Quantum Hall spin liquids
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CHAPTER 5

QUANTUM HALL LIQUIDS OF SPIN-$\frac{1}{2}$ FERMIONS

Strongly correlated electrons in low dimensional systems are known to exhibit physical phenomena that are surprising and, at first sight, counterintuitive. Among these is the remarkable phenomenon of quantum number fractionalization: elementary excitations in strongly interacting many-electron systems can have quantum numbers (for spin and charge) that are fractions of those of the electron. This fractionalization can take the form of a separation of spin and charge degrees of freedom, or of a literal fractionalization of the electric charge of the electron.

The fractional quantum Hall effect has proven to be a rich playground of many-body physics. The elementary particles carry the quantum numbers of a fraction of an electron and their braiding and exclusion statistics interpolate between fermionic and bosonic. However, so far it seemed impossible to separate the spin degree of freedom from the charge. There is by now considerable literature on the description of abelian (spin-singlet) quantum Hall states in which the spin and charge bind together (in a so-called gluing theory) to form the physical excitations.

In this chapter$^1$, a series of quantum Hall spin liquids is proposed which displays a true separation of spin and charge. The presence of a pairing structure allows spinons and holons to move independently. We will also discuss a different class of non-abelian quantum Hall spin liquids, the so-called NASS states. These, however, do not exhibit a separation of spin and charge.

SPIN-CHARGE SEPARATION

In $D = 1$ spatial dimension, the separation of spin and charge is well understood. It is seen in explicit solutions of specific integrable model systems (Hubbard and supersymmetric $t$-$J$ models). The general framework of the Luttinger Liquid has made it

$^1$based on a collaboration with E. Ardonne, A.W.W. Ludwig and K. Schoutens [9]
clear that in 1+1 dimensions the separation of spin and charge is a generic feature, which does not require any fine tuning of the interactions among the electrons.

In spatial dimensions $D = 2$ or higher, spin and charge tend to confine and a separation of the two is only possible under very special conditions. It has been proposed that the key feature underlying the anomalous behavior of the cuprate high-$T_c$ materials is precisely a separation of spin and charge [3], and concrete scenario’s, based on $\mathbb{Z}_2$ or $U(1)$ gauge theories, have been put forward [82].

In this chapter, we discuss the separation of spin and charge in the quantum Hall (qH) regime. We propose a series of paired spin-singlet qH states, of filling factor $\nu = \frac{2}{2m+1}$, which are generalizations of the Moore-Read or pfaffian states for spin polarized electrons. The fundamental excitations over these states are spinons (with spin $\frac{1}{2}$ and zero charge) and holons (with zero spin and fractional charge $\pm \frac{1}{2m+1}$, in units of the charge of the electron). The braid statistics of these excitations are non-abelian, and thereby the paired spin-singlet states fall in the category of ‘non-abelian qH states’.

Even though the experimental relevance of the spin-charge separated state remains questionable, there is considerable theoretical motivation to understand the phenomenon of spin-charge separation.

**Spin-singlet states**

Before we present the non-abelian spin-singlet states, we briefly recall some facts about spin-singlet quantum Hall states. Despite the presence of strong magnetic fields in the qH regime, there is experimental motivation to study states that are not (fully) spin polarized (see e.g. [22]). In many qH systems, the energy scale for the Zeeman splitting is relatively low, and it can be further suppressed by the application of hydrostatic pressure. Using this technique, combined with a tilted field technique, spin transitions in the qH regime can be studied [43]. The simplest qH states that are singlets w.r.t. the $SU(2)$ spin symmetry are the Halperin states [31]

$$\mathcal{F}_{\text{Halperin}}^{(m+1, m+1, m)}(\{z_i^\uparrow, z_j^\downarrow\}) =$$

$$\Pi_{i<j}(z_i^\uparrow - z_j^\uparrow)^{m+1}\Pi_{i<j}(z_i^\downarrow - z_j^\downarrow)^{m+1}\Pi_{i=j}(z_i^\uparrow - z_i^\downarrow)^m,$$

where $z_i^\uparrow$ and $z_i^\downarrow$ are the coordinates of the spin up and spin down electrons, respectively, and $m$ is an even integer. The state eq. (5.2) has filling factor $\nu = \frac{2}{2m+1}$. Hierarchies of more general (abelian) spin-singlet states were studied in [72, 100, 49, 51].

It is important to stress that these conventional abelian spin-singlet qH states do not exhibit a separation of spin and charge. The excitations over such states are conveniently analyzed in terms of a ‘spin-charge decomposition’ [55, 10, 51] but this is subject to certain gluing conditions (expressing locality of the excitation w.r.t. the electrons), which exclude single spinons or holons from the (bulk) physical spectrum. The essential feature that liberates spin and charge in the paired states is
5.1. NASS QUANTUM HALL STATES

The (historically) first series of non-abelian quantum Hall states for spin-$\frac{1}{2}$ electrons is known by the name of 'non-abelian spin-singlet' (NASS) states [5]. These states are direct generalizations of the (bosonic) Halperin states and are similar to the $SU(4)_k$ states discussed in chapter 4. They live at filling factors $\nu = 2k/3$ and have the explicit form:

$$ N_{\text{NASS}}(\{z_i^1; z_i^2\}) = S \left( \prod_{\text{groups}} \Psi_{\text{Halperin}}^{(2,2,1)} \right) . \tag{5.2} $$

The $N = 2kn = 2N_1 = 2N_1$ particles are divided into $k$ groups, each with $n$ spin-up and spin-down particles. These states are $SU(2)$ singlets and have short range spin correlations. It is straightforward to construct the (ultra-local) $k + 1$-body Hamiltonians which have these states as unique ground states:

$$ H_{\text{NASS}}^k = V \sum_{i_1 < \cdots < i_{k+1}} \delta(z_{i_1} - z_{i_2}) \cdots \delta(z_{i_k} - z_{i_{k+1}}) . \tag{5.3} $$

To obtain (topological) information about this state, we need to identify the correct conformal field theory. The CFT was found by Ardonne and Schoutens [5] and Schoutens [5].
to be $SU(3)_k$, in fact by using the reversed construction. The electron operators have the form

\[ J^{(k)}_\alpha(z) = \psi^{(k)}_\alpha e^{i\bar{\beta} \cdot \vec{r}/\sqrt{k}}(z). \] (5.4)

The vectors $\bar{\beta}$ are indicated in the root lattice, fig. 5.1. The embedded $SU(2)$ affine Lie algebra, perpendicular to the charge direction, ensures that all correlators in this CFT are spin-singlets. The parafermions have conformal dimension $\Delta_\alpha = 1 - 1/k$. In figure 5.1, we have also indicated the operators ($\phi^\alpha$) that create quasi-holes. These holes, corresponding to flux $\Phi = \frac{1}{2k}\Phi_0$, carry charge $\frac{1}{3}$ and spin $\frac{1}{2}$. As one can see in the root lattice, only composites with integer spin can be neutral, such as $S^\pm$.

We will not pursue an in-depth treatment of these states, this can be found in ref. [5] for $k = 2$ and [6] for $k \geq 2$.

### 5.2 Spin-Charge Separated QH State

In search of a quantum Hall state which supports independent spinons and holons, it is illustrative to consider the fractionalization of charge in the paired $q$-paffan, spin polarized, states which were introduced in chapter 2. For the $q$-paffan states, Laughlin's gauge argument gives that the adiabatic insertion of a single flux quantum will produce an excitation of charge $\frac{1}{q}$. However, as in the case of BCS superconductors, the presence of the pairing condensate leads to a reduction of the elementary flux quantum by a factor of 2, and thereby the unit-flux Laughlin quasiparticles are separated into two constituents, each carrying a charge $\frac{1}{2q}$. In a similar way, conventional quasiparticles (carrying spin and charge) over a paired spin-singlet state can be, as shown below, separated into spinons and holons.

In the paired spin-singlet states that we propose here, the pairing takes place in the charge sector, irrespective of the spin of the electrons. This leads to a wave function

\[ \tilde{\Psi}^{(m)}_{\text{paired}}(\{z_i^\uparrow; z_j^\uparrow\}) = \text{PF} \left( \frac{1}{x_i - x_j} \right) \tilde{\Psi}^{(m+1,m+1,m)}_{\text{Halperin}}(\{z_i^\uparrow; z_j^\uparrow\}), \] (5.5)

where $x_i = z_i^\uparrow, z_j^\downarrow$, $m$ is now an odd integer and the filling factor is $\nu = \frac{2}{2m+1}$. Before discussing the excitations over this qH state, we interpret the wave function eq. (5.5) in terms of an associated conformal field theory (CFT).

### 5.3 Conformal Field Theory

Following the CFT-qH correspondence of chapter 2, one quickly finds that the CFT associated to the (bosonic) paired spin-singlet state at $m = 0$ is the (chiral) CFT based on the affine Kac-Moody algebra $SO(5)_1$. For this algebra, the eight currents associated to the roots of $SO(5)$ can be written in terms of spin and charge bosons $\varphi_{s,c}$ and a Majorana fermion $\psi$. [The assignment of spin and charge quantum...
numbers to the weights and roots of $SO(5)$ is indicated in fig. 5.2] For general $m$, the 'condensate' operators $\Psi$ and $\Delta$ are obtained from these currents by the substitution $\varphi_c \rightarrow \sqrt{2m+1} \varphi_c$:

$$
\Psi^\alpha = \psi e^{i \sqrt{2m+1} \varphi_s \pm \frac{i}{\sqrt{2}} \varphi_s}, \quad \overline{\Psi}^\alpha = \psi e^{-i \sqrt{2m+1} \varphi_s \pm \frac{i}{\sqrt{2}} \varphi_s},
$$

$$
\Delta_c = e^{i \sqrt{4m+2} \varphi_c}, \quad \overline{\Delta}_c = e^{-i \sqrt{4m+2} \varphi_c}, \quad \Delta_s^\alpha = e^{\pm i \sqrt{2} \varphi_s},
$$

with $\alpha = \uparrow, \downarrow$ referring to the spin eigenvalue $s_z = \pm \frac{1}{2}$ and $A = \uparrow \uparrow, \uparrow \downarrow, \downarrow \uparrow, \downarrow \downarrow$. The quantum numbers $q$ (charge) and $s_z$ are measured by the operators $Q = -i \sqrt{\frac{2}{2m+1}} \oint \frac{dz}{2\pi i} \partial \varphi_c$ and $S_z = \frac{1}{\sqrt{2}} \oint \frac{dz}{2\pi i} \partial \varphi_s$. The wave function eq. (5.5) is obtained as a correlator of $N$ spin-up electrons $\Psi^\uparrow$ and $N$ spin-down electrons $\Psi^\downarrow$, together with a neutralizing background charge. The CFT description makes it easy to identify the fundamental (quasi-particle) excitations. For $m = 0$ they are the operators that generate the spinor (4-dimensional) representation of the $SO(5)_1$ current algebra. For general $m$ these become

$$
\phi_c = \sigma e^{i \sqrt{4m+2} \varphi_c}, \quad \overline{\phi}_c = \sigma e^{-i \sqrt{4m+2} \varphi_c}, \quad \phi_s^\alpha = \sigma e^{\pm \frac{i}{\sqrt{2}} \varphi_s},
$$

where $\sigma(z)$ is the so-called spin field associated to the Majorana (Ising) fermion $\psi(z)$. Higher excitations, such as those constituting the vector representation, can be generated by bringing together two or more of the fundamental excitations. The expressions eq. (5.7) show that the fundamental excitations can be characterized as spinons $\phi_s^\alpha$ (spin-$\frac{1}{2}$ but no charge) and holons $\phi_c, \overline{\phi}_c$ (of charge $\pm \frac{1}{2m+1}$ and zero spin).
To illustrate the separation of spin and charge, we present explicit wave functions for excited states. We first consider an abelian excitation, with spin up \( s_z = \frac{1}{2} \) and charge \( \frac{1}{2m+1} \), at location \( w \). Its wave function takes the familiar form

\[
\prod_i (z_i - w)^j \tilde{\Psi}^{(m)}_{\text{paired}} .
\]

The important observation is now that, starting from this wave function, one can separate the locations of the spin and charge parts of this excitation, creating a spinon at position \( w_s \) and a holon at \( w_c \). In the corresponding wave function, the pfaffian factor in eq. (5.5) is replaced by (compare with [55])

\[
\text{Pf} \left( \frac{\Phi(x_i, x_j; w_c, w_s)}{x_i - x_j} \right) \prod_i (x_i - w_c)^{1/2} \prod_j (z_j - w_s)^{1/2} \prod_i (z_i - w_s)^{1/2} .
\]

where

\[
\Phi(x_i, x_j; w_c, w_s) = \left( \frac{x_i - w_c}{x_j - w_c} \right)^{1/2} + i \leftrightarrow j .
\]

That (5.9) in fact defines a well-behaved (see appendix 5.A) electronic wave function can be seen by noting that it is identical to

\[
\frac{1}{\prod_j (z_j^* - w_s)} \text{Pf} \left( \frac{(x_i - w_c)(x_j - w_s) + i \leftrightarrow j}{x_i - x_j} \right) .
\]

In the limit where \( w_s, w_c \to w \), spin and charge recombine and the wave function reduces to eq. (5.8).

The charge of the holon excitation equals \( \frac{1}{2} \Phi_0 \sigma_H \) (with \( \Phi_0 = \frac{\hbar}{e} \) the flux quantum), showing that the creation of a single holon involves the insertion of a half-quantum of magnetic flux, which is the canonical flux quantum in the presence of a pairing condensate. This flux insertion is accompanied by a vortex in the pairing condensate, and this brings in the factor \( \sigma(z) \) in the expressions eq. (5.7).

An important feature that is implied by the presence of spin-fields \( \sigma(z) \) in the expressions eq. (5.7) for the spinons and holons, is that the braid statistics of these excitations will be non-abelian. This feature is analogous to the non-abelian statistics of the charge \( \frac{1}{2q} \) excitations over the (spin-polarized) \( q \)-pfaffian state, and we refer to section 2.3 for a discussion.

### 5.4 State counting

**Deriving the interaction**

We can extract the interaction needed to produce the paired spin-singlet quantum Hall state as the exact ground state directly from the CFT. This procedure can be
carried out by the use of the operator product expansions (OPE's) of the theory. In fact, the interactions encode the exact same information of the CFT and it is possible to derive the OPE's from the interaction.

The OPE of two electron operators \( m = 1 \) is:

\[
\Psi^\dagger(z)\Psi^\dagger(w) \sim (z - w)e^{i\sqrt{6}\varphi_c + i\sqrt{2}\varphi_s}(w) + \ldots
\] (5.12)

\[
\Psi^\dagger(z)\Psi^\dagger(w) \sim e^{i\sqrt{6}\varphi_c}(w) + \ldots
\] (5.13)

so that two electrons can approach each other as close as the Pauli exclusion principle permits. There is no two-body interaction. If we approach with a third electron, however:

\[
e^{i\sqrt{6}\varphi_c + i\sqrt{2}\varphi_s}(0)\Psi^\dagger(z) \sim z^4\varphi_c e^{i3\sqrt{3/2}\varphi_c + i3/2\varphi_s}(0) + \ldots
\] (5.14)

\[
e^{i\sqrt{6}\varphi_c}(0)\Psi^\dagger(z) \sim z^3\varphi_c e^{i3\sqrt{3/2}\varphi_c + i3/2\varphi_s}(0) + \ldots
\] (5.15)

We see that there is no term proportional to \( z^2 \) in the first line, which implies that we need an interaction term which prohibits the closest approach of three spin-up electrons. The second line is found to imply two interaction terms, prohibiting the two closest approaches.

The Hamiltonian which has \( SO(5)_{1} \) as the unique zero-energy state for \( N_\varphi = \frac{3}{2}N - 3 \) is:

\[
H_{\text{paired}}^{(1)} = \sum_{i<j<k} V_0 P_{ijk} \left( \frac{3}{2}N_\varphi - 3, \frac{3}{2} \right) + V_1 P_{ijk} \left( \frac{3}{2}N_\varphi - 2, \frac{1}{2} \right)
+ V_2 P_{ijk} \left( \frac{3}{2}N_\varphi - 1, \frac{1}{2} \right),
\] (5.16)

where \( P_{ijk}(L, S) \) is the projector onto the 3-particle state with angular momentum \( L \) and spin \( S \). We can study the zero-energy eigenstates of this Hamiltonian, which should correspond to our CFT analysis.

We have carried out the reverse procedure, reconstruction of the CFT from the Hamiltonian, for an interaction closely related to the one presented here. When \( V_1 \) vanishes, the resulting Hamiltonian still defines a spin-singlet quantum Hall state at \( \nu = 2/3 \), however the conformal field theory now is \( SL(2|1)_{1} \) [77].

**Counting composites**

In a numerical analysis of the spin-charge separated state, we use the semi-positive Hamiltonian (5.16), which has the spin-charge separated state as the unique zero energy ground state of lowest angular momentum.

It is unfortunately not possible to see the separation of spin and charge in the zero-energy spectrum. This is due to the pole in the spinon wavefunction, which should be regularized. Upon regularization however, some of the correlations in the wavefunction, needed to vanish under (5.16), will be lost. The neutral spinon
therefore cannot be observed in the zero-energy spectrum. There is no fundamental reason that prohibits the existence of spinons within the lowest Landau level, only our decision to study zero-energy eigenstates of this particular Hamiltonian prevents their appearance.

Within the zero-energy eigenspace we should replace the spinon by an operator which does not create poles in the electronic wavefunction. Such an operator is easily found, by taking the composite of a spinon with a holon. It may look as though we are back to the spin-charge confined Halperin case, but the presence of individual holons complicates the analysis.

Let us first look at the Halperin states. The archetypical example is the $(1,1,0)$ state, the fully filled lowest Landau level. The number of $(\text{spin-}1/2)$ quasi-hole states is simply the number of states within the lowest Landau level with a given number of particles and flux:

$$\sum_{n_\uparrow, n_\downarrow} \binom{N_\uparrow + n_\uparrow}{n_\uparrow} \binom{N_\downarrow + n_\downarrow}{n_\downarrow},$$

where we sum over $n_\uparrow$ quasi-holes with spin down and $n_\downarrow$ holes with spin up. The constraint (indicated by the prime $'$ on the sum), which one can interpret as the (spin) singlet condition on a CFT correlator, is $N_\uparrow + n_\uparrow = N_\downarrow + n_\downarrow$. Furthermore $N_\uparrow + N_\downarrow = N$, the number of particles, and $n_\uparrow + n_\downarrow = n$, the amount of excess flux.

We can now introduce a composite operator which creates both an up and a down quasi-hole at the same position. This composite has charge 2 and spin 0. The existence of such an operator introduces an exclusion between up and down holes. We rewrite the above counting formula as

$$\sum_{n_\uparrow, n_\downarrow, m} \binom{N_\uparrow + n_\uparrow - n_\downarrow}{n_\uparrow} \binom{N_\downarrow + n_\downarrow - n_\uparrow}{n_\downarrow} \binom{N + m}{m}.$$  

We interpret $m$ as enumerating the number of composites. We see that the number of available orbitals for a $\uparrow$ ($\downarrow$) hole is reduced by the number of $\downarrow$ ($\uparrow$) holes.

**Final counting formula**

For the final counting formula we should take the degeneracies, present due to the parafermionic CFT, into account. Fortunately, we have already obtained the counting formula for the Ising model (Moore-Read state) in chapter 2. The number of states which can be obtained by only creating holons is equal to the Moore-Read result:

$$\#_{\text{MR}}(N, n_h) = \sum_{F, n_h} \binom{n_h/2}{F} \binom{(N - F)/2 + n_h}{n_h}.$$  

In the combined spectrum of charged spinons and holons, we note that the singlet composite which was introduced above can also be obtained as the fusion
of two holons. A natural solution to the counting problem now is to sum over all distributions of \( n_h, n_{\uparrow} \) and \( n_{\downarrow} \) with the constraints

\[
N_{\uparrow} + n_{\uparrow} = N_{\downarrow} + n_{\downarrow}
\]

\[
n_h + n_{\uparrow} + n_{\downarrow} = 2n
\]

with \( n \) the number of excess flux quanta. We also take into account the exclusion between up and down spinons and arrive at the following state-counting formula for quasi-particles over the \( SO(5) \) state:

\[
\sum_{n_h, n_{\uparrow}, n_{\downarrow}} \left( \frac{n_h}{2} \right) \left( \frac{(N - F)/2 + n_h}{n_h} \right) \left( \frac{N_{\uparrow} + n_{\uparrow} - n_{\downarrow}}{n_{\uparrow}} \right) \left( \frac{N_{\downarrow} + n_{\downarrow} - n_{\uparrow}}{n_{\downarrow}} \right)
\]

(5.22)

We have checked this formula, in exact diagonalization, for a variety of \( (N, N_{\phi}) \) combinations and found full agreement.

## 5.5 DISCUSSION

We have presented several quantum Hall spin liquids of spin-\( \frac{1}{2} \) fermions, including a series of paired spin-singlet states which display spin-charge separation. Unfortunately, the natural context to search for these states, the quantum Hall effect, does not seem to support these exotic states of matter. The pure Coulomb interaction favors the anti-parallel flux attached composite fermion state [100] at filling factor \( \nu = 2/3 \). Since no second order phase transition is possible between the paired spin-singlet state and this composite fermion state, it does not seem likely that a small deviation from the Coulomb interaction could change this situation.

If we interpret the spin of the electron as a layer index, then we find a description of a double layer quantum Hall system at \( \nu = 2/3 \). This system has been studied in detail by Moore and Haldane [56], who found three different regimes. Depending on the tunneling rate and the distance between the layers, either two decoupled \( \nu = 1/3 \) Laughlin liquids, the anti-parallel flux composite fermion state or the particle-hole conjugate of the \( \nu = 1/3 \) Laughlin state is found. It seems unlikely that there is room in their phase diagram for a fourth, the spin-charge separated, state.

The situation is reminiscent of the situation in the \( Z_2 \) theory of fractionalization [80]. The experimental context (high-\( T_c \) superconductors) does not seem to realize the fractionalization [81, 14]. There is, however, considerable theoretical motivation to understand fractionalization patterns.

The two theories of spin-charge separation are in fact closely related. The elementary excitations, spinors and holons, both introduce a vortex in the pairing condensate. In the \( Z_2 \) theory, the \( Z_2 \) charge of holons and spinons is liberated by a paired condensate of \( Z_2 \) flux. An important difference between the paired spin-singlet quantum Hall and the \( Z_2 \) case is that the \( p \)-wave pairing in the quantum Hall state introduces a zero mode at the core of the holon (and spinon). In the Senthil-Fisher scenario, vortices form \( s \)-wave pairs and the absence of a vison zero mode makes the braiding abelian.
A field theoretic description, which gives a 2D interpretation of the CFT results, is presented in the next chapter.
5.A QUASI-PARTICLES

In the case of the spin-charge separated state described in the previous chapter, there is a complication in the numerical analysis. As one can see from eq. (5.11), a pole is introduced in the electronic wavefunction when a spinon is present.

This is in fact a situation very similar to the quasi-particle over the Laughlin $\nu = 1/3$ state. When the CFT formalism is directly applied by using a vertex operator with charge $-1/3$, the wavefunction that follows is

$$\tilde{\Psi}_{1/3}^{1/3} = \prod_i \frac{1}{z_i - w} \prod_{i<j} (z_i - z_j)^3.$$ (5.A.1)

This makes it clear that the CFT connection breaks down at short distances. This was to be expected, since correlators in a conformal field theory are not aware of any length scale. Therefore, the topological properties that the wavefunctions have are present at all length scales.

Although the quasi-particle present in the excitation spectrum over a quantum Hall state has not been identified with an operator in the conformal field theory, we believe that the above operator does carry the correct topological information.

REGULARIZATION

The pole in the quasi-particle wavefunction can be regularized in various ways. Unfortunately, none of these procedures give rise to trial wavefunctions which approximate the true quasi-particle as well as the quasi-hole. The original proposal by Laughlin, for a quasi-particle located at the origin, is:

$$\tilde{\Psi}_{1/3}^{1/3} = \prod_i \left(2 \frac{\partial}{\partial z_i}\right) \prod_{i<j} (z_i - z_j)^3.$$ (5.A.2)

A different proposal, by Jeon and Jain [41], is based on the composite fermion approach to the construction of trial states. One starts with a wavefunction with amplitude outside the LLL,

$$\tilde{\Psi}_{\text{unprojected}} = \begin{vmatrix} z_1 & \cdots & z_N \\ 1 & \cdots & 1 \\ \vdots & \cdots & \vdots \\ z_1^{N-2} & \cdots & z_N^{N-2} \end{vmatrix} \prod_{i<j} (z_i - z_j)^2$$ (5.A.3)

and proceeds by projecting into the lowest Landau level. The complex coordinates are normal ordered, i.e. the $\tilde{z}_i$'s are moved to the left. Then, one replaces them as $\tilde{z}_i \rightarrow 2\theta_i$.

All regularization schemes give the same topological properties, which are our main interest. We expect, therefore, that a suitable procedure makes it possible to obtain regular electronic wavefunctions with well-defined neutral spinons.