The Formation of a Core Periphery Structure in Heterogeneous Financial Networks

in 't Veld, D.; van der Leij, M.; Hommes, C.

TI 2014-098/II
Tinbergen Institute Discussion Paper

The Formation of a Core Periphery Structure in Heterogeneous Financial Networks

Revision: May 2, 2019

Daan in 't Veld[1]
Marco van der Leij[1,2]
Cars Hommes[1]

[1] Faculty of Economics and Business, University of Amsterdam, and Tinbergen Institute, The Netherlands
[2] De Nederlandsche Bank, The Netherlands

The formation of a core-periphery structure in heterogeneous financial networks

Daan in 't Veld[a,b,*], Marco van der Leij[a,c,d,*], Cars Hommes[a,c]

[a] *CeNDEF, University of Amsterdam, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands*
[b] *SEO Amsterdam Economics, Roetersstraat 29, 1018 WB Amsterdam, The Netherlands*
[c] *Tinbergen Institute, Gustav Mahlerplein 117, 1082 MS Amsterdam, The Netherlands*
[d] *Research Department, De Nederlandsche Bank, Westeinde 1, 1017 ZN Amsterdam, The Netherlands*

May 2, 2019

Abstract

Recent empirical evidence suggests that financial networks exhibit a core-periphery network structure. This paper aims at giving an explanation for the emergence of such a structure using network formation theory. We propose a simple model of the overnight interbank lending market, in which banks compete for intermediation benefits. Focusing on the role of bank heterogeneity, we find that a core-periphery network cannot be unilaterally stable when banks are homogeneous. A core-periphery network structure can form endogenously, however, if we allow for heterogeneity among banks in size. Moreover, size heterogeneity may arise endogenously if payoffs feed back into bank size.

Keywords: financial networks, core-periphery structure, network formation models, over-the-counter markets, interbank market
JEL classifications: D85, G21, L14

1. Introduction

The extraordinary events of 2007 and 2008 in which the financial system almost experienced a global meltdown, led to an increased interest in the role of financial networks, the network of trading relationships and exposures between financial institutions,[1] on systemic risk, the risk that liquidity or solvency problems in one financial institution spread to the whole financial sector. Building on pre-crisis work by Allen and Gale (2000) and Eisenberg and Noe (2001), an extensive body of theoretical, simulation and empirical research has shown that the structure of the network

☆The authors would like to thank Darrell Duffie, Ester Faia, Filomena Garcia, Michael Gofman, Matthew Jackson, Christian Julliard and Mariëlle Non for comments. We also thank participants at several workshops and conferences for their attendance and comments. The research has been supported by the Netherlands Organisation for Scientific Research (NWO) under the Complexity program *Understanding financial instability through complex systems* (grant number: 645.000.015). Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

*Corresponding authors: Daan in 't Veld, (`D.intVeld@seo.nl`), Marco van der Leij, (`M.J.vanderLeij@uva.nl`)

[1] In this paper we use the words 'financial institutions' and 'banks' interchangeably.

of interbank liabilities matters for the likelihood and the extent of financial contagion.[2]

Importantly, however, until recently almost all of this work assumed that the network of financial interconnections is *exogenously fixed*. This assumption ignores the fact that financial networks do not come out of the blue. Financial relations are formed consciously by financial institutions who borrow, lend and trade financial assets with each other in order to maximize profits. This is important, as a change in the risk or regulatory environment may incentivize financial institutions to rearrange their financial links. This change may in itself constitute a financial crisis, for example in the case of an interbank market freeze, which may be interpreted as a sudden shift from a connected to an empty financial interbank network.

It is therefore important to better understand the formation process of financial networks. A natural starting point is to try to explain stylized facts about financial network structure. Regarding financial networks, one consistent empirical finding is that financial networks of interbank markets have a structure close to a *core-periphery structure*, which is defined as a connected network that has two tiers, a core and a periphery, the core forming a fully connected clique, whereas peripheral banks are only connected to the core (Borgatti and Everett, 1999). For example, Peltonen et al. (2014) find such a structure for financial networks in derivative markets, Di Maggio et al. (2017) in the corporate bond markets, and Craig and Von Peter (2014) and in 't Veld and van Lelyveld (2014) in interbank markets in respectively Germany and The Netherlands. Moreover, in 't Veld and van Lelyveld (2014) show that a core-periphery network structure fits the actual Dutch interbank network better than alternative network structures, such as a scale free network or a nested split graph.[3]

This paper aims to contribute to our understanding of why such core-periphery networks are formed. This aim is similar to recent work by Castiglionesi and Navarro (2016), Bedayo et al. (2016), Farboodi (2017), and Chang and Zhang (2019). Differently from those papers, we aim to understand if such a core-periphery structure may arise *endogenously from ex-ante identical banks*.

We present a network formation model of a financial market with an explicit role for *intermediation*. The model builds on work of Goyal and Vega-Redondo (2007). They show that, starting from ex-ante identical agents, the star network with a single intermediating counterparty, quickly arises in an environment in which relations are costly.[4] Intuitively, this result arises from network effects for intermediation; it becomes more attractive to link to an intermediator if the intermediator has already many links. However, in practice, rather than simple stars, we observe core-periphery net-

[2]See Glasserman and Young (2016) for a comprehensive review, and Gai et al. (2011), Elliott et al. (2014) and Acemoglu et al. (2015) for seminal papers.

[3]See Cohen-Cole et al. (2015) and König et al. (2014) for theoretical financial network models that have nested split graphs as an outcome.

[4]A star network is a network in which one and only one node, the center, is connected to all periphery nodes, and no other links exist. Formally, the star may be considered a trivial case of a core-periphery network. Our interest is in core-periphery networks that are not star networks.

works that have *multiple banks* in the core. Moreover, core banks tend to form a fully connected *clique*. Such core-periphery networks are not stable in the framework of Goyal and Vega-Redondo (2007). An important reason for that is their assumption of perfect competition for intermediation benefits. This assumption drives payoffs to zero for any network in which there are two or more members in the core.[5] On the other hand, *imperfect* competition for intermediation benefits allow for positive payoffs for competing core players, which may allow for the possibility of non-trivial stable core-periphery networks.

In order to see if core-periphery networks are stable in an environment of imperfect competition for intermediation benefits, we therefore extend the framework of Goyal and Vega-Redondo (2007). First, we propose a simple model of interbank overnight lending, in which banks receive positive or negative liquidity shocks on a daily basis, creating trading opportunities for banks on the interbank overnight lending market. We assume that trade can only take place between costly long-term trading relationships, allowing for the possibility of intermediation benefits. Intermediators compete for these benefits, but unlike Goyal and Vega-Redondo (2007) competition is *imperfect*, opening up the possibility that multiple core players benefit from intermediation. With the benefits from this trading network as a second stage, we then consider first-stage network formation of long-term trading relationships. Apart from the usual equilibrium concept of pairwise stability (Jackson and Wolinsky, 1996), we also consider the stronger concept of *unilateral stability*. This concept allows for deviations of deleting or adding multiple links.

We ask ourselves if the core-periphery network is stable in this model. To our surprise, in general, the answer is no. We provide three results on that. First, Proposition 2 shows that a core-periphery network, in which the set of connections of one core player contains the set of another core player, is not pairwise (let alone unilaterally) stable. The intuition behind this result is as follows: a stable core-periphery network implies that periphery banks prefer to trade indirectly via intermediating core banks, rather than trade directly. However, given that periphery and core banks have identical technologies, the core bank with the larger set of connections should have an incentive to trade indirectly via peripheral banks as intermediators as well. Hence, the core player does not have an incentive to maintain all direct trading relationships in the core, in contradiction to the definition of a core-periphery network.

Second, Proposition 3 states that, when the periphery becomes very large compared to the core, a core-periphery network cannot be unilaterally stable. For large enough networks, the payoff inequality between core and periphery banks becomes unsustainable large, as intermediation benefits for core banks grow quadratically with the number of periphery banks. Periphery banks therefore have an incentive to enter the core, even if competition between intermediators reduces their

[5] Babus and Hu (2017) consider a financial network formation model that also builds on Goyal and Vega-Redondo (2007). In their model an interlinked star network with 2 members in the core may be stable. However, similarly as in Goyal and Vega-Redondo (2007) core-periphery networks with 3 or more nodes in the core are not stable in the model of Babus and Hu (2017).

benefits.

The first two results, still leave open the possibility for unilaterally stable core-periphery networks under specific conditions. Theorem 1, however, shows that, in a dynamic setting, stable core-periphery networks are unlikely to arise. In particular, we show that, in a simple dynamic model a la Kleinberg et al. (2008), best-response dynamics never converge to a core-periphery network. Interestingly, instead of core-periphery networks, we find that *multipartite* networks may be a stable outcome. These type of networks are two- or multi-tiered as well; however, unlike core-periphery networks, they do not have links within a (core) tier.[6]

Key to these findings is that banks are ex ante identical; periphery banks obtain the same trading opportunities and have the same intermediation technology as core banks . This puts a limit on inequality, and therefore excludes the stability of a core-periphery network. We therefore investigate the role of heterogeneity in our model. We analyze a version with two types of banks, big banks and small banks, and allow big banks to have more frequent trading opportunities. Proposition 4 shows that for sufficiently large differences between big and small banks, it becomes beneficial for large banks to have direct lending relationships with all other large banks in the core, such that the core-periphery network becomes a stable structure.

Finally, we show that this heterogeneity between banks, and in fact a stable core-periphery network, may arise *endogenously* with ex ante identical banks, if one allows for a feedback loop from inequality in payoffs to inequality in size. This process works as follows. Starting from identical banks, best-response dynamics may converge to an unequal multipartite network, such that one side earns more than the other side of the network. Due to the feedback from payoff to size, the banks on the side that earn more increase their scale, until it finally becomes attractive for the largest banks to trade directly, forming a core-periphery network structure.

We now review the literature on financial network formation. One may make a distinction between papers that are more concerned with the trade offs between contagion, risk sharing, efficiency and stability (Acemoglu et al., 2015; Babus, 2016; Cabrales et al., 2017), and papers that (among other things) rationalize the formation of a core-periphery structure in financial networks (Bedayo et al., 2016; Castiglionesi and Navarro, 2016; **?**; Farboodi, 2017; Wang, 2018). Our paper belongs to the second category. It is interesting to note the role of heterogeneity in these papers. In papers of the first category (Cabrales et al., 2017; Acemoglu et al., 2015) ex-ante homogeneity is assumed, and indeed, the resulting efficient or stable networks do not correspond to core-periphery networks at all.[7] On the other hand, in papers of the second category heterogeneity usually plays a

[6]Next to stability we also investigate efficiency. Theorem 2 characterizes the efficient networks as the empty network or the star network. The core-periphery network and (stable) multipartite network are never efficient networks.

[7]In Babus (2016) it is assumed that there are 2 regions with negatively correlated shocks, such that a bipartite network arises.

key role. In Farboodi (2017) banks are heterogeneous in their investment opportunities, and they compete for intermediation benefits. A core-periphery network is formed with investment banks forming the core, as they are able to offer better intermediation rates. In Bedayo et al. (2016) intermediation also plays a key role, with intermediaries bargaining sequentially and bilaterally on a path between two traders. Here agents are heterogeneous in their time discounting. They find that a core-periphery network is formed with impatient agents in the core. In Castiglionesi and Navarro (2016) heterogeneity in investments arises endogenously. Some banks invest in safe projects, and others in risky projects. Links are created as a coinsurance to liquidity shocks. Safe banks link freely with each other, but the incentives to link to risky banks is limited, leading to a core-periphery like structure.[8,9]

Overall, the main message of our paper is that bank heterogeneity matters for the formation of financial core-periphery networks, and that, in order to understand the financial system and its (systemic) risks, it is crucial to understand which types of heterogeneity and which mechanisms are driving the core-periphery network structure. This is particular relevant, because inefficiency results tend to depend on the particular type of heterogeneity.

This paper is organized as follows. In Section 2 we introduce our model with the basic network structures, the pay-off function and the stability concepts. Our main results are presented separately for homogeneous traders (Section 3) and heterogeneous traders (Section 4). In Section 5 we provide an application of our model to the interbank market of the Netherlands. Section 6 concludes.

2. Model

Our goal is to model the formation of a network of *long-term* trading relationships between banks. We denote the set of banks by N, and the number of banks by n. There are two stages. In the first stage, at time $t = 0$, banks form an undirected network, g, of these trading relationships. Denote by $g_{ij} = g_{ji} = 1$ the existence of a trading relationship, and by $g_{ij} = g_{ji} = 0$ the absence of it. After forming their long-term trading relationships, liquidity trade takes place through these relationships in an infinite number of periods, $t = 1, 2, \ldots$. Payoffs from forming trading relationships at time

[8] In the model of Castiglionesi and Navarro (2016) periphery bank may form links with each other. This contradicts the definition of a core-periphery network of Borgatti and Everett (1999), Craig and Von Peter (2014) and in 't Veld and van Lelyveld (2014), which we follow. In **?** a hierarchical multi-tier financial system is formed in which the most stable banks intermediate for other banks and have many trading relations. In their model, banks do not have incentives to link with other banks in the same tier, such that the equilibrium structure in **?** is a multipartite network. Again, this structure violates the definition of a core-periphery network that we apply.

[9] Wang (2018) has a model with ex-ante identical traders in which some of them act as dealers who manage an asset inventory and provide price quotes. He shows that core-periphery networks as well as bipartite networks can be equilibrium outcomes. There are other papers in the social science literature that explain core-periphery networks. These network formation models are typically concerned with optimal effort levels to account for peer effects. Galeotti and Goyal (2010) and Hiller (2017) provide conditions under which core-periphery networks are the only stable network structure. See also Persitz (2016) who adopts heterogeneity in the connections model of Jackson and Wolinsky (1996). This literature cannot easily be translated to financial networks, because of the different interpretation of links as (channels for) financial transactions.

0 depend on expected present value trade benefits from these liquidity trades and the costs of maintaining relationships.

2.1. Basic structures

Before discussing the payoff structure of the model, we first define the relevant network structures around which our analysis revolves. Denote the *empty network*, g^e, as the network without any links, i.e. $\forall i, j \in N : g_{ij} = 0$, and the *complete network*, g^c, as the network with all possible links, i.e. $\forall i, j \in N : g_{ij} = 1$. A *star network*, g^s, has a single player, the center of the star, that is connected to all other nodes, while no other links exist, i.e. $\exists i$ such that $\forall j \neq i : g_{ij} = 1$ and $\forall j, k \neq i : g_{jk} = 0$.

A *core-periphery network* is a network, in which the set of agents can be partitioned in a core and a periphery, such that all agents in the core are completely connected within and are linked to some periphery agents, and all agents in the periphery have at least one link to the core, but no links to other periphery agents.

Definition 1. *A network g is a core-periphery network, if there exists a set of core agents $K \subset N$ and periphery agents $P = N \setminus K$, such that:*

(a) $\forall i, j \in K : g_{ij} = 1$, *and* $\forall i, j \in P : g_{ij} = 0$*;*

(b) $\forall i \in K : \exists j \in P$ *with* $g_{ij} = 1$, *and* $\forall j \in P : \exists i \in K$ *with* $g_{ij} = 1$.

This definition follows Borgatti and Everett (1999), Craig and Von Peter (2014) and in 't Veld and van Lelyveld (2014). See Figure 1 for an example of an core-periphery network.

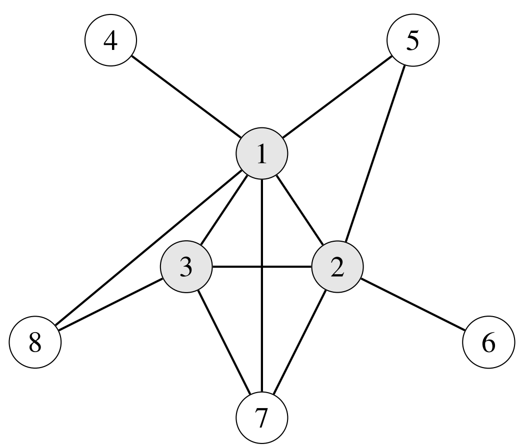


Figure 1: A core-periphery network with $n = 8$ players, of which $k = 3$ are in the core

A special case of a core-periphery network is the *complete core-periphery network*, where each agent in the core K is linked to *all* agents in the periphery P: $\forall i \in K$ and $\forall j \in P$ it holds that $g_{ij} = 1$. See Figure 2 for an example. We denote a complete core-periphery network with $k = |K|$ agents in the core as $g_{com}^{CP(k)}$.[10] In Section 4, we use this special case to show the stability of core-periphery networks under heterogeneous traders.

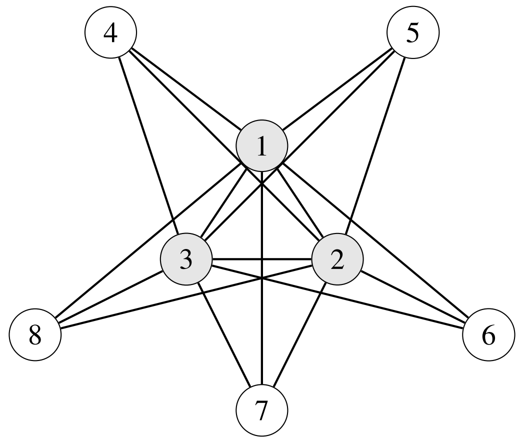


Figure 2: A complete core-periphery network with $n = 8$ players, of which $k = 3$ are in the core

Finally, a (complete) *multipartite network* is a network in which the agents can be partitioned into q groups, i.e. $N = \{K_1, K_2, \ldots, K_q\}$, such that nodes do not have links within their group, but are connected to (all) nodes outside their own group. Formally, in a complete multipartite network it holds that $\forall m \in \{1, 2, \ldots, q\} : \forall i \in K_m$ we have $\forall j \in K_m : g_{ij} = 0$ and $\forall j \notin K_m : g_{ij} = 1$. All relevant multipartite networks in this paper are *complete* multipartite networks and therefore, for brevity, we will drop the word 'complete'. Multipartite networks will be denoted as $g_{k_1,k_2,\ldots,k_q}^{mp(q)}$, where $k_m \equiv |K_m|$ is the size of the m-th group. Multipartite networks are called *balanced* if the group sizes are as close as possible to each other, i.e. $|k_m - k_{m'}| \leq 1$ for all m, m'. Figure 7 in Section 3.2 presents examples of multipartite networks that arise in our model.

2.2. Trading Benefits

We now turn to the actual model. At $t = 0$, banks form a network of long-term trading relationships. Given the network, liquidity trade takes place through these relationships in an infinite number of periods, $t = 1, 2, \ldots$.

We first discuss the payoffs in a single trading period. We assume that trades cannot take place outside the long-term trading relationships. Hence, in each period a (directed) trading network arises which is a subnetwork of the (undirected) relationship network g. Trade in each period occurs as follows. At the beginning of period $t \in \{1, 2, \ldots\}$ each bank i receives a random unexpected liquidity shock $S_{it} \in \{-s, 0, s\}$, independently distributed with probabilities $P[S_{it} = -s] = P[S_{it} = s] =$

[10]Borgatti and Everett (1999) call this architecture a perfect core-periphery network. By Definition 1, empty, star and complete networks are special cases of core-periphery networks with cores of size $k = 0$, $k = 1$ and $k = n$ respectively. A complete core-periphery networks with $k = n - 1$ is also identical to a complete network. In discussing our results we will make clear when we are speaking of non-trivial core-periphery networks with $k \in \{2, 3, ..., n-2\}$.

γ_i and $P[S_{it} = 0] = 1 - 2\gamma_i$.[11] Without loss of generality, we set $s = 1/2$.

A liquidity shock has a temporary effect on the liquidity position of the bank. That is, in that period t, the bank may have a liquidity surplus or deficit relative to a liquidity target, set by the Central Bank or by internal risk management procedures, but at the beginning of the next period, each bank is again in a neutral position. The effect of liquidity shocks are thus temporary, as in the intermediate run banks are able to manage their liquidity position by increasing or decreasing their asset size, for example by extending more or less loans. Liquidity positions do not carry over to the next period, and this excludes the possibility of speculation or storage of liquidity. Hence, our model differs from the search and matching models of over-the-counter markets, as introduced by Duffie et al. (2005), in which trading positions remain until a suitable trading partner is found.

If bank i has a positive liquidity shock, it may set aside the resulting liquidity surplus at the Central Bank, receiving interest rate $\underline{r}$. On the other hand, if the bank receives a negative shock, it may borrow liquidity from the Central Bank at cost $\bar{r}$. In practice, Central Banks set a positive interest rate wedge $\bar{r} - \underline{r} > 0$, such that trading opportunities arise between banks that receive a positive liquidity shock and banks that receive a negative liquidity shock. The main motivation for Central Banks to do so, and hence to have an active liquidity interbank market in the first place, is the belief that a private interbank market is better to able to monitor banking risks and allocate liquidity than a Central bank is able to (Rochet and Tirole, 1996). Without loss of generality, we set $\bar{r} - \underline{r} = 1$.

In general, in each period there may be multiple banks with a positive and multiple banks with a negative liquidity shock, leading to a myriad of potential trades. In order to keep the analysis tractable, we analyze the model in case the only relevant trading opportunities arise from one (and only one) bank, say i, receiving a positive shock and one (and only one) other bank, say j, receiving a negative shock. This situation occurs when the probability of a liquidity shock becomes small, and is formerly worked out in Proposition 1 of Section 2.3.

Now, suppose that bank i receives a positive shock, $S_{it} = 1/2$, bank j receives a negative shock, $S_{jt} = -1/2$, and all other banks receive no shock, $S_{kt} = 0$ for all $k \neq i, j$. We assume that a liquidity trade between i and j can only be realized if i and j have a direct (long-term) trading relationship, $g_{ij} = 1$, or an indirect trading relationship through one or more middlemen, who are directly connected to both i and j. Denote the set of these middlemen in g as $M_{ij}(g) = \{k : g_{ik} = g_{jk} = 1\}$, and the number of middlemen as $m_{ij} \equiv m_{ij}(g) = |M_{ij}(g)|$. For trade to be realized, we require that these middlemen are directly connected to i and j. In network parlor, it implies that trade between i and j can only be realized if the network distance, that is the shortest path length, between i and

[11] Above assumptions impose a strong symmetry in the model as regards to the treatment of positive and negative liquidity shocks. We do this to keep the model analytically tractable. However, in actual interbank markets, asymmetry in the effect of positive and negative liquidity shocks does play an important role. We leave it to future research to determine the effect of this asymmetry on interbank network formation.

j in g is at most 2. We make this assumption to simplify notation. However, we emphasize that our main results – core-periphery networks are generally unstable under homogeneity, but can be stable if agents are heterogeneous – do *not* depend on this assumption; the proposition also holds if i and j are connected at distance more than 2. This is shown in the proofs of Propositions 2 and 3, in Appendix C.

If the network distance between i and j is at most 2, trade is realized, and agents divide the surplus of 1 in the following way. Let $v_k^{ij}(g,\delta)$ be the share that k receives if i receives a positive shock and j a negative shock. Apart from the network structure g, we let the share that each agent obtains depend on a parameter $\delta \in [0,1]$. This parameter captures the level of *competition* between the number of middlemen m_{ij} of a certain trade between i and j. If $\delta = 0$, then middlemen collude and act as if there were only a single middleman between i and j. If $\delta = 1$, then middlemen engage in Bertrand competition whenever $m_{ij} > 1$; then banks i and j share the surplus.

More concretely, if i and j are directly connected, then $v_i^{ij}(g,\delta) = f_l(0,\delta) > 0$ and $v_j^{ij}(g,\delta) = f_b(0,\delta) > 0$, such that $f_l(0,\delta) + f_b(0,\delta) = 1$. Note that the level of competition (between middlemen) is irrelevant for this trade, so $f_l(0,\delta)$ and $f_l(0,\delta)$ are independent of δ.

If i and j are indirectly connected in g by m_{ij} middlemen, then the bank with a positive liquidity shock i (the lender) receives a share of $v_i^{ij}(g,\delta) = f_l(m_{ij},\delta)$ and the bank with a negative liquidity shock j (the borrower) receives a share of $v_j^{ij}(g,\delta) = f_b(m_{ij},\delta)$. Each of the m_{ij} middlemen receives a share of $v_k^{ij}(g,\delta) = f_m(m_{ij},\delta)$ if $k \in M_{ij}(g)$ and 0 otherwise. Note that by definition:

$$f_l(m_{ij},\delta) + m_{ij} f_m(m_{ij},\delta) + f_b(m_{ij},\delta) = 1.$$

If there is one middleman, $m_{ij} = 1$, then we have $f_l(1,\delta) < f_l(0,\delta)$, $f_b(1,\delta) < f_b(0,\delta)$, and $f_m(1,\delta) = 1 - f_l(1,\delta) - f_b(1,\delta) > 0$, and then the shares are independent of δ. If there is more than one middleman, then the distribution over agents depends on the level of competition. If $\delta = 0$, the middlemen collude, and i and j obtain the same share as if there were one intermediator, $f_l(m_{ij},0) = f_l(1,\delta)$ and $f_b(m_{ij},0) = f_b(1,\delta)$. The middlemen share the intermediation benefits evenly, i.e. $\forall m_{ij} \in \{2,\ldots,n-2\} : f_m(m_{ij},0) = f_m(1,\delta)/m_{ij}$.

If $\delta = 1$ and there are $m_{ij} > 1$ intermediaries, perfect competition drives the intermediary shares to 0, and $f_l(m_{ij},1) = f_l(0,\delta)$, $f_b(m_{ij},1) = f_b(0,\delta)$ and $f_m(m_{ij},1) = 0$. A further straightforward assumption is monotonicity with respect to the competitiveness and number of middlemen. Table 1 summarizes the assumed dependencies of the surplus distributions $f_l(\cdot)$, $f_b(\cdot)$ and $f_m(\cdot)$ to the parameters.

The effect of competition parameter δ on the division of the trade surplus and intermediation benefits can be thought of as arising from a bargaining process, in which δ is the discount factor of players, such that bargaining power of the end nodes (that is, the banks with the positive and negative liquidity shock) increases with δ and intermediation benefits decrease. We do not model

Share of payoffs for:	lender	borrower	middlemen
Number of middlemen m	$f_l(1,.) = f_l^1$ $\frac{\partial f_l}{\partial m} > 0$	$f_b(1,.) = f_b^1$ $\frac{\partial f_b}{\partial m} > 0$	$f_m(1,.) = f_m^1$ $\frac{\partial f_m}{\partial m} < 0$
Level of competition δ	$f_l(.,0) = f_l^1$ $\frac{\partial f_l}{\partial \delta} > 0$ $f_l(.,1) = f_l^0$	$f_b(.,0) = f_b^1$ $\frac{\partial f_b}{\partial \delta} > 0$ $f_b(.,1) = f_b^0$	$f_m(.,0) = \frac{f_m^1}{m_{ij}}$ $\frac{\partial f_m}{\partial \delta} < 0$ $f_m(.,1) = 0$

Table 1: Assumptions about the payoff shares $f_l(m_{ij},\delta)$, $f_b(m_{ij},\delta)$ and $f_m(m_{ij},\delta)$.

this bargaining process explicitly. However, our assumptions on $f_l(\cdot)$, $f_b(\cdot)$ and $f_m(\cdot)$ generalize an explicit Rubinstein-type of bargaining process developed by Siedlarek (2015).[12] From his bargaining protocol the distribution of the surplus is given by

$$f_l(m,\delta) = f_b(m,\delta) = \frac{m-\delta}{m(3-\delta)-2\delta} \text{ and } f_m(m,\delta) = \frac{1-\delta}{m(3-\delta)-2\delta}. \tag{1}$$

It is easily checked that (1) satisfies the assumptions we made on $f_l(\cdot)$, $f_b(\cdot)$ and $f_m(\cdot)$. Below we will use this explicit function to illustrate our (more general) results in Figures 6, 9 and 11.

Figure 3 gives examples of payoff shares received by different agents involved in a trade between i and j for different levels of competition δ and different numbers of middlemen m_{ij}, given equations (1). In Appendix A, we give a detailed explanation of the specification of payoffs by Siedlarek (2015).

2.3. Network payoffs

We now turn to the payoff from network formation in period $t = 0$. In period zero, payoffs from network formation are determined by the expected net present value of payoffs from trades in subsequent periods minus the net present value of the costs of maintaining trading relationships.

Let $S_t = \{S_{it}\}_{i \in N}$ and shock $s_t \in \{-1/2, 0, 1/2\}^n$ a realisation of S_t. Let $\beta \in [0,1)$ be the discount factor. Let $b(s_t, g, \delta) \in [0,1]^n$ the vector of trade benefits of the n agents given network $g \in G$, and shock realisation s_t. Let $\tilde{c}$ be the cost of maintaining a link for one period. Then the payoff function of bank i is given by

$$\tilde{\pi}_i(g,\delta,\tilde{c}) = \sum_{t=1}^{\infty} \beta^t \left(E[b_i(S_t, g, \delta)] - \eta_i(g)\tilde{c} \right),$$

where $\eta_i(g) = |N_i^1(g)|$ the number of links of i in g.

[12] In this bargaining process, δ has the usual interpretation of a discount factor, such that, if δ is higher, intermediary agents are forced to make more competitive offers to end players.

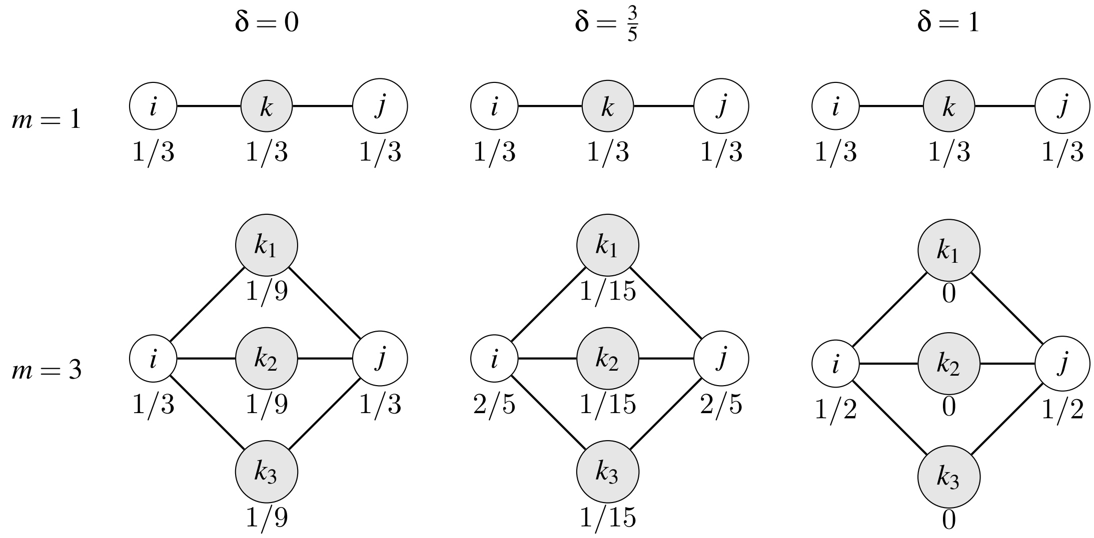


Figure 3: Examples of payoff shares received by endnodes i and j and intermediaries k, depending on the parameters δ and m under the specification of f_b, f_l and f_m in equation (1), as in Siedlarek (2015).

We write the probability of a liquidity shock as $\gamma_i = \rho\alpha_i$. ρ is a scaling factor, and α_i captures potential heterogeneity among banks. We assume that bigger banks (in terms of asset size) have higher α_i, that is, they receive more trade opportunities. In the homogeneous case, $\alpha_i = 1$ for all i. If we let ρ become small, then the payoffs can be approximated as follows:

Proposition 1. *Let $N_i^d(g)$ denote the set of nodes at distance d from i in network g. Define $f_e(\cdot) \equiv (f_l(\cdot) + f_b(\cdot))/2$. The quadratic approximation of firm i's payoff function around $\rho = 0$ is given by*

$$\tilde{\pi}_i(g, \delta, \tilde{c}) = \frac{\beta\rho^2}{1-\beta}\pi_i\left(g, \delta, \frac{\tilde{c}}{\rho^2}\right) + O\left(\rho^3\right),$$

where the function $\pi_i(g, \delta, c)$ is given by

$$\pi_i(g,\delta,c) = \underbrace{\sum_{j \in N_i^1(g)} \left(\frac{1}{2}\alpha_i\alpha_j - c\right)}_{\textit{direct trade}} + \underbrace{\sum_{j \in N_i^2(g)} \alpha_i\alpha_j f_e(m_{ij}(g), \delta)}_{\textit{indirect trade}} + \underbrace{\sum_{k,l \in N_i^1(g) | g_{kl}=0} \alpha_k\alpha_l f_m(m_{kl}(g), \delta)}_{\textit{intermediation benefits}} \tag{2}$$

Proof. See Appendix B. $\square$

Proposition 1 says that, for ρ small, the payoff function can be approximated by

$$\tilde{\pi}_i(g, \delta, \tilde{c}) \approx \frac{\beta\rho^2}{1-\beta}\pi_i(g, \delta, \frac{\tilde{c}}{\rho^2}),$$

in which the only trade opportunities that matter are those in which one and only one bank has a positive liquidity shock and one and only one bank a negative liquidity shock. The benefits from these trade opportunities can be partitioned in benefits from trading directly minus the cost of maintaining a link, benefits from trading indirectly, and benefits from intermediating between two banks. Trade opportunities arising from multiple banks receiving a shock, occur with infinitesimal probability. In the rest of the paper we therefore consider the function $\pi_i(g, \delta, c)$ as in (2) with $c \equiv \frac{\tilde{c}}{\rho^2}$, as the payoff function from network formation. The payoff function of Goyal and Vega-Redondo (2007) is a special case with homogeneity ($\alpha_i = 1\ \forall i$) and perfect competition ($\delta = 1$).

Also, in the rest of the paper we fix $f_e^1 = f_m^1 = \frac{1}{3}$. In other words, we assume that middlemen with a monopoly position between pairs of lenders and borrowers get a share of one third of their trades. As $f_e(\cdot) \equiv (f_l(\cdot) + f_b(\cdot))/2$, this assumption does not impose restrictions on bargaining power between borrowers and lenders. The assumption simplifies the exposition without significantly changing the results.[13]

We now comment on the interpretation of these payoffs. First, the undirected links in this model should be interpreted as established preferential trading relationships. We assume that trade opportunities can only be realized if the agents are linked directly or indirectly through intermediation by mutual trading relationships. This is a strong, but not implausible assumption. The existence of preferential trading relationships has been shown by Cocco et al. (2009) in the Portuguese interbank market and by Bräuning and Fecht (2017) in the German interbank market. Afonso and Lagos (2015) document how some commercial banks act as intermediaries in the U.S. federal funds market. A bank that attempts to borrow outside its established trading relationships may signal that it is having difficulties to obtain liquidity funding and, as a consequence, may face higher borrowing costs. Hence, banks have incentives to use their established trading relationships.

Second, it is assumed that the preferential trading relationship comes at a fixed cost of c. This cost follows from maintaining mutual trust and from monitoring, i.e. assessing the other bank's risks. In principle, it is possible that these costs are not constant over banks, e.g. economies of scale may decrease linking costs in the number of relationships that are already present. The possible heterogeneity in linking costs is most likely smaller than the heterogeneity in trading surpluses, and we therefore assume that all banks pay an equal cost for a trading relationship.

2.4. Network stability concepts

Given the setup of the payoffs discussed above, we analyze which networks arise if agents form links strategically at time $t = 0$. Here we assume that, in order to establish a link between two agents, both agents have to agree and both agents face the cost of a link, a version of network formation that is called *two-sided network formation.*[14] Network formation theory has developed stability or equilibrium concepts to analyze the *stability* of a network. Here, stability does not refer to systemic

[13] In the formulas in Theorem 1 and Proposition 5, the assumption is visible in the recurring factor $\frac{1}{3}$.
[14] See Goyal (2009) for a text book discussion.

risk, but to the question whether an agent or a pair of agents has an incentive and the possibility to modify the network in order to receive a higher payoff.

There are many stability concepts, which differ in the network modifications allowed. For an overview of these stability concepts, we refer to Jackson (2005) or Goyal (2009). For our purposes we consider two stability concepts.

The first concept is *pairwise stability*, a standard concept in the literature (Jackson and Wolinsky, 1996). A network is pairwise stable if, for all the links present, no player benefits from deleting the link, and for all the links absent, one of the two players does not want to create a link. Denote the network $g+g_{ij}$ as the network identical to g except that a link between i and j is added. Similarly, denote $g-g_{ij}$ as the network identical to g except that the link between i and j is removed. While focusing on comparing networks, we drop the arguments of δ and c in the function $\pi(\cdot)$. Then the definition of pairwise stability is as follows:

Definition 2. *A network g is* pairwise stable *if for all $i,j \in N$, $i \neq j$:*

(a) if $g_{ij}=1$, then $\pi_i(g) \geq \pi_i(g-g_{ij}) \wedge \pi_j(g) \geq \pi_j(g-g_{ij})$;

(b) if $g_{ij}=0$, then $\pi_i(g+g_{ij}) > \pi_i(g) \Rightarrow \pi_j(g+g_{ij}) < \pi_j(g)$.

The concept of pairwise stable networks only allows for deviations of one link at a time. This concept is often too weak to draw distinguishable conclusions, i.e. in many applications including ours, there are many networks that are pairwise stable.

In our application, we consider it relevant that agents may consider to propose many links simultaneously in order to become an intermediary and establish a client base. The benefits from such a decision may only become worthwhile if the agent is able to create or remove more than one link. This leads us naturally to the concept of *unilateral stability*, originally proposed by Buskens and van de Rijt (2008).

A network is unilaterally stable if no agent i in the network has a profitable unilateral deviation: a change in its links by either deleting existing links such that i benefits, or proposing new links such that i and all the agents to which it proposes a new link benefit. Denote g^{iS} as the network identical to g except that all the links between i and every $j \in S$ are altered by $g^{iS}_{ij} = 1-g_{ij}$, i.e. are added if absent in g or are deleted if present in g.

Definition 3. *A network g is* unilaterally stable *if for all i and for all subsets of players $S \subseteq N \backslash \{i\}$:*

(a) if $\forall j \in S: g_{ij}=1$, then $\pi_i(g) \geq \pi_i(g^{iS})$;

(b) if $\forall j \in S: g_{ij}=0$, then $\pi_i(g^{iS}) > \pi_i(g) \Rightarrow \exists j : \pi_j(g^{iS}) < \pi_j(g)$.

Note that unilateral stability implies pairwise stability, that is, a network that is unilaterally stable is also pairwise stable, but not vice versa. This can be easily verified by considering subsets S that consist of only one node $j \neq i$.[15]

Apart from analyzing the stability of networks, for policy considerations it is also relevant to consider efficient networks. As usual in the literature, we define a network efficient, if it maximizes the total sum of payoffs of the agents.

Definition 4. *A network g is* efficient *if there is no other network g', such that*

$$\sum_{i \in N} \pi_i(g') > \sum_{i \in N} \pi_i(g).$$

3. Results for homogeneous banks

We first analyze the model for the baseline homogeneous case where all pairs generate the same trade surplus, $\alpha_i = 1$ for all $i \in N$.[16] Section 3.1 contains our main result that core-periphery networks are not unilaterally stable under the assumption of homogeneity. To find what structures arise in this homogeneous case, if not core-periphery networks, we investigate in Section 3.2 a best-response dynamic process. In Section 3.3 we relate these outcomes to the efficient networks.

3.1. Stability of the core-periphery structure

We consider the two stability concepts described in Section 2.4, pairwise and unilateral stability. We first derive the conditions under which, in the homogenous case, core-periphery networks are not pairwise stable.[17]

Proposition 2. *Let $\alpha_i = 1$ for all $i \in N$, and let $c > 0$ and $0 < \delta < 1$ be given. Suppose that g is a core-periphery network with $K \subset N$ the set of core agents, and that for the number of core agents $k = |K|$ it holds that $2 \leq k \leq n-3$. Denote the set of agents connected to some player i as N_i with size $n_i = |N_i|$. If there are two core agents $i, j \in K$ with $N_i \supseteq N_j$ and $n_j \geq k+1$, then g is not pairwise stable.*

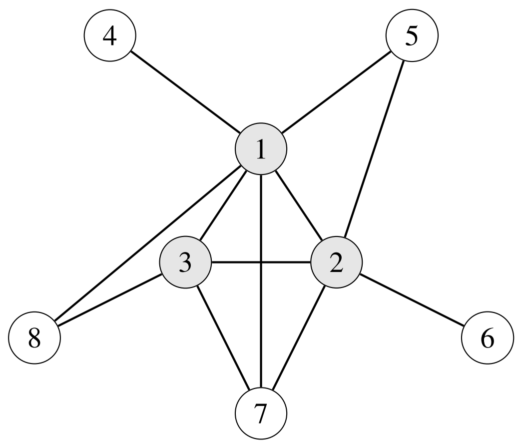
Proof. See Appendix C. □

[15] Our definition of unilateral stability is slightly less restrictive than the definition in Buskens and van de Rijt (2008). While we are following Buskens and van de Rijt (2008) in considering deviations of simultaneously deleting *or* proposing multiple links, we do not allow simultaneously deleting *and* proposing of multiple links. This adaptation eases the exposition, but does not affect our results qualitatively.

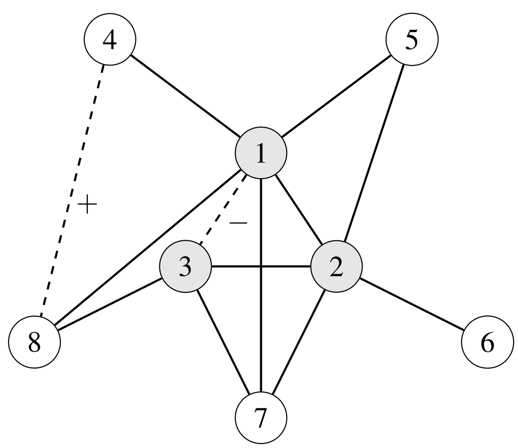
[16] The normalisation $\alpha = 1$ in the homogeneous case goes without loss of generality. If $\alpha_i = \alpha$ for all i with $\alpha > 0$, then the payoffs in Proposition 1 are proportional to α when costs c are considered as a fraction of α.

[17] We emphasize that the results in this section do not depend on the assumption that trade surpluses between i and j are only realized if the path length between i and j is less than 3, as shown in Appendix C.

The intuition behind Proposition 2 is illustrated by the following example. Suppose that the network in Figure 1 (repeated in Figure 4a) is pairwise stable. A stable core-periphery network would imply that all periphery banks, for example 4 and 8 do *not* have an incentive to trade directly with each other, and at the same time, all core banks *do* have an incentive to trade directly, for example 1 and 3. Figure 4b shows these two deviations from the core-periphery structure. The reason that this structure is not pairwise stable, is because periphery agent 4 has only one core agent as intermediators (namely 1), while agent 1 has three potential intermediators to connect to core bank 3 (namely 2, 7 and 8). After all, periphery banks may act as intermediators between core banks. Given that two periphery banks trade indirectly, then two core banks have an incentive to do the same. Hence, core bank 1 should delete its link with 3, contradicting the pairwise stability of the network.



(a) A core-periphery network.



(b) Two potential deviations to the structure.

Figure 4: A core-periphery network with $n = 8$ and $k = 3$, and two potential deviations to the structure, one adding a link (+) and one deleting a link (−).

The condition in Proposition 2 that there exist two core agents i and j with $N_i \supseteq N_j$ and $n_j \geq k+1$, guarantees that core bank i, after deleting a link with core bank j, retains the same access to the periphery. In the example, $N_1 \supseteq N_3$, so that player 1 can delete its link with 3, as all periphery players connected to 3 are also connected to 1. Player 1 may not wish to delete its link to 2, however, as 2 provides access to periphery player 6. Note that the condition in Proposition 2 is fairly mild. For a core-periphery networks to be pairwise stable, *all* core agents are necessarily connected to a periphery agent with a single link. In this situation, every core player has local monopoly power. Empirically, relevant interbank structures typically show levels of connectivity between core and periphery that exclude this local monopoly power.[18]

Even core-periphery networks not excluded by Proposition 2 are often unstable. So the condition is sufficient but not necessary. We now consider unilateral deviations, in which agents are allowed to add or delete multiple links. We show that all core-periphery networks are unstable, as long as there are enough periphery banks.[19]

Proposition 3. *Let the payoff function be homogeneous ($\alpha_i = 1$ for all $i \in N$) and let $c > 0$ and $0 < \delta < 1$ be given. Then, there is a function $F(c, \delta)$, such that, if $n - k > F(c, \delta)$, a core-periphery network with k core and $n - k$ periphery players is not unilaterally stable.*

Proof. See Appendix C. □

The intuition is that, for $n - k$ large enough, periphery banks have an incentive to enter the core; e.g. player 4 in Figure 5. Because we allow for multiple links to be added simultaneously, peripheral players can take a share of the intermediation benefits by replicating the position of core players. Unequal payoffs between core and periphery players makes the core-periphery networks unstable.

3.2. A dynamic process

So far, core-periphery networks were shown to be unstable under homogeneous agents, as long as the number of periphery agents is sufficiently large. The question remains what happens in small-sized networks, and, if not core-periphery networks, what other kind of network architectures do arise? This motivates us to consider a simple dynamic process of network formation and analyze its stable states.

In particular, we consider a round-robin best-response-like dynamic process as in Kleinberg et al. (2008). We order nodes $1, 2, \ldots, n$ and starting from the empty graph, in this order nodes

[18] The Proposition considers core-periphery networks with $n - k \geq 3$, i.e. the number of periphery agents is at least three, which is generally satisfied for actual financial networks. If $n - k = 2$, and the set of connections of core player j is a subset of the connections of core player i are both connected to two distinct core agents, then given δ there is a unique value of $c > 0$ for which stability cannot be excluded.

[19] Propositions 2 and 3 still leave open the possibility for unilaterally stable core-periphery networks with local monopoly power and small network size. Indeed, in Appendix E we do give examples of such stable core-periphery networks. Theorem 1 shows that these cannot be the outcome of best-response dynamics.

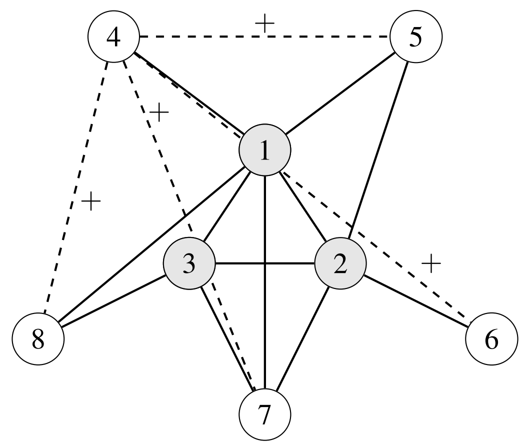


Figure 5: A core-periphery network with $n = 8$ and $k = 3$, and a potential deviations to the structure with multiple links added (+).

consecutively try to improve their position by taking a *best feasible action*. An action of player i is defined as feasible if i either proposes links to a subset of players S such that every $j \in S$ accepts a link with i, or if i deletes a subset of its links. The best feasible action of player i is then the feasible action that leads to the highest payoff for i. The formal definition of a (best) feasible action is given in Appendix C. Note that a unilaterally stable network is a network in which all players choose a best feasible action.[20]

After node n chooses its best feasible action the second round starts again with node 1, again each player consecutively choosing its best feasible action. Next, the third round start, and so on, until convergence. The process converges if $n-1$ consecutive players cannot improve their position.

The assumptions about the dynamic process allow for a sharp characterisation of stable network structures. The advantage of starting in an empty network is that, initially, nodes can only add links to the network. A fixed round-robin order limits the number of possibilities that has to be considered for every step in the process. Simulations for our model indicate that the results below also hold for a random order of agents.[21] Excluding border line cases, we find that the dynamics always converge to a unilaterally stable network whose architecture is well described for every choice of the model parameters.[22]

Theorem 1 below shows which network structures result from the dynamics. The empty, star and complete networks are prominent for large parameter regions. In the remaining parameter regions

[20]If a player's best feasible action is not unique, then we assume that a player chooses randomly from the set of best feasible actions. In our theorems we will only focus on parameter regions in which the best feasible action is always unique.

[21]Houy (2009) considers a dynamic framework for the model of Goyal and Vega-Redondo (2007), in which randomly drawn agents make best responses to the network.

[22]In principle, a dynamic process may lead to cycles of improving networks (cf. Jackson and Watts, 2002; Kleinberg et al., 2008), but Theorem 1 shows that this is not the case in our model.

the attracting steady state cannot be a core-periphery network (in line with Propositions 2 and 3) and turns out to be a multipartite network. The theorem singles out one special type of multipartite networks, called maximally unbalanced bipartite networks, for parameters satisfying condition IV, while parameters under condition V can lead to various types of multipartite networks.

Theorem 1. *Consider the homogeneous baseline model with $\alpha_i = 1$ for all $i \in N$. From an empty graph, the round-robin best-feasible-action dynamics converge to a unilaterally stable network with the following network architecture for the following parameter regions:*

I: *for* $c > \frac{1}{2} + \frac{1}{6}(n-2)$
the empty network,

II: *for* $c \in \left(\frac{1}{6} + (n-3)\min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}, \frac{1}{2} + \frac{1}{6}(n-2)\right)$
the star network,

III: *for* $c < \frac{1}{2} - f_e(n-2,\delta)$
the complete network.

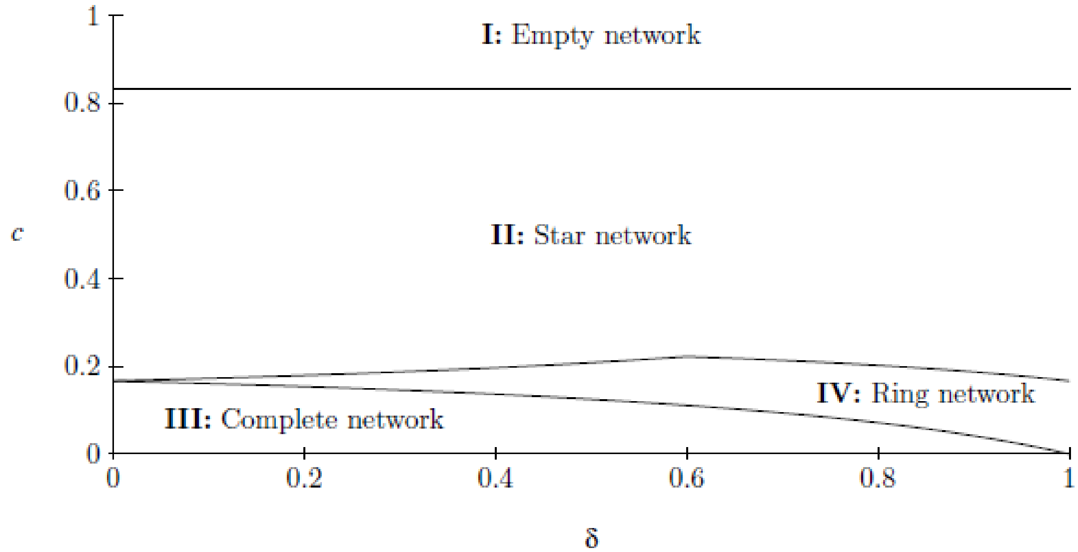
IV: *for* $c \in \left(\frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2}f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}, \frac{1}{6} + (n-3)\min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}\right)$
the maximally unbalanced bipartite network $g^{mp(2)}_{2,n-2}$,

V: *for* $c \in \left(\frac{1}{2} - f_e(n-2,\delta), \frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2}f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}\right)$
a multipartite network $g^{mp(q)}_{k_1,k_2,\ldots,k_q}$ *with* $q \geq 2$ *and* $|k_m - k_{m'}| < n-4$ *for all* $m, m' \in \{1,2,..,q\}$.

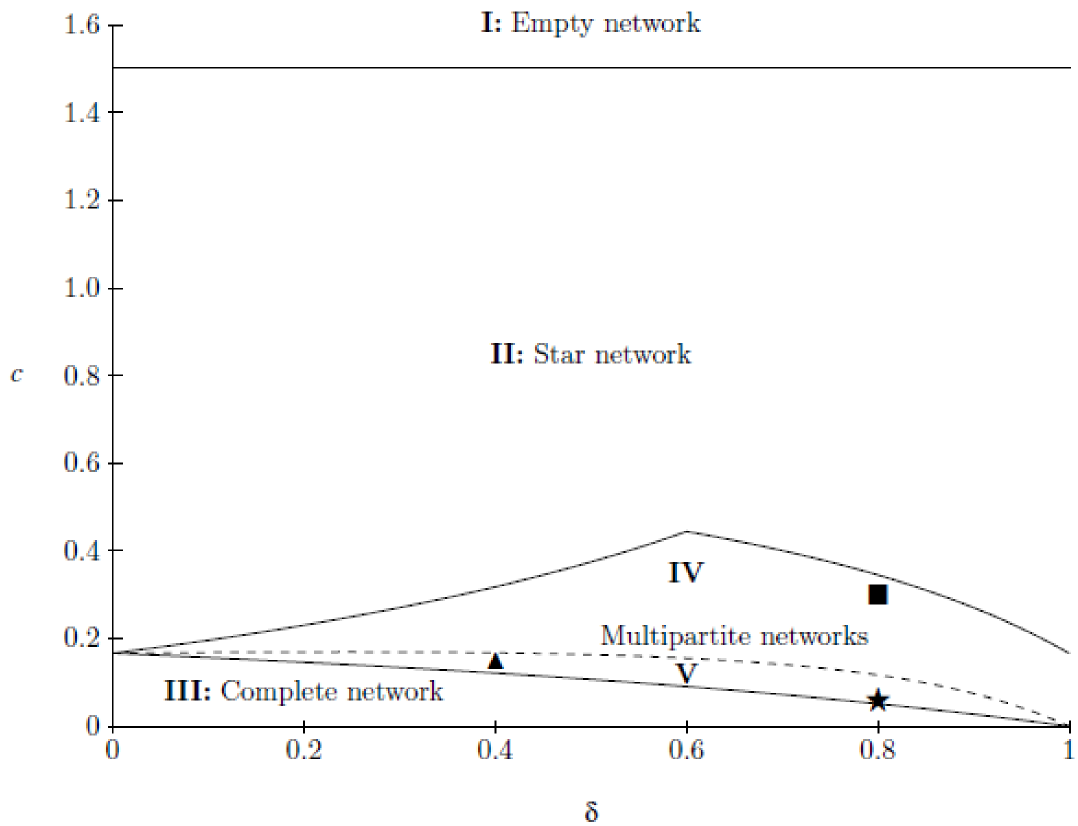
Proof. See Appendix C. □

Figure 6 illustrates the parameter regions specified by Theorem 1 for $n = 4$ and $n = 8$ under the specification of f_e and f_m in equation (1), as in Siedlarek (2015). The possible network outcomes range intuitively from empty to complete networks as the cost of linking decreases. The star is an important outcome in between empty and complete networks, but cannot be a unilaterally stable outcome for intermediate competition δ and relatively low c. For intermediate δ the incentives to enter the core and the incentives for periphery players to accept the proposal of the new core player are both high. The parameter area between complete networks and stars gives multipartite networks as the stable outcomes, and this area increases with n.

It is noteworthy that for large n the set of attained multipartite networks is quite diverse. Possible outcomes for $n = 8$ are the maximally unbalanced bipartite network $g^{mp(2)}_{2,6}$, but also, for example, an unbalanced bipartite network $g^{mp(2)}_{3,5}$ or a balanced multipartite network $g^{mp(3)}_{2,3,3}$ consisting of 3 groups. Figure 7 presents these three examples of multipartite networks and the values of δ and c for which they are formed.

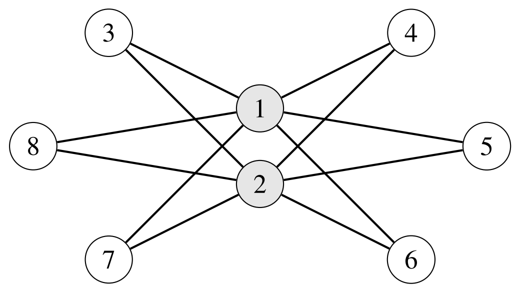


(a) $n = 4$

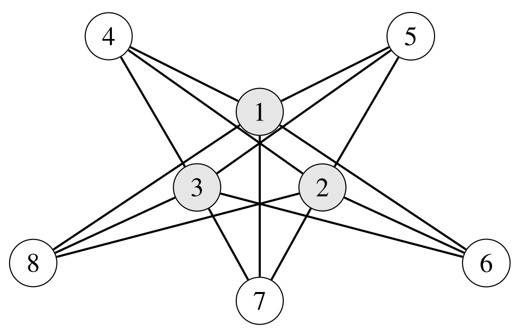


(b) $n = 8$

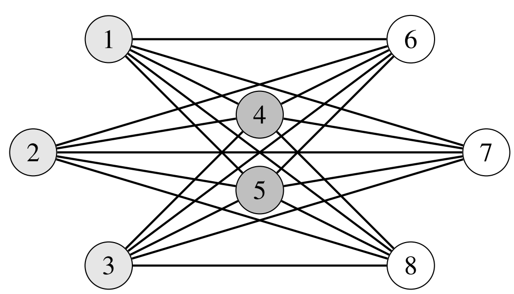
Figure 6: Attained equilibria after best-feasible-action dynamics from an empty network in (δ, c)-space for $\alpha_i = 1$ for all $i \in N$ and $n \in \{4, 8\}$ under the specification of f_e and f_m in equation (1). The roman numbers correspond with those in Theorem 1. The symbols (■, ▲ and ★) correspond to examples of multipartite networks in Figure 7.



(a) Black square ■: $(\delta, c) = (0.8, 0.3)$. The maximally unbalanced bipartite network: $g^{mp(2)}_{2,6}$.



(b) Black triangle ▲: $(\delta, c) = (0.4, 0.15)$. An unbalanced bipartite network: $g^{mp(2)}_{3,5}$.



(c) Black star ★: $(\delta, c) = (0.8, 0.06)$. A balanced multipartite network: $g^{mp(3)}_{2,3,3}$

Figure 7: Examples of attained multipartite networks after best-feasible-action dynamics from an empty graph for $n = 8$ players. The symbols (■, ▲ and ★) correspond to locations in the (δ, c)-space in Figure 6.

A comparison with the results of Goyal and Vega-Redondo (2007) can easily be made by setting $\delta = 1$. We observe that their assumption of perfect competition $\delta = 1$ is a special case for which a complete network is not stable. In their model agents in a complete network always have incentives to remove links even for arbitrary low linking costs, because intermediated trades along multiple middlemen gives them exactly the same share of the surplus.

More importantly, we show that the star is less stable when competition is imperfect compared to the case of Goyal and Vega-Redondo (2007). For $\delta < 1$ and a relatively low c multipartite networks arise instead of stars. Multipartite networks arise for larger regions of parameter choices if n increases; see Figure 11a for the results for $n = 100$. Interestingly, multipartite networks were found by Buskens and van de Rijt (2008) and Kleinberg et al. (2008) as the main equilibrium architecture. The current analysis shows that empty, star, complete and multipartite networks can all arise within a network formation model with intermediation and imperfect competition. Our model thus reproduces earlier results of homogeneous network formation models and places them in a more general perspective.

Figure 8 shows the possible routes of the dynamics for $n = 4$, depending on the remaining parameters c and δ. For $n = 4$, the only possible multipartite network is one that consists of $q = 2$ groups of 2 nodes, which coincides with a ring of all 4 players. The steps towards this multipartite network are as follows. Starting from an empty network, in the first round a star is formed. One of the periphery players then has an incentive to join the core, such that in the second round a complete core-periphery network is formed. By Proposition 2, this core-periphery network is not stable, as core players have an incentive to trade indirectly with each other. Hence, the link within the core is dropped and the multipartite network is formed.

3.3. Efficient networks

In this section we compare the stable networks with efficient networks. Following the results of Goyal and Vega-Redondo (2007), minimally connected networks, i.e. with $n-1$ links, are efficient for $c < \frac{n}{4}$. Networks are efficient if all trade surpluses are realized irrespective of the distribution of these trading surpluses. For higher c it is efficient to have no network at all, i.e. an empty network. Because of the assumption that two agents only trade if they are at distance 1 or 2, the network should not have a maximal distance higher than 2, leaving the star as the unique efficient minimally connected network. This is summarized in the following theorem.

Theorem 2. *If the payoff function $\pi(g, \delta, c)$ is given as in Proposition 1 with $\alpha_i = 1$ for all $i \in N$, then:*

(a) for $c \geq n/4$, the empty network *is efficient;*

(b) for $c \leq n/4$, the star network *is efficient;*

(c) no other network structure than the empty or star network is efficient.

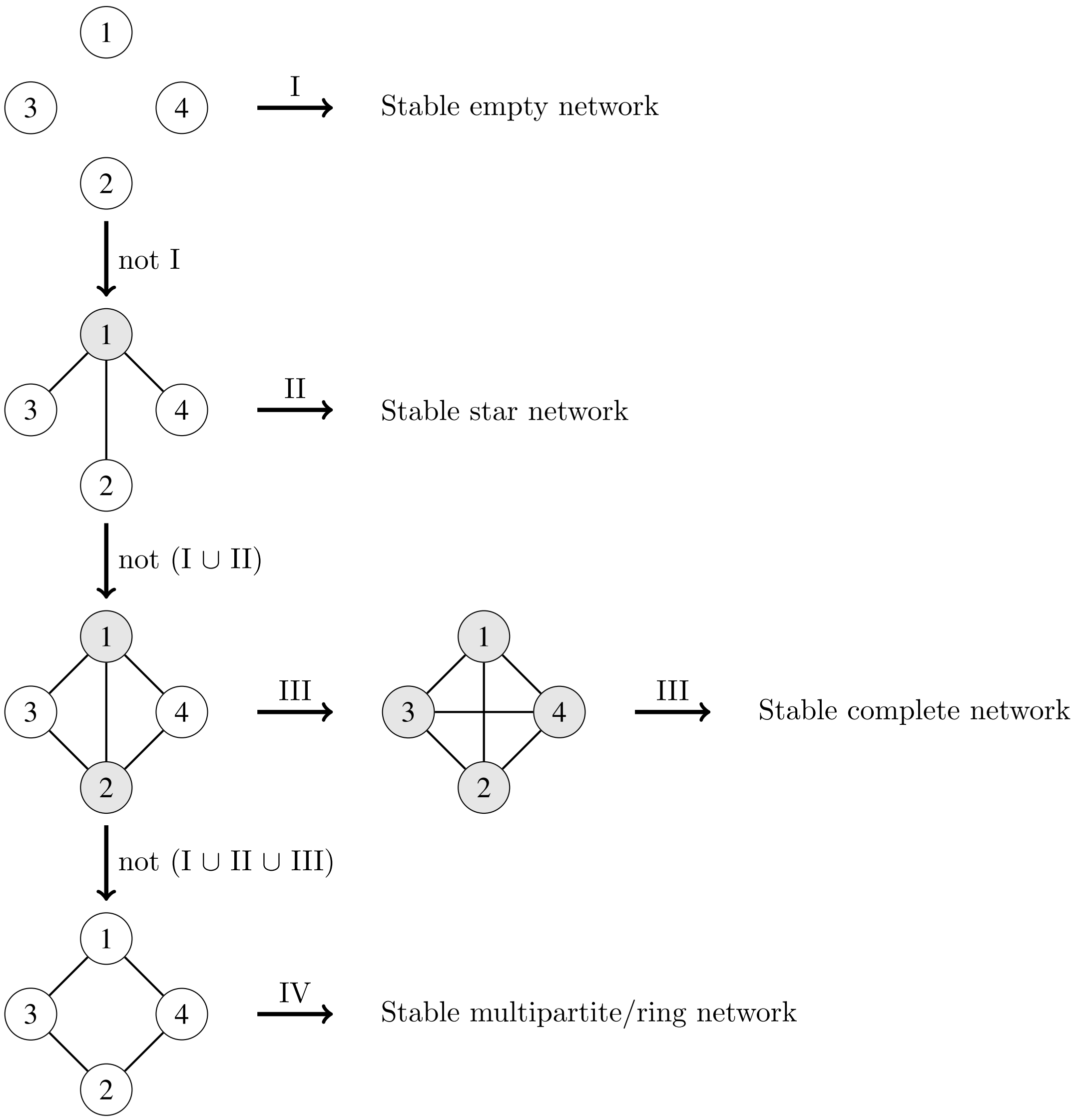


Figure 8: Map of the dynamics from an empty graph for $n = 4$ leading to one out of 4 stable possible structures. The roman numbers correspond with the conditions as in Theorem 1 under which the best-feasible-action route follows the direction of the arrows.

Theorem 2 implies that core-periphery networks with $k \geq 2$ are not efficient.

Comparing the results in Theorem 2 to Theorem 1, we observe that the star network is always efficient, whenever the star network is attained in the dynamic round-robin best-feasible-action process. However, for both low costs and an area of high costs c, the dynamic process leads to an inefficient outcome. When c is reasonably high ($\frac{1}{2} + \frac{1}{6}(n-2) < c < \frac{n}{4}$), the dynamic process converges to the empty network, even though the star network is efficient. This is because of the marginal benefits from maintaining a link in the star network are unevenly distributed; the center benefits more than the periphery, such that the center may have an incentive to create link with the periphery, but not vice versa.

For low costs, under conditions III, IV or V in Theorem 1, the dynamic process converges to either multipartite or to complete networks, whereas the star network is the efficient network. In this case, networks are overconnected. The star network, although efficient, is not stable, as the center extracts high intermediation rents, which the other players try to circumvent.

The upperbound on c of area IV, below which stable networks are not efficient, is increasing in n as shown in Figures 6 and 11a. So for relatively large network sizes, stable networks can be expected to be overconnected.

4. Results for heterogeneous banks

We found that core-periphery networks were not stable, when agents are ex-ante identical. Instead, typically multipartite networks are formed, in which banks are connected to members of other groups, but not within their own group. In real interbank markets we do observe links within the core. We now try to explain this discrepancy.

Key in this result in Section 3 is the assumed homogeneity; periphery banks have the same capabilities as core banks in terms of profit generation, intermediation or linking, such that they can easily replace or imitate a core bank. In practice, we see large differences between banks, in particular banks in the core are much bigger than banks in the periphery (Craig and Von Peter, 2014). It is natural to think that these big banks have a strong incentive to have tight connections within the core as well as to the periphery for intermediation reasons.

For this reason we analyze the consequences of heterogeneity within our model. In Section 4.1 heterogeneity is introduced in terms of exogenous differences in bank size. In Section 4.2, heterogeneity is endogenized by extending the dynamic process to include feedback of profits on trading opportunities.

4.1. *Exogenous heterogeneity*

We start by considering exogenous heterogeneity in the trading opportunities of banks within our model. We interpret this heterogeneity in trading opportunities as arising from differences in bank size. We introduce two types of banks, k big banks and $n-k$ small banks, and we assume that the probability of receiving a random unexpected liquidity shock is proportional to bank size. If a bank i is big we assume $\alpha_i = \alpha \geq 1$, and if i is small we assume $\alpha_i = 1$.[23] So the difference in size is captured by a parameter $\alpha \geq 1$ quantifying the relative size of a big bank. In this subsection we take their size as exogenously given. Given heterogeneity in the size of banks, we will show that a stable core-periphery network can form.

The number of big banks, k, has become a new exogenous parameter, and we consider a complete core-periphery network where the core consists of the k big banks. Proposition 4 states that the complete core-periphery network can be unilaterally stable under heterogeneity.

Proposition 4. *Consider the model with the following form of heterogeneity: $\alpha_i = \alpha$ for $i \in \{1,2,...,k\}$ ('big banks'), and $\alpha_i = 1$ for $i \in \{k+1, k+2, ..., n\}$ ('small banks'). Then given a level of heterogeneity $\alpha > 1$, the complete core-periphery network with k big banks is unilaterally stable if:*

$$\text{VII:} \qquad c \in \Bigg(\tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\},$$
$$\min\Big\{ \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1) f_m(k,\delta),$$
$$\min_{l \leq k}\big\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\big\}\Big\}\Bigg).$$

Proof. See Appendix D. □

Region VII in Proposition 4 gives the parameter combinations for which complete core-periphery networks are unilaterally stable. The lower bound for c in region VII does not depend on α. The upperbound for c does depend on α. If the condition for the lower bound is satisfied, the upperbound implicitly defines a minimum level of heterogeneity required for a stable complete core-periphery network. This is made explicit in the following corollary.

Corollary 1. *Let the payoff function be heterogeneous as in Proposition 4 and let $c > 0$ and $0 < \delta < 1$ be given. Then, if*

$$c \geq \frac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\frac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\},$$

there exists an $\overline{\alpha} > 1$, such that for all $\alpha > \overline{\alpha}$ the complete core-periphery network with k big banks is unilaterally stable.

[23] The normalisation $\alpha_i = 1$ for small banks goes without loss of generality. Compare footnote 16.

Proof. The upperbound for c in region VII in Proposition 4 is at least linearly increasing in α. Given $c > 0$ and $0 < \delta < 1$, region VII is nonempty if α is large enough. $\square$

For $\delta \downarrow 0$, $\delta \uparrow 1$ and $k \to \infty$, the required level of heterogeneity $\overline{\alpha}$ is arbitrarily close to 1, as shown in Appendix D.

Proposition 4 combined with Propositions 2 and 3 implies that heterogeneity is crucial in understanding core-periphery networks. Complete core-periphery networks are never stable under homogeneous players, but can be stable for arbitrary small levels of heterogeneity.

Heterogeneity also affects the results of the dynamics. Starting from an empty graph, complete core-periphery networks can arise because players replicate the position of the central players and become intermediators for periphery players. If the relative size α is sufficiently large and condition VII of Proposition 4 is fulfilled, the result will be unilaterally stable. Proposition 5 below specifies the seven possible attracting stable networks for the special case of $n = 4$ and $k = 2$ big banks. In this dynamic process, it is assumed that the k large banks are the first in the round-robin order. Given $n = 4$, $k = 2$ and any choice of the other parameters c, δ and α (except for borderline cases), the proposition shows that the dynamics converge to a unique unilaterally stable network.[24]

Proposition 5. *Consider the model with $n = 4$, of which $k = 2$ big banks having size $\alpha > 1$ and $n - k = 2$ small banks having size 1. From an empty graph, the round-robin best-feasible-action dynamics starting with the two big banks converge to the following unilaterally stable equilibria:*

I: *for* $c > \max\left\{\frac{1}{2}\alpha^2, \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}\right\}$
the empty network,

II: *for* $c \in \left(\frac{1}{6}\alpha + \min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}, \min\left\{\frac{5}{6}\alpha + \frac{1}{6}, \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}\right\}\right)$
the star network,

III: *for* $c < \frac{1}{2} - f_e(2,\delta)$
the complete network,

IV: *for* $c \in \left(\min\left\{\alpha^2(\frac{1}{2} - f_e(2,\delta)), \frac{1}{6}\alpha + f_m(2,\delta), \frac{1}{6}\alpha + f_e(2,\delta) - \frac{1}{3}\right\}, \frac{1}{6}\alpha + \min\{\frac{1}{2}f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}\right)$
the multipartite (ring) network $g^{mp(2)}_{2,2}$

V: *(other multipartite networks do not exist for $n = 4$),*

[24] Under heterogeneous players the results of the dynamic process depend on the order in which agents choose their best feasible actions. If agents are randomly drawn to make best feasible actions, multiple attracting steady states may exist. Using simulations, we found that for certain parameter values in VII a multipartite ring network can arise. However, for a large subset of the parameter region VII, the complete core-periphery remains the unique attracting steady state even under a random order of agents.

VI: *for* $c \in \left(\min\left\{\frac{5}{6}\alpha+\frac{1}{6}, \frac{1}{6}\alpha^2+\frac{5}{9}\alpha+\frac{1}{9}\right\}, \max\left\{\frac{1}{2}\alpha^2, \frac{1}{6}\alpha^2+\frac{5}{9}\alpha+\frac{1}{9}\right\}\right)$
the 'single pair' network g *with* $g_{12} = 1$ *and* $g_{ij} = 0$ *for all* $(i, j) \neq (1,2)$,

VII: *for* $c \in \left(\frac{1}{2} - f_e(2,\delta), \min\left\{\alpha^2(\frac{1}{2} - f_e(2,\delta)), \frac{1}{6}\alpha + f_m(2,\delta), \frac{1}{6}\alpha + f_e(2,\delta) - \frac{1}{3}\right\}\right)$
the complete core-periphery network $g_{com}^{CP(2)}$
(other core-periphery networks do not exist for $n = 4$*).*

Proof. See Appendix D. □

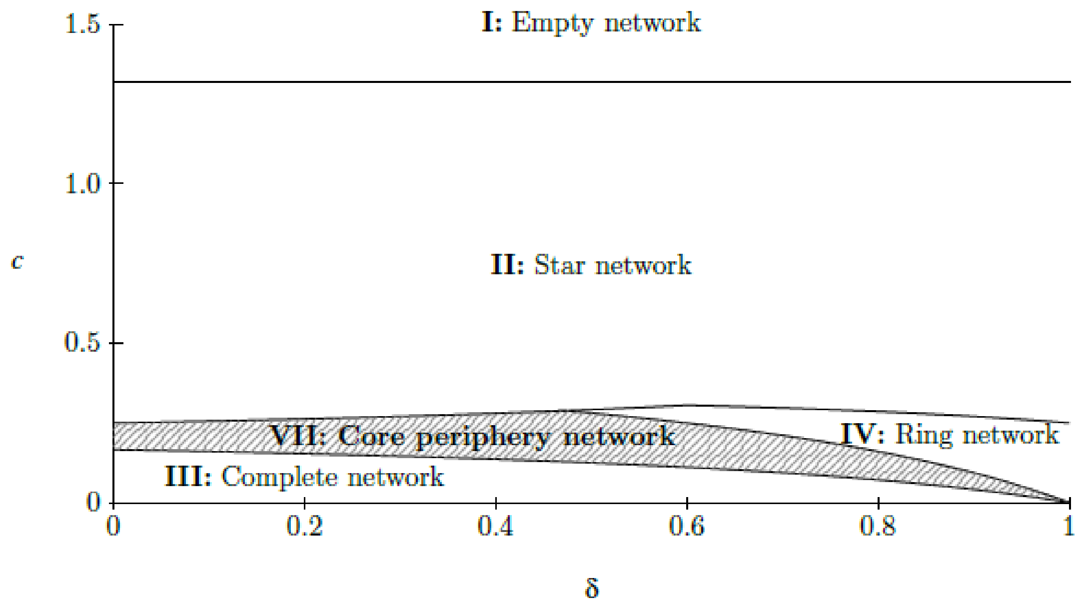
Figure 9 illustrates the different network outcomes for $n = 4$, $k = 2$ and two levels of heterogeneity, $\alpha = 1.5$ and $\alpha = 2$. Proposition 5 introduces a new (simple) type of network, called a 'single pair' network, in which only the two big banks are linked and the small banks have no connections. The parameter region VI is nonempty if the level of heterogeneity is sufficiently high: $\alpha > \frac{1}{6}(5+\sqrt{37}) \approx 1.85$. As observed in Figure 9a, for $\alpha = 1.5$ the regions of empty and star networks share a border in the (δ, c)-diagram: $c = \frac{1}{6}\alpha^2 + \frac{5}{9}\alpha + \frac{1}{9}$. For a larger value like $\alpha = 2$ in Figure 9b, the single pair network arises under condition VI. This type of network structure in which only part of the nodes is connected intuitively arises because some connections are more worthwhile.

Let us now discuss the main network outcome of interest, namely core-periphery networks. The condition under which the complete core-periphery network is the attracting steady state is in Figure 9 indicated by the shaded regions. As expected, this region increases with the level of heterogeneity α. Also observe that for complete core-periphery networks to arise it is necessary that competition is less than fully perfect, i.e. $\delta < 1$. For the special case of $\delta = 1$ (as considered by Goyal and Vega-Redondo, 2007) core-periphery networks are never unilaterally stable, not even under large heterogeneity. Finally, complete core-periphery networks arise for a larger set of parameters if the number of players is larger than the minimal network size of $n = 4$. See Figure 11b for the shaded region VII given $n = 100$, $k = 15$ and $\alpha = 10$.

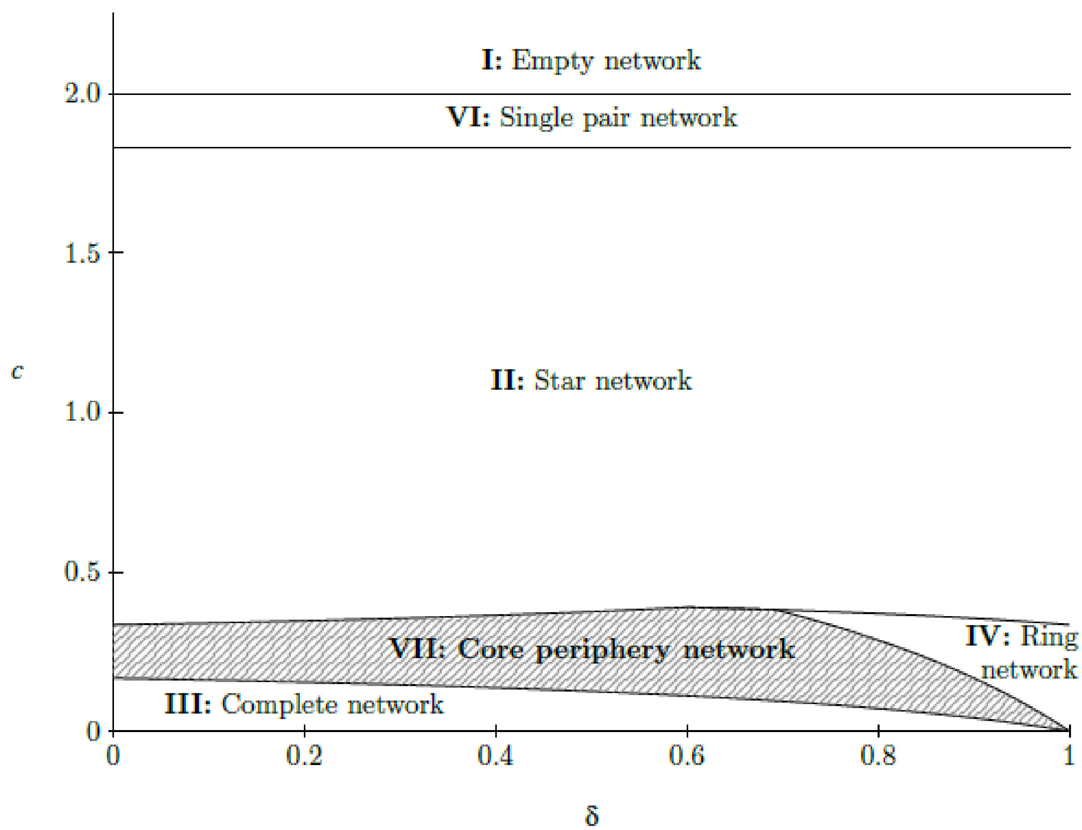
4.2. Endogenous heterogeneity

From the analysis in the above subsection it follows that heterogeneity is a necessary condition for a stable core-periphery network. We have shown that under the assumption of ex ante heterogeneity in trading opportunities, stable core-periphery networks arise for large regions in the parameter space. The assumption of heterogeneity is quite realistic, given the amount of heterogeneity between banks in practice.

Nevertheless, it is of interest whether the formation process in itself may generate sufficient payoff differences between banks as to form an endogenous core-periphery network structure. To this end we extend the dynamic process to allow for feedback of profits on bank size. Using simulations, we



(a) $\alpha = 1.5$



(b) $\alpha = 2$

Figure 9: Attained equilibria after best-response dynamics from an empty network in (δ, c)-space for $n = 4$, $k = 2$ and $\alpha \in 1.5, 2$ under the specification of f_e and f_m in equation (1). The roman numbers correspond with those in Proposition 5. In the shaded area, the complete core-periphery network with $k = 2$ big banks is the unique unilaterally stable outcome of the dynamics.

will show that core-periphery networks can be the outcome of a dynamic process, when bank size is updated according to profits.

The extended dynamic process starts off at round $\tau = 1$ as the homogeneous baseline model with $\alpha_i^{(1)} = 1$ for all $i \in N$. From an empty graph, round-robin best-feasible-action dynamics converge to the empty network, the star network, the complete network or multipartite networks, as in Theorem 1. In this attained network the profits are $\pi_i^{(1)}$. From then on, at the beginning of rounds $\tau = 2, 3, 4, \ldots$ trading opportunities are updated as

$$\alpha_i^{(\tau)} = \frac{\pi_i^{(\tau-1)}}{\min_k[\pi_k^{(\tau-1)}]} \tag{3}$$

Trade is assumed to be proportional to size. In this updating, the additional assumption is made that size is proportional to interbank profits. The trading opportunities are rescaled with $\min_k[\pi_k]$ to assure that trade surpluses of the smallest banks remain normalized to $\alpha_i = 1$.

Having updated the trading opportunities $\{\alpha_i\}_{i \in N}$ at the beginning of round τ, we perform a new round of the round-robin best-feasible-action dynamics, leading to potentially a new network structure and new payoffs. In the next round, the trading opportunities are again updated using (3, and so on.

We simulate this dynamic process. As a stopping rule for the simulations, we impose that the process stops after $\mathcal{T} = 25$ rounds, or before at round τ if the trading opportunities do not alter any more given a tolerance level $\Delta\alpha$, i.e. if $|\alpha_i^{(\tau)} - \alpha_i^{(\tau-1)}| < \Delta\alpha$ for all $i, j \in N$. We choose $\Delta\alpha = 0.1$. Theoretically, it is possible that the system exhibits recurring cycles. In our simulations presented below, we have checked that the dynamics converge within the tolerance level for all $\delta \leq 0.8$. For δ close to 1, dynamics do not converge within 25 rounds and we cannot exclude cyclic behavior in that case.

Figure 10 plots the results for $n = 8$ and $\Delta\alpha = 0.1$ after 1 round (left panel) and after 25 rounds $\mathcal{T} = 25$ (right panel), using a 25x25 grid in the (δ, c)-space ($\delta \in [0, 1]$ and $c \in [0, 0.4]$). The black regions correspond to complete networks, green regions to star networks, blue and purple to multipartite networks and different shades of red to core-periphery networks. In the case of $\tau = 1$, banks are still homogeneous, that is, $\alpha_i^{(1)} = 1$ for all $i \in N$. The left panel of Figure 10 therefore repeats Figure 6b. As we have seen in Subsection 3.2, in that case, core-periphery networks do not occur.

The right panel of Figure 10 shows the results after a maximum of $\mathcal{T} = 25$ updates in the parameters α_i. The black regions remain unchanged, as the banks are in symmetric positions. All banks obtain the same payoffs, and trading opportunities remain equal to 1. Also the star network cannot change: profit feedback increases the trading opportunities α_i of the center of the star $i = 1$. Links with the center thus generate higher payoffs, and will not be severed. Reversely, links between periphery players do not generate more, so no links are added. Hence, if a star network is formed

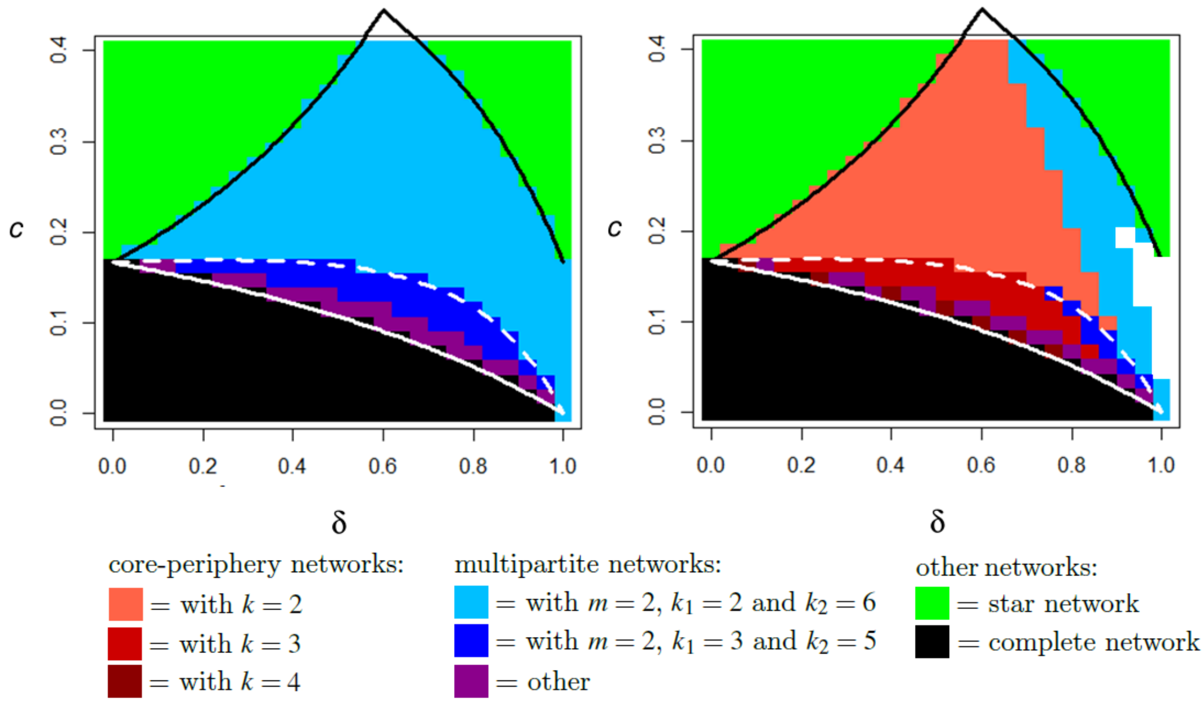


Figure 10: Attained equilibria after best-feasible action dynamics from an empty graph in (δ, c)-space for $n = 8$, extended with profit feedback using $\Delta\alpha = 0.1$ and $\mathcal{T} = 1$ (left panel) or $\mathcal{T} = 25$ (right panel). The black regions correspond to complete networks, green regions to star networks, blue and purple to multipartite networks and different shades of red to core-periphery networks.

under homogeneity after the first round, then the network architecture remains a star network after any future rounds.

This is not the case, however, if in the first round a multipartite network is formed. In that case, we observe significant changes after 25 rounds. In fact, in many simulations, we observe a transition from multipartite networks to core-periphery networks. This happens intuitively. For example, if after round 1 a bipartite network with two banks in tier 1 and six banks in tier 2 are formed ($g_{2,6}^{mp(2)}$, lighter blue region), the payoffs of the tier-1 banks is much higher than the payoffs of the banks in tier 2. This is, because the two tier-1 banks intermediate between the six tier-2 banks, receiving much more intermediation benefits than the tier-2 banks. With the feedback mechanism specified in the dynamics, these higher payoffs for the tier-1 banks result in higher trading opportunities α_i, such that in the end the two banks in tier-1 have an incentive to form a direct link. The network architecture then converts into a core-periphery network. Similarly, many multipartite networks in the darker blue regions evolve into red regions with a core-periphery networks architecture.[25]

We conclude that for many parameter values, core-periphery networks can arise endogenously in the extended model with ex ante homogeneous banks and feedback of the payoffs on trade surpluses.

5. Applying the model to the Dutch interbank market

To gain some insight into the general applicability of the model, we calibrate the model to the Dutch interbank market. The network structure of the interbank market in the Netherlands, a relatively small market with approximately 100 financial institutions, has been investigated before by in 't Veld and van Lelyveld (2014). The attracting networks in the dynamic homogeneous model with $n = 100$ are indicated in Figure 11a. Under homogeneous banks our model predicts multipartite networks for most parameter values. In contrast, in 't Veld and van Lelyveld (2014) found that the observed network contains a very densely connected core with around $k = 15$ core banks.

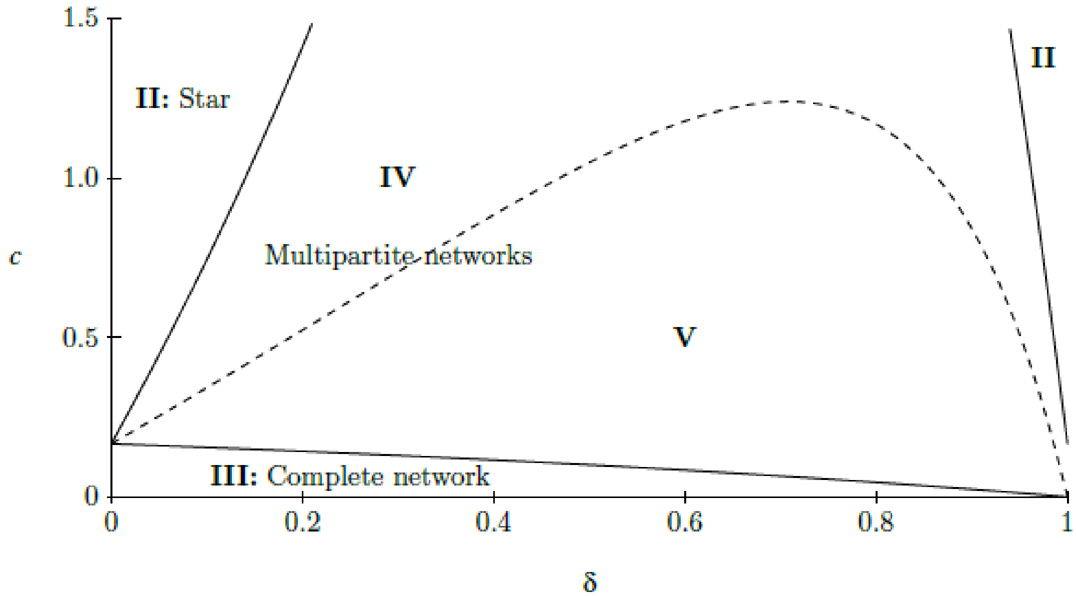
We choose a level of heterogeneity $\alpha = 10$ to capture in a stylized way the heterogeneity of banks in the Netherlands. In reality banks in the core as well as in the periphery of the Dutch banking system are quite diverse. A few very large banks reach a total asset size of up to €1 *trillion*, while the asset value of some investment firms active in the interbank market may not be more than a few million euro. The median size of a core bank of in 't Veld and van Lelyveld (2014) lies around €8 billion. For periphery banks the median size is approximately €300 million (see Fig. 9, in 't Veld and van Lelyveld, 2014, for the plotted distribution of total asset size over core and periphery banks). Ignoring the exceptionally large size of some core banks, a relative difference of $\alpha = 10$ seems a reasonable order of magnitude. The larger α, the wider the area in which the core-periphery network is stable.

[25] For δ close to 1, the results have to be interpreted with care. The reason is that given the high level of competition, central players may earn less than peripheral players. After the updating of trading opportunities central players may be induced to remove several links, and cycles with different players taking central positions may occur.

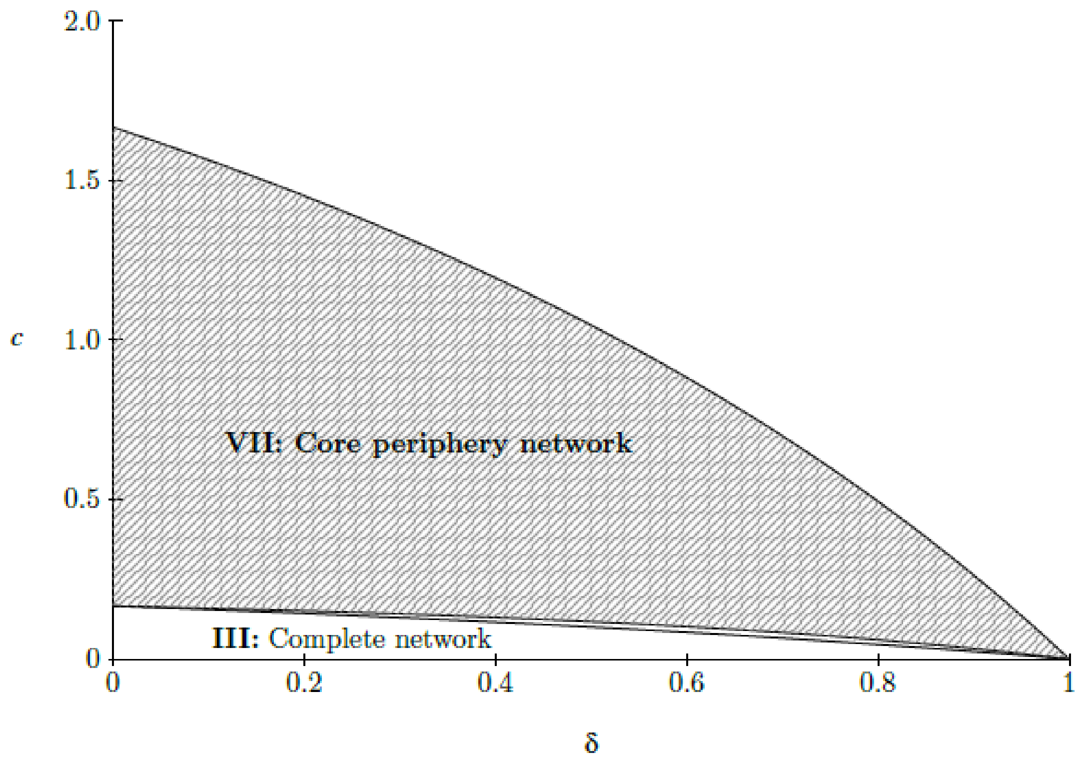
Figure 11b shows the parameter region given by Proposition 4 for which a complete core-periphery network of the fifteen big banks is a unilaterally stable network. The stability of the complete network does not depend on α as also indicated in Figure 11b. These complete networks and complete core-periphery networks would also be the outcome of best-feasible-action dynamics starting with the big banks.[26] The observed core-periphery structure in the Netherlands can be reproduced for many reasonable choices of linking costs c and competitiveness δ.

This application suggests that our model is suitable to explain stylized facts of national or perhaps even international interbank networks. It should be noted that observed core-periphery networks are not necessarily complete core-periphery networks, which can be explained as follows. Empirical studies of interbank markets often rely on either balance sheet data measuring the total exposure of one bank on another, or overnight loan data specifying the actual trades. We interpret the undirected links in our model as established preferential lending relationships, which are typically unobserved in practice. Given a theoretically complete core-periphery network of lending relationships, trades and exposures are executed on the same structure of connections; see Section 2.2 on how we model trade benefits. The empirically observed core-periphery structure from these trade networks is a subset of the unobserved relationship network, and hence, it can have less than complete connections between core and periphery depending on the realisations of trade opportunities. In any case, the densely connected core of a subset of the banks is a well-documented empirical fact that is reproduced by our model.

[26] A full description of the outcomes of these dynamics, specifying all other areas in Figure 11b, is missing. This would require a generalisation of Proposition 5 for all $n \geq 4$. Simulations show that for linking costs so high that CP networks are not stable, many different structures arise. As none of them are core-periphery networks, we restrict ourselves to Proposition 4 and Proposition 5.



(a) Homogeneous banks ($\alpha_{ij} = 1$ for all $i, j \in N$)



(b) Heterogeneous banks: $k = 15$ big banks (with relative size $\alpha = 10$) and $n - k$ small banks

Figure 11: Application of the model to the Dutch interbank market. Attained equilibria after best-response dynamics from an empty network in (δ, c)-space for $n = 100$, $k = 15$ and $\alpha \in 1, 10$ under the specification of f_e and f_m in equation (1). The roman numbers correspond with those in Theorem 1 and Proposition 4. In the shaded area, the complete core-periphery network with $k = 15$ big banks is the unique unilaterally stable outcome of the dynamics.

6. Conclusion

In this paper we propose a way to explain the formation of financial networks by intermediation. We focus on the core-periphery network because it is found to give a fair representation of the complex empirical structures, while at the same time being relatively simple and intuitively appealing. In our model brokers strive to intermediate between their counterparties and compete with each other. Our results suggest that heterogeneity is crucial, that is, the core-periphery structure of the interbank market cannot be understood separately from the heterogeneity and inequality in the intrinsic characteristics of banks.

We explore these results further in a dynamic extension of the model. We endogenize heterogeneity by updating the size of each bank with the payoffs received from trades in the network. Better connected banks receive higher payoffs by intermediation, which could feed back on the balance sheet and thus on future trade opportunities of these banks. We find that core-periphery networks arise endogenously in the extended model with ex ante homogeneous agents and feedback of the network structure on trade surpluses.

We would like to make three suggestions for future research. First, in our model we treat the effect of negative and positive liquidity shocks in a symmetric way. However, in actual interbank markets, the effect of negative liquidity shocks are likely to be more severe, in particular for small banks. This may explain a well known fact in the interbank market that, on average, large bank tend to be net borrowers, whereas small banks tend to be net lenders (Allen et al., 1989; Furfine, 1999; Cocco et al., 2009). An extension of the model to include asymmetric effects of positive and negative liquidity shocks may shed light on this phenomenon.

Second, research that analyzes financial contagion typically take the balance sheet and network structure as exogenously fixed. However, this forces them to make arbitrary assumptions on the size of the balance sheets, that is, total (interbank) assets and liabilities, when analyzing the effect of heterogeneity or core-periphery structure on financial contagion. For example, Nier et al. (2007) keep the size of banks' balance sheet constant, when analyzing the effect of a two-tier system on financial contagion. Our research suggests that one cannot impose such arbitrary regularities, and instead one has to think carefully on how heterogeneity in balance sheets and heterogeneity in network structure co-evolve. This point was made by Glasserman and Young (2016) as well. In this paper, we do not model the balance sheets. Introducing balance sheets in our model, would be one potential direction for future research.

Third, our model does not involve any risk of banks defaulting. Hence, our model is a model of a riskless interbank market, such as the interbank overnight loan market approximately was before the financial crisis of 2007. However, we could analyze the effects of an increase of default risk on overnight interbank lending as well by introducing a probability that a bank and its links default. Doing so allows us to understand network formation in stress situations. This seems relevant in light of findings that the fit of the core-periphery network in the interbank market deteriorated during the recent financial crisis (in 't Veld and van Lelyveld, 2014; Martínez-Jaramillo et al., 2014; Fricke and Lux, 2015).

References

Acemoglu, D., Ozdaglar, A., Tahbaz-Salehi, A., 2015. Systemic risk and stability in financial networks. American Economic Review 105 (2), 564–608.

Afonso, G., Lagos, R., 2015. Trade dynamics in the market for federal funds. Econometrica 83, 263–313.

Allen, F., Gale, D., 2000. Financial contagion. Journal of Political Economy 108, 1–33.

Allen, L., Peristiani, S., Saunders, A., 1989. Bank size, collateral, and net purchase behavior in the federal funds market: empirical evidence. Journal of Business, 501–515.

Babus, A., 2016. The Formation of Financial Networks. RAND Journal of Economics 47 (2), 239–272.

Babus, A., Hu, T.-W., 2017. Endogenous Intermediation in Over-the-Counter Markets. Journal of Financial Economics 125 (1), 200–215.

Bedayo, M., Mauleon, A., Vannetelbosch, V., 2016. Bargaining in endogenous trading networks. Mathematical Social Sciences 80, 70–82.

Borgatti, S. P., Everett, M. G., 1999. Models of core/periphery structures. Social Networks 21, 375–395.

Bräuning, F., Fecht, F., 2017. Relationship Lending in the Interbank Market and the Price of Liquidity. Review of Finance 21 (1), 33–75.

Buskens, V., van de Rijt, A., 2008. Strategic network formation with structural holes. American Journal of Sociology 114, 37–407.

Cabrales, A., Gottardi, P., Vega-Redondo, F., 2017. Risk-sharing and contagion in networks. Review of Financial Studies 30 (9), 3086–3127.

Castiglionesi, F., Navarro, N., 2016. (in)efficient interbank networks. Cahiers du GREThа 2016-13, GREThа.

Chang, B., Zhang, S., 2019. Endogenous market making and network formation. mimeo, University of Wisconsin-Madison and London School of Economics.

Cocco, J. a. F., Gomes, F. J., Martins, N. C., 2009. Lending Relationships in the Interbank Market. Journal of Financial Intermediation 18 (1), 24–48.

Cohen-Cole, E., Patacchini, E., Zenou, Y., 2015. Static and dynamic networks in interbank markets. Network Science 3 (01), 98–123.

Craig, B., Von Peter, G., 2014. Interbank tiering and money center banks. Journal of Financial Intermediation 23 (3), 322–347.

Di Maggio, M., Kermani, A., Song, Z., 2017. The value of trading relationships in turbulent times. Journal of Financial Economics 124 (2), 266–284.

Duffie, D., Gârleanu, N., Pedersen, L. H., 2005. Over-the-Counter Markets. Econometrica 73, 1815–1847.

Eisenberg, L., Noe, T. H., 2001. Systemic Risk in Financial Systems. Management Science 47 (2), 236–249.

Elliott, M., Golub, B., Jackson, M. O., 2014. Financial networks and contagion. American Economic Review 104 (10), 3115–3153.

Farboodi, M., 2017. Intermediation and voluntary exposure to counterparty risk. Mimeo, Princeton University.

Fricke, D., Lux, T., 2015. Core–periphery structure in the overnight money market: evidence from the e-mid trading platform. Computational Economics 45 (3), 359–395.

Furfine, C. H., 1999. The microstructure of the federal funds market. Financial Markets, Institutions & Instruments 8 (5), 24–44.

Gai, P., Haldane, A., Kapadia, S., 2011. Complexity, Concentration and Contagion. Journal of Monetary Economics 58 (5).

Galeotti, A., Goyal, S., 2010. The law of the few. American Economic Review 100 (4), 1468–92.

Glasserman, P., Young, H. P., 2016. Contagion in financial networks. Journal of Economic Literature 54 (3), 779–831.

Goyal, S., 2009. Connections: An Introduction to the Economics of Networks. Princeton University Press.

Goyal, S., Vega-Redondo, F., 2007. Structural holes in social networks. Journal of Economic Theory 137 (1), 460–492.

Hiller, T., 2017. Peer effects in endogenous networks. Games and Economic Behavior 105, 349–367.

Houy, N., 2009. Structural holes in social networks: A remark. Journal of Economic Theory 144 (1), 422–431.

in ’t Veld, D., van Lelyveld, I., 2014. Finding the core: Network structure in interbank markets. Journal of Banking & Finance 49, 27–40.

Jackson, M. O., 2005. A survey of network formation models: Stability and efficiency. In: Demange, G., Wooders, M. (Eds.), Group Formation in Economics: Networks, Clubs and Coalitions. Cambridge University Press, Cambridge, UK, pp. 11–57.

Jackson, M. O., Watts, A., 2002. The evolution of social and economic networks. Journal of Economic Theory 106 (2), 265–295.

Jackson, M. O., Wolinsky, A., 1996. A strategic model of social and economic networks. Journal of Economic Theory 71 (1), 44–74.

Kleinberg, J., Suri, S., Tardos, E., Wexler, T., 2008. Strategic network formation with structural holes. Proceedings 9th ACM Conference on Electronic Commerce, 480–489.

König, M. D., Tessone, C. J., Zenou, Y., 2014. Nestedness in networks: A theoretical model and some applications. Theoretical Economics 9 (3), 695–752.

Martínez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benítez, B., Solórzano-Margain, J. P., 2014. An empirical study of the mexican banking system’s network and its implications for systemic risk. Journal of Economic Dynamics and Control 40, 242–265.

Merlo, A., Wilson, C. A., 1995. A stochastic model of sequential bargaining with complete information. Econometrica 63 (2), 371–99.

Nier, E., Yang, J., Yorulmazer, T., Alentorn, A., 2007. Network Models and Financial Stability. Journal of Economic Dynamics and Control 31 (6), 2033–2060.

Peltonen, T. A., Scheicher, M., Vuillemey, G., 2014. The network structure of the cds market and its determinants. Journal of Financial Stability 13, 118–133.

Persitz, D., 2016. Core-periphery r&d collaboration networks. Tech. rep., Tel Aviv University.

Rochet, J. C., Tirole, J., 1996. Interbank Lending and Systemic Risk. Journal of Money, Credit, and Banking 28 (4), 733–762.

Siedlarek, J.-P., 2015. Intermediation in networks. Working Paper 15-18, Federal Reserve Bank of Cleveland.

Wang, C., 2018. Core-periphery trading networks. mimeo, Wharton School, University of Pennsylvania.

Appendix A. Siedlarek's payoff function

Siedlarek (2015) applies the bargaining protocol introduced by Merlo and Wilson (1995) to derive the distribution of a surplus for a trade that is intermediated by competing middlemen. The underlying idea is that one involved agent is selected to propose a distribution of the surplus. If the proposer succeeds in convincing the other agents to accept his offer, the trade will be executed. Otherwise, the trade will be delayed and another (randomly selected) agent may try to make a better offer. A common parameter $0 \leq \delta \leq 1$ is introduced with which agents discount future periods, that results in the level of competition as used in our paper. The relation between the discount factor δ and the level of competition, is that if agents are more patient, then intermediaries are forced to offer more competitive intermediation rates when having the chance to propose a distribution, as trading partners are more willing to wait for the opportunity to trade with alternative intermediaries. As a special case, for $\delta = 1$ the surplus is distributed equally among the essential players as in Goyal and Vega-Redondo (2007).

More formally, assume that the set of possible trading routes is known to all agents (i.e. complete information). Each period a route is selected on which the trade can be intermediated, and additionally one player (the 'proposer') on this route that proposes an allocation along the entire trading route.[27] Any state, which is a selection of a possible path and a proposer along that path, is selected with equal probability and history independent. The question now is: what is the equilibrium outcome, i.e. the expected distribution of the surplus, taking into account that every agent proposes optimally under common knowledge of rationality of other possible future proposers? Siedlarek (2015) shows that the unique Markov perfect equilibrium is characterized by the following payoff function for any player i in a certain state:

$$f_i = \begin{cases} 1 - \sum_{j \neq i} \delta E_j[f_j] & \text{if } i \text{ is the proposer in this state} \\ \\ \delta E_i[f_i] & \text{else if } i \text{ is involved in this state} \end{cases} \tag{A.1}$$

This equation shows that the proposer can extract all surplus over and above the outside option value given by the sum of $E_j[f_j]$ over all other players $j \neq i$. All (and only) the players along the same route have to be convinced by offering exactly their outside option.

$$\textcircled{i} \longrightarrow \textcircled{k} \longrightarrow \textcircled{j}$$

Figure A.12: A trading route with one intermediary

In the simple example of intermediary k connecting i and j (see Figure A.12), each of the three

[27] Siedlarek (2015) assumes that only *shortest* paths are considered, an assumption not explicitly made by Goyal and Vega-Redondo (2007). However, in the model Goyal and Vega-Redondo (2007) only essential players, who by definition are part of shortest paths, receive a nonzero share of the surplus.

players has equal probability of becoming the proposer and proposes an equal share to the other two, so the equilibrium distribution of the surplus will simply be the equal split $(f_i, f_k, f_j) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ for all δ.

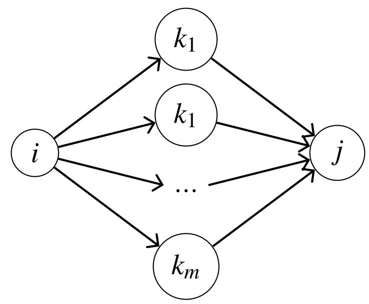


Figure A.13: A trading network with m competing intermediaries

In case m intermediaries compete for the trade between i and j (as in Figure A.13), the extracted intermediation rents are reduced. Siedlarek (2015) shows that:

$$f_i = f_j = f_e(m,\delta) \equiv \frac{m-\delta}{m(3-\delta)-2\delta} \tag{A.2}$$

$$f_k = f_m(m,\delta) \equiv \frac{1-\delta}{m(3-\delta)-2\delta} \tag{A.3}$$

As mentioned before, this distribution of payoffs satisfies our assumptions on $f_l(m,\delta)$, $f_b(m,\delta)$ and $f_m(m,\delta)$. The distribution of Siedlarek (2015) is used as the leading example in the results.

In a general network with longer intermediation chains $d > 2$, the payoffs cannot be calculated directly, but depend on specifics of (part of) the network. The vector of payoffs $\overrightarrow{F}$ can be calculated indirectly from the network g using the following notation:

- s: The number of shortest paths that support the trade,
- d: The length of the shortest paths (i.e. the number of required intermediaries plus one),
- $\overrightarrow{P}$: $(n \times 1)$-vector of shortest paths that run through any player,
- K: $(n \times n)$-diagonal matrix of times that any player receives an offer,
- S: $(n \times n)$-off-diagonal matrix of times that two players share a path.

As in any given state the distribution of profits is as given in equation (A.1), the vector of payoffs that averages over all possible states of the world is:

$$\overrightarrow{F} = \frac{1}{s \cdot d}(\overrightarrow{P} - \delta(S-K)\overrightarrow{F}) \tag{A.4}$$

If $\delta < 1$ this can be solved to:

$$\overrightarrow{F} = (s \cdot d \cdot I_n + \delta(S-K))^{-1} \overrightarrow{P} \tag{A.5}$$

In Appendix E we use this general formula to investigate whether core-periphery networks can be stable when intermediation paths of lengths $d = 3$ are allowed.

Appendix B. Proofs of Section 2

Proof of Proposition 1. Writing out the payoff function, we obtain

$$\begin{aligned}
\tilde{\pi}_i(g,\delta,\tilde{c}) &= \sum_{t=1}^{\infty} \beta^t \left(\left(\sum_{s_t \in \{-1,0,1\}^n} P[S_t = s_t] b_i(s_t, g, \delta) \right) - \eta_i(g)\tilde{c} \right) \\
&= \frac{\beta}{1-\beta} \left(\left(\sum_{s_t \in \{-1,0,1\}^n} P[S_t = s_t] b_i(s_t, g, \delta) \right) - \eta_i(g)\tilde{c} \right) \\
&= \frac{\beta}{1-\beta} \left(\left(\sum_{s_t : n_t^+ = n_t^- = 1} P[S_t = s_t] b_i(s_t, g, \delta) + \sum_{s_t : n_t^+ + n_t^- \geq 3} P[S_t = s_t] b_i(s_t, g, \delta) \right) - \eta_i(g)\tilde{c} \right),
\end{aligned}$$

where $n_t^- = |s_{it} < 0|$, $n_t^+ = |s_{it} > 0|$, and $n_t^0 = n - n_t^+ - n_t^-$. Note that $b_i(s_t, g, \delta) = 0$ if $n_t^+ = 0$ or $n_t^- = 0$, as in that case, there is no trade. Let $A = \max_i \alpha_i$. S_t has a multinomial distribution, and hence $P[S_t = s_t]$ is bounded by $P[S_t = s_t] < A^n \rho^{n_t^+ + n_t^-}$. As $b_i(s_t, g, \delta) < 1$, and $|s_t : n_t^+ + n_t^- \geq 3| < 2^n$ we have

$$\lim_{\rho \downarrow 0} \frac{1}{\rho^3} \sum_{s_t : n_t^+ + n_t^- \geq 3} P[S_t = s_t] b_i(s_t, g, \delta) < (2A)^n,$$

that is, benefits from periods in which three banks or more receive a liquidity shock are of order $O(\rho^3)$. Hence,

$$\tilde{\pi}_i(g,\delta,\tilde{c}) = \frac{\beta}{1-\beta} \left(\left(\sum_{s_t : n_t^+ = n_t^- = 1} P[S_t = s_t] b_i(s_t, g, \delta) \right) - \eta_i(g)\tilde{c} \right) + O(\rho^3).$$

Suppose that j receives a positive shock and k a negative shock. Then the benefit of i is $b_i(s_t, g, \delta) = sv_i^{jk}(g,\delta) = v_i^{jk}(g,\delta)/2$, as explained in the main text. Hence the payoff function becomes

$$\begin{aligned}
\tilde{\pi}_i(g,\delta,\tilde{c}) &= \frac{\beta}{1-\beta}\left[\left(\sum_j \sum_{k\neq j} P[S_{jt}=1, S_{kt}=-1, \forall l \neq j,k : S_{lt}=0] v_i^{jk}(g,\delta)/2\right) - \eta_i(g)\tilde{c}\right] + O(\rho^3) \\
&= \frac{\beta}{1-\beta}\left[\left(\sum_j \sum_{k\neq j} \rho^2 \alpha_j \alpha_k \left(\prod_{l\neq j,k}(1-2\rho\alpha_l)\right) v_i^{jk}(g,\delta)/2\right) - \eta_i(g)\tilde{c}\right] + O(\rho^3) \\
&= \frac{\beta}{1-\beta}\left[\left(\sum_j \sum_{k\neq j} \rho^2 \alpha_j \alpha_k v_i^{jk}(g,\delta)/2\right) - \eta_i(g)\tilde{c}\right] \\
&\qquad - \frac{\beta}{1-\beta}\left[\sum_j \sum_{k\neq j} \rho^2 \alpha_j \alpha_k \left(1 - \prod_{l\neq j,k}(1-2\rho\alpha_l)\right) v_i^{jk}(g,\delta)/2\right] + O(\rho^3) \\
&= \frac{\beta\rho^2}{1-\beta}\left[\left(\sum_j \sum_{k\neq j} \alpha_j \alpha_k v_i^{jk}(g,\delta)/2\right) - \eta_i(g)\frac{\tilde{c}}{\rho^2}\right] + O(\rho^3),
\end{aligned}$$

as

$$\rho^2\left(1 - \prod_{l\neq j,k}(1-2\rho\alpha_l)\right) = O(\rho^3).$$

The benefits for bank i from direct trade are:

$$\sum_{j\in N_i^1(g)} \alpha_i\alpha_j \left(v_i^{ij}(g,\delta)/2 + v_i^{ji}(g,\delta)/2\right) = \sum_{j\in N_i^1(g)} \alpha_i\alpha_j \frac{f_l^0 + f_b^0}{2} = \sum_{j\in N_i^1(g)} \frac{1}{2}\alpha_i\alpha_j,$$

as $f_l^0 + f_b^0 = 1$. Note that we count two trades for i and j, one time i having a liquidity surplus, and one time i having a liquidity deficit. Similarly, the benefits from indirect trade are given by

$$\sum_{j\in N_i^2(g)} \alpha_i\alpha_j \left(v_i^{ij}(g,\delta)/2 + v_i^{ji}(g,\delta)/2\right) = \sum_{j\in N_i^2(g)} \alpha_i\alpha_j f_e(m_{ij}(g),\delta),$$

where $f_e(\cdot) = (f_l(\cdot) + f_b(\cdot))/2$, and for $d > 2$:

$$\sum_{j\in N_i^d(g)} \alpha_i\alpha_j \left(v_i^{ij}(g,\delta)/2 + v_i^{ji}(g,\delta)/2\right) = 0.$$

Finally, intermediation benefits are given by

$$\sum_{k,l\in N_i^1(g)|g_{kl}=0} \alpha_k\alpha_l \left(v_i^{kl}(g,\delta)/2 + v_i^{kl}(g,\delta)/2\right) = \sum_{k,l\in N_i^1(g)|g_{kl}=0} \alpha_k\alpha_l f_m(m_{kl}(g),\delta),$$

for all pairs k, l at distance 2 and i in between. For all other pairs, there are no benefits to i. The payoff function then follows. $\square$

Appendix C. Proofs of Section 3

Definition of best feasible action. We first introduce the concept of best feasible (unilateral) action of a certain player i. An action of player i is feasible if its proposed links to every $j \in S$ are accepted by every $j \in S$, or if i deletes all links with every $j \in S$. This action changes the network g into g^{iS}. The best feasible action is formally defined as follows.

Definition 5. *A feasible action for player i in network g is represented by a subset $S \subseteq N \backslash \{i\}$ with:*

(a) $\forall j \in S : g_{ij} = 1$, *or:*

(b) $\forall j \in S : g_{ij} = 0$ *and* $\pi_j(g^{iS}) \geq \pi_j(g)$.

A best feasible action S^* *for player i in network g is a feasible action in g that gives i the highest payoffs:*

$$\forall S \subseteq N \backslash \{i\} \text{: } \pi_i(g^{iS^*}) \geq \pi_i(g^{iS}).$$

A network g is unilaterally stable if and only if, for all players, $S^* = \emptyset$ is a best feasible action in g. In other words, no player has an incentive to take a feasible action that changes the network.

We continue with the proofs of Section 3.

Proof of Proposition 2. Suppose g is a core-periphery network, such that there are two core agents $i, j \in K$ with $N_i \supseteq N_j$ and $n_j = |N_j| \geq k+1$. Note that by definition of a core-periphery network: $g_{ij} = 1$, as both i and j are in the core.

For the marginal benefit of any two peripheral players i_1 and i_2 to connect directly is, it holds that:

$$\pi_{i_l}(g + g_{i_1 i_2}) - \pi_{i_l}(g) \geq \left(\frac{1}{2} - f_e(k, \delta)\right) - c \tag{C.1}$$

for $l = 1, 2$, as two peripheral players have by definition at most k intermediaries.

For the marginal benefit of core player i to delete its link to j, it holds that:

$$\pi_i(g - g_{ij}) - \pi_i(g) \geq c - \left(\frac{1}{2} - f_e(k, \delta)\right). \tag{C.2}$$

The value $\left(\frac{1}{2} - f_e(k, \delta)\right)$ equals the difference in payoff i receives from its link to j. After the removal of their direct link , player i is indirectly linked to j via at least k intermediaries, that is, $k-2$ core and at least 2 periphery agents to which both i and j are connected. Note that i and j must have at least 2 periphery agents in common as $n_i \geq n_j = |N_j| \geq k+1$.

Suppose now, contrary to the propositioin, that g is pairwise stable. In order for the network to be pairwise stable, it is required that $\pi_{i_l}(g+g_{i_1 i_2})-\pi_{i_l}(g)\leq 0$ and $\pi_i(g-g_{ij})-\pi_i(g)\leq 0$ hold simultaneously, which implies that

$$\pi_{i_l}(g+g_{i_1 i_2})-\pi_{i_l}(g)=\pi_i(g-g_{ij})-\pi_i(g)=\left(\frac{1}{2}-f_e(k,\delta)\right)-c=0. \tag{C.3}$$

Hence, $c=\left(\frac{1}{2}-f_e(k,\delta)\right)$.

Consider now a third periphery node $i_3\in P$. This node exists, since $k\leq n-3$. Node i_3 cannot be linked to both i or j, as in that case, i and j have at least 3 periphery neighbors in common, such that the marginal benefit for i to delete its link with j would be

$$\pi_i(g-g_{ij})-\pi_i(g)\geq c-\left(\frac{1}{2}-f_e(k+1,\delta)\right)>c-\left(\frac{1}{2}-f_e(k,\delta)\right),$$

which would violate (C.3). Consider now the marginal benefit for i_1 and i_3 of creating a direct link. As i_3 is linked to at most $k-1$ core nodes, the marginal benefit is

$$\pi_{i_l}(g+g_{i_1 i_3})-\pi_{i_l}(g)\geq\left(\frac{1}{2}-f_e(k-1,\delta)\right)-c>\left(\frac{1}{2}-f_e(k,\delta)\right)-c,$$

for $l=1,3$. This violates (C.3). Hence, g is not pairwise stable if $0<\delta<1$. □

Proof of Proposition 3. Proposition 2 was concerned with complete core-periphery networks. For general core-periphery networks, we now derive a lower bound, for which it is a best feasible action of a periphery player to enter the core.

Consider a periphery player i proposing $l_i=n-k-1$ links in order to reach all other periphery players. The marginal benefits[28] for i of this action are bounded by:

$$M_i(g^{CP(k)},+l_i)\geq-(n-k-1)(c-\frac{1}{2}+f_e^1)+\binom{n-k-1}{2}f_m(2,\delta). \tag{C.5}$$

Then positive marginal benefits for i are implied by:

$$\begin{aligned} M_i(g^{CP(k)},+l_i) &> 0 \\ \Leftarrow n-k &> F_1(c,\delta)\equiv 2+\frac{c-\frac{1}{2}+f_e^1}{\frac{1}{2}f_m(2,\delta)} \end{aligned} \tag{C.6}$$

[28]Throughout the appendix, marginal benefits of an action S by player i for player j are defined as:

$$M_j(g,S)\equiv\pi_j(g^{iS})-\pi_j(g). \tag{C.4}$$

Suppose g is a core-periphery network, such that there exist two distinct periphery agents $i_1,i_2\in P$ that are connected by two distinct core agents $j_1,j_2\in K: g_{i_1 j_1}=g_{i_1 j_2}=g_{i_2 j_1}=g_{i_2 j_2}=1$. Let $k=|K|$ be the number of core nodes. Note that by definition of a core-periphery network: $g_{i_1 i_2}=0$ and $g_{j_1 j_2}=1$.

For the deviation of l_i links to be executed, the peripheral players $j \in (P \setminus i)$ also have to agree with the addition, that is, they should not receive a lower payoff. The marginal benefits for j are bounded by:

$$M_j(g^{CP(k)}, +l_i) \geq -c + (n-k-2)(f_e(2,\delta) - f_e^1) \tag{C.7}$$

Positive marginal benefits for j are implied by:

$$\begin{aligned} M_j(g^{CP(k)}, +l_i) &\geq 0 \\ \Leftarrow n-k &> F_2(c,\delta) \equiv 2 + \frac{c}{f_e(2,\delta) - f_e^1} \end{aligned} \tag{C.8}$$

Combining the conditions for i (C.6) and j (C.8), a sufficient lower bound for the number of periphery nodes $n-k$ for any core-periphery network to be unilaterally unstable is:

$$n-k > F(c,\delta) \equiv \max\{F_1(c,\delta), F_2(c,\delta)\} \tag{C.9}$$

□

Remarks on Proposition 3. In the derivation of this sufficient lower bound on n, we have not made any assumptions about the path lengths on which trade is allowed. It is sufficient to consider the parts of the marginal benefits for i and j that depend on the constant costs c and the new intermediation route is formed between $j,l \in (P \setminus i)$.

A core-periphery network is generally unstable because the inequality between core and periphery becomes large for increasing n, and a periphery player can always benefit by adding links to all other players. An important assumption for this result is therefore that multiple links can be added at the same time. It is crucial that $\delta > 0$ because for $\delta = 0$ intermediated trade always generates $f_e(m,0) = f_e^0$ $\forall m$, i.e. additional intermediation paths do not increase profits for endnodes. Moreover it is crucial to impose $\delta < 1$ because for $\delta = 1$ intermediation benefits disappear, i.e. $f_m(m,1) = 0$ $\forall m > 1$.

Proof of Theorem 1. To proof this Theorem we start with two Lemma's, followed by the main part of the proof that uses these Lemma's.

In an empty network, the best feasible action for a player is to connect either to all other players or to none, as shown in the following lemma.

Lemma 1. *Consider the homogeneous baseline model with $\alpha_i = 1$ for all $i \in N$. If the empty network g^e is not unilaterally stable, then the best feasible action for a player in g^e is to add links to all other nodes.*

Proof of Lemma 1. The marginal benefits of adding l links to an empty network are:

$$M_i(g^e, +l) = l(\frac{1}{2} - c) + \binom{l}{2}\frac{1}{3} \tag{C.10}$$

As this function is convex in l, maximizing the marginal benefits over l results in either $l^* = 0$ or $l^* = n-1$. □

In a complete core-periphery network, if a peripheral agent has an incentive to add a link it is optimal to connect to all other players, thereby entering in the core himself.

Lemma 2. *Consider the homogeneous baseline model with $\alpha_i = 1$ for all $i \in N$, and suppose that the complete core-periphery network $g_{com}^{CP(k)}$ (including the star network) with $k \in \{1, 2, ..., n-2\}$ is not unilaterally stable because a peripheral player i can deviate by adding one or more links. Then the best feasible action for i in $g_{com}^{CP(k)}$ is to add links to all $l = n-k-1$ other periphery players.*

Proof of Lemma 2. The marginal benefits of a periphery member for replicating the position of a core member of a complete core-periphery networks with k core members are:

$$M_i(g_{com}^{CP(k)}, +l) = l(\frac{1}{2} - f_e(k, \delta) - c) + \binom{l}{2} f_m(k+1, \delta) \tag{C.11}$$

As this function is convex in l, the maximum of this function is reached at $l^* = 0$ or $l^* = n-k-1$. □

Now we have proven Lemma's 1 and 2, we continue the proof of Theorem 1.

I: By Lemma 1, after the move of player $i = 1$ the network is either empty or a star. In case it is empty the process converges to an empty network, as all other nodes face the same decision as $i = 1$. Player 1 has no incentive or possibility to form a star g^s with 1 the center if $\pi_1(g^s) < 0$, that is, if

$$c > \frac{1}{2} + \frac{1}{6}(n-2),$$

and part I directly follows.

II: If

$$c < \frac{1}{2} + \frac{1}{6}(n-2), \tag{C.12}$$

then player 1 forms a star. Note that this is a feasible action, because $\pi_1(g^s) > 0$ implies $\pi_j(g^s) > 0$ for $j \neq 1$.

If player 2 does not want to add a subset of links, then the other players also do not want to, in which case the star is the outcome of the dynamic process (by part I, deleting her link is not profitable for player 2). If player 2 has an incentive to add links, then by Lemma 2 she will add all $n-2$ links in order to form a complete core-periphery network $g_{com}^{CP(2)}$ with 1 and 2 in the core.

This happens if $\pi_2(g_{com}^{CP(2)}) > \pi_2(g^s)$ and $\pi_j(g_{com}^{CP(2)}) > \pi_j(g^s)$ for all $j = 3, \ldots, n$, that is,

$$c < \frac{1}{6} + \frac{1}{2}(n-3)f_m(2,\delta) \text{ and } c < \frac{1}{6} + (n-3)(f_e(2,\delta) - \frac{1}{3}). \tag{C.13}$$

On the other hand, if either $c > \frac{1}{6} + \frac{1}{2}(n-3)f_m(2,\delta)$ or $c > \frac{1}{6} + (n-3)(f_e(2,\delta) - \frac{1}{3})$, then neither player 2 nor any of the subsequent players change the network, and part II directly follows.

III: Under equation (C.13), after player 1 and 2 have chosen their links, a core-periphery network with 2 core players is formed. Player 3 does not delete any of her links under the second part of equations (C.12) and (C.13). By the same argument using Lemma 2, each next player $i \in \{3, 4, ..., k\}$ adds links to all nodes not yet connected to i, as long as costs c are low enough. This might lead to a stable complete network, if the last two nodes have an incentive to form a link (which will be proposed by player $i = n-1$), i.e.

$$c < \frac{1}{2} - f_e(n-2,\delta), \tag{C.14}$$

and part III directly follows.

For parameters not satisfying I, II or III, the first round of best feasible actions results in a complete core-periphery network with $1 < k < n-1$ core members. This complete core-periphery network is not stable by Lemma 2. Because the $k+1$-th node did not connect to other periphery nodes, adding links in the periphery cannot be beneficial. Therefore the first core bank $i = 1$ must have an incentive to delete at least one within-core link. The marginal benefit of deleting l core links in the network is:

$$M_i(g_{com}^{CP(k)}, -l) = l(c - \frac{1}{2} + f_m(n-l-1,\delta)). \tag{C.15}$$

As $M_i(g_{com}^{CP(k)}, -0) = 0$ and $M_i(g_{com}^{CP(k)}, -1) > 0$, there is a unique choice $l_1^* > 0$ of the optimal number of core links to delete. It will become clear that, for parameters outside I ∪ II ∪ III, attracting networks are multipartite networks of various sorts, depending on the choice l_1^*.

An important case is that a complete core-periphery network with $k = 2$ has arisen after the first round. For $k = 2$ the only possible solution is $l_1^* = 1$. A (complete) bipartite network arises with a small group of 2 players and a large group of $n-2$ players, the maximal difference in group size possible for bipartite networks. We denote such a network as $g_{2,n-2}^{mp(2)}$.

The case of $k = 2$ happens if the third player $i = 3$ does not enter the core, either because entering is not beneficial for himself or because some other periphery players j does not accept the offer of i. For $k = 2$ a positive marginal benefit of entering the core implies:

$$\begin{aligned} M_i(g_{com}^{CP(2)}, +(n-2)) &> 0; \\ \Rightarrow c &< \frac{1}{2} - f_e(2,\delta) + \frac{1}{2}(n-4)f_m(3,\delta), \end{aligned} \tag{C.16}$$

and the periphery player j has an incentive to accept the offer if

$$\begin{aligned} M_j(g^{CP(2)}_{com}, +(n-2)) &\geq 0; \\ \Rightarrow c &\leq \frac{1}{2} - f_e(2,\delta) + (n-4)(f_e(3,\delta) - f_e(2,\delta)); \end{aligned} \tag{C.17}$$

So if after the first round the result is $k = 2$ and the third player has not entered the core, it must be the case that:

$$c \geq \frac{1}{2} - f_e(2,\delta) + (n-4)\min\{\frac{1}{2}f_m(3,\delta), f_e(3,\delta) - f_e(2,\delta)\}. \tag{C.18}$$

The parameter values for which the maximally unbalanced network $g^{mp(2)}_{2,n-2}$ is the attracting steady state are given by condition IV.

Alternatively, under the remaining condition V, the first round has resulted in a complete core-periphery network with $k > 2$. This network cannot be stable, and the first core bank $i = 1$ has an optimal choice of $0 < l_1^* \leq k-1$ links to delete depending on the parameters. First consider that the core bank deletes all its within-core links, i.e. $l_1^* = k-1$. This action necessarily implies that the linking costs exceed the loss in surplus associated with having an indirect connection to other core banks via the periphery rather than a direct connection:

$$c > \frac{1}{2} - f_e(n-k,\delta). \tag{C.19}$$

Given this high level of linking costs, the next core banks $i = \{2,...,k\}$ have the same incentive to delete all their within-core links. The attracting network is therefore a multipartite network $g^{mp(2)}_{k_1,k_2}$ with two groups of size $k_1 = k$ and $k_2 = n-k$.

Finally, consider the case $0 < l_1^* < k-1$.[29] Denote $K_1 \subset K$ as the set of core banks with at least one missing within-core link after the best feasible action of $i = 1$, including players $i = 1$ itself. The size of this set of banks is $k_1 = l_1^* + 1$. The best feasible action of $i = 1$ necessarily implies:

$$c > \frac{1}{2} - f_e(n-k_1,\delta). \tag{C.20}$$

Given this high level of linking costs, all next banks $i \in K_1$ with connections to some other $j \in K_1$ with $j > i$ have the same incentive to delete every link g_{ij}. These banks do not have an incentive to remove any further link, because the number of intermediators for indirect connections to banks $j \in K_1$ would become less than $n-k_1$. If the latter were worthwhile, $i = 1$ would have chosen a higher number l_1^*. Therefore K_1 becomes a group of players not connected within their group, but

[29]This is the only case in which the best feasible action is not unique in a parameter region with nonzero measure: the first core banks $i = 1$ has $\binom{k}{l_1^*}$ best feasible actions deleting l_1^* links from the core. The resulting multipartite networks can consist of different sets $K_1, K_2, ..., K_{q-1}$, but all are isomorphic.

completely connected to all players outside their group.

Players $i \in (K \setminus K_1)$ are connected to all other players $j \in N$. Because of the lower bound on c in equation (C.20), these players have an incentive to remove links to some j as long as the number of intermediators for the indirect connections to j stays above $n-k_1$. This will lead to other groups $K_2, ..., K_{q-1}$ of players not connected within their group, but completely connected to all players outside their group. The remaining group K_q was the original periphery at the end of the first round. The result of the best-feasible-action dynamics if $0 < l_1^* < k-1$ is a multipartite network $g^{mp(q)}_{k_1,k_2,...,k_q}$ with $q \geq 3$.

For all parameters under condition V, the resulting network is multipartite with $q = \lfloor \frac{k}{k_1} \rfloor + 1$ groups. These multipartite networks are more balanced than $g^{mp(2)}_{2,n-2}$, i.e. have group sizes $|k_m - k_{m'}| < n-4$ for all $m, m' \in \{1, 2, .., q\}$. □

Appendix D. Proofs of Section 4

Proof of Proposition 4. We start with possible deviations of a peripheral player by adding links to other periphery players. Note that the change in payoffs by these deviations do not depend on α, because they only concern trade surpluses between small banks. Hence Lemma 2 holds also in this heterogeneous setting: if adding one link improves the payoff of a small periphery player, the best feasible action is to connect to all other small banks.

Starting from a complete core-periphery network with the k big banks in the core, if one peripheral player adds links to all other periphery players, the core is extended to $k+1$ players, k big banks and 1 small bank. For the new core member i to have positive marginal benefits of supporting all new links, it is required that

$$\begin{aligned} M_i(g^{CP(k)}_{com}, +(n-k-1)) &> 0 \\ \Rightarrow c &< \frac{1}{2} - f_e(k,\delta) + \frac{1}{2}(n-k-2) f_m(k+1,\delta) \end{aligned} \tag{D.1}$$

and every remaining peripheral player $j \neq i$ requires

$$\begin{gathered} M_j(g^{CP(k)}_{com}, +(n-k-1) \geq 0 \\ \Rightarrow c \leq \tfrac{1}{2} - f_e(k,\delta) + \tfrac{1}{2}(n-k-2) + (n-k-2)(f_e(k+1,\delta) - f_e(k,\delta)). \end{gathered} \tag{D.2}$$

Conversely, the deviation of player i is *not* a best feasible action if c exceeds the minimum of the two values in equations (D.1) and (D.2):

$$c \geq \frac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\frac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}. \tag{D.3}$$

Given a complete core-periphery networks and a sufficiently high c satisfying (D.3), it is not beneficial for periphery players to add links.

We will now derive an $\overline{\alpha}$, such that for all $\alpha > \overline{\alpha}$ the complete core-periphery network with k big banks is unilaterally stable. In order to derive $\overline{\alpha}$ we consider the possible deletion of one or multiple links by either core or periphery players. After deriving $\overline{\alpha}$, we will rewrite the condition of stable complete core-periphery networks in terms of c.

First, consider a core player i. Player i can delete links with other core players and/or links with periphery players. The marginal benefit of deleting l^c core links and l^p periphery links is:

$$\begin{aligned} M_i(g_{com}^{CP(k)}, -(l^c + l^p)) &= l^c\Big(c - \alpha^2\big(\frac{1}{2} - f_e(n - l^c - l^p - 1, \delta)\big)\Big) \\ &\quad + l^p\Big(c - \alpha\big(\frac{1}{2} - f_e(k - l^c - 1, \delta)\big) - (2n - 2k - l^p - 1) f_m(k, \delta)\Big) \\ &\equiv M_i^c + M_i^p \end{aligned} \tag{D.4}$$

The marginal benefit of deleting $l^c + l^p$ links can be separated in benefits from deleting links with the core M_i^c and benefits from deleting links with the periphery M_i^p. The cross-over effects of deleting links with both groups of banks are negative: M_i^c is decreasing in l^p and M_i^p is decreasing in l^c. To find the conditions under which the core player does not want to delete any link, it is therefore sufficient to consider deletion of links in each group separately.

The marginal benefit of deleting l^c core links is:

$$M_i(g_{com}^{CP(k)}, -l^c) = l^c\Big(c - \alpha^2\big(\frac{1}{2} - f_e(n - l^c - 1, \delta)\big)\Big) \tag{D.5}$$

If $M_i(g_{com}^{CP(k)}, -l^c) > 0$, it must hold that $M_i(g_{com}^{CP(k)}, -1) > 0$, because $f_e(n - l^c - 1, \delta)$ decreases in l^c. Player i thus has a beneficial unilateral deviation if

$$\begin{aligned} & M_i(g_{com}^{CP(k)}, -l^c) > 0 \\ \Rightarrow\ & M_i(g_{com}^{CP(k)}, -1) > 0 \\ \Leftrightarrow\ & \alpha < \sqrt{c/(\frac{1}{2} - f_e(n-2, \delta))} \end{aligned} \tag{D.6}$$

The marginal benefit for a core player i of deleting l^p links with the periphery is :

$$M_i(g_{com}^{CP(k)}, -l^p) = l^p\Big(c - \alpha\big(\frac{1}{2} - f_e(k - 1, \delta)\big) - (2n - 2k - l^p - 1) f_m(k, \delta)\Big) \tag{D.7}$$

If $M_i(g_{com}^{CP(k)}, -l^p) > 0$, it must be a best feasible action to choose $l^p = n - k$ and delete all links with the periphery, as the function is convex in l^p. Player i thus has a beneficial unilateral deviation if

$$
\begin{aligned}
& M_i(g_{com}^{CP(k)}, -l^p) > 0 \\
\Rightarrow \quad & M_i(g_{com}^{CP(k)}, -(n-k)) > 0 \\
\Leftrightarrow \quad & \alpha < \frac{c - \frac{1}{2}(n-k)(n-k-1) f_m(k,\delta)}{\frac{1}{2} - f_e(k-1,\delta)}
\end{aligned}
\tag{D.8}
$$

Second, consider a player $i \in P$ in the periphery. This marginal benefit for a periphery bank $i \in P$ to delete l links is positive if:

$$
\begin{aligned}
& M_i(g_{com}^{CP(k)}, -l) = l\Big(c - \alpha\big(\frac{1}{2} - f_e(k-l,\delta)\big)\Big) - (n-k-1)(f_e(k,\delta) - f_e(k-l,\delta)) > 0 \\
\Leftrightarrow \quad & \alpha < \frac{c - \frac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))}{\frac{1}{2} - f_e(k-l,\delta)}
\end{aligned}
\tag{D.9}
$$

The network is unilaterally stable if neither of these three deviations is beneficial, i.e. if α exceeds all values given in equations (D.6), (D.8) and (D.9):

$$
\begin{aligned}
\overline{\alpha} = \max\Big\{ \quad & \sqrt{c/(\frac{1}{2} - f_e(n-2,\delta))}, \frac{c - \frac{1}{2}(n-k)(n-k-1) f_m(k,\delta)}{\frac{1}{2} - f_e(k-1,\delta)}, \\
& \max_{l \leq k}\Big(\frac{c - \frac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))}{\frac{1}{2} - f_e(k-l,\delta)}\Big)\Big\}
\end{aligned}
\tag{D.10}
$$

By rewriting we get given a sufficiently large level of heterogeneity $\alpha > 1$ the following condition for unilaterally stable core-periphery networks in terms of linking costs:

$$
\begin{aligned}
c \in \Bigg(\quad & \tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}, \\
& \min\Big\{ \quad \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1) f_m(k,\delta), \\
& \qquad \min_{l \leq k}\big\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\big\}\Big\}\Bigg).
\end{aligned}
$$

□

Remarks on Proposition 4. Notice that, for given c and δ and for n sufficiently large, the level of heterogeneity $\overline{\alpha}$ as given in (D.10) equals

$$
\sqrt{c/(\frac{1}{2} - f_e(k-1,\delta))}. \tag{D.11}
$$

For such a value of α, the complete core-periphery network with k big banks in the core is unilaterally stable if condition (D.3) is satisfied. To minimize $\overline{\alpha}$ we take the smallest vale of c satisfying (D.3).

A lower bound for $\overline{\alpha}$, given values of δ, k and sufficiently large n, is thus given by:

$$\overline{\alpha} \geq \overline{\alpha}_{min} \equiv \sqrt{\frac{\frac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\frac{1}{2}f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}}{\frac{1}{2} - f_e(k-1,\delta)}}. \tag{D.12}$$

Using the assumptions we made about the distribution of intermediated trades in Section 2.2, one can verify that

$$\lim_{k\to\infty} \overline{\alpha}_{min} = \lim_{\delta\downarrow 0} \overline{\alpha}_{min} = \lim_{\delta\uparrow 1} \overline{\alpha}_{min} = 1, \tag{D.13}$$

showing that an arbitrary small level of heterogeneity can be sufficient to have an unilaterally stable core-periphery network.

Proof of Proposition 5. Starting in an empty network, the first big bank $i=1$ has two relevant options: either connect to all players or connect only to big bank 2. Linking to only part of the small banks cannot be a best feasible action, because linking to an additional small bank pays off positive additional intermediation benefits (similar to the proof of Lemma 1). Player 1's payoffs depending on its action S are:

$$\pi_1(S) \quad = \begin{cases} 0 & \text{if } S = \emptyset \\ \frac{1}{2}\alpha^2 - c & \text{if } S = \{2\} \\ \frac{1}{2}\alpha^2 - c + 2(\frac{1}{2}\alpha - c) + 2\frac{1}{3}\alpha + \frac{1}{3} & \\ \quad = \frac{1}{2}\alpha^2 + \frac{5}{3}\alpha + \frac{1}{3} - 3c & \text{if } S = \{2,3,4\} \end{cases} \tag{D.14}$$

Under condition I, player 1's best feasible action is not to add any links. The resulting network is empty. In this case the network must be unilaterally stable. The reason is that second big bank faces the same decision as $i=1$, and small banks have strictly lower payoffs from adding links, so no player will decide to change the network structure.

Under condition VI, the best feasible action is to add only one link to big bank 2. The second big can choose from the same resulting networks as $i=1$ could, so does not add or remove links. Small banks have strictly lower payoffs from adding links and also do not change the structure. So under condition VI the resulting network with only one link, namely $g_{12}=1$, is stable.

If the first player adds links to all three other players, a star network is formed. Then the best feasible action for the second big bank $i=2$ is either to do nothing or to add links to both periphery players. If 2 does nothing, the periphery nodes 3 and 4 will likewise decide not to add links, and the star network is the final, stable outcome. For 2 to have positive marginal benefits of supporting two links, it is required that

$$\begin{aligned} M_2(g^s,+2) &> 0 \\ \Rightarrow c &< \frac{1}{6}\alpha + \frac{1}{2} f_m(2,\delta), \end{aligned} \tag{D.15}$$

and each peripheral player $j \in \{3,4\}$ requires

$$\begin{aligned} M_j(g^s,+2) &\geq 0 \\ \Rightarrow c &\leq \frac{1}{6}\alpha + f_e(2,\delta) - \frac{1}{3}. \end{aligned} \tag{D.16}$$

Conversely, the star network is stable if the deviation of player 2 is *not* beneficial to 2 and/or a peripheral player j, i.e. if c exceeds the minimum of the two values in equations (D.15) and (D.16):

$$c \geq \frac{1}{6}\alpha + \min\{\frac{1}{2} f_m(2,\delta), f_e(2,\delta) - \frac{1}{3}\}, \tag{D.17}$$

leading to condition II.

Consider the case that both big banks have added all possible links. Then there is a possibility that the dynamic process leads to a complete network if node 3 links to node 4. This happens if c is so small that the share $f_e(2,\delta)$ from intermediated trade between 3 and 4 can be raised to $\frac{1}{2}$ by creating a direct link. The complete network is therefore stable under condition III.

For parameters not satisfying I, II, III or VI, the first round of best feasible actions results in a complete core-periphery network with 2 core members. This complete core-periphery network can be stable because of the heterogeneity between core and periphery banks with $\alpha > 1$. As stated in Proposition 4, this occurs under condition VII. For $n = 4$ and $k = 2$ condition VII reduces to:

$$\begin{aligned} c \in \Bigg(& \tfrac{1}{2} - f_e(k,\delta) + (n-k-2)\min\{\tfrac{1}{2} f_m(k+1,\delta), f_e(k+1,\delta) - f_e(k,\delta)\}, \\ & \min\Big\{ \alpha^2(\tfrac{1}{2} - f_e(n-2,\delta)), \alpha(\tfrac{1}{2} - f_e(k-1,\delta)) + \tfrac{1}{2}(n-k)(n-k-1) f_m(k,\delta), \\ & \qquad \min_{l \leq k}\{\alpha(\tfrac{1}{2} - f_e(k-l,\delta)) + \tfrac{n-k-1}{l}(f_e(k,\delta) - f_e(k-l,\delta))\}\Big\}\Bigg) \\ \in \Bigg(& \tfrac{1}{2} - f_e(2,\delta), \min\{\alpha^2(\tfrac{1}{2} - f_e(2,\delta)), \tfrac{1}{6}\alpha + f_m(2,\delta), \tfrac{1}{6}\alpha + f_e(2,\delta) - \tfrac{1}{3}\}\Bigg). \end{aligned}$$

Notice that when this condition is fulfilled, the second player always adds the two links to the periphery (cf. condition (D.17)).

Finally, in the remaining region IV, the complete core-periphery network cannot be stable. Because the node $i = 3$ node did not connect to the last periphery node, adding links cannot be a best feasible action. Therefore the first core bank $i = 1$ must have an incentive to delete the link with 2. The attracting steady state is the multipartite (ring) network consisting of the groups $\{1,2\}$ and $\{3,4\}$. □

Appendix E. Longer intermediation chains

In this appendix we generalize the model to allow for intermediation paths of lengths longer than two. We will analyze the general model for lengths up to distance three and under homogeneity (i.e. $\alpha_i = 1$ for all i, j), and check whether a (incomplete) core-periphery network may be stable.

Before rewriting a more general form than the payoff function in Proposition 1 formally, we repeat that the proofs of Proposition 2 and 3 do not require any assumptions on the path lengths on which trade is allowed; see the remarks in Appendix C on these propositions. Proposition 2 states that complete core-periphery networks are not pairwise (or unilaterally) stable. Proposition 3 states that incomplete core-periphery networks are not unilaterally stable if n is sufficiently large. Also Proposition 4, specifying a level of heterogeneity sufficient for a complete core-periphery network to be unilaterally stable, holds for longer intermediation paths.

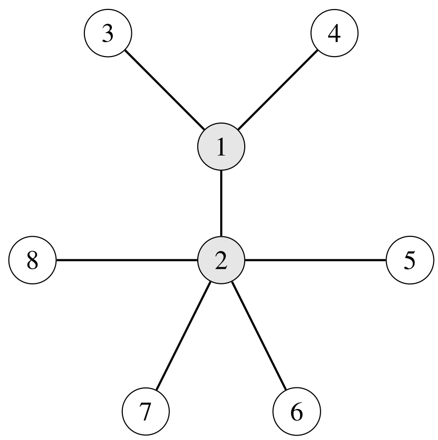
We introduce new, more general notation for intermediation over longer path lengths: $F_e(g, \{i, j\}, \delta)$ denote the shares for the endnodes in the pair i and j; and $F_m(g, \{i, j, k\}, \delta)$ denotes how much middleman k receives. In Appendix A it is explained how such an distribution can be derived for long intermediation chains in the example of Siedlarek (2015). Note that if i and j at length three are assumed to generate a surplus, middlemen involved in a trade between i and j do not necessarily earn the same: if k lies on more of the shortest paths than k', k will earn more than k'. For this reason (part of) the graph g must be given as an argument in the function F_e and F_m. The payoff function becomes:

$$\pi_i(g) = \eta_i(g)\left(\frac{1}{2} - c\right) + \sum_{j \in N_i^3(g)} F_e(g, \{i, j\}, \delta) + \sum_{\substack{k,l \in N_i^2(g) \\ g_{kl} = 0, d_{kl} \leq 3}} F_m(g, \{k, l, i\}, \delta), \tag{E.1}$$

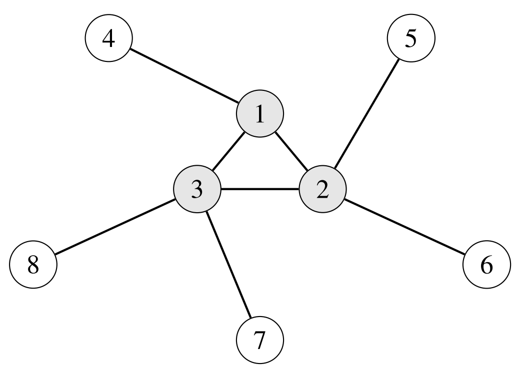
where $N_i^r(g)$ denotes the set of nodes at distance r from i in network g, $\eta_i(g) = |N_i^1(g)|$ the number of direct connections of i, and d_{kl} the distance between nodes k and l.

In incomplete core-periphery networks, shortest paths of three may exist between some periphery players, which were previously assumed not to generate any trading surplus. By allowing intermediation chains of three we found that some core-periphery networks can become unilaterally stable. For $n = 8$, we found that $k = 2$ and $k = 3$ are the only possibly stable core sizes. See Figure E.14 for two examples of networks that are stable for the given parameter values when paths of three are allowed. These two network structures are not stable in the baseline model.

We can safely interpret these examples as low-dimensional exceptions to the rule that the core-periphery structure in homogeneous networks is generally unstable. The examples in Figure E.14 show that for $n = 8$ incomplete core-periphery networks with core sizes of $k = 2$ and $k = 3$ can be stable. For n sufficiently large, however, core-periphery networks are always unstable as stated by Proposition 3.



(a) $(\delta, c) = (0.8, 1.1)$. A minimally connected core-periphery network with $k = 2$.



(b) $(\delta, c) = (0.5, 0.9)$. A minimally connected core-periphery network with $k = 3$.

Figure E.14: Examples of unilaterally stable core-periphery networks after allowing for intermediation chains of length 3, for $n = 8$ players and given certain (δ, c).

Moreover, even though exceptionally for small n core-periphery networks can be unilaterally stable, they are never the outcome of a dynamic process as described in Section 3.2. For $n = 8$, the core-periphery networks with $k = 2$ and $k = 3$ were found to be stable in parameter regions where the star networks is stable as well, cf. region II in Figure 6b. Exploring the parameter space by simulations, we found that this was always the case for such stable core-periphery networks. By Lemma 1, the star is created as a first step in the dynamic process whenever the initial empty network is not stable. Therefore the star network is the outcome of a dynamic process even when exceptional (low-dimensional) networks are unilaterally stable as well given the parameter values. This implies that the dynamic results of Theorem 1 do not depend on the assumption of maximal intermediation paths of length two, as was already shown for the static results of Propositions 2, 3 and 4.