Chapter 3

Lifecycle model of educational choice

3.1 Introduction

One may choose from different ways to get educated in a certain job or profession. One possibility is to attend a full-time vocational education and the other way is to choose for a combination of part-time working and part-time vocational education. For some professions both educational tracks exist. We are interested in the determinants of the choice between these two tracks. We have constructed a theoretical model which models the decision process and which might provide insight into the factors determining educational choice.

We start with a person who wants to be schooled in a certain job (or type of job) and has to choose between full-time vocational education (FVE) and dual vocational education (DVE) (combined working and learning) in order to become qualified. We assume that the education choice is determined by the expected net value of lifetime income from each educational type (abbreviated to ELI\textsubscript{j} where the subscript j refers to educational type). The educational type which yields the highest ELI\textsubscript{j} is chosen.

There are some differences between FVE and DVE. The main difference is that students who attend FVE have to spend about 40 hours a week on going to school and studying whereas students who are in the dual track work 3-4 days a week and go to school 1-2 days a week. Study durations may differ between the DVE and the FVE. In FVE students do not go to school all the time, they also have practical periods. Dual students usually are employed and receive a salary during their training period. Full-time students' income consists of scholarships, educational loans, money from relatives and wages earned at sidelines. The curriculum of the full-time education usually contains relatively
many theoretical and general subjects preparing students for a relatively wide range of occupations whereas the curriculum of the DVE is typically smaller, preparing students for a smaller range of jobs. This might make dual students more sensitive to fluctuations in the labor market since it is more difficult for them to switch to another job.

The outline of this chapter is as follows. Section 3.2 introduces the basic model. This model is simple and contains only a few basic variables, like income while in training, starting wage, cumulative reward growth, discount factor and study duration, determining $ELI_j$. The marginal effects of these basic variables on the $ELI_j$'s and on the choice of educational type people make is shown. Section 3.3 extends the basic model. Here we also take the probability to drop out of the education into account. Dropping out make people enter the 'quit-track' at the labor market with its accompanying reward profile over time instead of the 'education type j track' with his specific reward profile. Section 3.4 gives a summary and concludes.

### 3.2 Basic model

We consider two educational tracks which prepare for the same profession. FVE is denoted by type $f$, and dual DVE is denoted by type $d$. An individual chooses the school type $j$ which yields the highest expected net value of lifetime income, denoted by $ELI_j$. We assume, for simplicity, that things like leisure, hours worked and hours studied are irrelevant. It is important to include income received/earned during the training period in the analysis because income during the training period depends on the type of training; income of students from different types of training have different components; dual students only receive salary while full-time students do not. Their income during their training period consist of several components, like scholarships, money from their parents, salary from sidelines and study loans. This is reflected in equation 3.1:

$$ELI_f = \int_0^{T_f} Be^{-\rho t} dt + \int_{T_f}^T R_f(t)e^{-\rho t} dt =$$

$$\frac{B}{\rho} (1 - e^{-\rho T_f}) + \int_{T_f}^T R_f(t)e^{-\rho t} dt$$

and

$$ELI_d = \int_0^T R_d(t)e^{-\rho t} dt$$
Equation 3.1 consists of two parts, namely expected income derived during the training period of education type j and expected income derived during the remaining working period. \( R_j(t) \) denotes the education specific expected reward at time t from education j. The discount factor \( \rho \) makes expected income in the future weight less the farther away in time. The training period of education j starts at \( t = 0 \) and stops after \( \tau_j \) years. So the study durations may differ between the two tracks. T stands for the number of years until the year of retirement. Full-time students study in the period \([0, \tau_f]\). We assume for simplicity that during this period a constant yearly income \( B \) is received\(^1\). They work in the period \([\tau_f, T]\) and we assume that at \( t=\tau_f \) they have a job and earn a starting reward \( R_{0f} = R_f(\tau_f) > 0 \). Dual students work in the period \([0, T]\). At \( t=0 \) the dual students earn a starting wage \( R_{0d} = R_d(0) > 0 \) and is higher than \( B \) would have been.

The level of the rewards changes over time. One receives the starting reward until \( t=t_1 \), the point in time of the first reward change. At \( t = t_2 \) the second reward change takes place. At \( t = t_n \) the last reward change occurs, resulting in the reward \( R_j(t = t_n) \) which one receives until retirement at \( t = T \). This may result in reward profiles over the whole working period as in figure 3.1. We think that the reward profiles presented there are quite realistic: during the training period full-time students receive a relatively low constant income whereas the dual students earn a relatively high income which increases during the training period. For both groups reward rises sharply during the first part of the working life then it starts rising less fast and eventually during the last part of someone's working life it increases very slowly/stays constant until retirement. At a certain point in time \( T^* \) the reward of the full-time students may equal/become higher than the reward of the dual students i.e. \( R_f(T^*) \geq R_d(T^*) \). If \( T^* \) exists the gap between the expected net present value of income generated until \( T^* \) in case of a full-time education (abbreviated to \( \text{ELI}_f(T^*) \)) instead of a dual education ( \( \text{ELI}_d(T^*) \)) starts to decrease. There may exist a point in time \( T^{**} \) where the \( \text{ELI}_f(T^{**}) \) is equal to/becomes higher than the \( \text{ELI}_d(T^{**}) \). If \( T^{**} \) exists the individual chooses full-time education and otherwise (s)he chooses dual education.

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\(^1\)B might be negative if study loans are larger than the other sources of income. Individuals can transfer some of R to the training period \( t<\tau_f \). Under the assumption of perfect capital markets and separability of earning and spending this has no effect on \( \text{ELU}_f \).
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Figure 3.1: Education specific reward profiles
3.2. **BASIC MODEL**

We decompose the function $R_j(t)$ into two parts. $R_j(t)$ is equal to the starting reward $R_{0j}$ times a growth factor $\lambda_j(t)$, i.e. $R_j(t) = R_{0j} \lambda_j(t)$ with $\lambda_j(t) = \frac{R_j(t)}{R_{0j}}$. Now $R_{0j}$ is a constant and $\lambda_j(t)$ is a real discontinuous function. We do this because later on in this chapter we want to analyze the effects of different starting rewards and different growth profiles on educational choice. This changes 3.1 and 3.2 into:

$$ELI_f = \frac{B}{\rho} (1 - e^{-\rho T_j}) + \int_{\tau_f}^{T_f} R_{of} \lambda_f(t) dt$$  (3.3)

$$ELI_d = \int_0^T R_{0d} \lambda_d(t) e^{-\rho t} dt$$  (3.4)

In order to solve the integral in equations 3.3 and 3.4 we use the first mean value theorem of integration which says:

**Theorem 3.1 (the first mean value theorem of integration)**

Let $f$ be a real continuous function on the interval $[a, b]$ and let $g$ be a real integrable function on $[a, b]$ and either $g \geq 0$ or $g \leq 0$. Then there is an $\xi \in (a, b)$ such that

$$\int_a^b f(x)g(x)dx = f(\xi) \int_a^b g(x)dx$$

The proof of this theorem can be found in many basic mathematical analysis text books. We define the following functions $f$ and $g$

$$f(t) = R_{0j} e^{-\rho t}, \text{ for } t \in [\tau_j \leq t \leq T]$$  (3.5)

$$g(t) = \lambda_j(t), \text{ for } t \in [\tau_j \leq t \leq T]$$  (3.6)

It is clear that $f$ is a real continuous function. Now we have to show that $g$ is an integrable function on $[\tau_j, T]$. The function $\lambda_j(t)$ is a real discontinuous function. According to the following theorem (proof see Almering et. al. 1993) $g(t)$ is a Riemann integrable function:
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**Theorem 3.2**

If \( f: [a, b] \rightarrow R \) is bounded and if there are only a finite number of points in \([a, b]\) where \( f \) is not continuous then \( f \) is Riemann integrable on \([a, b]\).

Define \( \tau_{wij} \) as the point in time someone starts working. The function \( \lambda_j(t) \) takes on the following values in the intervals \([\tau_j, T]\):

\[
\lambda_j(t) = \begin{cases} 
1 & \text{if } t \in [\tau_{wij}, t_1) \\
\frac{R_j(t_i)}{R_{0j}} & \text{if } t \in [t_i, t_{i+1}), \ i \in \{1, 2, \ldots, n - 1\} \\
\frac{R_n(t_n)}{R_{0j}} & \text{if } t \in [t_n, T]
\end{cases}
\]

The function \( \lambda_j(t) \) is discontinuous in the points \( t_i \). There are \( n \) such points in \([\tau_{wij}, T]\) so \( n \) is finite. The values \( \lambda_j(t) \) takes on in \([\tau_{wij}, T]\) are finite, since \( R_{0j} \neq 0 \). So we may conclude that \( \lambda_j(t) \) is a Riemann integrable function. Now we have shown that \( g \) is an integrable function on \([\tau_{wij}, T]\) we may apply the first mean value theorem on integration on equation 3.3 and with \( f \) and \( g \) defined as in equations 3.5 and 3.6:

Applying Theorem 3.1 and 3.2 gives the following expressions for the expected net present value of lifetime income of the two types of education:

\[
ELI_f = \int_0^{\tau_f} R_f(t) e^{-\rho t} dt + \int_{\tau_f}^{T} R_f(t) e^{-\rho t} dt
\]

\[
= \int_0^{\tau_f} Be^{-\rho t} dt + e^{-\rho \xi_f} R_{0f} \lambda_f^* \\
= \frac{B}{\rho} (1 - e^{-\rho \xi_f}) + e^{-\rho \xi_f} R_{0f} \lambda_f^*
\]

with \( \xi_f \in (\tau_f, T) \)

\[
ELI_d = \int_0^{\tau_d} R_d(t) e^{-\rho t} dt + \int_{\tau_d}^{T} R_d(t) e^{-\rho t} dt
\]

\[
= e^{-\rho \xi_d} R_{0d} \lambda_d^*
\]

with \( \xi_d \in (0, T) \)
Here $\lambda_j^*$ is the solution of $\int_{t_j}^{T} \lambda_j(t)dt$ and reflects the cumulative reward growth (CRG$_j$) over the working period. Note that we have not made any assumptions about the reward profile over time. By using CRG$_j$ this is not necessary. Now we have got a very simple expression for the ELI$_j$. With equation 3.7 and 3.8 in hand we can derive the marginal effects of the discount factor $\rho$, the start reward $R_{0j}$, the cumulative growth $\lambda_j^*$ and study duration on ELI$_j$. Below the derivatives of the basic variables on the ELI$_j$'s are shown:

\[
\frac{\partial \text{ELI}_d}{\partial \rho} = -\xi_d e^{-\rho \xi} R_{0d} \lambda_d^* < 0
\]

\[
\frac{\partial \text{ELI}_f}{\partial \rho} = -\frac{B}{\rho^2} \left[1 - e^{-\rho \tau_f}\right] + \frac{B \tau_f}{\rho} e^{-\rho \tau_f} - \xi_f e^{-\rho \xi} R_{0f} \lambda_f^* < 0
\]

\[
\frac{\partial \text{ELI}_d}{\partial \tau_d} = e^{-\rho \xi} R_{0d} \frac{\partial \lambda_d^*}{\partial \tau_d}
\]

\[
\frac{\partial \text{ELI}_f}{\partial \tau_f} = \frac{\partial e^{-\rho \xi}}{\partial \tau_f} R_{0f} \lambda_f^* + e^{-\rho \xi} \left(\lambda_f^* \frac{\partial R_{0f}}{\partial \tau_f} + R_{0f} \frac{\partial \lambda_f^*}{\partial \tau_f}\right)
\]

\[
\frac{\partial \text{ELI}_j}{\partial B} = \frac{1}{\rho} (1 - e^{-\rho \tau_f}) > 0
\]

\[
\frac{\partial \text{ELI}_j}{\partial R_{0j}} = e^{-\rho \xi} \lambda_j^* > 0, \ j = d, f
\]

\[
\frac{\partial \text{ELI}_j}{\partial \lambda_j^*} = e^{-\rho \xi} R_{0j} > 0, \ j = d, f
\]

The results are as expected. The marginal effect of the discount factor $\rho$ on ELI$_j$ is negative for both dual and full-time education. For the latter this is not clear immediately. However, we have simulated the value of the expression $e^{-\rho \tau_f} (\rho \tau_f - 1) - 1$ using reasonable values of $\rho$ ($\rho \in (0, 0.20)$) and $\tau_f$ ($\tau_f \in [3, 8]$). The simulated values of $e^{-\rho \tau_f} (\rho \tau_f - 1) - 1$ were all negative and because $-\xi_f e^{-\rho \xi} R_{0f} \lambda_f^*$ is also negative the derivative of $\rho$ on ELI$_f$ stays negative. If we link this finding with the choice between FVE and DVE this may indicate that people with a high discount factor choose for dual training because of the relatively high earnings during the training period compared to the income during the full-time training period.

In order to say something about the marginal effect of study duration on lifetime earnings we make use of theorem 3.3 (see appendix) which says that in case of a real continuous decreasing function $f$ on an interval $[a,b]$ an increase in the lower bound $a$ in the interval $[a,b]$ increases the value of $\xi$ which makes the
following hold: $ \int_a^b f(x)g(x)dx = f(\xi) \int_a^b g(x)dx, \xi \in (a,b)$. It is not clear what the sign of the marginal effect of study duration on lifetime earnings will be, because a longer study duration not only decreases the length of the working life (after the training period) but, according to human capital theory, it may also increase someone's human capital. The shorter working life decreases $\text{ELI}_f$ according to theorem 3.3. This effect is not present in $\text{ELI}_d$ because $\tau_{wd} = 0$. Increases in human capital may result in higher starting rewards for the full-timers (not for the dual students because they start working at $\tau_{wd} = 0$) and a change in CRG. The sign of the change in CRG is not clear because of two opposite effects; the shorter working life (for dual students a shorter working life in which they are fully qualified) leads to a decrease in CRG, but an increase in human capital may lead to better job opportunities/higher productivity which increase CRG.

The marginal effect of the yearly income $B$ during the school period of full-time students has a positive effect on lifetime income; this is as expected. The starting reward $\text{RO}_f$ has a positive effect on $\text{ELI}_f$. This is just what we expected; a high starting wage results in a higher $\text{ELI}_f$. Furthermore we see that the marginal effect of CRG on $\text{ELI}_f$ is positive. If you make career your lifetime income will increase.

Educational type $j$ is chosen when it yields the highest $\text{ELI}$ of the two educational types. If we denote the difference in lifetime income by choosing FVE instead of DVE by $I$ we get:

$$ I = \text{ELI}_f - \text{ELI}_d = \frac{B}{\rho} (1 - e^{-\rho \tau_f}) + e^{-\rho \xi_f} \text{RO}_f \lambda_f^* - e^{-\rho \xi_d} \text{RO}_d \lambda_d^* $$

If $I$ is positive (negative) then FVE (DVE) is chosen because it yields the highest expected net present value of lifetime income. The marginal effects of the different components of equation 3.9 on $I$ are quite obvious. More interesting may be the effect of the difference in the basic variables on $I$. Assume

$$ \text{RO}_d = \varepsilon_r \text{RO}_f, \varepsilon_r > 0 $$
$$ \lambda_d^* = \varepsilon_{\lambda} \lambda_f^*, \varepsilon_{\lambda} > 0 $$

Here the $\varepsilon_k$'s reflect the relative magnitude of the dual starting reward and the dual CRG compared to their full-time counterparts$^2$. These $\varepsilon_k$'s are individual

$^2$We have decided not to investigate the effect of different study durations because we knew beforehand that the sign of the effect would be unclear, just as with the marginal effect of study duration on the $\text{ELI}_j$'s.
3.3 Including graduation probabilities

3.3.1 Including graduation probabilities

Students who start an education may drop out after some time. The graduation probability is an important variable. In the Netherlands the graduation probabilities in e.g. higher education (which is full-time education) lie around 60-70% (see e.g. De Jong & Meijer, 1990). This shows that the probability that people end up in the drop-out track is considerable. With ordinary probit models you can see which factors are important in graduating or not and whether there are differences in the effects of variables on the graduation probability of the two different education types. Cognitive ability may play a more important role (in magnitude and maybe also in significance) in the full-time education than in the DVE graduation, where practical and social skills may be more important than in full-time education. These quitters have another life time income than those who graduate. This is because they can not practise the job for which they went to school whereas the graduates can. So the quitters face a starting reward $R_{0q}$ and a CRG $\lambda^*_q$ which differ from their graduate counterparts.

Full-time students, respectively dual students may pass with probability $P_f$, respectively $P_d$. We assume that people choosing between dual and full-time education take these graduate probabilities into account when they maximize their ELI.

The ELI of the working period in case of quitting equals

$$I = ELI_f - ELI_d =$$

$$\frac{B}{\rho} (1 - e^{-\rho T_f}) + e^{-\rho T_f} R_{0f} \lambda^*_f - e^{-\rho T_d} e^{\varepsilon_r \varepsilon_\lambda} R_{0f} \lambda^*_f$$

Differentiating $I$ with respect to the $\varepsilon_k$'s shows that the marginal derivatives of the relative magnitudes $\varepsilon_r$ and $\varepsilon_\lambda$ on $I$ are negative

$$\frac{\partial I}{\partial \varepsilon_r} = -\varepsilon_\lambda e^{-\rho T_d} R_{0f} \lambda^*_f < 0$$

$$\frac{\partial I}{\partial \varepsilon_\lambda} = -\varepsilon_r e^{-\rho T_d} R_{0f} \lambda^*_f < 0$$

If the dual starting wage rises compared to the starting wage of full-time students it becomes more attractive to choose the DVE and the same holds for the cumulative wage growth. This is caused by the increase of $ELI_d$ relative to $ELI_f$. 

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Full-time students, respectively dual students may pass with probability $P_f$, respectively $P_d$. We assume that people choosing between dual and full-time education take these graduate probabilities into account when they maximize their ELI.

The ELI of the working period in case of quitting equals
where $E(\tau_{jq})$ stands for the expected time the quitter has spend in education. This expectation lies somewhere between the start of education and the time someone would have needed to graduate. With equation 3.13 and the graduate probabilities $P_f$ and $P_d$ in hand the general formula for the ELI$_j$'s are now

$$ELI_j = \int_0^{E(\tau_{jq})} R_j(t)e^{-\rho t} dt + P_j \int_{E(\tau_{jq})}^T R_j(t)e^{-\rho t} dt + \int_{E(\tau_{jq})} J E(T_{jq})) f T (1 - P_j) / R_{jq} e^{-\rho t} dt$$

We want to concentrate on the effect of the graduation probabilities and the ELI$_{jq}$ of quitters on the ELI$_j$ of educational type $j$. By plugging equation 3.13 into equation 3.14 we get the following education specific ELI$_j$'s:

$$ELI_j = P_f \left( \frac{B}{\rho} (1 - e^{-\rho T_f}) + R_{0f} \lambda_f^* e^{-\rho \xi_f} \right)$$

$$+ (1 - P_f) \left( \frac{B}{\rho} (1 - e^{-\rho E(\tau_{fq})}) + R_{0fq} \lambda_{fq}^* e^{-\rho \xi_{fq}} \right)$$

with $\xi_f \in (\tau_f, T)$ and $\xi_{fq} \in (E(\tau_{fq}), T)$

$$ELI_d = P_d \left( e^{-\rho \xi_d} R_{0d} \lambda_d^* \right) +$$

$$\left( 1 - P_d \right) \left( \int_{E(\tau_{dq})}^{E(\tau_{dq})} R_{dq}(t)e^{-\rho t} dt + \int_{E(\tau_{dq})}^T R_{dq}(t)e^{-\rho t} dt \right)$$

$$= P_d \left( e^{-\rho \xi_d} R_{0d} \lambda_d^* \right) + \left( 1 - P_d \right) \left( 1 - e^{-\rho E(\tau_{dq})} \right) R_{0d} \lambda_d^{**} + R_{0dq} \lambda_{dq}^* e^{-\rho \xi_{dq}}$$

with $\xi_d \in (0, T)$ and $\xi_{dq} \in (E(\tau_{dq}), T)$

The terms in brackets after the graduation probabilities $P_d$ and $P_f$ are the same as before. New are the terms in brackets after the quit probabilities. The first
expression reflects expected income during the training period until one quits and the second expression reflects expected income during the working period in case of quitting. The term $\lambda_d^{**}$ is the resulting CRG of quitters during their training period when using theorem 3.2 on equation 3.16.

We want to concentrate on the effect of the differences in the variables on educational choice just as in equation 3.8. We do that for the new variables in our model i.e. the graduation probabilities, the start reward of the quitters and the CRG of the quitters. Therefore, we define:

\[
P_d = \varepsilon_p P_f, \varepsilon_p > 0
\]

\[
R_{0dq} = \varepsilon_{rq} R_{0fq}, \varepsilon_{rq} > 0
\]

\[
\lambda_{dq}^* = \varepsilon_{\lambda q} \lambda_{fq}^*, \varepsilon_{\lambda q} > 0
\]

and put this in the following expression for $I$:

\[
I = ELI_f - ELI_d =
\]

\[
P_f \left( \frac{B}{\rho} \left( 1 - e^{-\rho \tau_f} \right) + R_{0f} \lambda_f^* e^{-\rho \xi_f} \right) +
(1 - P_f) \left( \frac{B}{\rho} \left( 1 - e^{-\rho \tau_{f(q)}} \right) + R_{0f} \lambda_f^* e^{-\rho \xi_{f(q)}} \right) -
\varepsilon_p P_f e^{-\rho \xi_d} R_{0d} \lambda_d^*

- (1 - \varepsilon_p P_f) \left( (1 - e^{-\rho E(\tau_{dq})}) R_{0d} \lambda_d^{**} + \varepsilon_{rq} R_{0fq} \varepsilon_{\lambda q} \lambda_{fq}^* e^{-\rho \xi_{dq}} \right)
\]

with $\xi_f \in (\tau_f, T)$, $\xi_{f(q)} \in (E(\tau_{f(q)}), T)$, $\xi_d \in (0, T)$ and $\xi_{dq} \in (E(\tau_{dq}), T)$

The derivatives of these new variables on $I$ are given below. The sign of the derivative of $\varepsilon_p$ on $I$ depends, loosely speaking, on whether the expected lifetime income from quitting exceeds the expected lifetime income of graduating in education type d; if expected lifetime income of quitting the DVE is higher (lower) than completing it the sign will be negative (positive). If the probability of graduating from DVE increases relative to the probability of completing full-time education then the relative value of choosing FVE instead of DVE will decrease (increase) if completing the DVE leads to a higher (lower) expected lifetime income than quitting the DVE. It is likely that completing the DVE results in a higher expected lifetime income than dropping out of DVE. So we expect $\partial I/\partial \varepsilon_p$ to be positive.
\[ \frac{\partial I}{\partial \varepsilon_p} = P_f (-e^{-\rho \xi_d} R_{0d} \lambda_d^* + (1 - e^{-\rho E(\tau_{dq})}) R_{0d} \lambda_d^{**} + \varepsilon_{rq} R_{0fq} \lambda_f q e^{-\rho \xi_d q}) \]

\[ \frac{\partial I}{\partial \varepsilon_{rq}} = -(1 - \varepsilon_p P_f) e^{-\rho \xi_d q} \varepsilon_{rq} \lambda_f q R_{0fq} \lambda_f^* < 0 \]

\[ \frac{\partial I}{\partial \varepsilon_{\lambda q}} = -(1 - \varepsilon_p P_f) e^{-\rho \xi_d q} \varepsilon_{rq} R_{0fq} \lambda_f^* < 0 \]

The effects of increases in \( \varepsilon_{rq} \) and \( \varepsilon_{\lambda q} \) are clear; they increase the relative income from dual education to the income from full-time education. This is as expected, because increases in \( \varepsilon_{rq} \) and \( \varepsilon_{\lambda q} \) just mean that the relative starting reward and the relative cumulative wage growth of quitters from DVE have increased compared to those from the full-time educated dual quitters.

### 3.4 Summary and concluding remarks

In this chapter a theoretical model is presented which models the educational choice of individuals; people can choose between two types of vocational education, namely full-time vocational education and dual vocational education. The individual's choice is based on choosing the school type which generates the highest expected net present value of lifetime income (ELI\(_j\)).

We have developed a simple model in which the ELI\(_j\) is the sum of the expected net present value of lifetime income generated by this education during the training period and the expected net present value of lifetime income generated during the remaining working life of the individual. A nice feature of our model is that no assumptions are made about the reward profile over time. The ELI\(_j\) of an education depends on four key variables, namely discount factor, study duration, education specific start reward and growth capacity CRG. We have shown that starting reward and growth capacity increase ELI\(_j\) and that an increasing discount factor decreases ELI\(_j\). The effect of study duration on ELI\(_j\) is unclear because this variable not only influences ELI\(_j\) directly through \( e^{-\rho \xi_j} \) but it is also likely to influence the starting reward and CRG indirectly and determining the sign of the overall effect of study duration on ELI\(_j\) is not possible.

In order to be able to predict the variable \( I \) is defined denoting the difference in expected net present value of lifetime income by choosing FVE instead of DVE. We have assumed that differences in start reward and CRG between the two education types stem from the personal characteristics of the individual; if
the dual starting reward increases relative to the full-time starting reward the DVE becomes more attractive. The same holds for the CRG.

In an extension of the basic model we have incorporated the possibility that someone drops out of the education and earns the 'quit start reward' and faces the 'quit cumulative growth reward'. These variables are still education specific. Just as in the basic model we have ascribed differences in the graduate probabilities, the quit start rewards and the quit cumulative growth reward to individual characteristics. If by a change in the personal characteristics the relative quit starting reward of the DVE increases the DVE becomes more attractive relative to the FVE. The same holds for the quit CRG. The sign of the effect of a relative increase of the graduate probability in DVE on the relative attractiveness of FVE versus DVE is undetermined; it depends on whether expected lifetime income in case of quitting is higher or lower than the expected lifetime income in case of graduating in dual; DVE becomes relatively more attractive if expected lifetime income in case of completing it is higher than dropping out of it.
3.5 Appendix to chapter 3

Theorem 3.3

Let \( f : [a, b] \rightarrow \mathbb{R} \) be a real continuous strictly decreasing function, \( a, b \geq 0 \), \( a > b \) and let \( g \) be a real integrable function on \( [a, b] \) and, \( g \geq 0 \) or \( g \leq 0 \), then there exist \( \xi' \) and \( \xi \) with \( \xi' > \xi \) such that

\[
\int_a^b f(x)g(x)dx = f(\xi) \int_a^b g(x)dx, \quad \xi \in (a, b) \tag{a 3.1}
\]

\[
\int_{a+\Delta}^b f(x)g(x)dx = f(\xi') \int_{a+\Delta}^b g(x)dx, \quad \xi' \in (a + \Delta, b) \text{ and } \Delta > 0 \tag{a 3.2}
\]

with \( \int_a^b g(x)dx > \int_{a+\Delta}^b g(x)dx > 0 \)

Proof

The left-hand side of equation (a 3.1) can be split up in two parts for any \( 0 < \Delta < b - a \)

\[
\int_a^b f(x)g(x)dx = \int_a^{a+\Delta} f(x)g(x)dx + \int_{a+\Delta}^b f(x)g(x)dx \tag{a 3.3}
\]

and according to the first mean value theorem there exist \( \xi \in (a, b), \xi' \in (a, a + \Delta), \xi''(a + \Delta, b) \) such that

\[
f(\xi) \int_a^b g(x)dx = f(\xi') \int_a^{a+\Delta} g(x)dx + f(\xi'') \int_{a+\Delta}^b g(x)dx \tag{a 3.4}
\]

and splitting up \( \int_a^b g(x)dx = \int_a^{a+\Delta} g(x)dx + \int_{a+\Delta}^b g(x)dx \) and rearranging things a bit gives

\[
(f(\xi) - f(\xi')) \int_a^{a+\Delta} g(x)dx = (f(\xi'') - f(\xi)) \int_{a+\Delta}^b g(x)dx \tag{a 3.5}
\]

and subsequently

\[
\frac{f(\xi) - f(\xi')}{f(\xi'') - f(\xi)} = \frac{\int_{a+\Delta}^b g(x)dx}{\int_a^{a+\Delta} g(x)dx} > 0 \tag{a 3.6}
\]
According to equation a 3.6 the following must hold:

I) \( f(\xi) - f(\xi') > 0 \) and \( f(\xi'') - f(\xi) > 0 \) or

II) \( f(\xi) - f(\xi') < 0 \) and \( f(\xi'') - f(\xi) < 0 \)

I is not true because \( f(\xi) - f(\xi') > 0 \) implies (note that \( f \) is a decreasing function) \( \xi < \xi' \), and \( f(\xi'') - f(\xi) > 0 \) implies \( \xi'' < \xi \). Combining this gives \( \xi'' < \xi < \xi' \) which is not true because \( \xi' < \xi'' \).

II is true and proofs our theorem; \( f(\xi) - f(\xi') < 0 \) implying \( \xi > \xi' \) and \( f(\xi'') - f(\xi) < 0 \) implying \( \xi'' > \xi \). This results in \( \xi' < \xi < \xi'' \) which proofs that \( \xi < \xi'' \).
CHAPTER 3. LIFE CYCLE MODEL: ECONOMIC BEHAVIOR

3.5. Appendix to Chapter 3

Proof

The left-hand side of equation (3.1) may be split up in two parts for any 
\( 0 < \Delta < b - a \)

\[
\int_{a}^{b} f(x)g(x)dx = \int_{a}^{a+\Delta} f(x)g(x)dx + \int_{a+\Delta}^{b} f(x)g(x)dx
\]

and according to the first mean value theorem there exist \( \xi \in (a, a+\Delta), \xi' \in (a+\Delta, b) \) such that

\[
f(\xi) \int_{a}^{a+\Delta} g(x)dx = f(\xi') \int_{a+\Delta}^{b} g(x)dx
\]

and splitting up \( \int_{a+\Delta}^{b} g(x)dx \) and rearranging things a bit gives

\[
(f(\xi) - f(\xi')) \int_{a+\Delta}^{b} g(x)dx = f(\xi) \int_{a+\Delta}^{b} g(x)dx - f(\xi') \int_{a+\Delta}^{b} g(x)dx
\]

and subsequently

\[
\frac{f(\xi) - f(\xi')}{f(\xi')} = \frac{\int_{a+\Delta}^{b} g(x)dx}{\int_{a+\Delta}^{b} g(x)dx} = \gamma(1)
\]