Direct measurement of the W boson mass in $e^+ e^-$ collisions at LEP
Mulders, M.P.

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Chapter 6

2D Ideogram analysis

In this chapter a comprehensive description is given of the 2D Ideogram analysis, applied to all DELPHI LEP2 data taken at 172, 183 and 189 GeV. Both the fully-hadronic and semi-leptonic channel are used. The analysis of the fully-hadronic channel presented here has been published in the DELPHI 189 GeV W mass paper [3]. The application to the semi-leptonic channel should be regarded as a feasibility study to show that the 2D Ideograms are suitable for use in the semi-leptonic channel as well.

The analysis is described in the following steps:

- The choice of data sets and run quality selection is discussed in section 6.1
- The selection of events and treatment of particles, which is different for
  - the fully-hadronic channel (section 6.2)
  - and the semi-leptonic channel (section 6.3)
- The construction of an event ideogram containing all measured kinematical information about the event (section 6.4).
- The extraction of an event likelihood curve from the ideogram through analytical convolution (6.5).
- The calibration of the analysis using Monte Carlo simulation (6.6).
- Cross-checks of the statistical properties (6.7).
- Summary (6.8).

6.1 Data sets used

Data run quality selection

The analysis described here was applied to DELPHI data taken in the period from October 1996 to November 1998 at the centre-of-mass energies shown in Table 6.1. Since DELPHI had a policy to only record data when at least a minimum configuration of the DELPHI sub-detectors was
operational, the data on tape is generally of good quality. More than 99% of the data on tape was used in the fully-hadronic analysis, requiring the TPC data taking to be > 90% efficient per run file, and checking the DELANA status flag for TPC data acquisition and high-voltage for each event. For the semi-leptonic channel additional requirements were put on the operational state of the calorimeters, requiring the HPC and the EMF to be > 95% efficient. During the 1997 data taking, a known problem in the HPC electronics caused a small fraction (~0.85 %) of the events to have no HPC information at all. All events (q\bar{q}q\bar{q} and q\bar{q}l\nu) were rejected that could possibly have been affected by this.

<table>
<thead>
<tr>
<th>Luminosity weighted mean ( \sqrt{s} ) (GeV)</th>
<th>( \int \mathcal{L} ) (pb(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>October-November 1996</td>
<td>172.3</td>
</tr>
<tr>
<td>July-November 1997</td>
<td>182.7</td>
</tr>
<tr>
<td>May-November 1998</td>
<td>188.6</td>
</tr>
</tbody>
</table>

Table 6.1: *DELPHI data sets on which the results in this thesis are based*

Monte Carlo simulation

Monte Carlo (MC) generators were used to produce simulated event samples at the nominal centre-of-mass energies corresponding to each of the data sets mentioned above. The main generator for WW-like and ZZ-like 4-fermion final states was EXCALIBUR, while q\bar{q}\gamma background processes were simulated using PYTHIA. Both generators were interfaced with DELPHI-tuned JETSET fragmentation (section 2.5), supplemented by the full DELSIM detector simulation (section 3.3). The reference samples used for the calibration of the analysis and checks of the statistical performance are listed in Table 6.2.

6.2 Fully-hadronic event selection

Particle reconstruction

Not all reconstructed particles in the events that are read from DST are of good quality. Since the optimal treatment of dubiously reconstructed tracks and energy clusters depends on the type of physics analysis, a customised particle selection is required at analysis level. For the fully-hadronic \( W \) mass analysis the November 1999 version of the DELPHI analysis software was used with the following standard particle selection criteria for LEP2 hadronic analyses:

For charged particles:

- track momentum \( |p| > 200 \text{ MeV}/c \)
- relative momentum error \( \Delta|p|/|p| < 1 \)
Table 6.2: DELPHI simulation sets used for the analysis in this thesis.

<table>
<thead>
<tr>
<th></th>
<th>(\sqrt{s}) (GeV)</th>
<th>(m_W) (GeV/(c^2))</th>
<th>(\mathcal{L}) (fb(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYTHIA WW</td>
<td>172.0</td>
<td>80.35, 79.85</td>
<td>1.15, 1.37</td>
</tr>
<tr>
<td>PYTHIA ZZ</td>
<td></td>
<td>78.35 - 83.35</td>
<td>13 \times 0.17</td>
</tr>
<tr>
<td>PYTHIA qq(\gamma)</td>
<td>79.35, 80.85, 81.35</td>
<td>3 \times 0.20</td>
<td>1.68, 0.519</td>
</tr>
<tr>
<td>EXCALIBUR 1.06 WW/ZZ</td>
<td>183.0</td>
<td>80.35, 79.35, 81.35</td>
<td>7.20, 7.18, 3.589</td>
</tr>
<tr>
<td>PYTHIA qq(\gamma)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EXCALIBUR 1.08 WW/ZZ</td>
<td>189.0</td>
<td>80.35, 79.35, 81.35</td>
<td>18.52, 10.08, 10.11</td>
</tr>
<tr>
<td>PYTHIA qq(\gamma)</td>
<td>188.0*</td>
<td></td>
<td>9.418</td>
</tr>
</tbody>
</table>

* this MC sample was generated at the start of the year when the average LEP centre-of-mass energy for this run (188.6 GeV) was not yet precisely known.

- impact parameter in \(r\phi < 4\) cm
- impact parameter in \(z \cdot \sin \theta < 4\) cm
- track length > 30 cm, i.e. rejecting tracks with only ID and VD hits associated.

For neutral particles:

- To distinguish genuine energy deposits from detector noise, a minimum energy was required depending on the calorimeter (500 MeV in the HPC, 400 MeV in the FEMC, 900 MeV in the HAC and 300 MeV in the STIC).

In addition to these standard quality criteria an extra cut was applied to eliminate possible energy depositions from off-momentum beam electrons [70],

- rejecting all particles with a polar angle outside the range \(3^\circ < \theta < 177^\circ\)

and a special treatment was included to protect the analysis against particles reconstructed with an unphysically high momentum \((p > E_{\text{beam}})\). Studies have shown that the 'straight' tracks reconstructed in such cases are often indeed caused by charged particles of fairly high momentum. Therefore charged particles with a momentum larger than 60 GeV/c were rescaled to 10 GeV/c — a value based on the momentum spectrum of particles in a hadronic event. Furthermore charged particles with a momentum between 10 and 60 GeV/c and a relative momentum error larger than 0.3, were rescaled to 10 GeV/c, and for neutral particles any excess of associated energy above 100 GeV was discarded.
Event selection

For the fully-hadronic channel a rather simple event selection based on sequential cuts was used, aiming for a reasonable purity without loss of efficiency for W events that contain useful mass information and without introducing correlations with the mass. The purity of the selection is not crucial for the analysis, because later the likelihood will include the estimated event purity on an event-by-event basis.

First of all a sample of hadronic events was selected requiring more than 13 charged particles and a total visible energy exceeding 0.575\sqrt{s} (Figure 6.1). Then the following cuts were applied, the effects of which are illustrated in Figure 6.2:

- A jet clustering was done using the DURHAM algorithm with a $y_{\text{cut}}$ fixed to 0.002. All jets were required to be of ‘good quality’, defined by the following two criteria:
  
  - an invariant mass of the jet larger than 1 GeV/c
  - at least 3 particles

  If necessary, clustering was continued to a higher value of $y_{\text{cut}}$ until all resulting jets satisfied these criteria.

- Events with less than 4 jets were rejected and events with 6 jets or more were re-clustered to 5 objects.

- The effective centre-of-mass energy $\sqrt{s}$ was estimated using the SPRIME+ package (section 5.4.2, page 77) and was required to be larger than $\sqrt{s} - 28$ GeV, mainly in order to remove $q\bar{q}\gamma$ background from ‘radiative returns to the Z’. To improve the purity of the photon identification inside SPRIME+, only candidate photons with less than 3 particles within a cone of opening angle $25.8^\circ = \arccos(0.9)$ were considered to be ISR photons.
Figure 6.2: The main \( q\bar{q}q\bar{q} \) event selection cuts. The sample shown consists of 189 GeV data and MC. In each of the plots all event selection cuts are used, except for the cut on the variable that is shown and without cut on the estimated purity (section 6.2).
Events containing clear b-quark candidates were rejected, requiring the DELPHI combined b-tag variable (section 3.5, page 46) to be smaller than 2.0. This cut removed 16.9% of the ZZ and 6.4% of the qqγ background while reducing the signal efficiency by only 0.2% (numbers for $\sqrt{s} = 189 \text{ GeV}$).

As visible in Figure 6.2 these cuts are quite loose, i.e. they were chosen to allow a high efficiency for $\text{WW} \rightarrow \text{qqqq}$ events ($92.3\pm0.3\%$ at $\sqrt{s} = 189 \text{ GeV}$).

4C kinematic fit

A 4C kinematic fit was applied to the remaining events, enforcing conservation of energy and momentum. In all fits asymmetric transverse jet errors were used, following the procedure explained in section 4.2.

Event purity

The fitted jets define the topological variable $D_{\text{pur}}$ that is used to distinguish 4-fermion from 2-fermion (+2 hard gluons) final states and estimate the purity of the event (ZZ events also have a 4-fermion signature and are treated as signal without W mass information).

$$D_{\text{pur}} = \rho_{\text{fit}}^{\text{jet}} \cdot E_{\text{fit}}^{\text{jet}} \cdot \sqrt{\frac{\theta_{\text{fit}}^{\text{jet}} \cdot E_{\text{fit}}^{\text{jet}}}{100 \text{ rad} \cdot \text{GeV}}}$$

(6.1)

where $E_{\text{fit}}^{\text{jet}}$ and $E_{\text{fit}}^{\text{jet}}$ are the smallest and the one-but-smallest fitted jet energies and $\theta_{\text{fit}}^{\text{jet}}$ and $\theta_{\text{fit}}^{\text{jet}}$ are the smallest and one-but-smallest fitted angle between two jets. $D_{\text{pur}}$ tends to be high for 4-fermion processes and low for 2-fermion processes as gluon radiation at low angles and small energies is preferred. The purity, i.e. the signal-to-(signal+background) ratio $P_{4f}(D_{\text{pur}})$, is fitted as a function of $D_{\text{pur}}$ as shown in Figure 6.3 using a simple parametrisation given by

$$P_{4f}(D_{\text{pur}}) = \frac{(A \cdot D_{\text{pur}})^2 + (B \cdot D_{\text{pur}})^3}{1 + (A \cdot D_{\text{pur}})^2 + (B \cdot D_{\text{pur}})^3}$$

(6.2)

resulting in the fitted parameters shown in Table 6.3.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>172 GeV</td>
<td>0.06091</td>
<td>0.04821</td>
<td>0.09844</td>
<td>0.04644</td>
</tr>
<tr>
<td>183 GeV</td>
<td>0.05679</td>
<td>0.08075</td>
<td>0.09463</td>
<td>0.07718</td>
</tr>
<tr>
<td>189 GeV</td>
<td>0.03458</td>
<td>0.09266</td>
<td>0.08870</td>
<td>0.07920</td>
</tr>
</tbody>
</table>

Table 6.3: Fitted parameters for the purity parameterisation given by equation (6.2) at different centre-of-mass energies.
Figure 6.3: Distribution of the $D_{\text{pur}}$ variable for 4-jet (top left) and 5-jet (top right) events separately, for 189 GeV data and MC. The legend is the same as in Figure 6.2. In the middle plots the parameterisations of the $P^{4f}(D_{\text{pur}})$ purity are plotted, used to extract the estimated event purity, for which the resulting distributions are shown in the bottom plots. The arrows indicate the final cut in the $q\bar{q}q\bar{q}$ event selection.
As a final $q\bar{q}q\bar{q}$ cut all events with an estimated purity $P^{4f}$ below 25% were rejected (see Figure 6.3). This hardly affects the analysis as events with low estimated purity already obtain a low effective weight in the W mass measurement, but it reduces the use of computing resources, proportional to the number of selected events, by 23%.

All remaining events were then used to extract the W mass, provided that at least one of the equal-mass constrained fits (next section) converges. A small fraction of the events for which this turned out not to be the case were kept in the sample, but obtained a flat likelihood curve. The numbers of selected events and the expectation from Monte Carlo are listed in Table 6.4, and Figure 6.4 shows a few distributions of relevant variables of the selected events.

<table>
<thead>
<tr>
<th>Event Type</th>
<th>$\sqrt{s}$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>172</td>
</tr>
<tr>
<td>$q\bar{q}q\bar{q}$</td>
<td>51.2</td>
</tr>
<tr>
<td>$q\bar{q}\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$q\bar{q}\mu\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$q\bar{q}\tau\nu$</td>
<td>0.4</td>
</tr>
<tr>
<td>Other 4f</td>
<td>0.0</td>
</tr>
<tr>
<td>$q\bar{q}^\gamma$ and other 2f</td>
<td>19.7</td>
</tr>
<tr>
<td>Total</td>
<td>71.9</td>
</tr>
<tr>
<td>Data</td>
<td>73</td>
</tr>
</tbody>
</table>

| $q\bar{q}q\bar{q}$ efficiency | 86.6 % | 87.8 % | 89.7 % |
| $q\bar{q}q\bar{q}$ purity | 71.2 % | 72.7 % | 75.5 % |

Table 6.4: Expected number of selected events in the $q\bar{q}q\bar{q}$ channel according to simulation at different centre-of-mass energies compared to the number of events selected from data.
Figure 6.4: Distribution of some relevant $q\bar{q}q\bar{q}$ event variables, for 189 GeV data and MC. The jet broadness values $B_b$ and $B_c$ are the eigenvalues of the momentum tensor given in equation (4.12). The $3C$ and $4C$ $\chi^2$'s refer to the constrained fits with and without the assumption of an unseen ISR photon in the beam-pipe. A large difference $\chi^2_{4C} - \chi^2_{3C}$ indicates an imbalance of momentum in the $z$ direction, most likely caused by ISR, a neutrino, or bad reconstruction of the event.
6.3 Semi-leptonic event selection

Particle reconstruction

The feasibility study for Ideograms in the semi-leptonic channel was based on the analysis framework provided by the WW physics package WWANA 6.10 [53] using the standard SKELANA 2.0 particle selection [52].

Event selection

The event selection for the semi-leptonic channel was designed to be simple and optimised for a high 'inclusive' efficiency for all semi-leptonic channels combined. Cross-feed between the different semi-leptonic channels was not considered to be a special concern as this is taken into account in the analysis later. Lepton tags, energies and energy resolution were taken from the WWANA 6.10 package (section 3.4), and the events were treated as follows:

- Only events with a charged particle multiplicity between 4 and 40 were considered, to reduce the background from di-lepton events and fully-leptonic or fully-hadronic WW events.

- Lepton candidates were searched for in a polar angular range from 5° to 175°, satisfying either one (or both) of the following two signatures:
  - A single isolated track tagged as electron or muon with a momentum larger than 20 GeV/c, with an energy less than 10 GeV from other particles in a cone with a 10° opening angle, and an angle with the nearest jet larger than 5°.
  - A narrow, low-multiplicity jet (at most 5 particles of which at least 1 but at most 3 charged) obtained when clustering the event in 3 jets, with a minimum momentum of 5 GeV/c and satisfying the same isolation criteria as above.

- In case more than one lepton was found, the candidates were arranged in order of preference based on the product of momentum and isolation angle (w.r.t. the nearest charged track with momentum larger than 1 GeV/c).

- Events with at least one lepton candidate were classified as follows:
  - If one of the candidates was a muon the event was classified as $q\bar{q}\mu\nu$ event.
  - Otherwise if an electron candidate was available the event was treated as $q\bar{q}e\nu$.
  - Otherwise the event was classified as $q\bar{q}\tau\nu$.

For events passing the pre-selection, the hadronic part of the event (after excluding the lepton) was clustered in the natural number of jets. And finally only events were accepted with an estimated event purity $P_{\text{event}}$ larger than 25%, where $P_{\text{event}}$ signifies the estimated probability that it was a semi-leptonic WW event. Its calculation is discussed in the next paragraph.
Calculation of the semi-leptonic event purity

The purity was estimated using a likelihood ratio based on 10 event variables:

1. Charged multiplicity $n_c$
2. Sphericity $S \equiv \frac{3}{2}(\lambda_2 + \lambda_3)$, where $\lambda_2$ and $\lambda_3$ are the two smallest eigenvalues of the sphericity tensor
   \[ S_{\alpha\beta} = \sum_i P_i^\alpha P_i^\beta \left/ \sum_i |P_i|^2 \right. \]
3. Estimated effective centre-of-mass energy $\sqrt{s} / \beta$
4. Size of the missing momentum $|\mathbf{p}_{\text{miss}}|$
5. The cosine of the polar angle of the missing momentum vector $\cos\theta_{\text{miss}}$
6. Size of the lepton momentum $|\mathbf{p}_{\text{lepton}}|$
7. Isolation angle $\theta_{\text{iso}}$ of the lepton with respect to the closest particle with an energy of more than 1 GeV
8. Product of the cosine of the polar angle of the lepton momentum and its charge: $Q_{\text{lepton}} \cdot \cos\theta_{\text{lepton}}$
9. Fitted polar angle $\theta_{W^-}$ of the $W^-$
10. Angle between the two jets $\theta_{ij}$ when the event is forced into a 2 jets + lepton configuration

For each lepton channel the distributions of signal and background for each of these variables were normalised to 1, and the signal over background ratios $(s/b)_i$ were plotted and parametrised. The overall signal-to-background likelihood ratio was then estimated using a simplified expression:

\[ \left( \frac{s}{b} \right)_{\text{total}} = \frac{\sigma_{\text{signal}}^{\text{acc}}}{\sigma_{\text{background}}^{\text{acc}}} \cdot \prod_{i=1}^{10} \left( \frac{s}{b} \right)_i \cdot a_{\text{corr}} \]

where $\sigma_{\text{signal}}^{\text{acc}}$ and $\sigma_{\text{background}}^{\text{acc}}$ are the accepted cross-section for signal and background after the event pre-selection, and $a_{\text{corr}}$ a correction factor to be determined by hand for each lepton channel and $\sqrt{s}$ in order to take into account part of the correlations between the likelihood variables. In absence of correlations $a_{\text{corr}}$ should equal 1. From the overall signal to background ratio it is straightforward to calculate the estimated purity

\[ P_{\text{event}} = \frac{s}{s + b} = \frac{\left( \frac{s}{b} \right)_{\text{total}}}{\left( \frac{s}{b} \right)_{\text{total}} + 1} \]

In order to cross-check the result and determine $a_{\text{corr}}$, the $s/(s + b)$ ratio obtained from MC simulation (≡ 'true' purity) was plotted as a function of the estimated purity. As visible in Figure 6.5, the agreement between estimated and true purity is quite reasonable. The numerical values found for $a_{\text{corr}}$ ranged from about 0.02 in the electron and tau channel to unity in the muon channel. Distributions of the variables used are shown in Figure 6.6, while Figure 6.7 shows the resulting
(\(\frac{2}{3}\)), parameterisations obtained for the tau channel as an example. Table 6.5 and 6.6 give an overview of the selection efficiencies and expected number of events determined using the Monte Carlo samples listed in Table 6.2.

| process | Efficiency (%) for selection as | | Selection purity (%) in channel |
|---|---|---|---|---|
|   | muon | electron | tau | (not selected) | muon | electron | tau |
| \(q\bar{q}\mu\nu\) | 88.3 | 0.1 | 2.7 | 8.8 | 87.2 | 0.1 | 3.7 |
| \(q\bar{q}\tau\nu\) | 0.2 | 68.3 | 9.7 | 21.7 | 0.2 | 74.8 | 14.2 |
| other 4f | 7.5 | 6.8 | 35.9 | 49.8 | 7.4 | 7.0 | 49.5 |
| \(q\bar{q}\gamma\) | 0.1 | 0.2 | 0.6 | 98.8 | 3.1 | 2.8 | 5.4 |
| \(q\bar{q}\gamma\) | 0.0 | 0.4 | 0.5 | 99.1 | 2.0 | 15.3 | 27.2 |

Table 6.5: Selection efficiencies and purities for the \(q\bar{q}\nu\) channel at \(\sqrt{s} = 189\) GeV

<table>
<thead>
<tr>
<th>Number of events</th>
<th>Expected</th>
<th>Observed in data</th>
</tr>
</thead>
<tbody>
<tr>
<td>172 GeV (\mu)-channel</td>
<td>18.3</td>
<td>17</td>
</tr>
<tr>
<td>e-channel</td>
<td>15.1</td>
<td>15</td>
</tr>
<tr>
<td>(\tau)-channel</td>
<td>11.3</td>
<td>18</td>
</tr>
<tr>
<td>183 GeV (\mu)-channel</td>
<td>164.9</td>
<td>129</td>
</tr>
<tr>
<td>e-channel</td>
<td>109.4</td>
<td>121</td>
</tr>
<tr>
<td>(\tau)-channel</td>
<td>74.5</td>
<td>86</td>
</tr>
<tr>
<td>189 GeV (\mu)-channel</td>
<td>373.7</td>
<td>338</td>
</tr>
<tr>
<td>e-channel</td>
<td>367.4</td>
<td>372</td>
</tr>
<tr>
<td>(\tau)-channel</td>
<td>267.2</td>
<td>280</td>
</tr>
</tbody>
</table>

Table 6.6: Number of events selected in the \(q\bar{q}\nu\) channel at different energies compared to the number of events expected from MC simulation.
Figure 6.5: Purity distributions obtained for the different channels at a centre-of-mass energy of 189 GeV.
Figure 6.6: Distributions of the variables used for the purity estimation in the \( q\bar{q}\ell\nu \) channels after full selection — including the final cut requiring \( P_{\text{event}} > 0.25 \). The points with error bars indicate 189 GeV data, while the histograms include \( Z\gamma \) (shaded) and WW+ZZ simulation.
Figure 6.6, cont'd
Figure 6.7: The parameterisations of the likelihood ratios used in the calculation of the estimated purity in the tau channel at $\sqrt{s} = 189$ GeV. The parameterisation as a function of $\theta_{jj}$ was deliberately chosen such as not to follow the structure of the peak, because of its direct correlation with the W mass.
6.4 Kinematical event reconstruction

Now that the events have been selected, their WW purity estimated, and the constrained fits have been prepared, the next step is to fully reconstruct the kinematics of the event under the assumption that two heavy vector bosons were produced with masses $m_1$ and $m_2$. The aim is to produce the 2-dimensional Ideogram $p(event|\vec{m}', I)$ (equation (5.16)), representing the relative likelihood (from the goodness-of-fit) that the event is kinematically compatible with this assumption as a function of the two masses $m_1$ and $m_2$.

6.4.1 Equal-mass constrained fit

In order to obtain a first impression of the mass information in the events, 1-dimensional mass spectra are made using a 5C equal mass constrained fit. Both in the $q\bar{q}l\nu$ and the $qqq\bar{q}$ channel more than one hypothesis is possible. For each hypothesis a 5C fit is performed and the most probable solution is plotted.

Fully-hadronic channel

For the 1-dimensional mass plot in the $qqq\bar{q}$ channel only the hypotheses without ISR and using the DURHAM clustering algorithm are considered, thus reducing the number of hypotheses to 3 possible jet pairings in a 4-jet event, and 10 in a 5-jet event. For each jet pairing $k$ a constrained fit is performed, leading to a different $\chi^2_{3C,k}$ per pairing, depending on its compatibility with the equal-mass constraint. In addition to the $w_k^{\text{gluon}} \equiv 1/k_T$ pairing probability for 5-jet events (section 5.2), jet charges $Q_{\text{jet}}^j$ are used to further improve the choice of the correct jet pairing. The jet charge is defined as

$$Q_{\text{jet}}^j \equiv \sum_{i=1}^{n_c^j} q_i \cdot \left( \frac{|P_{\text{par}}|}{1\text{GeV}/c} \right)^{\kappa}$$

with $\kappa = 0.5$, and summing over all $n_c^j$ charged particles $i$ in jet no. $j$ where $q_i$ is the charge of particle no. $i$ and $P_{\text{par}}$ its momentum parallel to the axis of jet $j$. For each jet pairing $k$ the measured boson charges $Q_k^{W1}$ and $Q_k^{W2}$ are calculated as the sum over the jet charges in each $W$ boson (see the distribution shown in Figure 6.8). The probability $p_{W^+}$ that $W_1$ corresponds to $W^+$ is extracted as a function of $\Delta Q_k \equiv Q_k^{W1} - Q_k^{W2}$. Then the distribution of the $W^+$ production angle $P_{\theta_+}(\theta_{W^+})$ (section 2.1) is used to determine the relative likelihood for each jet pairing:

$$w_k^{\text{charge}} = p_{W^+}(\Delta Q_k) \cdot P_{\theta_+}(\theta_{W_1}^k) + (1 - p_{W^+}(\Delta Q_k)) \cdot P_{\theta_+}(\pi - \theta_{W_1}^k)$$

In Figure 6.9 one mass per event is plotted, choosing the jet pairing with the lowest $\chi^2_{\text{total,qqqq}}$ where

$$\chi^2_{\text{total,qqqq}} = \chi^2_{5C} - 2 \cdot \ln(w_k^{\text{charge}}) - 2 \cdot \ln(w_k^{\text{gluon}})$$

thus using the extra information from jet charge and hard gluon emission.
The jet pairing closest to the true generated W bosons is chosen using MC information, and the distribution of measured charge difference of the respective bosons $\Delta Q_k$ is plotted for 189 GeV WW simulation and fitted with a Normal distribution (top left). The fitted Gaussian is used to calculate $p(W_1 = W^+)$ (top right), representing the probability that a supposed W boson with a measured charge difference $\Delta Q_k$ with respect to the other W boson in the event indeed is the positively charged W boson, provided the respective jet pairing $k$ is the 'correct' jet pairing. The bottom plots contain the same distributions of $\Delta Q_k$ for data and MC, but choosing the most probable jet pairing in the analysis, without using generator level information.
Figure 6.9: Mass spectra from a constrained fit using an equal-mass constraint at different energies. The jet pairing is chosen that gives the lowest $\chi^2_{\text{total,qqq}}$ (equation (6.8)).
Figure 6.10: The fraction of $q\bar{q}\tau\nu$ events in each of the $q\bar{q}\ell\nu$ channels after full selection shown as a function of the lepton momentum at 189 GeV. The functions shown were used to give relative weights to the tau and the non-tau hypotheses in the analysis.

**Semi-leptonic channel**

As shown before (see Table 6.5) cross-feed occurs between the different $q\bar{q}\ell\nu$ channels. Whereas the cross-feed between the electron and the muon channel is small, this is not true between the tau channel and both of the other channels. Mis-identification of tau events as $q\bar{q}\mu\nu$ or $q\bar{q}\ell\nu$ is to be expected when the tau decays leptonically, resulting in an isolated electron or muon. Cross-feed in the opposite direction is dominated by electrons showering as a result of secondary interactions with 'dead' material at 90 degrees and in the gaps between barrel and endcaps (at 40 and 140 degrees), resulting in multiple tracks. When these are not recognised as an electron by the REMCLU package they might consequently be reconstructed as a tau jet. This effect is believed to be the cause of the narrow peaks in Figure 6.7 for the purity as a function of $Q_{\text{lepton}} \cdot \cos\theta_{\text{lepton}}$. To take this cross-feed into account, each event is fitted according to two hypotheses:

1. The lepton was a tau. In this case two additional neutrinos were lost, decreasing the kinematical information carried by the lepton. The constrained fit treats the lepton as a tau.

2. The lepton was an electron or a muon. In the $q\bar{q}\ell\nu$ and the $q\bar{q}\mu\nu$ channel the corresponding assumption for the lepton is used. In the $q\bar{q}\tau\nu$ channel, for this hypothesis the supposed tau candidate is defined as an electron in the fit.

The relative probability $w_k^{\tau/\text{non-}\tau}$ of each hypothesis $k = 1, 2$ to be correct is estimated as a function of the lepton momentum, using parameterisations extracted from MC simulation as shown in Figure 6.10 for $\sqrt{s} = 189$ GeV. An effective $\chi^2$ is calculated as

$$\chi^2_{k,\text{total},q\bar{q}\ell\nu} = \chi^2_{5C,k} - 2 \cdot \ln(w_k^{\tau/\text{non-}\tau})$$  \hfill (6.9)

and for each event the solution with the lowest $\chi^2_{k,\text{total},q\bar{q}\ell\nu}$ is plotted as in Figure 6.11 and 6.12.
Figure 6.11: Mass spectra from a constrained fit using an equal-mass constraint, for the semi-leptonic muon and electron channel at different energies.
Figure 6.12: Mass spectra from a constrained fit using an equal-mass constraint, for the tau channel (left) and all semi-leptonic channels combined (right) at different $\sqrt{s}$ energies.
6.4.2 Calculating the 2D ideograms

As explained already in chapter 5 the Ideogram analysis does not use equal-mass plots to extract the W mass. Instead, for each event a W mass likelihood curve is computed, taking into account the full kinematic information about the two masses $m_1$ and $m_2$ of the W bosons in the event. The likelihood curve is extracted from a 2-dimensional ideogram representing the goodness-of-fit for each value of $m_1$ and $m_2$, obtained from kinematic constrained fits. The constrained fits all apply the error parameterisation given in equation (4.13) and (4.14), exploiting the information about jet broadness and undetected jet energy. The exact procedure depends on the analysis channel.

Fully-hadronic Ideogram

In case of a fully-hadronic event one ideogram is made for every jet pairing (max. 10), three clustering algorithms and a possible ISR hypothesis (additional factor 2), leading to a maximum of 60 ideograms per event.

The preferred way (proposed in section 5.3) to determine the likelihood that an event is compatible with two invariant masses $m_1$ and $m_2$ is by means of a 6C fit, as in equation (5.13), where both masses are fixed and used as constraints in the fit:

$$p(event | m_1, m_2, I) = P_{6C}(jets | m_1, m_2, \sqrt{s}, \sum \vec{p})$$

(6.10)

To cover the region of interest in the 2-dimensional $(m_1, m_2)$ mass plane, however, would require a large number of fits for each of the ideograms. Fortunately the constrained fit in the $q\bar{q}q\bar{q}$ channel is well behaved and the corresponding ideogram can generally be described by a single, Gaussian solution. Therefore, the number of fits (and amount of CPU time needed) can be significantly reduced by using a Gaussian approximation based on a single 4C fit per ideogram.

For each of the (up to 60) hypotheses mentioned above a 4C fit is performed, rendering a $\chi^2_{4C}$, two fitted boson masses $m^{fit}_1$ and $m^{fit}_2$, the estimated errors on these masses $\sigma_{m_1}$ and $\sigma_{m_2}$ and the correlation coefficient $\rho_{12}$. Then the $\chi^2(m_1, m_2)$ as a function of the two masses $m_1$ and $m_2$ is approximated as

$$\chi^2_{\text{hypothesis}}(m_1, m_2) \approx \chi^2_{4C} + (m - m^{fit})^T V^{-1} (m - m^{fit})$$

(6.11)

where

$$V = \begin{pmatrix} \sigma^2_{m_1} & \sigma_{m_1} \sigma_{m_2} \rho_{12} \\ \sigma_{m_1} \sigma_{m_2} \rho_{12} & \sigma^2_{m_2} \end{pmatrix}$$

$$m = \begin{pmatrix} m_1 \\ m_2 \end{pmatrix}$$

$$m^{fit} = \begin{pmatrix} m^{fit}_1 \\ m^{fit}_2 \end{pmatrix}$$

In case the 4C fit gives a $\chi^2_{4C}$ larger than the number of degrees of freedom NDF= 4, the whole ideogram $\chi^2_{\text{hypothesis}}(m_1, m_2)$ is rescaled with a factor NDF/$\chi^2_{4C}$ in order to take into account
non-Gaussian resolution effects, similar to the procedure used by the Particle Data Group. The $\chi^2_{IC}(m_1, m_2)$ distribution thus obtained is interpreted as a probability distribution:

$$P_{\text{hypothesis}}^{\text{qqqq}}(m_1, m_2) dm_1 dm_2 \sim \exp \left( -\frac{1}{2} \chi^2_{\text{hypothesis}}(m_1, m_2) \right)$$

(6.12)

and normalised on the kinematical region so that

$$\int_{m_{\text{min}}}^{m_{\text{max}}} \int_{m_{\text{min}}}^{m_{\text{max}}} P_{\text{hypothesis}}^{\text{qqqq}}(m_1, m_2) dm_1 dm_2 = 1$$

(6.13)

with $m_{\text{min}} = 60 \text{ GeV}/c^2$ and $m_{\text{max}} = 110 \text{ GeV}/c^2$.

In the $qqq\bar{q}$ channel sufficient information is available to apply an ISR correction on an event-by-event level. As explained in section 5.4.3, for those events more than 1.5 sigma away from $p_z = 0$, additional ideograms are calculated for the hypothesis that an ISR photon escaped undetected in the beam-pipe. The relative probability $w_j^{\text{ISR}}$ of the two hypotheses ($j = 1, 2$) was extracted from MC.

For the W mass measurement $m_1$ and $m_2$ are equivalent, and the choice of which of the two boson masses is assigned to $m_1$ and which to $m_2$ was arbitrary. Now in order to make the jet-charge information visible in the ideogram (and use it later to measure the difference between $m_{W^+}$ and $m_{W^-}$), the ideogram is 'symmetry broken' according to the calculated jet-charge weights:

$$P_{\text{hypothesis}}^{W^+W^-}(m_{W^+}, m_{W^-}) dm_{W^+} dm_{W^-} =$$

$$\left[ p_{W^+}(\Delta Q_k) \cdot P_{\text{hypothesis}}^{\text{qqqq}}(m_1, m_2) + (1 - p_{W^+}(\Delta Q_k)) \cdot P_{\text{hypothesis}}^{\text{qqqq}}(m_2, m_1) \right] dm_1 dm_2$$

(6.14)

Since no equal-mass constraint is used the different jet pairings have an equal a-priori probability looking only at the energy and momentum constraints. Using the information from jet charge and gluon emission as described before, the ideograms for the different jet pairings are weighted and subsequently added to give the combined 'DURHAM' ideogram:

$$P^{W^+W^-}_{\text{DURHAM}}(m_{W^+}, m_{W^-}) dm_{W^+} dm_{W^-} =$$

$$\sum_{k=1}^{n_{\text{pairing}}} \sum_{j=1}^{n_{\text{ISR}}} u_k^{\text{charge}} \cdot u_k^{\text{gluon}} \cdot w_j^{\text{ISR}} \cdot \Phi_{k,j}^{W^+W^-}(m_{W^+}, m_{W^-}) dm_{W^+} dm_{W^-}$$

(6.15)

This ideogram was made using the jets found by the DURHAM clustering algorithm. In order to identify and empirically treat clustering ambiguities, this whole procedure is repeated using the CAMJET algorithm and the DICLUS algorithm (sections 4.1 and 5.4.2). The event is clustered again, forcing it into the same number of jets as found by the DURHAM algorithm, and the whole procedure described in this section (6.4) is repeated. Finally the (normalised) ideograms are added with equal weight, giving

$$p(\text{event} | \vec{m'}, I) d\vec{m'} \sim P^{\text{qqqq}}_{\text{TOT}}(\vec{m}) d\vec{m'} =$$

$$\frac{1}{3} \left[ P^{W^+W^-}_{\text{DURHAM}}(\vec{m'}) + P^{W^+W^-}_{\text{CAMBRIDGE}}(\vec{m'}) + P^{W^+W^-}_{\text{DICLUS}}(\vec{m'}) \right] d\vec{m'}$$

(6.16)

with $\vec{m'} \equiv (m_{W^+}, m_{W^-})$. A few examples are shown in Figure 6.13.

---

1These values of $m_{\text{min}}$ and $m_{\text{max}}$ have been chosen such that CPU processing time is minimised while ensuring a numerically reliable result and a good calibration curve (section 6.6) in a reasonable W mass range e.g. from 75 to 85 GeV/c$^2$. 

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Figure 6.13: Examples of Ideograms constructed for 4 different events from 189 GeV MC simulation. The two Ideograms at the right-hand side include the hypothesis of collinear ISR, while the other two events (left) are well balanced in $p_z$ and were not treated with ISR hypothesis. The first three sigma contours are shown. The corresponding W mass likelihood curves are shown in Figure 6.15.

Semi-leptonic Ideogram

In the $q\bar{q}\ell\nu$ channel only two ideograms per event are made, corresponding to the tau and non-tau lepton hypotheses, as mentioned before (page 104). But while the number of hypotheses is smaller than in the $q\bar{q}q\bar{q}$ channel, the construction of the $q\bar{q}\ell\nu$ ideograms is more elaborate. Here the Gaussian approximation applied in the $q\bar{q}q\bar{q}$ channel cannot be used, because in a significant fraction of the $q\bar{q}\ell\nu$ events the presence of the invisible neutrino leads to involved non-Gaussian ideogram shapes and double solutions (see section 5.5). In this case the full $\chi^2_{6C}$ from a 6C fit is needed.

To obtain the 6C fit $\chi^2$ in the whole $m_1, m_2$ plane turned out not to be trivial. Especially
in events with double solutions, near the boundary between the $m_1$, $m_2$ regions corresponding to the different solutions, convergence is an issue. Even if the fit converges it often requires an increased number of iterations and may converge to the 'second' solution, i.e. not the solution with the lowest $\chi^2$ of the two for that particular $(m_1, m_2)$ point.

Therefore a special interpolation algorithm was developed, which reduces the number of fits needed, improves the speed of the fits, enhances their rate of convergence and helps each fit to converge to the global $\chi^2$ minimum instead of the 'second' solution. The technical details of this algorithm are described below:

- The 6C fits were performed on a rectangular grid in the $(m_1 + m_2)$ and $(m_1 - m_2)$ direction with a grid spacing decreasing towards the kinematical limit.

- The constrained fit package was altered to make it possible for the user to define the 'fitted' particle (= jets + lepton) 4-vectors to be used as a starting configuration for the iteration procedure in each fit. This feature offers the possibility to help the fit converge to the correct solution in a smaller number of iteration steps (see below).

- Starting with the $(m_1, m_2)$ point found by a 4C fit leaving both masses free, fits were performed on the grid. Every next fitting point was randomly chosen from the neighbouring points of the point with the lowest $\chi^2_{6C}$ that had already been fitted and still had at least one 'free' (= not yet fitted) neighbour. As start of the new fit the fitted particle configuration of this lowest 6C $\chi^2$ neighbouring point was used. For each successful fit the particle configuration was saved and kept for later. When a fit did not converge, an unphysically high value $\chi^2 = 10,000$ was assigned. This step was repeated until no fitted point with a $\chi^2_{6C} < 9$ with free neighbouring points remained.

- To obtain the $\chi^2$ for an arbitrary point $(m_1, m_2)$, a 10-point interpolation was applied when 10 neighbouring fits converged, otherwise a 3-point interpolation was used, or no value was returned at all.

This algorithm proved to be very successful. The resulting $\Delta \chi^2_{6C}(m_1, m_2)$ distribution is transformed into a probability distribution analogously to the fully-hadronic channel:

$$P_{\text{hypothesis}}(m_1, m_2)dm_1dm_2 \sim \exp\left(-\frac{1}{2} \chi^2_{6C}(m_1, m_2)\right)$$

(6.17)

followed by a normalisation of the ideogram as in equation (6.13):

$$\int_{m_{\text{min}}}^{m_{\text{max}}} \int_{m_{\text{min}}}^{m_{\text{max}}} P_{\text{hypothesis}}(m_1, m_2)dm_1dm_2 = 1$$

(6.18)

on the same kinematical region defined by $m_{\text{min}} = 60$ GeV/c$^2$ and $m_{\text{max}} = 110$ GeV/c$^2$. Then the tau and the non-tau ideograms were weighted according to their relative probabilities $w_{\tau/\text{non-}\tau}$ and summed to give the combined ideogram probability

$$p(\text{event} | \bar{m}', I)d\bar{m}' \sim P_{\text{TOT}}(\bar{m}')d\bar{m}' = \left[w_{\tau/\text{non-}\tau} \cdot P_{\tau}^{-\text{hyp}}(\bar{m}') + w_{\text{non-}\tau/\tau} \cdot P_{\text{non-}\tau}^{-\text{hyp}}(\bar{m}')\right]d\bar{m}'$$

(6.19)

with $\bar{m}' \equiv (m_1, m_2)$. A few examples are shown in Figure 6.14.
Figure 6.14: Examples of Ideograms for typical $q\bar{q}\nu$ events, from MC simulation at $\sqrt{s} = 189$ GeV. The first 3 sigma contours of the probability density are shown. The variations in shape are related to different configurations of the lepton and the jets in the event. The distribution can be Gaussian and more or less elongated (top left and right), or non-Gaussian due to the double solution induced by the missing neutrino (bottom left) or when both lepton hypotheses (tau and non-tau) obtain a significant probability (bottom right).

6.5 W mass, width and $\Delta m_{W^+W^-}$ extraction

For each event a likelihood curve $L_{\text{event}}(m_W, \Gamma_W)$ was extracted using equation (5.16):

$$L_{\text{event}}(m_W, \Gamma_W) = \int_{m_{\text{min}}}^{m_{\text{max}}} \int_{m_{\text{min}}}^{m_{\text{max}}} P_{\text{TOT}}^{\text{q\bar{q}\nu}}(m') \cdot [P_{\text{event}} \cdot S(m'|m_W, \Gamma_W) + (1 - P_{\text{event}}) \cdot B(m')] \, dm'$$

where $P_{\text{TOT}}^{\text{q\bar{q}\nu}}(m')$ represents the Ideogram function $P_{\text{TOT}}^{\text{q\bar{q}\nu}}$ or $P_{\text{TOT}}^{\text{q\bar{q}\nu}}$, described in the previous section. The measurement of $m_W$ and the $\Gamma_W$ is based directly on these likelihood curves. The
extraction method of the difference of the $W^+$ and $W^-$ is slightly more complicated and is discussed separately, later in this section.

**Construction of the event likelihood**

The physics probability density functions $S$ and $B$ in equation (6.20) are normalised to one. The background function is assumed to be flat $B(\vec{m}') = B$, while the signal function is the product of a phase space function and a factor containing Breit-Wigners:

$$
S(m_1, m_2, m_W, \Gamma_W) = \frac{1}{8} \sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2} \cdot [\sigma_{\text{WW}}^{\text{accept}} \cdot \text{BW}_{\text{WW}}(m_1, m_2, m_W, \Gamma_W) + \sigma_{\text{ZZ}}^{\text{accept}} \cdot \text{BW}_{\text{ZZ}}(m_1, m_2, m_Z, \Gamma_Z)],
$$

(6.21)
to be explained in more detail below; for the $q\bar{q}q\bar{q}$ and the $q\bar{q}l\nu$ channel separately:

- In the $q\bar{q}q\bar{q}$ channel, the background function $B$ describes the mass distribution expected from $q\bar{q}\gamma$ background. The proper background distributions can be taken and parameterised from MC simulation, but it was found that for all practical purposes this function can be assumed to be flat on the region of integration. For analyses that use ideograms close to the kinematical limit [71] a more precise description is needed.

The 4-fermion distribution contains a contribution from correct jet pairings and a contribution from wrong pairings, which also is assumed to be flat. The correct-pairing part contains relativistic Breit-Wigners for the WW and ZZ contribution, weighted according to their relative accepted cross-sections $\sigma_{\text{WW}}^{\text{accept}}$ and $\sigma_{\text{ZZ}}^{\text{accept}}$, extracted for every centre-of-mass energy from the MC simulation. This is actually a simplification of the more involved expression including the probability of the other jet-pairings to give the masses assumed to be wrong combinations. This simplification is only allowed if the wrong-pairing distribution is rather flat.

Preliminary MC studies have shown that a proper inclusion of the wrong pairing background would improve the mass resolution by 1% or less. Event-by-event investigations of the jet structure show that not all $q\bar{q}q\bar{q}$ events have a jet clustering which corresponds well to the parton shower truth. This was taken into account by reducing the effective purity of the events by a factor $\epsilon_{\text{clus}}$. This procedure is justified by also this component of wrong pairings being very flatly distributed in the $(m_1, m_2)$ plane. The effective event purity $P_{\text{event}}$ thus becomes $P_{\text{event}} = \epsilon_{\text{clus}} \cdot P^{4f}$ where $P^{4f}$ was the estimated 4-fermion purity. A value of $\epsilon_{\text{clus}} = 0.7$ was found to give good pull distributions for all of the centre-of-mass energies. The tuning procedure used to determine $\epsilon_{\text{clus}}$ is described in section 6.7.

- In the $q\bar{q}l\nu$ channel a contribution from ZZ will not give a significant mass peak, as the fit hypothesis (jets, 1 lepton and 1 neutrino) is likely to be wrong, and the cross-section is small. Therefore $\sigma_{\text{ZZ}}^{\text{accept}}$ was taken to be zero in the likelihood expression. With only one possible jet pairing and a jet clustering that is less likely to cause problems that can affect the reconstructed mass, the effective event purity $P_{\text{event}}$ was defined to be equal to the estimated $q\bar{q}l\nu$ purity obtained from equation (6.5).
The expression used for the 2-dimensional Breit-Wigner is the product of two 1-dimensional Breit-Wigners with $s$-dependent width (section 2.2):

$$BW_{WW}(m_1, m_2, m_W, \Gamma_W) = BW_W(m_1, m_W, \Gamma_W) \cdot BW_W(m_2, m_W, \Gamma_W)$$

(6.22)

with

$$BW_W(m, m_W, \Gamma_W) \propto \frac{\Gamma_W m^2}{(m^2 - m_W^2)^2 + (m^2 \frac{\Gamma_W}{m_W})^2}$$

(6.23)

and a similar expression for the ZZ term $BW_{ZZ}(m_1, m_2, m_Z, \Gamma_Z)$. Since the Breit-Wigners depend on the boson masses and decay widths $m_W, \Gamma_W, m_Z$ and $\Gamma_Z$, the event likelihood (6.20) can be calculated as a function of these parameters by varying the corresponding parameter while ensuring that the BW functions stay normalised on the integration area.

The event likelihood $\mathcal{L}(m_W)$ is calculated in steps of 0.5 GeV/c$^2$ in $m_W$ (0.4 GeV/c$^2$ for the $q\bar{q}\nu\nu$ channel) and $\mathcal{L}(\Gamma_W)$ in steps of 0.2 GeV/c$^2$. Examples are shown in Figure 6.15.

![Figure 6.15: Event likelihood curves corresponding to the Ideograms shown in Figure 6.13.](image)

The logarithms of the event likelihood curves

$$L_{\text{event}}(m_W) \equiv -2 \cdot \ln (\mathcal{L}_{\text{event}}(m_W))$$

$$L_{\text{event}}(\Gamma_W) \equiv -2 \cdot \ln (\mathcal{L}_{\text{event}}(\Gamma_W))$$

(6.24)

are saved and kept for later combination and analysis.
Extraction of $\Delta m_{W^-W^+}$

For the measurement of the difference of the $W^+$ and the $W^-$ boson masses also a 1-dimensional likelihood was extracted. The average $W$ mass was fixed, and the $W^+$ and $W^-$ masses in the 2D Breit-Wigner were varied according to: $m_{W^+} = 80.35 \text{ GeV}/c^2 + \frac{1}{2} \Delta m_{W^+-W^-}$ and $m_{W^-} = 80.35 \text{ GeV}/c^2 - \frac{1}{2} \Delta m_{W^+-W^-}$. By convoluting this with the $W^+ W^-$ de-symmetrised ideograms in the $qq\bar{q}q$ channel the likelihood as a function of the difference of the masses $\Delta m_{W^+-W^-}$ was extracted and saved in steps of 1 GeV/$c^2$. The effect on the measurement of fixing the average mass to 80.35 GeV/$c^2$ is negligible because the correlation between the sum of the masses and the difference of the masses should be extremely small, and because the sum of the masses is measured more than one order of magnitude more precisely than the difference. Any residual effect is taken into account in the estimation of the systematic errors.

A similar approach in the $q\bar{q}l\nu$ channel would not be optimal, due to the fact that the resolution on the mass difference is significantly worse, while on the other hand the $W^+/W^-$ separation is much better. For the $\Delta m_{W^+-W^-}$ measurement in this channel a $W$ mass likelihood $\mathcal{L}_{\text{event}}(m_W)$ was extracted, using only the tau-hypothesis. By using the tau-hypothesis, the mass information extracted from the event is almost fully determined by the hadronic system, while the lepton is only used to determine the sign of the $W$ bosons. Thus the $W$ mass difference $\Delta m_{W^+-W^-}$ can be measured by dividing the total event sample in a $W^+$ and a $W^-$ sample and by then using the difference of the measured masses. Any remaining correlations inside the events are taken into account in the calibration of the analysis (section 6.6).

Overall likelihood

As the events are independent, the overall likelihood curve can be obtained by taking the product of the event likelihood curves, or equivalently, adding the negative log likelihood (or $\Delta \chi^2$) curves that were saved (see equation (6.24)), e.g. for the $W$ mass:

$$\Delta \chi^2_{\text{overall}}(m_W) \equiv L_{\text{overall}}(m_W) = \sum_{\text{event}=1}^{n_{\text{event}}} L_{\text{event}}(m_W)$$

(6.25)

The ‘bin’ of $m_W$ with the lowest value of $\Delta \chi^2_{\text{overall}}$ is looked for, and a parabola is interpolated through this $\Delta \chi^2$ value and the neighbouring two bins. The error on the fitted mass is calculated from the second derivative at the minimum:

$$\sigma_{m_W}^{m_{\text{fit}}} = \sqrt{\frac{2}{\alpha^2}} \quad \text{with} \quad \alpha = \frac{\partial^2 L_{\text{overall}}(m_W)}{\partial m_W^2} \bigg|_{m_W=m_{\text{fit}}}$$

(6.26)

This is equivalent to the $m_W$ range over which $\Delta \chi^2_{\text{overall}}$ changes from 0 to 1 if the curve is parabolic which turns out to be a good assumption for the mass measurement in all channels at all energies. The fitted value for $m_W$ will be called $m_{\text{fit}}$ from now on.

In Figure 6.16 some examples of likelihood curves are shown. It was found for the measurement of the width, that the parabolic approximation could lead to numerical instabilities of the order of 3 MeV/$c^2$ due to the asymmetry in the likelihood curves. This was solved by using 3rd order polynomial interpolation through the 4 bins closest to the minimum in the $\Gamma_W$ negative log likelihood curves.
6.6 Final calibration using simulation

The mass thus obtained from the overall likelihood (6.25) relies on analytical event likelihood expressions based on energy and momentum conservation, a simplified jet-resolution parameter-
isation and a BW mass dependence modified by phase-space. Although these analytical likelihoods do not at all take into account the full complexity of the DELPHI detector and only partly correct for ISR, this ‘raw’ mass measurement turns out to be accurate to better than 1% (0.2% in the $qar{q}qar{q}$ channel) over a broad $\sqrt{s}$ range, even without further calibration.

In order to reach the precision aimed for, however, Monte Carlo simulation is needed to correct for more complicated detector effects and the influence of ‘higher order’ physics effects like ISR radiation. The calibration can be done directly using MC reweighting or by using a linear transformation based on independent MC samples as described in the following two paragraphs.

**Independent Samples at different generated mass**

The MC samples listed in Table 6.2 were generated using different values of $m_W$ in order to check how the bias on the measured W mass changes as a function of the W mass. The difference between the fitted and generated mass as a function of the generated mass is plotted (see Figure 6.17 and 6.18) and a slope $a$ and bias $b$ were obtained from a linear fit using the following relation:

$$m_{\text{fit}} - m_{\text{gen}} = a \cdot (m_{\text{gen}} - 80.35 \text{ GeV}/c^2) + b$$

(6.27)

In all cases the fit is satisfactory, indicating that the calibration curve is consistent with being linear within MC statistics for all energies. Figure 6.19 shows the fitted bias and the slope as a function of $\sqrt{s}$. The increase of the positive bias with centre-of-mass energy is expected from ISR radiation. The ISR treatment in the $qar{q}qar{q}$ channel (partly) corrects for this effect.

**Continuous MC reweighting**

The calibration curve can also be obtained without assuming a linear behaviour (or any other functional relation) by using a MC reweighting technique. The resulting reweighting curve compared to the straight line fit of the independent MC samples is shown in Figure 6.20. It provides an extra linearity cross-check for the W mass measurement.

For the $\Gamma_W$ and the $\Delta m_{W^+W^-}$ measurement MC reweighting was the only way to obtain the calibration curve, since only samples generated with $\Gamma_W = 2.07$ GeV/$c^2$ and $\Delta m_{W^+W^-} = 0$ were available. Figure 6.20 shows the calibration curve for the $\Gamma_W$ measurement at 189 GeV as an example.

### 6.7 Checks of the statistical properties

Not only the fitted mass, also the error has to be cross-checked using Monte Carlo simulation, as the analytical likelihood expression used is only a simplification of the true (unknown) likelihood.

**resampling**

The analysis was applied many times on MC samples corresponding to the same integrated luminosity as the data in order to check whether the estimated statistical errors agreed with the spread in the measured quantity ($m_W$, $\Gamma_W$, $\Delta m_{W^+W^-}$). In order to obtain a sufficiently precise result on this test, a large number of samples was needed. For $n$ independent samples, the estimated
Figure 6.17: Calibration curves for muon and electron channel showing the difference of the fitted mass $m_{\text{fit}}$ and generated W mass $m_{\text{gen}}$ as a function of the generated W mass. The definition of the fitted slope $a$ and bias $b$ is given in equation (6.27).
Figure 6.18: Calibration curves for tau and hadronic channel. The definition of the fitted slope $a$ and bias $b$ is given in equation (6.27).
Figure 6.19: Slope and bias (see Figure 6.17 and 6.18) as a function of $\sqrt{s}$.

Figure 6.20: The calibration curves obtained from MC reweighting for $m_W$ (left) and $\Gamma_W$ (right) in the $q\bar{q}q\bar{q}$ channel at $\sqrt{s} = 189$ GeV. The statistical errors indicated for reweighting results are highly correlated from point to point. Two different methods to calculate the event weights (section 4.3) are shown as a cross-check of the linear fit to independent samples used to calibrate the $m_W$ result. For the right-hand plot only the main MC sample (with $m_{\text{gen}} = 80.35$ GeV/c$^2$ and $\Gamma_{\text{gen}} = 2.07$ GeV/c$^2$) was used.

Precision on the statistical error is given by $\Delta \sigma/\sigma \sim \sqrt{\frac{1}{2n}}$, so in order to reach a precision better than 1%, more than 5,000 samples are needed. However, in practice the available MC statistics was typically limited to about 100 times the size of the data. Therefore a resampling method was used, using events more than once. The size of each sample was determined using Poissonian statistics with a mean equal to the expected number of WW and $q\bar{q}\gamma$ events, picking the events
randomly from the available sets of MC simulation.

**Pull distribution and tuning of the analysis**

Of interest for the estimation of the statistical error is the pull distribution, with the pull defined as:

\[
pull \equiv \frac{m_W^{\text{measured}} - m_W^{\text{generated}}}{\sigma^{\text{estimated}}} (6.28)
\]

where \(\sigma^{\text{estimated}}\) is the statistical error estimated from the likelihood curve (6.26). Equivalently pull distributions can be made for \(\Gamma_W\) and \(\Delta m_{W^+W^-}\). A few examples are shown in Figure 6.21. In order to optimise the likelihood model used in this analysis, two parameters \(f_{qgqg}\) (a scaling factor for the input errors of the constrained fit in equation (4.14)) and \(\epsilon_{\text{clus}}\) (clustering efficiency in \(P_{\text{event}} = \epsilon_{\text{clus}} \cdot P^{4f}\) on page 112) were tuned so that

- the RMS of the pull distribution for the mass became one.
- the bias \(\Gamma_{\text{fit}} - \Gamma_{\text{gen}}\) on the measured \(\Gamma_W\) became zero for \(\Gamma_{\text{gen}} = 2.07\) GeV/c^2.

The values for \(\epsilon_{\text{clus}}\) and \(f_{qgqg}\) found for the fully-hadronic channel were 0.7 and 1.1 respectively, at \(\sqrt{s} = 189\) GeV. The same tuning was also used for the analysis at other centre-of-mass energies, with satisfactory pull and bias — as shown in Table 6.7. Both parameters are highly correlated to the estimation of the error on the mass but hardly influence the bias on the mass. In the semi-leptonic channel only the scaling factor for the input errors in the constrained fit \(f_{qgqg}\) was tuned (to \(f_{qgqg} = 1.4\)), optimizing the overall agreement for the pulls in all channels. As visible in Table 6.7 the \(\Gamma_W\) measurement in the semi-leptonic channels after tuning shows a significant positive bias in all channels.

![Pull distributions for \(m_W\) (left), \(\Gamma_W\) (middle), and \(\Delta m_{W^+W^-}\) (right) for the fully-hadronic channel at \(\sqrt{s} = 189\) GeV; fitted with a Normal distribution. Each plot contains 100,000 samples.](image-url)
2D IDEOGRAM ANALYSIS

<table>
<thead>
<tr>
<th></th>
<th>RMS of $m_W$ pull</th>
<th>Bias on $\Gamma_W$(GeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>172</td>
<td>183</td>
</tr>
<tr>
<td>$qq\bar{q}q$</td>
<td>1.05 ± 0.09</td>
<td>0.032 ± 0.023</td>
</tr>
<tr>
<td>$qqe\mu$</td>
<td>1.06 ± 0.03</td>
<td>0.502 ± 0.057</td>
</tr>
<tr>
<td>$q\bar{q}\mu\mu$</td>
<td>1.01 ± 0.15</td>
<td>0.700 ± 0.047</td>
</tr>
<tr>
<td>$q\bar{q}\tau\nu$</td>
<td>0.98 ± 0.09</td>
<td>0.712 ± 0.089</td>
</tr>
</tbody>
</table>

Table 6.7: Width of the $m_W$ pull distribution and $\Gamma_W$ bias ($= \Gamma_{fit} - \Gamma_{gen}$ for $\Gamma_{gen} = 2.07$ GeV/c$^2$) obtained after tuning of the analysis (see text).

Pull as a function of estimated error

For small event samples (like the 172 GeV data sample) the accuracy of the measurement can show significant variations depending on the amount of 'luck' in the composition of the data sample that is actually obtained. In those cases the estimated statistical error from the data is a better estimator of the statistical accuracy than the average statistical error expected from MC simulation, provided that the method used to calculate this error is reliable. This can be tested by plotting the RMS of the pull distribution as a function of the estimated error per sample. An example is shown for the $W$ mass in the $qq\bar{q}q$ channel at $\sqrt{s} = 172$ GeV in Figure 6.22. Such plots were only used as a cross-check. For all measurements presented, the quoted errors have been corrected for the average width of the pull obtained from pull distributions as shown in Figure 6.21 (and listed in Table 6.7).

Figure 6.22: Distribution of the estimated error per sample for 1 million MC samples (left) and the RMS of the pull as a function of the estimated error (right) for the $W$ mass in the $qq\bar{q}q$ channel at $\sqrt{s} = 172$ GeV. The arrows indicate the estimated error actually obtained with the DELPHI data. As the analysis was tuned at 189 GeV (see text), the behaviour of the pull is not optimal. The dashed line indicated the average width of the pull used for the correction of the quoted error.
6.8 Summary

An overview of the results obtained with the 2D Ideogram analysis is shown in Table 6.8. The results for the W mass, width and difference of $m_{W^+}$ and $m_{W^-}$ are shown with statistical errors only, after full calibration. All quoted results and errors were corrected for bias and slope of the calibration curve and width of the corresponding pull distribution. In the next chapter a study of the systematic errors is presented, and the overall combination is done in chapter 8.

<table>
<thead>
<tr>
<th>Channel</th>
<th>Calibrated results</th>
<th>172 GeV</th>
<th>183 GeV</th>
<th>189 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$m_{W}$ (GeV/c$^2$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}qq$</td>
<td>80.001 ± 0.475</td>
<td>80.224 ± 0.190</td>
<td>80.466 ± 0.106</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\mu\nu$</td>
<td>80.370 ± 0.671</td>
<td>80.626 ± 0.309</td>
<td>80.048 ± 0.213</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\ell\nu$</td>
<td>80.307 ± 0.971</td>
<td>80.661 ± 0.443</td>
<td>80.115 ± 0.290</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\tau\nu$</td>
<td>81.25 ± 1.27</td>
<td>80.075 ± 0.577</td>
<td>80.075 ± 0.332</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\Gamma_{W}$ (GeV/c$^2$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}qq$</td>
<td>1.61 (+1.32) (-1.01)</td>
<td>2.39 (+0.45) (-0.39)</td>
<td>2.08 (+0.23) (-0.21)</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\mu\nu^*$</td>
<td>2.72 (+0.99) (-0.75)</td>
<td>1.98 (+0.52) (-0.46)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\ell\nu$</td>
<td>0.4 (+1.6) (-1.6)</td>
<td>3.0 (+2.0) (-1.2)</td>
<td>3.7 (+0.8) (-0.7)</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\nu^*$</td>
<td>3.8 (+3.8) (-1.9)</td>
<td>2.83 (+0.95) (-0.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\ell\nu$</td>
<td>0.4 (+1.6) (-1.6)</td>
<td>3.00 (+0.72) (-0.58)</td>
<td>2.75 (+0.41) (-0.37)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$m_{W^+} - m_{W^-}$ (GeV/c$^2$)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}qq$</td>
<td>7.2 ± 19.4</td>
<td>3.57 ± 3.29</td>
<td>2.06 ± 2.22</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\mu\nu^*$</td>
<td>-0.87 ± 3.08</td>
<td>-1.67 ± 1.59</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\ell\nu$</td>
<td>5.3 ± 7.7</td>
<td>-4.38 ± 3.48</td>
<td>-6.66 ± 1.98</td>
<td></td>
</tr>
<tr>
<td>$q\bar{q}\tau\nu^*$</td>
<td>-7.7 ± 6.8</td>
<td>2.38 ± 2.29</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* At $\sqrt{s} = 172$ GeV the three semi-leptonic channels were combined for $\Gamma_{W}$ and $\Delta m_{W^+ - W^-}$ before calibration in order to avoid large statistical fluctuations.

Table 6.8: Results obtained with the 2D Ideogram analysis; with statistical errors only.