Direct measurement of the W boson mass in $e^+ e^-$ collisions at LEP
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Chapter 2

W pair production at LEP

The crossing of the WW production threshold in July 1996 marked the beginning of the LEP2 phase. The first LEP2 energy was chosen to be 161 GeV, just above 2 times the W mass, at that time known to 160 MeV/c\(^2\) precision from direct measurements and predicted with an accuracy of about 50 MeV/c\(^2\) from a global SM fit to LEP1 and SLC data. Figure 2.1 shows the behaviour of the WW production cross-section as a function of the e\(^+\)e\(^-\) centre-of-mass energy.

Figure 2.1: **DELPHI** measurements of the e\(^+\)e\(^-\) \(\rightarrow\) W\(^+\)W\(^-\) production cross-section compared with the Standard Model prediction given by the YFSWW [26] and RacoonWW [27] programs. The shaded band indicates the uncertainty on the theoretical calculations. Results for \(\sqrt{s} > 190\) GeV are preliminary.
**W pair production**

The production of on-shell (i.e. stable) W bosons already demonstrates some of the most important features of W pair production at LEP, as will be discussed in section 2.1. The W pair events thus produced have allowed measurements of the W mass via two different methods:

- The threshold measurement, based on a SM fit to the cross-section near threshold.
- The direct kinematic reconstruction: deriving the W mass from the invariant mass of its decay products.

The cross-section method was used at $\sqrt{s} = 161$ GeV since it has its optimal statistical sensitivity at the threshold. At all other energies, however, direct reconstruction is the preferred method. Its performance is rather independent of the centre-of-mass energy, as long as it is at least a few GeV above threshold to allow both W bosons to be on-shell. This thesis concerns a direct measurement based on the 172 GeV, 183 GeV and 189 GeV DELPHI data sets, covering approximately 1/3 of the available statistics.

**W decay**

The W boson can decay either into a lepton and a neutrino or into a quark anti-quark pair, followed by subsequent hadronisation into stable particles observed as jets. The combination of 2 W's in one event thus leads to 3 different event topologies, with the following branching ratios:

- 45.6% fully-hadronic ($q\bar{q}q\bar{q}$)
- 43.9% semi-leptonic ($q\ell\nu$)
- 10.5% fully-leptonic ($\ell\nu\nu$)

The doubly resonant production of W bosons, followed by their decay, gives rise to a double Breit-Wigner shape of the differential cross-section as a function of the two W boson invariant masses. It is the shape of this differential cross-section which is used to extract the W mass. This is discussed in more detail in section 2.2.

**QCD and jets**

At LEP, QCD processes are purely restricted to the final state, after the hard scale EW process has taken place and only if quarks were produced. Quarks are never observed as individual particles. Due to QCD confinement, the hadronic decay gives rise to jets of particles in the final state. Section 2.5 will concentrate on the phenomenological aspects of the radiation of gluons and the formation of jets. QCD is also responsible for the main source of background processes in the fully hadronic channel, to be discussed in section 2.6.
Other phenomenological aspects

The reconstruction of the invariant mass of the decay products is based on the crucial property of each event that at all times total energy and momentum are conserved. This is also true for complex decay processes like the hadronic decay into jets. In chapter 4 to 6 it will be explained how, using jet clustering and a constrained fit, even in those complex final states the invariant mass of the W bosons can be measured and improved considerably using the knowledge of energy and momentum conservation.

In the majority of the events, the constraints give a vast improvement. But when, occasionally, an Initial State Radiation (ISR) photon escapes undetected inside the beam-pipe, the constraints are incomplete and lead to an erroneous value of the fitted mass. It is therefore important to understand this effect with sufficient precision. Fortunately it is well described by QED (section 2.3) and included in the Monte Carlo generators (section 2.4).

Another possible complication emanates from Final State Interferences (FSI) between the W bosons. When such cross-talk occurs, the simple picture of two bosons decaying independently no longer holds, and the direct correspondence of the invariant mass of the decay products with the invariant mass of the W bosons is lost. These possible cross-talk effects are discussed in section 2.7.

2.1 On-shell W pair production at LEP

The three dominant diagrams for the production of stable W bosons at LEP are shown in Figure 2.2. This set of three Charged Current diagrams is often referred to as ‘CC03’. The corresponding tree level amplitude can be written as [29]:

\[ \mathcal{M}(\sigma, \lambda, \bar{\lambda}) = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_\nu \quad (2.1) \]

where the \(e^-\) and \(e^+\) helicities are given by \(\sigma/2\) and \(-\sigma/2\) (in the massless limit \(m_e \ll \sqrt{s}\)) and \(\lambda\) and \(\bar{\lambda}\) denote the \(W^-\) and \(W^+\) helicities, which can have values -1, 0 or 1 as the W bosons are massive particles with spin 1.

Choosing the \(z\)-axis along the \(e^-\) flight direction, and the \(x\)-axis along the \(W^-\) transverse momentum, the leading angular dependences can be expressed in terms of the \(d^{3\alpha}\) functions [11]
Figure 2.3: Distribution of \( \cos \theta \), where \( \theta \) is the production angle of the W bosons (left), from EXCALIBUR [28] simulation at \( \sqrt{s} = 189 \text{ GeV} \). While the W production is clearly peaked in the forward directions, the distribution of the fermions after the decay of the W bosons (next section) remains almost independent of the polar angle (right plot).

shown in Table 2.1, with the reduced matrix elements \( \tilde{M} \) defined in such a way that:

\[
\tilde{M}(\sigma, \lambda, \bar{\lambda}; \theta) = \sqrt{2\sigma e^2} \left[ \tilde{M}_\gamma + \tilde{M}_Z + \tilde{M}_\nu(\theta) \right] d_{\sigma, \lambda, \bar{\lambda}}(\theta) \tag{2.2}
\]

\[
\tilde{M}_\gamma = -\beta A_{\lambda\bar{\lambda}}, \quad \tilde{M}_Z = +\beta A_{\lambda\bar{\lambda}} \left[ 1 - \delta_{\sigma,-1} \frac{1}{2\sin^2 \theta_W} \right] \frac{s}{s - m_Z^2}, \quad \tilde{M}_\nu = \delta_{\sigma,-1} \frac{1}{2\beta \sin^2 \theta_W} \left[ B_{\lambda\lambda} - \frac{1}{1 + \beta^2 - 2\beta \cos \theta} C_{\lambda\bar{\lambda}} \right]
\]

where \( \beta = \sqrt{1 - 4m_W^2/s} \) is the W velocity, \( \theta \) is the production angle of the W bosons with respect to the positive z axis, \( \delta_{ij} \) is the Kronecker delta function, and \( J_0 = \max(1, |\lambda - \bar{\lambda}|) = 1, 2 \) is the minimum angular momentum contributing to a given helicity combination. The coefficients \( A_{\lambda\lambda}, B_{\lambda\bar{\lambda}} \) and \( C_{\lambda\bar{\lambda}} \) are given in Table 2.1.

Just above threshold \( (\beta \ll 1) \) the differential cross-section is given by [30]:

\[
\frac{d\sigma}{d\cos \theta} = \frac{\alpha^2}{s} \frac{\beta}{4\sin^4 \theta_W} \left[ 1 + 4\beta \cos \theta \frac{3\cos^2 \theta_W - 1}{4\cos^2 \theta_W - 1} + \mathcal{O}(\beta^2) \right] \tag{2.3}
\]

where the leading term \( \propto \beta \) comes from the t-channel \( \nu \) exchange diagram only. Thus, for small values of \( \beta \) the differential cross-section is essentially angular independent, while for increasing values of \( \sqrt{s} \) other angular dependent terms become important.
The differential cross-section as a function of the production angle $\theta$ is one of the main input variables for the study of the couplings of the vector bosons among themselves (Trilinear Gauge Couplings, TGC). For the W mass measurement it is not important, but the knowledge of this distribution does play a minor role in the analysis, providing extra information in the choice of the correct jet pairing (section 6.4, page 101). The overall angular distribution for $\sqrt{s} = 189$ GeV is shown in Figure 2.3.

The total Born cross-section has a threshold behaviour proportional to $\beta$ as shown in Figure 2.4 where the 6 interference terms (from the square of the matrix element separated in the 3 terms shown in equation (2.1)) are shown separately. At high energies the individual terms diverge, but the total cross-section is well behaved, thanks to the precise Gauge cancellations prescribed by the Standard Model.

![Figure 2.4](image.png)

Figure 2.4: The $e^+e^- \rightarrow W^+W^-$ production cross-section as a function of $\sqrt{s}$. The partial cross-sections corresponding to the 6 interference terms from the CC03 Born level diagrams are shown separately.
Figure 2.5: Dominant lowest order diagrams for the process $e^+e^- \rightarrow W^+W^- \rightarrow \bar{f}_1f_2\bar{f}_3f_4$.

### 2.2 Unstable W's and 4-fermion production

In the previous section the production of stable W bosons was described. In reality, however, W bosons are unstable particles whose properties are analysed only through their decay products. It is therefore important to take the whole 4-fermion production process into account and describe the W bosons as resonances with a finite width, leading to the following expression for the leading-order cross-section for off-shell $W^+W^-$ production:

$$\sigma(s) = \int_{0}^{s} ds_1 \int_{0}^{(\sqrt{s} - \sqrt{s_1})^2} ds_2 \rho(s_1)\rho(s_2)\sigma_0(s, s_1, s_2)$$  \hspace{1cm} (2.4)

where $s_1$ and $s_2$ are the virtualities of the two W bosons and $\sigma_0$ reduces to the on-shell Born cross-section for $s_1 = s_2 = m_W^2$. The function $\rho(s)$ is described by a relativistic Breit-Wigner:

$$\rho(s) = \frac{1}{\pi} \frac{s}{(s - m_W^2)^2 + m_W^2\Gamma^2(s)}$$  \hspace{1cm} (2.5)

where $\Gamma(s)$ is given by:

$$\Gamma(s) \equiv \frac{s}{m_W^2} \Gamma(m_W^2), \text{ where } \Gamma(m_W^2) \equiv \Gamma_W$$  \hspace{1cm} (2.6)

This definition of the Breit-Wigner function and corresponding decay width is called the 's-dependent width' or 'running width' definition. At LEP1 the Z lineshape was fitted according to this definition and the same convention is used for the presentation of LEP2 W mass results. An alternative definition, equally well motivated from a theoretical point of view, is the 'fixed width' definition:

$$\rho(s) = \frac{1}{\pi} \frac{\Gamma}{m^2} \frac{s}{(s - m^2)^2 + m^2\Gamma^2}$$  \hspace{1cm} (2.7)

Near the pole both Breit-Wigner shapes are equivalent, provided that the mass and the width parameters satisfy the following transformation:

$$\bar{m} = m_W/\sqrt{1 + \frac{\Gamma_W^2}{m_W^2}} \approx m_W \left(1 - \frac{1}{2} \frac{\Gamma_W^2}{m_W^2}\right) \approx m_W - 26.9 \text{ MeV}/c^2$$  \hspace{1cm} (2.8)

$$\bar{\Gamma} = \Gamma_W/\sqrt{1 + \frac{\Gamma_W^2}{m_W^2}} \approx \Gamma_W \left(1 - \frac{1}{2} \frac{\Gamma_W^2}{m_W^2}\right) \approx \Gamma_W - 0.7 \text{ MeV}/c^2$$  \hspace{1cm} (2.9)
Figure 2.6: Example of a single W production diagram (left) and a neutral current doubly-resonant ZZ process (right). These diagrams interfere with WW diagrams leading to the same 4-fermion final states.

The width $\Gamma_W$ is predicted by the Standard Model. At Born level, neglecting the masses of the fermions which are all small except for the top quark and the bottom quark, the width of the W boson is calculated as:

$$\Gamma_{W}^{\text{Born}} = \sum_{i,j} \Gamma_{Wf_i f_j} = \sum_{i,j} N_C^f \frac{m_W}{6 \sin^2 \theta_W} |V_{ij}|^2 \approx \frac{3 \alpha m_W}{2 \sin^2 \theta_W}$$ (2.10)

where the sum includes the 3 leptonic decay modes (with $N_C^f = 1$ and $|V_{ij}| = \delta_{ij}$) and all decays into quarks with $m_{f_i} + m_{f_j} < m_W$. This excludes decay modes that contain a top quark, and therefore reduces the occurrence of bottom quarks in the decay through the small values of the CKM matrix elements $|V_{ub}| \approx 0.004$ and $|V_{cb}| \approx 0.04$ (section 1.1). With a colour factor $N_C^f = 3$ the three dominating contributions to the decay width become: $\Gamma_{W_{l\nu}} \approx \Gamma_{W_{ud}} \approx \Gamma_{W_{cs}} \approx \frac{1}{3} \Gamma_W$. A prediction of $\Gamma_W$ slightly more precise than equation (2.10) is given by the improved Born approximation [30]:

$$\Gamma_W \approx \frac{3 G_F m_W^3}{2 \pi \sqrt{2}} \left(1 + \frac{2 \alpha_s (m_W^2)}{3 \pi} \right) \approx \left( \frac{m_W}{80.35 \text{ GeV}/c^2} \right)^3 \cdot 2.09 \text{ GeV}/c^2$$ (2.11)

with a precision better than 0.5%. This is more than accurate enough for our purposes, since the direct measurement of the W width presented here has a statistical precision of 7%.

The CC03-like 4-fermion diagrams shown in 2.5 do not form a complete subset of Feynman diagrams, since other leading order diagrams can lead to the same final state. Two examples of such diagrams that are not of the doubly-resonant form, $e^+e^- \rightarrow W^+W^- \rightarrow \bar{f}_1 f_2 \bar{f}_3 f_4$, are shown in Figure 2.6. These ‘single W’ and ZZ diagrams contribute to (pseudo) backgrounds for the analysis presented here. To take this into account all leading order EW diagrams producing WW-type 4-fermion final states were included in the MC simulation (section 2.4). The presence of ZZ background is relevant for the W width measurement, but the effects of 4-fermion backgrounds on the W mass measurement are negligible.

\footnote{Except in the case of the $q\bar{q}\ell\nu\ell$ channel, where mass shifts up to 50 MeV/c$^2$ were observed [31].}
It should be mentioned that the inclusion of the finite width of the W also has important effects for the cross-section. The behaviour near threshold becomes more smooth, and the total cross-section decreases by about 5%. These effects are of no importance for the direct W mass measurement, however.

2.3 Radiative corrections

The leading order EW diagrams discussed so far did not include any higher order corrections. In particular photonic (QED) corrections can play an important role. Corrections to be considered are: Initial State Radiation (ISR) of photons from the incoming $e^+$ and $e^-$; the exchange of virtual photons between the charged W bosons (Coulomb correction); and Final State Radiation (FSR) of photons from the 4 fermions in the final state.

The FSR photons can generally be combined with the lepton or jet they belong to, and do not play an important role in the analysis presented here. Therefore this will not be discussed further. The Coulomb correction (see Figure 2.7) can be included as a correction to the Born cross-section [32] $\approx \frac{\alpha}{v} \delta_{\text{Coul}}$, where $v$ is the relative velocity of the two W bosons. This correction becomes important near the WW production threshold where it has a singularity ($v \to 0$). The width of the W reduces the singularity at threshold to an effect of about 6% on the total cross-section, falling off with increasing $\sqrt{s}$ to about 2% at 190 GeV.

![Figure 2.7: Illustration of radiative corrections to the off-shell WW cross-section.](image)

**Initial State Radiation**

The most important correction to WW production comes from Initial State Radiation. ISR photons can carry away a significant amount of energy from the incoming electron and positron, thus reducing the effective centre-of-mass energy $\sqrt{s'}$ available for the hard scattering process. If the constraints used in the W mass measurement assume that the full centre-of-mass energy $\sqrt{s}$ is available, the reconstructed mass is shifted by

$$\Delta m \approx \langle E_{\gamma} \rangle m_W / \sqrt{s} \quad (2.12)$$
where the average radiated ISR photon energy $\langle E_{\gamma} \rangle$ varies from 1 to 3 GeV in the $\sqrt{s}$ range from 170 to 200 GeV. To good approximation the correction due to the emission of real photons in the initial state can be written as the product of a ‘radiator function’ and a cross-section modified by weak corrections (by a factor $\equiv (1 + \delta_{WW})$) — this is called factorisation. A convenient way to include the photonic corrections is by means of the ‘structure function’ formalism [28, 33]. In this method, the probability that the colliding electron (positron) has a longitudinal momentum fraction $x_1 (x_2)$ is described by a structure function, and the leading-order cross-section for off-shell $W^+W^-$ production (2.4) is modified to:

$$\sigma(s) = \int_0^s ds_1 \int_0^{(\sqrt{s}-\sqrt{s_1})^2} ds_2 \rho(s_1) \rho(s_2) \cdot \int_0^{x_{\text{max}}} dx F(x, s) \sigma_0(s', s_1, s_2) \left( 1 + \frac{\alpha}{\pi} \delta_{WW} + \frac{\alpha}{v} \delta_{\text{Coul}} \right)$$

(2.13)

where $x$ is defined by $s' = (1 - x)s$ and the radiator function $F(x, s)$ is given by [32]:

$$F(x, s) = \beta x^{\beta - 1} \left[ 1 + \frac{3}{4} \beta - \frac{\beta^2}{24} \left( \frac{1}{3} \ln \left( \frac{s}{m_e^2} \right) + 2\pi^2 - \frac{37}{4} \right) \right] - \beta \left( 1 - \frac{x}{2} \right)$$

$$+ \frac{\beta^2}{8} \left[ 4(2 - x) \ln \left( \frac{1}{x} \right) + \frac{1 + 3(1 - x)^2}{x} \ln \left( \frac{1}{1 - x} \right) - 6 + x \right]$$

(2.14)

with

$$\beta = \frac{2\alpha}{\pi} \left( \ln \left( \frac{s}{m_e^2} \right) - 1 \right)$$

(2.15)

Alternative methods to describe the ISR spectrum are the QED parton shower (QEDPS [34]) and the YFS exponentiation method [30, 35]. All these methods are able to describe the ISR spectrum with great precision, corresponding to an uncertainty of just a few MeV$/c^2$ on the W mass [36]. The numerical importance for the W mass depends on the beam energy via the available phase space for ISR radiation. This corresponds to the integration limit $x_{\text{max}}$ in equation (2.13) and is graphically illustrated in Figure 2.8.

**Non-factorisable $O(\alpha)$ corrections**

In the recent LEP200 MC workshop, however, one of the main topics of discussion was the necessity also to take non-factorisable corrections into account. Ideally one would like to take into account the complete set of $O(\alpha)$ electroweak corrections. For the on-shell case the $O(\alpha)$ corrections have been calculated, but for the much more complicated off-shell case this work has not been completed so far. As in WW events — contrary to $Z^0$ events at LEP1 — charged particles are present during the whole reaction, virtual photons can connect at all places in the diagrams, leading among others to these non-factorisable corrections. These corrections were found to be responsible for a 2% change in the predicted WW production cross-section. A possible effect of 10 MeV on the W invariant mass peak was reported in a theoretical study based on RacoonWW [27], which remains to be confirmed with a (more) realistic W mass analysis.
Figure 2.8: The $z$ momentum of the 4-fermion system due to the recoil from ISR photons as generated by PYTHIA simulation at 172 GeV (left) and EXCALIBUR simulation at 189 GeV (right). The distributions are compared to the leading term $\beta x^{\beta-1}$ from equation (2.14), indicated by the solid line. The phase space available when both $W$ bosons are on-shell, $\sqrt{s'} > 2m_W$, is indicated by the dashed lines.

2.4 ElectroWeak simulation models

Models to simulate the EW physics processes described above have been implemented in different programs, which can be divided into two distinct classes:

Semi-analytical programs such as GENTLE and BBC [36] are designed to do the most precise calculations, incorporating (almost) all ideas available in the literature. They provide precise theoretical calculations of specific physics processes.

Monte-Carlo event generators are more suitable for the experimental situation. By generating events with simulated particles and typically interfaced to hadronisation packages (next section) and full detector simulation (next chapter), they enable the investigation of all experimental aspects relevant for a physics analysis. Due to the extra complications induced by the requirement to generate (large numbers of) events in a reasonable amount of computing time, these MC generators do not always reach the same theoretical precision as the semi-analytical programs. Recently, however, W-physics generators like YFSWW [26] and RacoonWW [27] have managed to include state-of-the-art theoretical knowledge in a generator.

In the analysis presented here, EXCALIBUR [28] was used as main Monte Carlo generator to simulate the 4-fermion signal ($WW + ZZ$). It takes all leading order 4-fermion diagrams into account. In order to treat the large number of possible diagrams and their interfer-
ences efficiently, EXCALIBUR makes use of helicity amplitude techniques which work under the assumption of massless fermions. Internally the program uses the fixed-width Breit-Wigner definition, but the standard DELPHI interface routine takes care of the transformation (2.9), ensuring that all physics results correspond to the running-width definition. For the inclusion of ISR, the default treatment in EXCALIBUR is based on the structure function approach according to (2.13), producing only collinear photons. For final states without \( e^+e^- \) pairs, however, the DELPHI implementation includes ISR with finite transverse momentum using the QEDPS [34] package.

This covers the simulation of 4 fermions (plus photons). However, for final states containing (anti-)quarks this is not the end of the story. In those events QCD plays a striking role. But it has been shown that QCD graphs, except for FSR from quark pairs, do not interfere with the EW graphs to any sizeable extent, and are therefore treated separately with standard Monte Carlo QCD programs, to be discussed in the next section.

### 2.5 QCD phenomena and jet formation

The quark and anti-quark produced in the hadronic decay of a W boson \( W \rightarrow q\bar{q} \) are never observed as free particles; due to colour confinement they give rise to jets of particles observed in the detector. At high energy scales the fragmentation of the \( q\bar{q} \) pair into high energy partons (gluons and \( q\bar{q} \) pairs) is described — in principle — by perturbative QCD. At lower energy scales, soft gluon radiation is less well described (the non-perturbative phase), and has to be modelled using phenomenological models. In the last phase, the partons produced have to form stable final state particles — the hadrons, leptons and photons that can be detected in the detector, as two or more jets (per W boson). This fragmentation process is depicted in figure 2.9.

In \( WW \) events the starting configuration of the fragmentation process is well defined: a \( q\bar{q} \) pair with a centre-of-mass energy of \( \sqrt{s_{qq}} \sim m_W \). The probability for the radiation of a single gluon can be calculated perturbatively. The exact \( O(\alpha_s) \) result is given by:

\[
\frac{d^2\sigma}{dx_1 dx_3} = \frac{2\alpha_s}{3\pi} \frac{x_1^2 + x_3^2}{(1 - x_1)(1 - x_3)}
\]

(2.16)

where \( x_1 \) and \( x_3 \) are the energy fractions of the quark and anti-quark after emission, defined as \( x_i \equiv 2E_i/\sqrt{s_{qq}} \) in the centre-of-mass system of the original \( q\bar{q} \) pair. A useful effective approximation, which has been used in some models and in several QCD studies [37], is given by:

\[
\frac{d\sigma}{dk_T^{\text{gluon}}} \sim \frac{\alpha_s(k_T^{\text{gluon}})}{k_T^{\text{gluon}}}
\]

(2.17)

where the transverse momentum of the gluon \( k_T^{\text{gluon}} \) is defined with respect to the original \( q\bar{q} \) pair. The differential cross-section becomes infinitely large when either the gluon is collinear with one of the outgoing quarks (either \( x_1 \) or \( x_3 \) becomes 1), or the gluon momentum goes to zero (both \( x_1 \) and \( x_3 \) approach 1). In both cases \( k_T^{\text{gluon}} \) approaches zero. If the gluon is required to be well-separated — experimentally observable as a separate jet \( (k_T^{\text{gluon}} > y_{\text{cut}}) \) — the divergences can be integrated out, and the corresponding cross-section for the emission of an extra gluon
is suppressed approximately by a factor $\alpha_s$. The value of $\alpha_s$ ranges between $\approx 0.1$ and $\approx 0.3$ depending on the transverse momentum scale of the gluon. With two W bosons the probability to see an extra jet doubles, leading to a sizeable fraction of 5-jet WW events. Indeed in 30% - 50% of the events at least one extra jet is seen, depending on the jet resolution $y_{\text{cut}}$ chosen (section 5.4.3, Figure 5.7). Examples of such multi-jet events are shown and will be discussed later; e.g. in pictures 3.9 and 4.1.

The subsequent radiation of additional gluons down to the typical scale of hadrons ($\sim 1$ GeV), cannot be calculated exactly using perturbative QCD. Instead one has to rely on phenomenological models, implemented in Monte Carlo programs. Fortunately these models have been tested and tuned with great precision exploiting the 4 million hadronic $Z^0$ events (per experiment) produced at LEP1. Only a fraction of the tuned parameters have real physical meaning. So to a certain extent these models can be regarded as ‘templates’, which through optimization on $Z^0$ data have come to reproduce the $Z^0$ decays to a high precision. It is important to stress that the models are built in such a way that they are able to predict the evolution from the scale of the $Z^0$ (91.2 GeV) to the W (80.4 GeV) with very high and reliable precision. The Monte Carlo programs used deal with the jet formation in the following three stages (as illustrated in Figure 2.9):

1. To describe the perturbative emission of high energy gluons JETSET [38], ARIADNE [39] and HERWIG [40] implement a Parton Shower (PS) approach. They all apply the dipole formula (2.16) for the emission of the first (i.e. highest $k_T$) gluon. In ARIADNE the emission continues according to the dipole formula (with analogous expressions for the emission from $qg$ and $gg$ dipoles), while in the other programs the Parton Shower is based on repetitive emission from single partons: $q \rightarrow qg$, $g \rightarrow q\bar{q}$, and $g \rightarrow gg$. The Colour Dipole
Model (CDM) approach used in ARIADNE is formulated in terms of Lorentz invariants and reproduces the experimentally observed angular ordering of gluons in a natural way. The other programs have to add the angular ordering conditions 'by hand' in order to avoid considerable double-counting and emission of too many gluons. These MC programs do not only cover the parton evolution but also the competing FSR from quarks and anti-quarks. Since $\alpha_s > \alpha_{QED}$, it effectively means that the QED FSR is suppressed. The final effect of this feature is very small, because the final jet properties are not very sensitive to the type of radiation responsible for the parton shower.

2. Several different models exist to describe the non-perturbative phase. The JETSET program uses the Lund string model, where the partons produced in the perturbative Parton Shower are connected by colour strings. When this string breaks up, hadrons are formed with a width of the momentum distribution transverse to the string direction of the order of 0.3 GeV/c. Numerous tunable parameters ensure that a good description of the data can be obtained. The colour dipoles produced by ARIADNE are also interfaced to the Lund string model.

Another approach, employed by the HERWIG MC program, is the cluster fragmentation model. In this model the remaining gluons are made to decay into $qq$ pairs, which form colourless clusters of different masses. Depending on their mass these clusters can decay into clusters of smaller mass or directly into hadrons. Both the string model and the cluster fragmentation model are well able to reproduce most of the available experimental data.

3. Finally resonances and particles that have been produced, if unstable, decay into stable particles. Here decay tables are used that contain masses, branching ratios, quantum numbers etc. of the particles.

The program used in the main analysis to describe the jet fragmentation is JETSET 7.409 with PS, tuned with $Z^0$ events from DELPHI LEP1 data [41]. The DELPHI-tuned HERWIG fragmentation and ARIADNE CDM approaches have been considered as well, to study possible systematic effects.

### 2.6 QCD background

Not only WW signal, but also background can have 4 jets. In fact the most significant irreducible background for the WW signal in the $q\bar{q}q\bar{q}$ channel is formed by $e^+e^- \rightarrow Z(\gamma) \rightarrow q\bar{q}gg(\gamma)$, where the hadronic decay of the Z boson obtains a ($\geq$)4-jet signature due to the radiation of two high $k_T$ final state gluons. The $q\bar{q}\gamma$ cross-section, with $\sigma \approx 100$ pb, is more than 10 times larger than the $q\bar{q}q\bar{q}$ cross-section (Figure 2.1). However, the requirement that two hard gluons be radiated supresses this background by two orders in $\alpha_s$. In practice about 1/50 of the total $q\bar{q}\gamma$ cross-section is selected in the $q\bar{q}q\bar{q}$ channel. The corresponding $O(\alpha_s^2)$ matrix elements have been calculated, but in this analysis the QCD background was simulated using PYTHIA 5.722 + JETSET 7.409 with PS, with an estimated uncertainty of about 5% on the 4-jet rate in terms of the accepted cross-section.
Fortunately the influence of such processes on the W mass measurement is small. The effect of the uncertainty in cross-section and possible deviations in differential distributions ('shape') are taken into account in the systematics studies in chapter 7.

2.7 Final State Interference phenomena

One aspect of the fragmentation models that cannot be tested and tuned on Z⁰ events is the possible final state cross-talk between the two W systems. In the q̅q̅q̅q̅ channel the two decaying hadronic systems can have a significant space-time overlap, since the distance between the decay vertices (1/T_W ~ 0.1 fm) is much smaller than the typical hadronisation size of 1-10 fm. This means that cross-talk between the two decaying hadronic systems cannot be excluded. The precise mechanism and significance of these effects, however, is largely unknown. In that respect LEP2 provides a beautiful laboratory to study the interaction of two super-imposed hadronising systems in a clean environment.

The two final state cross-talk phenomena of interest are Bose-Einstein Correlations and Colour Reconnection. Their physics background and possible effect on the W mass measurement will be discussed in the following.

Bose-Einstein Correlations

Correlations within pairs or multiplets of identical bosons in the final state are a well-known quantum-mechanical phenomenon. In astronomy, it is known as the Hanbury Brown-Twiss (HBT) effect for incoherent emission of photons (e.g. from stars). The analogous effect has been observed in hadronic, heavy ion and e⁺e⁻ collisions. However, in most observations except for the heavy-ion collisions, source sizes of ≈ 1 fm are seen, which appears (too) small compared to the event size at the time of hadron formation (typically several fm).

An alternative model, in the framework of the Lund string model, was proposed by Andersson and Ringnér [42]. In this model the correlations follow as a coherent effect related to the symmetrisation of the quantum-mechanical amplitude for particle production from the Lund string. Its predictions are in agreement with the expected source size and correlation strength seen in Z⁰ events. A fundamental prediction of the model is that only bosons from the same string are subjected to BEC, which means that this type of BEC does not lead to cross-talk between W bosons, unless Colour Reconnection happens at parton level.

Experimentally the correlations can be observed by investigating two-particle correlations between like-sign pions. By comparing the correlations in q̅q̅q̅q̅ events with the correlations in a reference sample of mixed q̅q̅q̅q̅ events, a model-independent measurement can be performed. Recent results from all four LEP experiments show that the correlations between W bosons are strongly suppressed compared to correlations within W (or Z⁰) bosons. In fact no evidence for correlations between W's is seen at all [43]. This could be an indication in favour of the coherent scenario.

This means that BEC from different W bosons might come only from the incoherent HBT effect, which occurs at a larger length scale. Therefore the effect on the mass measurement is expected to be small. Unfortunately the coherent BE model has not yet been implemented in a MC generator. The other models available are not based on quantummechanics, but implement in var-
ious ways the enhancement of identical bosons that form pairs close in phase space. Such effects could change the way the decay products of the W bosons are mixed in the jet reconstruction, and thus affect the W mass measurement. This will be addressed in more detail in chapter 7.

**Colour Reconnection**

Cross-talk via the strong interaction is known as Colour Reconnection (CR). The effect is expected to enhance particle production in phase space regions ‘outside’ the EW W bosons, and reduce it inside the W boson domains. This would actually change the invariant mass of the decay systems of the supposed W bosons. As mentioned already QCD does not provide a well-defined description of this effect, but several observations can be made:

**Perturbative QCD:** Since the W bosons are typically separated by a distance \(1/\Gamma_W\) at the time of decay, only virtual gluons with an energy less than \(\Gamma_W\) can participate in the cross-talk. For the leading perturbative CR the exchange of two colour-matched gluons is required, giving an additional suppression factor of \(\alpha_s^2/(N_C^2-1)\), where \(N_C = 3\) is the number of colours in QCD. Therefore the effect from CR in the perturbative phase is expected to be small and indeed calculations have shown that the consequence for the measurement of the W mass is below 5 MeV/c^2 [44].

**Non-perturbative QCD:** Hadronisation at distance scales \(\sim 1 \text{ fm}\) is not independent. Coherent gluon emission from both hadronic systems is to be expected, leading to final state interference. To study CR effects in the non-perturbative phase, one has to rely on the available phenomenological models. These models are not directly based on first principles, but implement CR in a semi-classical way inside existing fragmentation models (JETSET, HERWIG and ARIADNE all have a built-in CR option).

As mentioned already, LEP2 provides interesting data for dedicated studies of Colour Reconnection effects themselves. These studies are ongoing [45], and will hopefully lead to an improved understanding of the physics, and give constraints on the models to be used.

**Possible effect of cross-talk on the W mass measurement**

More details about the different phenomenological models and experimental constraints will be presented in chapter 7, together with their predicted effects on the W mass measurement. In principle all predictions and models that have not been proven to be wrong are taken seriously. The effects of BEC appear to be less severe than CR. In both cases our understanding of the models and the experimental measurements of the effect is improving, but currently they still constitute major contributions to the systematic uncertainty in the qqqq channel.
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