Quantum Hall spin liquids
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CHAPTER 1

INTRODUCTION

Condensed matter physics has provided us with many astounding effects. Even though the microscopic equations that govern these systems have been known for decades by now, dating back to the early (non-relativistic) days of quantum mechanics, it continues to hold many surprises. The first of these, superconductivity and superfluidity, date back to the beginning of the twentieth century when the laboratory of Kamerlingh Onnes set new low temperature records at an amazing pace. The explanations of these phenomena were much later traced down to the heart of quantum mechanics, the existence of two different types of particles: fermions and bosons.

Bosons were in essence known to Max Planck, but were named after Satyendranath Bose, an Indian physicist who rederived Planck's result by purely statistical means. Soon after that, Albert Einstein realized that at low enough temperatures, all bosons would occupy the same one-particle state and Bose-Einstein Condensation (BEC) was born.

A fundamental property of fermions is their half-integer spin. Wolfgang Pauli originally postulated this additional quantum number, while Ralph Kroning and George Uhlenbeck and Samuel Goudsmit proposed that it is an intrinsic property of the electron. The relation between statistics and spin (i.e. bosons have integer spin, while fermions have half-integer spin) is fundamental from a theoretical point of view. The only known theory which is able to describe relativistic quantum mechanics, quantum field theory, is only consistent, in three dimensions, with exactly this spin-statistics relation.

In one and two dimensions, the difference between bosons and fermions is more subtle. From a theoretical point of view, the 1D interacting Fermi gas is equivalent to a free bosonic field! Even two dimensional many-particle systems of bosons or fermions can be very similar, under extreme conditions. Electrons, in the presence of strong magnetic fields, form states of matter, quantum liquids, almost identical to states formed by atoms in rapidly rotating traps. In this thesis we will analyze these strongly interacting systems, and observe that it is natural to consider both bosons and fermions in one 'unifying' framework.
1.1 QUANTUM CONDENSATES AND LIQUIDS

Superfluidity, which occurs when $^4$He is cooled below 4 Kelvin, turned out to be the first manifestation of Bose-Einstein condensation. Due to the high density, however, the interactions are strong and only a small fraction of the particles share the same state. Despite this, superfluid Helium shows the characteristic features of a true Bose-Einstein condensate. In particular, when a rotation is imposed upon the system, it creates localized regions where the rotation is stored. These regions, vortices, support a well-defined quantized amount of rotation (vorticity).

Quantum mechanics also provides a beautiful framework for the description of metals. Electrons can move freely in such solids and minimize their energy by occupying the orbitals with the lowest energy. In this Fermi gas, all the dynamics is located near the threshold of occupied levels, the Fermi energy. If the electrons near the Fermi energy are attracted to each other, they form (bosonic) Cooper pairs. These pairs subsequently Bose-condense and form a superconductor. This charged BEC expels magnetic field, which is in a very precise way analogous to the rotation of a superfluid. The charge of the condensate ($2e$) fixes the quantization of the magnetic flux to be $\Phi_0 = h/2e$. A (charged) superfluid defines a complex order parameter whose phase winds (once) when a vortex is encircled. A condensate can support a current without dissipating energy, again due to the existence of the complex order parameter. The resistance vanishes!

These quantum condensates have set the stage for 'conventional' condensed matter physics. They led to the discovery of various order parameters, classical fields that describe the local order in a system. These order parameters depend locally on the underlying particles, greatly facilitating the description of the qualitative (and even quantitative) behavior.

Over the past few decades, however, it has become clear that there are systems which do not lend themselves to such a description. In particular, incompressible states of matter that support quasi-particles with highly exotic properties have been observed in experiments. All excitations, including density waves, cost a finite energy. We will refer to these states of matter as quantum liquids.

The first, most extensively studied, liquid in this category is the state proposed by Robert Laughlin to describe the fractional quantum Hall effect [48]. It describes the behavior of strongly interacting electrons in two dimensions. An important distinction from more classical systems is that there is no local order parameter. There is a high degree of order, however, dubbed topological order [95]. One of the consequences is the phenomenon of charge fractionalization: the charge of a quasi-particle over the original Laughlin state is $q = e/3$. The density of electrons in this state is tightly fixed to $1/3$ of the magnetic flux density.

In the context of lattice spin models, several (quantum) spin liquids have been proposed. One of these, the chiral spin liquid, was put forward by Vadim Kalmykov and Robert Laughlin [42] in the context of high-temperature superconductivity. It has become apparent that the Heisenberg model on the square lattice (supposed to describe the high-$T_c$ material 'parent compounds') in fact has anti-ferromagnetic order and there is no (experimental) sign of chiral symmetry breaking upon doping.
However, it has become clear that in *frustrated* spin models, i.e. when the number of ‘classical ground states’ is macroscopic, the ground state generically is a spin liquid. The chiral spin liquid, for example, is believed to be the ground state of the triangular Heisenberg model [42].

It is possible to construct ‘spin liquid’ ground states that respect time reversal symmetry. An often used simplifying assumption is that short-range singlet bound spins can be represented by *dimers*, particles which live on the links of a lattice and do not touch. When these dimers are put on the triangular lattice, there is a finite region in parameter space where the ground state is a dimer-liquid [54]. On the square lattice, in contrast, only at a precisely tuned point (the Rokhsar-Kivelson point [75]), a liquid exists. This is however a critical point between two ordered phases.

In this thesis, I will present some *quantum Hall spin liquids*. These generalizations of the Laughlin liquid concern spin-full electrons (atoms) in a magnetic field (rotating trap). There will be a particular emphasis on the topological properties of these states, such as quasi-particle quantum numbers and ground state degeneracies.

### 1.2 Rotating BEC’s

Weakly interacting bosons have in recent years received an intense burst of research interest. Although the program to create Bose-Einstein condensates has been active for many years, since the first successful creation by Cornell [4] many groups have contributed greatly to our understanding of these gases. A new program, entered a few years ago, entails the rotation of these condensed gases. Several techniques are employed to create the rotation. One of these is to perform the evaporative cooling, needed to reach the low temperatures which are necessary for a BEC, on a rotating cloud of atoms. When the condensate forms, vortices will be induced, arranged in triangular lattices. A different technique is to spin an already condensed gas by deforming the trap and rotating the deformation. This allows, for example, a detailed study of the dynamics involved in the creation of the vortex lattice.

Theoretical analysis predicts that, at very high rotation speeds, the vortex lattice will undergo *quantum melting*. Beyond this melting point the vorticity is spread uniformly over the system and a series of quantum liquids will form. In several laboratories, a quest for these ultra-high rotation speeds is pursued [79, 15]. By expelling atoms with low angular momentum, the lowest Landau level has already been reached convincingly by the group of Cornell [79]. Using a laser to stir the condensate and a confining potential which prevents escape, Bretin *et al.* [15] are even able to go beyond the point where harmonic confinement is cancelled by the centrifugal force.

A different program in BEC experiments is the usage of optical traps. These traps liberate the (hyperfine) spin degree of freedom of spin-full atoms [34], as there is no polarizing magnetic field. This allows a variety of new phenomena, such as skyrmions [34], monopoles [90] and π-disclinations [103]. There are two different regimes [34], depending on the sign of the spin-dependent interaction $c_2$. 

If this interaction is repulsive \((c_2 > 0)\), the system tends to minimize the total spin and we speak of the anti-ferromagnetic or “polar” regime. If, on the other hand, the interaction is attractive \((c_2 < 0)\) the system will tend to maximize the total spin. This is the ferromagnetic regime. Examples of such systems are spin-1 Bose-Einstein condensates (BEC) which can be realized by trapping atoms such as \(^{87}\text{Rb}\) \((c_2 < 0)\) [11, 78, 18] and \(^{23}\text{Na}\) \((c_2 > 0)\) [89]. In both cases, the ratio \(\gamma = c_2 / c_0\) of the spin-dependent \((c_2)\) and the spin-independent \((c_0)\) parts of the (contact) interaction is small. For \(^{87}\text{Rb}\), \(\gamma \approx -0.005\) [46] while for \(^{23}\text{Na}\) the ratio is \(\gamma \approx 0.05\) [21].

Although the experimental effort is currently focussed on rotating scalar condensates, we wish to anticipate the (fast) rotation of spin-1 atoms. In the presence of the internal degrees of freedom, associated with the spin-states of spin-1 atoms, a sequence of quantum ground states is expected, similar to the scalar series. Additional structure is provided by the spin, and by the presence of parameters such as \(\gamma\).

In chapter 3, we will give a more detailed analysis of slowly rotating spin-1 bosons in rotating traps. Although the discussion is limited to the lowest Landau level, many of the (2-dimensional) features such as spin-textures are present. The approximation allows us to derive exact results and provide extensive results in the mean-field approximation. The high rotation regime, where the condensate has melted into a quantum liquid, is analyzed in chapter 4. We find three series of spin-1 quantum liquids which display phenomena such as non-abelian braiding and spin fractionalization.

### 1.3 The Quantum Hall Effect

In the 80’s, the evolution of the semiconductor industry led to increasingly clean samples of semiconducting heterostructures. In such structures, two different materials are grown on top of each other. At the interface a 2D “inversion” layer is formed in which charged particles (electrons or holes) can move freely. The two materials have a different band structure, such that doped particles will prefer to reside in one of the two. Far away from the interface, \(p\) or \(n\)-type donor atoms are deposited. The doped electrons will feel a Coulomb attraction to this layer of donor atoms, effectively creating a potential well near the interface where the electrons are trapped.

Imposing a magnetic field perpendicular to the interface leads to the well known “Hall effect”. In the classical Hall effect, a current will be deflected by the Lorentz force. When a finite voltage difference perpendicular to both the current and the magnetic field is formed by the deflected charges, the electric field cancels the Lorentz force. In this stationary setting, one can easily calculate the Hall resistance, defined by \(\Delta V_H = R_H I\), to be:

\[
R_H = \frac{n_\Phi}{n} \frac{h}{e^2},
\]

where we have expressed the magnetic field strength in terms of the density \(n_\Phi\) of
1.3. THE QUANTUM HALL EFFECT

Figure 1.1: Schematic setup of a quantum Hall experiment. Figure taken from [23]

flux quanta. The density of electrons is $n$. We will refer to the ratio $n/n_\phi$ as $\nu$, the filling factor.

The quantum Hall effect is the observation that, in sufficiently clean samples, this Hall resistance is quantized to high precision. An important role is played by the disorder in this system. Without disorder, the above expression would always be valid since we could obtain it by a Lorentz transformation of a static configuration! To explain the observed plateau, one can use the gauge argument due to Laughlin[47]. Imagine that the sample has the topology of a punctured disc. We assume that all states near the Fermi energy are localized. The flux through the puncture is adiabatically increased by one flux quantum. After this procedure, we can gauge this flux away by a singular gauge transformation. The new state should be equivalent to the original ground state, with an integer number of electrons transported from one edge to the other. This integer, say $p$, fixes the Hall conductance to $\sigma_H = 1/R_H = p \frac{e^2}{h}$. The quantization of the conductance is very precise, nowadays up to 1 part in $10^8$.

The above argument uses in a fundamental way the knowledge of the charge of a localized particle, the electron. This is not surprising, as we can understand the essence of the integer quantum Hall effect in terms of non-interacting electrons in the presence of disorder. A very different subject, however, is the fractional quantum Hall effect. From an experimental point of view, this effect is largely equivalent to the integer case. Except that the existence of a plateau at rational $\sigma_H = \frac{p}{q} \frac{e^2}{h}$ implies the transport of fractionally charged particles! When Tsui, Störmer and Gossard first observed plateaus in the Hall resistance at rational values [91], it was immediately clear that these can not be understood in terms of non-interacting electrons.

The existence of a plateau in the Hall resistance and, even more important, the vanishing of the longitudinal resistance, imply the presence of a mobility gap. For a range of Fermi energies, there should be no low-energy charge-carrying states. Since at a fractional filling of a Landau level, no such gap exists for non-interacting electrons, a different approach is needed. Interactions play a vital role in the formation of the necessary gap.

We will follow the route used by Laughlin to address the strongly interacting
many-body problem for the $\nu = \frac{1}{3}$ quantum Hall plateau. That is, we start from
the microscopic Hamiltonian:

$$H = \frac{1}{2m} \sum_i (p_i - eA_{i}^{EM})^2 + \sum_{i<j} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|},$$

where $m$ is the mass of the electron, $e$ the charge and $A^{EM}$ the external electromagnetic field. The single-particle states will arrange themselves in Landau levels, macroscopically degenerate bands which are separated by an energy gap $\hbar \omega_c = \hbar eB/m$. In the symmetric gauge, the states in the lowest Landau level take the simple form of analytic functions of $z = x + iy$ times a gaussian.

We then proceed by writing down a trial wavefunction:

$$\Psi(z_1, \ldots, z_N) \propto \prod_{i<j} (z_i - z_j)^3 e^{-\sum_{i} |z_i|^2/4\lambda^2}.$$  (1.3)

This Ansatz has many desirable properties. First of all, it actually describes a collection of particles at filling $\nu = \frac{1}{3}$. This can be seen by noting that, when $N-1$ particles are fixed, the wavefunction has $3(N-1)$ nodes. So this particle sees $3(N-1)$ flux quanta. Second, it describes a liquid, with short-range screening. The amplitude $Z = \int d|z_i| |\Psi|^2$ is the partition function for a classical 2-dimensional one-component plasma at high temperature. And finally, electrons very effectively repel each other, due to the $(z_i - z_j)^3$ Jastrow factor. The conclusive evidence was provided by exact diagonalization studies, where it was found that this trial wavefunction has an excellent overlap with the true ground state.

Based on this success, other states were rapidly proposed to explain the fractional quantum Hall effect at fillings $\nu = p/q$, seen in later experiments. Haldane proposed the *hierarchical construction* [29], in which quasi-particles over a Laughlin state again form a Laughlin liquid. A particularly successful phenomenological explanation of the stability of fractions was found by Jain [40], in the form of the *composite fermion* (CF) scheme. In this scheme an even number of flux quanta are bound to an electron and the composite fermions exhibit the integer quantum Hall effect in an effectively reduced magnetic field.

The composite fermion approach and in particular its field theoretic description have received a firm footing by a direct derivation from the microscopies by Baranov, Pruisken and Škorić [63, 64]. Their theory takes both disorder and long-range interactions into account and successfully yields the continuously varying tunneling exponents, observed in experiment.

**THE $\nu = \frac{5}{2}$ QUANTUM HALL EFFECT**

The one eminent deviation from the composite fermion liquid is the incompressible state seen at $\nu = \frac{5}{2}$ [98, 24]. All CF states share the property of an *odd denominator*, which makes it clear that this state does not fit into the existing framework. The first Landau level is filled and does not contribute to the dynamics. Effectively, then,
the filling factor is \( \nu' = \frac{1}{2} \). This is precisely the filling where composite fermions feel no magnetic field!

Among the states, proposed to explain this anomalous fraction, is the paired quantum Hall state of Moore and Read [55]. In this state, electrons are paired; the composite fermions form a BCS superconductor. Furthermore, when the temperature is raised to the point that the plateau disappears, Fermi liquid effects are seen. These indicate that the 'BCS transition' coincides with the plateau formation. Again, exact diagonalization studies have shown that the Moore-Read state is a successful trial wavefunction, confirming the first observation of a paired quantum Hall state. Many more details regarding this state will be presented in the next chapter.

In other half-filled Landau levels, many more examples of 'composite fermion Fermi liquid effects' have been seen. In the lowest Landau level, at \( \nu = \frac{1}{2} \) and \( \nu = \frac{3}{2} \), Shubnikov-de Haas oscillations are seen, indicating the presence of a Fermi surface. In Landau levels higher than the second the behaviour is anisotropic; a striped phase develops due to a density wave instability.

In chapters 5 and 6, we will extend the notion of pairing and find paired quantum Hall states for spin-\( \frac{1}{2} \) electrons. In particular, we find a quantum Hall state which displays spin-charge separation and a field-theoretic description of the liberated spin and charge degrees of freedom.