Dual views of string impurities. Geometric singularities and flux backgrounds

Duivenvoorden, R.J.

1

INTRODUCTION

GEOMETRY AND STRING THEORY

The language of geometry has proved remarkably adept to formulate the presently known fundamental physical theories. The general theory of relativity on the one hand, but also gauge theories such as the standard model of particle physics can be formulated in essentially differential geometric language.

String theory, as a candidate to provide a unified framework for the description of both gravitation and the other known fundamental forces should, and does contain both familiar gravitational and gauge theory in appropriate regimes. But this is not all. Crucially, in string theory, a theory in which the fundamental objects are extended, the rôle of geometry is quite different than in theories of point particles. Even the very notion of what we would mean by geometry can be very different than is familiar from 'ordinary' differential geometry.

In the perturbative approach[1] to string theory, to a large extent the rôle of geometry is taken over by worldsheet conformal field theory.

In many situations, the worldsheet conformal field theory has a target space interpretation. It is interpreted as describing the embedding of the string worldsheet in a spacetime background, which has an 'ordinary geometric' interpretation. Yet in many other cases worldsheet conformal field theories can have all the properties required of them to define a string 'background', yet no target space interpretation is apparent. This situation is possible because the definition of a conformal field theory can be made in ways very different than as a sigma model.

[1] We mean perturbation theory in the string worldsheet genus expansion. This is not to be confused with the α' expansion, a perturbation expansion in the string scale used in worldsheet conformal field theory. The term 'non-perturbative T-duality' which permeates the setup of this work, alludes to perturbation theory in α'

T-DUALITY

At least equally interesting, is the situation that two different sigma models can define isomorphic conformal field theories. Such isomorphic conformal field theories define equivalent string backgrounds which have different 'ordinary geometric' interpretations. Intuitively speaking, the reason why backgrounds that look different to a classical geometer may look indistinguishable to a string theorist, is that strings are not in general located at a 'point' in spacetime, but they trace out a curve. Thus, a closed string can wind around a closed curve.

T-duality essentially exchanges winding modes in the string worldsheet theory with momentum modes. The momentum modes, are nothing but the modes that a theory of point particles would also have. Consequently, T-duality exchanges the 'intrinsically stringy' part of geometry (probed by winding strings) in one background, with the 'ordinary' geometry (as probed by point particles) in the dual background.

The archetypal relation of T-duality is the $R \leftrightarrow 1/R$ duality of strings on a circle. On a circle of radius R, the winding modes have energy levels which are spaced with an energy difference proportional to R, while momentum modes are spaced with energy levels proportional to R^{-1}. On a circle of radius $1/R$, the level spacing of winding and momentum modes is interchanged.

IMPURITIES 1: GEOMETRIC IMPURITIES

T-duality can have more complicated implications in more complicated backgrounds. The backgrounds which we consider in this work can be called 'impurities'. There are two kinds of impurities which we distinguish.

The first kind is a 'geometric impurity'. In this case there are no background fields, other than a metric. The metric defines a singular geometry, more precisely, a geometry which preserves some spacetime supersymmetry[2] and which has an isolated singularity. Often an explicit metric of the background will not be known. Instead, we use other more implicit means to characterize the background geometry.

In chapter 2 different ways are discussed to characterize supersymmetric singular background geometries. Of the methods discussed, two play a prominent rôle later, in chapter 4. The first method is the characterization as a metric cone. In this method the differential geometry of the background is emphasized. Supersymmetry imposes a restriction on the holonomy of the background. The structure of a metric cone, together with restricted holonomy leads to differential geometric constraints on the base of the cone. In particular, it turns out that all of the bases of supersymmetric complex metric cones have a Killing vector field, which degenerates at the apex of the cone. This is interesting because when there is an isometry, it is usually possible to consider a T-dual background.

[2] We concern ourselves with complex geometry. It would be an interesting but quite separate undertaking to transform the methods discussed to a form suitable for supersymmetric singular geometries which admit no complex structure.

The second kind of characterization, describes the geometric impurity as a hypersurface. In this description, the differential geometry is less explicit. However, there are analytic, algebraic and topological properties which can be studied and have been studied by mathematicians.

Actually, the hypersurfaces which we consider, bear a similarity to metric cones. The affine hypersurfaces under consideration are defined by weighted homogeneous polynomials. These are equivariant under a $\mathbb{C}^*$ action. Compare this to the supersymmetric metric cones, they admit a scaling of the base, and the base has a Killing vector field, which in all cases that are discussed by us, has closed orbits, so defines a $U(1)$ action. So both the hypersurfaces and metric cones we use, admit a $\mathbb{R} \times U(1)$ action.

Describing a singularity as a hypersurfaces offers some advantages which a metric cone description does not have. First, deformations of the space, and more specifically desingularizations, which smooth out the space completely, are described as simple analytic deformations of the defining polynomial. Second, weighted homogeneous polynomials, used to describe the hypersurfaces, can also be used to describe conformal field theories, as Landau-Ginzburg theories. Both these properties are very important in the construction of backgrounds which are T-dual to the geometric impurities.

Impurities 2: Flux Impurities

The second kind of impurities can be called 'flux impurities'. These are, as the name indicates, sources of gauge field flux. So in backgrounds with flux impurities, there are other non-trivial background fields than just the metric.

The flux impurities are sources of Kalb-Ramond field. The archetypal example is a simple Neveu-Schwarz fivebrane. Also, these impurities create a non-constant dilaton: near the impurity the effective string coupling is large.

Localized Physics Near an Impurity

Intuitively speaking, because the string coupling grows large near a flux impurity, it may be possible to decouple the physics localized near this impurity by sending the string coupling asymptotically far from the impurity to zero. Then the bulk degrees of freedom, coupling to the 'localized' degrees of freedom through gravity, can decouple, and one can restrict attention to the degrees of freedom localized near the impurity alone.

Such decoupling limits have various interesting properties. First, typically the 'localized' physics has a holographic description. That is to say, the decoupled subsector of string theory in the original background, which contains just the 'localized' physics of the impurity, is equivalently described by the full string theory in another background (think of Anti-de Sitter backgrounds, and also of linear dilaton backgrounds).

Next, it happens often, as we will see, that these 'decoupling limit backgrounds' admit an exact worldsheet conformal field theory description, while such a description is unknown for the full, unscaled backgrounds with a local impurity.

On the other hand, geometric impurities also have localized physics. Essentially, it comes from branes wrapping vanishing cycles in the singularity. The notion of vanishing cycles is also useful to understand that geometric impurities are quite generic. One may start with a smooth geometric background. Such a background is usually one in a family of connected backgrounds, parametrized by moduli. At certain perfectly fine values of the moduli, a homology cycle in the geometric background may shrink to zero size. Then a singularity, or geometric impurity, develops. A scaling limit which isolates the physics localized at the singularity, typically involves tuning the size of a vanishing cycle to zero, while also scaling other parameters, usually the string coupling, analogous to the limit for flux impurities. Especially the hypersurface singularities are suited to such a scaling limit, as blowing up certain cycles corresponds to simply deforming the defining polynomial.

Duality between Geometric and Flux Impurities

Geometric impurities and flux impurities are related by T-duality. In practice it is difficult to explicitly carry out the duality transformation. A reason for this difficulty is, that worldsheet instanton effects are crucial to the duality. These worldsheet instantons break spacetime symmetries which seem to be present if one considers only the perturbative physics.

If one performs a perturbative analysis and dualizes a geometric impurity, it appears that the dual flux impurity has an isometry, which turns out not to be present in reality, when considering non-perturbative contributions to the duality. The prime example of such a duality, is that between IIA strings in an asymptotically Euclidean space, with an A_k singularity, and IIB string theory on $\mathbb{R}^{5,1} \times \mathbb{R}^3 \times S^1$, with a stack of $k+1$ Neveu-Schwartz fivebranes localized at a point in $\mathbb{R}^3 \times S^1$.

It is easier to consider duality in the 'near impurity background', rather than the full background, before zooming in on the impurity. The 'localization' of the flux impurity is of course crucial to get the correct 'decoupling limit' or, as we just referred to this limit, a 'near impurity limit'.

We will see exact worldsheet conformal field theory descriptions in the 'near impurity limit' of both the geometric and the dual flux impurity. Actually, in certain cases the worldsheet conformal field theory of the dual flux impurity will have an explicit construction and interpretation as a sigma model. In other cases, a geometric interpretation of the 'near flux impurity' worldsheet theory remains to be discovered.

Why Impurities?

What are the motivations for the study of 'geometric' and 'flux' impurities and the T-duality relation between the two?

First, there are the intriguing relations between the descriptions of geometric and flux impurities. We will find flux impurities which can be viewed as a background of the form

$$(\text{linear dilaton}) \times \frac{G}{H}$$

the right hand factor denotes a coset conformal field theory. When this is realized as a gauged WZW model and when the level of G is large, the target space can be approximated by a one-loop calculation in the gauged WZW model. This gives a target space $G/H = \tilde{L}$, where H acts as a vector gauging, $g \sim h^{-1}gh$. Note that this target space looks very different from the coset manifold G/H, where group elements are identified as $g \sim gh$.

We shall see intriguing cases that flux impurities of this kind are related, by T-duality and adjusting the moduli, to geometric impurities, that are described as follows. These impurities are metric cones on a base space L, where L is a fiber bundle

$$\begin{array}{ccc} S^1 & \xrightarrow{i} & L \\ & & \big\downarrow{\scriptstyle \pi} \\ & & Z \end{array}$$

with base Z, which is a homogeneous space $G/\left(H \times \Gamma\right)$, and Γ is a discrete subgroup of G, related to modular data of the coset model.

So there are some intriguing similarities and differences going on, which must point at some stringy geometric phenomena. It seems worthwhile to try and understand such stringy geometric aspects better.

There is also a quite different motivation. This is related to holographic duality: string theory in certain backgrounds is believed to be exactly equivalent to a non-gravitational theory in a spacetime that has one dimension less. The two types of string background that are widely believed to exhibit such behavior are linear dilaton backgrounds and Anti-de Sitter backgrounds.

Very generically, the flux impurities we find have a linear dilaton. The linear dilaton backgrounds are believed to holographically describe certain exotic quantum theories in dimensions $d \leq 6$: Little String Theories. These theories are non-local, and little is known about them. Clearly it would be highly interesting to better understand such unfamiliar quantum theories.

Also very generically, the linear dilaton backgrounds can be 'deformed' to AdS backgrounds. Therefore the flux impurities are of interest to study AdS backgrounds, and their holographic duals, which are conformal field theories.

Then why are the geometric impurities of interest? A fruitful way to gain knowledge about AdS/CFT, is to take certain non-dilatonic brane configurations, that is, impurities of a sort, and take a scaling limit which isolates the physics near the branes. This physics can be characterized in two different looking ways, one can think in terms of open string degrees of freedom, describing the physics on the worldvolume of the branes, or closed string degrees of freedom describing the dual, gravitational physics in the background near the branes, which is deformed by the branes. A lot can be learned about AdS/CFT by realizing the AdS background and the dual field theory through a brane setup. However, simple brane configurations only give a limited number of geometries AdS $\times$ N.

A lot of geometries can be obtained by also considering 'geometric impurities', that is, singularities. In particular, many interesting AdS/CFT realizations are possible by considering D3branes in a Calabi-Yau singularity. These produce AdS geometries of the form $\mathrm{AdS}_5 \times N$, where N is an Sasaki-Einstein manifold, which can be viewed as the base of a metric Calabi-Yau cone.

Apart from considering D3 branes, there is another way to get AdS backgrounds from geometric impurities, which is an important motivation for the study of these impurities and their T-duality. One can take a geometric impurity and put fundamental strings at the singularity. This is not a non-dilatonic background as such, but by performing a T-duality it becomes non-dilatonic. The dilaton contribution of the fundamental string, in the 'near impurity limit' compensates the linear dilaton that is generated by the T-duality. In this way many backgrounds of the form $\mathrm{AdS}_3 \times N_7$ might be realized, which cannot be obtained from other simple brane configurations. Therefore, we hope that the knowledge about T-duality of these impurities will also lead to a better understanding of holographic duality.

OUTLINE

The outline of this thesis is as follows.

In chapter 2 geometric impurities are discussed. Mainly two characterizations of supersymmetric (and complex) singularities are presented: metric cones with holonomy contained in $SU(n)$ on the one hand, and weighted homogeneous affine hypersurfaces on the other.

Differential geometric aspects of the metric cones are discussed. A particular rôle is played by quasi-regular Sasaki-Einstein manifolds. Many known examples are homogeneous spaces, or related to homogeneous spaces. Sasakian(-Einstein) manifolds have a characteristic Killing vector field, which is used to relate these spaces to quasi-smooth Kähler-Einstein varieties. These are the subject of study of algebraic geometers.

Weighted homogeneous polynomials can also be used to characterize supersymmetric complex singular hypersurfaces. Aspects of such hypersurfaces are discussed. As somewhat of an aside, some topological properties of such hypersurfaces are discussed. Weighted homogeneous polynomials also define Kähler varieties in weighted projective space. These can be interpreted as base spaces of Sasaki-Einstein circle fibrations. This establishes a connection between metric cones and affine hypersurfaces.

In chapter 3 various aspects are discussed of superconformal field theories, which are put to use later, in chapter 4, to describe strings in the background of impurities. Some particularly important constructions are Landau-Ginzburg models, which are defined through a weighted homogeneous polynomial and thus make contacts with hypersurface singularities. Also coset conformal field theories play a rôle, since the best understood dualities between geometric and flux impurities involve coset conformal field theories, which are actually coset models that are closely related to Landau-Ginzburg (and Kazama-Suzuki coset-) models. Of course an important class of conformal field theories is formed by sigma models.

Finally in chapter 3 non-conformal models are discussed which interpolate between sigma models on hypersurfaces, and Landau-Ginzburg theories. Models of this kind are employed to formulate the T-duality of impurities in chapter 4.

Chapter 4 begins with a discussion of geometric and flux impurities (in particular: five-branes) in string theory, and the 'near impurity geometry' and exact conformal field theories for 'near impurity' geometries. Next generalities of T-duality are discussed: classical T-duality rules, the rôle of degenerating isometries, and breaking of isometries in the dual model by worldsheet instantons. Finally, in section 4.4, T-duality for a large class of impurities is discussed. Agreement is found with the known result of hyper-Kähler surface singularities and ADE-throat geometries, and some further examples are discussed, and some final observations are made.

Basis for this Thesis and Other Work by the Author

Perhaps it may be difficult for the reader to separate original work by the author from previously known results obtained by others, which serve as a basis for the author's work, solely from the references throughout the body of this thesis. In order to draw a clearer picture of the original contribution of the author, we wish to spend a few paragraphs here, before commencing our exposition in the subsequent chapters.

The central objective and main work of the author presented in this thesis, is the proposition for a way to T-dualize singular supersymmetric string backgrounds. In the dualization of such backgrounds, worldsheet instantions contribute in a crucial way. It has proved very difficult to take into account these crucial contributions in a systematic fashion. The proposition entails a manner to take into account these contributions, using an intermediate 'half-dualized' model. This proposition is an essential original element of this thesis and it is presented in section 4.4. The relations between original, 'half-dualized' and fully dual model are phrased making use of a collection of notions and ideas from geometry and from (conformal) field theory and string theory. Separate elements of these had been known in circles of geometers or string theorists, but arguably not in the context provided by the proposition for T-duality.

From the various existing characterizations of singular supersymmetric string backgrounds, it is found that affine hypersurfaces provide a description that is directly connected to the dualization proposal. In particular, the affine hypersurfaces are defined by weighted homogeneous polynomials. These polynomials define the 'half-dual' intermediate models as Landau-Ginzburg field theories.

A connection between, on the one hand, the sigma model which describes string propagation on the singular background and, on the other hand, the 'half-dual' model is provided by embedding these models in a family of non-conformal 'worldsheet' field theories. These families of models are first presented in section 3.3, and the full connection with T-duality is presented in section 4.4. Essentially, within these families of non-conformal field theories the effect of worldsheet instantons is argued to be described in a concise fashion, as turning on a vacuum expectation value of a certain field. Earlier parts of chapters 3 and 4 are to a large extent intended to provide the necessary context, from existing literature, to arrive at the author's proposals regarding these families of theories and the T-duality.

There is a number of ideas from older literature that play an important part in the appreciation of the duality proposition. We name a few of these.

In chapter 2 mostly geometric ideas are discussed that have a place in earlier, to a large extent mathematical literature. These ideas include characterization of a class of supersymmetric backgrounds by means of weighted homogeneous polynomials. Such polynomials in turn appear in string theory as Landau-Ginzburg superpotential, which is a point of view not considered in the mathematical literature. On the other hand, for the very special ADE hyper-Kähler surface singularities, the connection between Landau-Ginzburg model and geometry was proposed by Ooguri and Vafa [19], without the here discussed duality prescription.

In section 3.4 particular Kazama-Suzuki coset models are reviewed. It has been strongly believed in existing literature, cited in that section, that some of these coset models have a Landau-Ginzburg description. Using these known correspondences it is possible to provide geometric interpretations of some models T-dual to some special singularities. These interpretations are discussed in chapter 4.

The ideas put forward in this thesis shall be presented in more condensed form in a forthcoming publication.

Finally, we should mention some work which has been done by the author in collaboration with Boels, de Boer and Wijnhout, [109, 110] which is not discussed in this thesis. This work deals with non-perturbative aspects of three- and four-dimensional gauge theories. This work is apparently quite unrelated to the work that is discussed in this thesis. However, there is an overarching theme common to that work as well as the work that has gone into this thesis, although any concrete aspects of this remain to be formulated.

Both the work discussed in this thesis and [109, 110] are connected to worldvolume descriptions of certain 'impurities' or brane configurations. On the one hand, gauge theories as considered in [109, 110] can be viewed as worldvolume theories of certain brane configurations. On the other hand, the geometric singularities discussed in this thesis can be used to construct AdS/CFT relations. In [109, 110] certain three-dimensional gauge theories are studied using Toda models. Curiously, these appear also in T-duals of geometric singularities, although these are T-duals performed in a somewhat different way than discussed in this thesis. It seems that many interesting U-duality relations might be studied through the use of families of non-conformal field theories which flow to various different conformal worldsheet cft's in the infrared. An example of the use of such families of theories is formed by the T-duality application that is central in this thesis.