes show significant time variation and react to the lagged changes in stock prices. Models that are close in spirit to the one analyzed here are DeLong et al. (1990a) and DeLong et al. (1990b). Previous attempts to estimate models that incorporated heterogeneous beliefs are Baak (1999) and Chavas (2000).

Estimation of the model to a century of yearly stock price data indicates that there is significant evidence of heterogeneity of beliefs and time variation in the market sentiment. Investors expect the deviation from the fundamentals to revert back to its long-run mean but at different speeds. A group expects slow mean reversion while the other group believes that the adjustment takes place fast. There is also evidence that investors switch from one belief to another explaining the time variation observed in stock prices. When the sample is extended to include the 90s the results change dramatically. When the dividends determine the fundamental value, we find that one group of investors expects a bubble in stock prices while the second group expects reversion to the mean. In the late 90s, the explosive expectations realized consistent gains and attracted most of the funds owned by investors. This lasted for some years and explains the rapid growth in stock prices experienced between 1995 and 2000. When earnings are used to determine the fundamental value, we find that one belief type has a unit root while the other group are mean reverting. This suggests that one group expects the mispricing to persist while the other group expects that it corrects in the long term.

The outline of the paper is as follows. In Section (6.2) the asset pricing model is presented and Section (6.3) discusses the estimation results. Finally, Section (6.4) concludes.

6.2 The Model

We consider a standard asset pricing model extended by Brock and Hommes (1998) to the case of investors with heterogeneous beliefs. Assume there are two assets available, a risky and a riskless asset. The riskless asset is in perfectly elastic supply and pays a constant return
r. Instead, the risky asset is in zero net supply. The price of the risky asset at the end of period $t$ is denoted by $P_t$ and pays a cash flow denoted by $Y_t$. We denote the excess return of the risky asset as $R_t = P_{t+1} + Y_{t+1} - (1 + r)P_t$. We assume that investors have heterogeneous beliefs about the distribution of future payoffs\(^1\). In particular, we assume that they can be classified into $H$ beliefs types. The expectation of investors type $h$ about the conditional mean and variance of the excess return are $E_{h,t}[R_{t+1}]$ and $V_{h,t}[R_{t+1}]$, for $h = 1, ..., H$. We assume that type $h$ agents have a myopic mean-variance demand function with risk aversion parameter $a_h$, given by

$$z_{h,t} = \frac{E_{h,t}[R_{t+1}]}{a_h V_{h,t}[R_{t+1}]}.$$  

(6.1)

In the rest of the model we suppose that all investors have the same risk aversion parameter, $a_h = a$, and that they have homogeneous and constant expectations about the conditional variance, $V_{h,t}[R_{t+1}] = \sigma^2$. The only heterogeneity we allow in the model concerns the beliefs about the future payoffs of the risky asset. We indicate the fraction of investors in the economy using the $h^{th}$ predictor at time $t$ by $n_{h,t}$. Under the assumption of zero net supply of the risky asset, the equilibrium equation is

$$\sum_{h=1}^{H} n_{h,t} \frac{E_{h,t}[P_{t+1} + Y_{t+1}] - (1 + r)P_t}{a \sigma^2} = 0,$$

(6.2)

and the equilibrium pricing equation is thus given by

$$P_t = \frac{1}{1 + r} \sum_{h=1}^{H} n_{h,t} E_{h,t}(P_{t+1} + Y_{t+1}),$$

(6.3)

where the price at time $t$ of the risky asset is given by a weighted (by the fractions) average of the beliefs of investors about next period pay-offs. The equilibrium price increases if an optimistic group represents a large fraction of investors. On the other hand, if a dominant portion of investors has pessimistic beliefs about future payoffs, they can drive the equilibrium price to lower levels. We assume that investors have homogeneous expectations about a constant growth rate of cash flows, that is $E_t(Y_{t+1}) = (1 + g)Y_t$. We can then rewrite the pricing equation in terms of Price-to-Cash Flows (PY) ratio, $P_t/Y_t = \delta_t$, as

$$\delta_t = \frac{1}{R^t} \left\{ 1 + \sum_{h=1}^{H} n_{h,t} E_{h,t}[\delta_{t+1}] \right\},$$

(6.4)

\(^1\)The term beliefs indicates that investors form their expectations in a non-rational manner. Empirical support for this hypothesis is Kandel and Pearson (1995). They provide evidence that analysts interpret differently the fundamental news contained in earnings announcements.
where \( R^* = (1 + r)/(1 + g) \). The fundamental valuation, denoted by \( \delta_t^* \), is assumed to be determined by the Gordon Model\(^2\), given by

\[
\delta_t^* = \frac{1 - g}{r - g} = m. \tag{6.5}
\]

Hence, the fundamental value of the PY ratio is constant and depends on the riskless interest rate \( r \) and the cash flow growth rate \( g \). We denote as \( x_t \) the deviation of the ratio from the Gordon valuation, that is \( x_t = \delta_t - m \). We now specify the mechanism used by agents to form their beliefs. We assume the fundamental value is known to all investors\(^3\). However, they have different beliefs about the persistence of the deviations from the fundamentals.

The expectations of the beliefs type \( h \) about next period valuation ratio is expressed as

\[
E_{h,t}[\delta_{t+1}] = \delta_t^* + f_h(x_{t-1}, ..., x_{t-l}), \tag{6.6}
\]

where \( \delta_t^* (= m) \) represents the fundamental component and \( f_h(\cdot) \) represents the expected transitory deviation of the PY ratio from \( m \). The model assumes that the transitory component depends only on \( l \) past deviations. The information available to investors at time \( t \) includes past cash flows and prices but does not include the contemporaneous price. In other words, we do not allow agents to react to the contemporaneous equilibrium price but only to realized prices. This assumption about the information available to traders was previously used by Hellwig (1982) and Blume et al. (1994) in a rational expectations setup. Another way of interpreting this assumption is that investors can only trade using market orders. At the beginning of the period they choose their optimal demand of the risky asset to submit to the market maker. At the end of period \( t \), the market maker fixes the equilibrium price \( P_t \) that clears the market. We can rewrite Equation (6.6) as

\[
E_{h,t}[x_{t+1}] = f_h(x_{t-1}, ..., x_{t-l}). \tag{6.7}
\]

We can interpret the function \( f(\cdot) \) as the belief of investors type \( h \) about the evolution of the transitory component in the asset price. Poterba and Summers (1988) showed that there is evidence to support the existence of a temporary mean reverting component in stock prices.

\(^2\)In order to simplify the presentation, we focus on the simple case of a constant interest rate \( r \) and dividends growth rate \( g \) (we also assume that \( r > g \)). In general, one can allow for a time varying fundamental processes and rewrite the model in deviations from such a fundamental. Campbell and Shiller (1989) showed that the deviations of the stock price from the fundamental value are difficult to explain when using different notions of time varying required interest rate.

\(^3\)This is a realistic assumption given the previously discussed survey evidence of Shiller (1989). An alternative approach is the model by Barberis et al. (1998) that is based on different expectations about the growth rate of cash flows.
We can now rewrite Equation (6.4) in term of deviations of the PY ratio from the fundamental valuation, \( m \), as follows

\[
x_t = \frac{1}{R^r} \sum_{h=1}^{H} n_{h,t} f_h(x_{t-1}, \ldots, x_{t-1}).
\] (6.8)

The interpretation of the equilibrium equation is clear: the adjustment of the deviation of the valuation ratio toward the fundamentals is slow if a majority of investors has persistent beliefs about it.

In addition to the evidence of persistent deviations from the fundamentals, there is also significant evidence of time variation in the sentiment of investors. This has been documented by Shiller (1999) using survey data. Also the model of Shleifer and Vishny (1997) generates a similar prediction. It assumes that investors evaluate their fund managers at regular intervals and withdraw their funds if the performance is poor. In the model analyzed here, agents switch among forecasting strategies according to their performances as measured by realized profits in the recent past. This evolutionary selection mechanism of beliefs has been introduced in Brock and Hommes (1997). The model analyzed here incorporates this intuition underlying the evolution of the fraction of investors. It assumes that at the beginning of period \( t \) the realized profits of each of the strategies are publically available. We denote by \( \pi_{h,t-1} \) the realized profits of type \( h \) at the end of period \( t-1 \), given by

\[
\pi_{h,t-1} = R_{t-1} z_{h,t-1},
\] (6.9)

where \( R_{t-1} \) indicates the realized excess return at time \( t-1 \) and \( z_{h,t-1} \) indicates the demand of the risky asset by belief type \( h \) at the beginning of time \( t-1 \). In other words, \( \pi_{h,t-1} \) represents the excess profit realized in the previous period by the \( h^{th} \) strategy. Notice that the realized profit is expressed in terms of quantities observed at the beginning of period \( t \).

We can express the excess return in terms of the PY ratio as

\[
R_{t-1} = (1 + g) \frac{1 + \delta_{t-1}}{b_{t-2}} - (1 + r).
\] (6.10)

This expression can be linearized for \( \delta_{t-1} \) and \( \delta_{t-2} \) around \( m \) to obtain

\[
R_{t-1} \approx (r - g) [x_{t-1} - R^r x_{t-2}],
\] (6.11)

which relates the ex-post excess return to the deviation of the PY ratio from the fundamental value. The demand \( z_{h,t-1} \) depends on the expectation formed at the beginning of period \( t-1 \). Hence, it is based on information available at the end of period \( t-2 \). The demand \( z_{h,t-1} \) is proportional to

\[
E_{h,t-2}[R_{t-1}] = (r - g) [f_h(x_{t-3}, \ldots, x_{t-l-2}) - R^r x_{t-2}],
\] (6.12)
The realized profit in Equation (6.9) is thus given by

$$\pi_{h,t-1} = (r - g)^2 [x_{t-1} - R^* x_{t-2}] [f_h(x_{t-3}, \ldots, x_{t-1-2}) - R^* x_{t-2}].$$  (6.13)

At the beginning of period $t$ investors compare the performances realized by the different beliefs and withdraw capital from those that performed poorly and move it to the winning strategies. The model assumes that the fractions $n_{h,t}$ evolve according to discrete choice probabilities, that is

$$n_{h,t} = \frac{\exp[\beta \pi_{h,t-1}]}{\sum_{k=1}^{H} \exp[\beta \pi_{k,t-1}]} = \frac{1}{1 + \sum_{k \neq h} \exp[-\beta \pi_{h,k}]}.$$  (6.14)

where $\beta > 0$ is called the intensity of choice and regulates the speed of transition from one belief to the others. $\Delta \pi_{h,k} = \pi_{h,t-1} - \pi_{k,t-1}$ indicates the difference in realized profits of belief type $h$ compared to type $k$. The fraction of investors using predictor $h$ at time $t$, $n_{h,t}$, increases (decreases) when it realizes higher (lower) profits with respect to the alternative predictors. If a type of investors has optimistic expectations about future values of the PY ratio that prove to be more profitable, then their fraction increases and they have a higher weight in the determination of future prices. Hence, the evolutionary mechanism in (6.14) captures the performance based selection of the winning beliefs in the recent past. It could also be interpreted as a mitigation of the assumption of bounded rationality. Investors form their expectations in a non-rational manner but withdraw from a belief if it performs poorly. So, they do not systematically make mistakes but, in a simple way, learn about the most profitable predictor in the recent past.

Figure (6.1) summarizes the timing of events assumed in the model. At the beginning of period $t$ for every belief type the fractions and the demands of the risky asset are determined based on past cash flows and prices. The fractions depend on the realized profits of the previous period, $\pi_{h,t-1}$. After all investors have submitted their orders, a market maker clears the market and reveals the equilibrium price of period $t$, $P_t$.

Brock and Hommes (1998) studied the deterministic dynamical system of Equation (6.8), (6.13) and (6.14) with various heterogeneous beliefs types, such as fundamentalists versus trend followers. They showed that the nonlinear evolutionary model may lead to multiple steady states, limit cycles or even chaotic price fluctuations. In the present application, we assume that the economy is characterized by two types of traders, that is $H = 2$. We assume they predict next period deviation by extrapolating past realizations in a linear fashion, that is

$$E_{h,t}(x_{t+1}) = f_h(x_{t-1}) = \phi_h x_{t-1}.$$  (6.15)

where, for ease of exposition, we consider one lag in the function $f_h(\cdot)$ and $\phi_h$ is the parameter that characterizes the strategy of type $h$. In a more general setup, we will consider $l$ lags in
the expectation of future deviations. The dynamical asset pricing model can then be written as

\[ R^*x_t = n_t(\phi_1 x_{t-1} + (1 - n_t)\phi_2 x_{t-1} + \epsilon_t) \quad (6.16) \]

where \( \phi_1 \) and \( \phi_2 \) indicate the coefficients of the two types of beliefs, \( n_t \) represents the fraction of investors that belongs to the first type of traders and \( \epsilon_t \) represents a disturbance term. The value of the parameter \( \phi_h \) can be interpreted in different ways. If it is positive and smaller than 1 it suggests that investors expect the stock price to revert back to the fundamental value. The closer \( \phi_h \) is to 1 the more persistent are the expected deviations. If the beliefs parameter is larger than 1, it implies that investors have explosive expectations about the deviations. In other words, they expect a bubble to occur and the asset price to diverge from the fundamentals. Instead, a negative value of \( \phi_h \) suggests that investors expect a reversal of the deviation in the next period.

In this simple example with 2 types and linear beliefs, the fraction of type 1 investors is

\[ n_{t-1} = \frac{1}{1 + \exp \left\{ -\beta^* \left[ (\phi_1 - \phi_2)x_{t-3}(x_{t-1} - R^*x_{t-2}) \right] \right\}} \quad (6.17) \]

where \( \beta^* = \beta(r - g)^2 \). The fraction depends on the difference in extrapolation rates of the 2 groups, the deviation from the fundamentals and the last period change in deviations. If we assume that the deviation is approximately constant, that is \( x_{t-1} \approx x_{t-2} \approx x_{t-3} \approx \bar{x} \), the fraction depends on the squared value of the deviation. If \( \phi_1 < \phi_2 \), the fraction is close to 0.5 for small deviations while it tends to 1 for large \( \bar{x} \). This suggests that when the first group has less persistent beliefs than the second one and deviations become large, their fraction increases toward 1. Hence, there is evidence that the more stabilizing expectations become active when they are most needed, that is, when the asset price is far away from the fundamentals.
Figure 6.2: Stock Price and Fundamental Value

Plots of the log of the stock price and (a) the log fundamental value when the dividends are used as cash flow, (b) when earnings are used. The period is from 1871 until 2001 (earnings are smoothed by a 10 years moving-average and the series starts in 1880).

6.3 Estimation Results

We estimate the model in equations (6.16) and (6.17). We consider $l$ lags in the linear beliefs functions. We used the dataset in Shiller (1989) of yearly observations from 1871 to 2001 on the S&P500 stock index and dividends and earnings as cash flows. In this case, the valuation ratios are the Price-to-Dividends (PD) and the Price-to-Earnings (PE) ratio. The excess return over the riskfree return is calculated using the 6 month rolling interest rate on commercial papers. Figure (6.2) shows the series of the real stock price and the fundamental value determined using dividends and earnings as cash flow.

We estimate the model from 1871 to 2001 and to the subsample of observations from 1871 to 1990. This is because the stock price runup of the late 90s might have significantly changed the parameter values. We estimate the model by nonlinear least squares (NLLS). We fix $g$ (the cash flow growth rate) and $r$ (the ex-ante return) to the sample average values. The parameters $(\phi_1', \phi_2', \beta')$ are estimated. For the estimated models, we indicate the $R^2$ of the regression, the value of the AIC selection criterion, the standard deviation of the residuals and the Ljung-Box test, $Q_{LB}$, for residuals autocorrelation of $4^{th}$ order.
PD ratio: 1871-1990  The PACF of the time series suggests positive autocorrelation up to the third lag. However, the estimated model seems correctly specified when only the first two lags are included. The estimation results are as follows.

\[
1.046 x_t = n_t [0.61 x_{t-1} - 0.05 x_{t-2}] + (1 - n_t) [0.95 x_{t-1} - 0.11 x_{t-2}] + \bar{\epsilon}_t
\]

\[
(0.14) \quad (0.03) \quad (0.09) \quad (0.06)
\]

\[
n_t = \left\{ 1 + \exp\left[-30.96(-0.34 x_{t-3} + 0.16 x_{t-4})|x_{t-1} - 1.046 x_{t-2}|\right]\right\}^{-1}
\]

\[
(49.53)
\]

\[
R^2 = 0.52, AIC = 3.93, \sigma_t = 4.12, Q_{LB}(4) = 0.17
\]

The coefficients are significantly different from zero at 10% level except for the estimated intensity of choice which has a large standard error. This is a common result in switching-type regression models due to the small variation of the fraction \(n_t\) caused by large changes in the \(\beta^*\). The residuals of the regression do not show significant evidence of autocorrelation. The largest root of the characteristic polynomial when \(n_t\) is equal to 0 and 1 are 0.81 and 0.51, respectively. The estimation results suggest that there is evidence to support the hypothesis of heterogeneous expectations. The two groups have mean reverting beliefs, that is, both groups expect the stock price to adjust toward the fundamentals. However, they are heterogeneous in the speed at which they expect the mean reversion to occur. One group believes the deviations to die out slowly while the other group that it is adjusted quickly. The first group expect the deviation to have a half-life of approximately 1 year.

Panel (a) of Figure (6.3) shows the time series of \(n_t\), the fraction of investors of the first type. The first group of investors are those expecting fast reversion to the fundamentals. The fraction switches frequently from 0 to 1 and in some periods it persist close to 1. The rapid switching is determined by the high estimated value of \(\beta^*\) such that investors respond quickly to differences in realized profits, \(\Delta \pi_{t-1}\). This suggests that there is high variability in the sentiment of investors about the force of the reversion back to the fundamentals. They rapidly switch from expecting mean reversion in the following year to a belief that the deviations will persist for a longer period. The switching function, the difference in realized profits \(\Delta \pi_{t-1}\), is displayed in panel (b). In most periods the investors of the first type, expecting stronger mean reversion, realize more profits compared to the other belief. The graph of \(\Delta \pi_{t-1}\) also shows that there are periods of large variability in the beliefs profitability. During the 30s and the 60s the difference in profits experienced quick transitions from positive to negative values. These were periods in which there was large uncertainty about the direction of the stock markets. The uncertainty translated into nervous investors moving from one belief type to the other. The plot in panel (d) shows the type 1 fraction \(n_t\) against the deviation from
Fig. 6.3: PD ratio: 1871-1990

Plots of (a) the time series of the fraction of investors of type 1, \( n_{t-1} \), (b) the performance measure \( \Delta \pi_{t-1} \) in time, (c) the scatter plot of \( n_{t-1} \) and \( \Delta \pi_{t-1} \) and (d) the fraction versus the deviation.

the fundamentals, \( x_t \). There is no clear pattern in the figure, but \( n_t \) gives the impression to be closer to 1 when the deviations are near zero. Instead, for large \( x_t \) the fraction is often close to zero. This indicates that the aggressive investors play a minor role when the PD ratio oscillates around its fundamental value but they become active to exacerbate deviations. However, the first group becomes active to drive the price back to the fundamentals when deviations are large.

**PD ratio: 1871-2001** In the full sample, \( x_t \) shows linear dependence in the first two lags and inclusion of these lags seems to account correctly for the dynamics in the series. The estimation results are as follows:

\[
1.04 x_t = n_t [0.65 x_{t-1} + 0.16 x_{t-2}] + (1 - n_t) [1.24 x_{t-1} - 0.12 x_{t-2}] + \tilde{\epsilon}_t
\]

\[
(0.18) \quad (0.06) \quad (0.12) \quad (0.08)
\]

\[
n_t = \{1 + \exp[-2.975 (-0.59 x_{t-3} + 0.28 x_{t-4})(x_{t-1} - 1.04 x_{t-2})]\}^{-1}
\]

\[
(1.560) \quad (6.21)
\]

\[
R^2 = 0.86, AIC = 4.138, \sigma_\epsilon = 4.589, Q_{LB}(4) = 0.14
\]
The coefficients are significant at 10% level and the residuals do not show linear autocorrelation. Also for the full sample there is evidence to support the heterogeneity of investors' beliefs about the degree of persistence of the deviations. The largest root of the characteristic polynomial when the fraction is equal to 0 and 1 are 1.13 and 0.84, respectively. The first belief type, for \( n_t = 1 \), is characterized by weaker mean reversion than in the subsample until 1990. Instead, the second type, for \( n_t = 0 \), expect a bubble in the stock price. It is characterized by explosive beliefs while in the subsample they also expected mean reversion.

An explanation for the change in the estimated parameters emerge from the plots in Figure (6.4). Panel (a)-(b) show the fraction of type 1 beliefs and the difference in realized profits of the 2 types. The plot of \( \Delta \pi_{t-1} \) shows that in the late 90s the profits realized by the second belief type, the trend extrapolators, was overwhelmingly large compared to the mean reverting expectations. This drove the fraction \( n_t \) very close to zero, that is, all investors were using the explosive belief. The result can be interpreted as follows: the dramatic increases in the stock price which occurred in the second half of the 90s confirmed the explosive expectations and they realized higher profits than the mean reverting beliefs. The higher profitability of the trend extrapolation strategy attracted more investors, and reinforced the upward boost in stock prices. The model suggests a novel interpretation of the

**Figure 6.4: PD ratio: 1871-2001**

Plots of (a) the time series of the fraction of investors of type 1, \( n_{t-1} \), (b) the performance measure \( \Delta \pi_{t-1} \) in time, (c) the scatter plot of \( n_{t-1} \) and \( \Delta \pi_{t-1} \) and (d) the fraction versus the deviation.
stock price runup of the 90s: a group of investors became very bullish about the stock price and expected the deviations from the fundamentals to persist (instead of adjust back). In addition, the explosive belief realized more profits than the mean reverting belief and more investors started using it. As a result, the fraction $n_t$ was close to zero (most investors were using the second belief type) and the equilibrium price increased very quickly because the market was dominated by traders expecting a bubble. However, in 2000 the mean reverting belief realized higher relative profits and in 2001 the fraction of type 1 investors jumped to 1, reinforcing prices to move back towards fundamentals.

The time series plot of the difference in profits, $\Delta \pi_{t-1}$, gives a clear intuition of what happened in the recent years. The difference has oscillated around zero with some peaks at the beginning of the 30s and 70s. However, the stock price movements of the late 90s is so rapid that expecting mean reversion is a losing strategy. A bubble was occurring in stock prices that was reinforced in its strength when a majority of investors had explosive beliefs.

**PE ratio: 1880-1990** For earnings we follow the approach of Campbell and Shiller (2001) and smooth the series with a 10 years moving average. Accordingly, the sample starts in 1880 instead of 1871. Also for the deviations from the PE ratio the best model specification includes two lags. The estimation results are the following:

$$1.07x_t = n_t[0.71x_{t-1} + 0.06 x_{t-2}] + (1 - n_t)[1.28 x_{t-1} - 0.41 x_{t-2}] + \bar{\epsilon}_t$$

$$n_t = \{1 + \exp[-2.765(-0.57x_{t-3} + 0.46x_{t-4})(x_{t-1} - 1.07x_{t-2})]\}^{-1}$$

$$R^2 = 0.62, AIC = 3.072, \sigma_t = 2.66, Q_{LB}(4) = 0.74$$

All the parameters are significant and also the intensity of choice, $\beta^*$, is significantly different from zero. When the fraction $n_t$ is 0 the largest root is equal to 0.64 while the root is 0.78 for the fraction equal to 1. This implies that the dynamics is characterized by two stable regimes. Contrary to the case of the PD ratio, the first belief type is more persistent than the second type. The results indicate that also for the PE ratio there is evidence of heterogeneity of beliefs characterized by mean reverting dynamics.

The fraction $n_t$ shown in panel (a) of Figure (6.5) shows that there is wide variation in the fraction but that in some periods investors are equally divided between the two types. Large variation in the fraction occurred at the end of the 20s and during the 30s, probably due to the high uncertainty created by the stock market crash of 1929. This feature was also clear in the estimation result concerning the PD ratio. Panel (d) shows the fraction of type 1
investors against the deviation from the fundamentals: it suggests that when deviations are small, the market is dominated by investors using the first belief. However, when indications that a trend is starting, the trend followers enter the market and drive the price further away from the fundamentals. When prices move against the trend the fundamentalists are activated again and contribute to correct the mispricing.

**PE ratio: 1880-2001** The estimation results for the full sample are given below.

\[
1.064 x_t = n_t [0.65 x_{t-1} + 0.11 x_{t-2}] + (1 - n_t) [1.50 x_{t-1} - 0.49 x_{t-2}] + \tilde{\epsilon}_t
\]

\[(0.22) \quad (0.05) \quad (0.16) \quad (0.16) \quad (6.24)\]

\[
n_t = \left[1 + \exp\left[-1.77 (-0.85 x_{t-3} + 0.60 x_{t-4}) (x_{t-1} - 1.064 x_{t-2})\right]\right]^{-1}
\]

\[(1.12) \quad (6.25)\]

\[
R^2 = 0.79, AIC = 3.21, \sigma_t = 2.86, Q_{LB}(4) = 0.76
\]

The model explains reasonably well the data. The parameter \(\beta^*\) is almost significant at 10\% level. As already mentioned, estimation of this parameter is quite difficult and the lack
of statistical significance for $\beta^*$ should not be interpreted as evidence against the nonlinear model. The residuals do not show evidence of autocorrelation according to the $Q_{LB}$ statistic.

The stock price runup seems to have affected also the deviations from the fundamental value when earnings are used as cash flows. When the fraction is 0, all investors are of the second type, the largest root of the characteristic polynomial is 1.02. Instead, when the fraction is 1, everybody is using the first belief, the root is 0.79. Compared to the results in the subsample we observe a significant increase in the persistence of the second type of beliefs. This supports the previous analysis for the PD ratio: the rapid stock price rise of the late 90s is accommodated by an increase in the persistence of the expectations of the aggressive investors. However, while for the PD ratio they expected a bubble in stock prices, here their beliefs are characterized by a unit root process. They expect the deviation at time $t + 1$ to be equal to the deviation at the beginning of period $t$. Hence, this expectations are also inconsistent with mean reversion in stock prices. Panel (a) of Figure (6.6) shows that

Figure 6.6: PE ratio: 1880-2001

Plots of (a) the time series of the fraction of investors of type 1, $n_{t-1}$, (b) the performance measure $\Delta \pi_{t-1}$ in time, (c) the scatter plot of $n_{t-1}$ and $\Delta \pi_{t-1}$ and (d) the fraction versus the deviation.

the fraction of investors of type 1, $n_t$, has large oscillations and rapid switching, for example during the 30s. In the late 90s the fraction is close to 0 in 1998 and 1999. This is clearly explained by the time series plot of $\Delta \pi_{t-1}$ in Panel (b): the second type achieved superior relative performances and all the investors switched to this expectation strategy. In 2000 the
mean reverting belief became the most profitable predictor again and the fraction jumped to 1.

6.4 Conclusion

A result that emerges from this analysis is the existence of time variation in the sentiment of investors. This was already suggested by Shiller (1999) who constructed a sentiment index from the answers of investors to a questionnaire survey. In this chapter, we use a simple asset pricing model with evolutionary switching of investors from one forecasting strategy to another based on their past performances. Therefore, the time variation in investors’ sentiment is explained by a boundedly rational feedback response to changes in stock prices. Investors become more bullish if in the recent past the bullish beliefs outperformed the other available beliefs. Evolutionary forces may thus reinforce deviations from fundamentals.

Until the beginning of the 90s, investors were expecting the transitory component of stock prices to be mean reverting. In other words, investors’ beliefs are consistent with the view that the stock price can take swings away from the fundamental value but revert to it in the long term. Investors also alternate between periods in which they believe the mean reversion to be stronger than other periods. This effect is again due to the evolutionary feature of the model to allow for switching of the investors from one type of beliefs to the other. This confirms the time series evidence of nonlinear mean reversion of Gallagher and Taylor (2001) and Chapter (5).

However, starting in 1996 the behaviour of stock prices was at odds with the evidence that when deviations are large they tend to revert back to their long run mean. From 1996 until 1999 the PD ratio indicated that the stock market was overvalued and it was likely to correct back to the fundamentals. Also the PE ratio gave the same indication, less clearly and somewhat later in time. Despite the common feeling among investors that stocks were overvalued, the market continued to grow by approximately 30% a year. The estimation of our model shows that the large majority of investors had explosive beliefs about the persistence of the deviations from the fundamentals. Apparently, investors neglected the role of fundamental news and continued to buy stocks for purely speculative reasons. The extraordinary performance of the bubble expectations convinced most investors to adopt this type of beliefs. The outcome of our model is consistent with the view that the mean reverting expectations had very limited capital to arbitrage the mispricing away and force the mean reversion to the fundamental value.