Chapter 6

Self-Organized Critical Behavior in Time Warp

We call things we don't understand complex, but that means we haven't found a good way of thinking about them.

—Tsutomu Shimomura

6.1 Self-Organized Criticality

Spatially extended dynamical systems, that is, systems with both temporal and spatial degrees of freedom, are common in physics, biology, and economics. The spatiotemporal behavior of these dynamic complex systems has been studied extensively, but there is still little understanding. In particular, two phenomena require some unifying underlying explanation, namely the temporal effect known as $1/f$ noise, and the emergence of spatial structures with scale-invariant, self-similar (fractal) properties.

Most of the time, equilibrium systems with short-range interactions, exhibit exponentially decaying correlations. Infinite correlations, i.e., scale invariance, can be achieved by fine-tuning some parameters (e.g., temperature) to a critical value. An example of such a system is the Ising spin model presented in Chapter 5.

Besides systems exhibiting critical behavior, a large class of non-equilibrium locally interacting, nonlinear systems spontaneously develop scale invariance. Such composite systems with many interacting degrees of freedom may evolve to a critical state in which minor events may trigger a chain reaction that can affect an arbitrarily large number of constituents of the system. This state is called self-organized criticality (SOC) (Bak et al. 1988). The probability of spontaneously generated structures or events, further called avalanches, of many different sizes $s$ show a power-law distribution

$$P(s) \sim s^{-\tau},$$

where $\tau$ is a critical exponent and most other observables of the system have no intrinsic time or length scale. This implies that we expect to observe scale
invariance, or power-law scaling, in the system. The absence of intrinsic length scale is attributed to SOC, where avalanches of all sizes contribute to keep the system perpetually in a critical state. This critical state is robust with respect to any small change in the rules of the system. The size of an avalanche can be defined in different ways. It can be measured by the number of relaxation steps needed for the chain reaction to stop or the total number of sites involved in the avalanche.

Many naturally occurring systems exhibit this kind of scaling- or self-similar behavior, examples are earthquakes, stock markets, and ecosystems (Turcotte 1999). The concept of SOC is developed to explain the behavior of such systems. Self-organized critical behavior was first investigated for sandpile models (Bak et al. 1988). In this cellular automata model, a particle is dropped onto a randomly selected lattice point. When a lattice point accumulates four particles, they are redistributed to the four adjacent lattice points, or in case of edge lattice points they are lost from the grid. Redistributions can lead to further instabilities and avalanches of particles in which many particles may be lost from the edges of the lattice. The average number of particles per lattice point is the density that fluctuates about a quasi-equilibrium value. One measure of the avalanche size is given by the number of particles lost by the lattice during a sequence of redistributions, or alternatively can be given by the number of lattice points that participate in the redistribution. Figure 6.1 shows the distribution of the avalanche sizes, including the power-law fitted to the distribution.

![Avalanche size distribution](image)

Figure 6.1: Avalanche size distribution in the sandpile model on a $100 \times 100$ lattice. The exponent of the fitted line is $-1.06$.

In this chapter a highly speculative conjecture is made, that the Time Warp dynamics can be characterized by self-organized criticality. The spatiotemporal behavior of the Time Warp method is investigated on the basis of the Ising spin model. The influence of temperature and various finite-size scaling effects such as lattice size, number of processors, and virtual time window size are
studied. If the conjecture is true, yet another hint is given that also problems in the field of parallel computing display behavior as is found in other complex systems (Macready et al. 1996; Schoneveld et al. 1997; Yuan et al. 2000).

6.2 Self-Organized Criticality in Time Warp Dynamics

6.2.1 Slowly Driven, Interaction-Dominated Threshold Systems

Self-organized critical behavior is found in slowly driven, interaction-dominated threshold systems (SDIDTs); if an SDIDT system exhibits power-laws without any apparent tuning then it is said to exhibit self-organized criticality (Jensen 1998). Interesting behavior arises because many degrees of freedom are interacting. In addition, the dynamics of the system must be dominated by the mutual interaction between these degrees of freedom, rather than by the intrinsic dynamics of the individual degrees of freedom.

An important characteristic of systems exhibiting SOC behavior is a separation of time scales. As stated before, it is required that such systems are slowly driven, that is perturbations occur on a much larger time scale than the diffusion or relaxation dynamics. The critical state in SOC systems is furthermore characterized by a stationary state where the driving forces balance the cascades. For example in the sandpile model, adding sand causes on the one hand the pile to grow, on the other hand avalanches. The dynamically stationary state is obtained at the "critical point" where these two effects exactly balance.

A highly speculative analogy could be made with Time Warp: adding events causes on the one hand the event rate to grow, on the other hand rollbacks to occur. Our experiments show that in Time Warp, the event rate eventually reaches a kind of stationary state with superimposed rollback cascade effects (see Fig. 6.2). We define two time scales in Time Warp: simulation time and protocol time. The simulation time in Time Warp is updated by the rate at which the system is driven. This driving rate is determined by the dynamics of the simulation. The protocol time, needed to process a rollback, is determined by machine specific parameters. A difference between conventional SOC systems and Time Warp is that there is no explicit separation of time scales: asynchronous updates and rollbacks may intervene. Also, rollbacks take place on separate processors instead of an entire lattice of sites.

The Ising spin model, presented in Section 5.3.1, is a critical system, that is for a certain parameter regime the system exhibits scale invariant structures. This makes the Ising spin system an ideal simulation model to study the influence of spatial correlation on the dynamical behavior of the Time Warp protocol. The many degrees of freedom that are interacting show emergent behavior, e.g., magnetization, as the temperature of the Ising spin system approaches $T_c^+$ (i.e., approach $T_c$ form high temperature $T$). This emergent behavior is the re-
To understand when “relaxation” in Time Warp occurs, we first have to explain how events are processed and can give rise to rollbacks in the Ising spin model. Events in the Ising spin model are attempts to flip a spin in the lattice. An attempted spin flip is accepted according to the Boltzmann probability distribution. Hence, not every event results in a spin flip, or equivalently in a state change. The parallel Ising spin simulation exploits the inherent parallelism by spatial domain decomposition. Each logical process in the parallel simulation represents a subdomain of the spin lattice. Events on the subdomain boundaries that result in a spin flip are communicated with the neighboring subdomain, i.e., logical process, by way of an event message.

The so-called relaxation in Time Warp occurs whenever the following three conditions are satisfied (threshold):

- if accepted event and;
- boundary event and;
- $\exists i \in \text{neighborhood}(local) : LVT_{\text{local}} < LVT_i$;

where $LVT_{\text{local}}$ is the simulation time on the local processor and $LVT_i$ is the simulation time on a neighboring processor $i$. For Ising spin simulations, simulation time and protocol time separate at low temperatures, when there are not that many spin flips, i.e., when the acceptance ratio of the Metropolis algorithm is low. For high temperatures many spin flips are accepted, and updates and rollbacks occur at comparable time scales.
6.2.2 Physical and Computational Critical Behavior

It is very important to note that, in fact, we are confronted with two kinds of critical behavior. The critical behavior of the first kind is a result of the Ising spin phase transition at the critical temperature \( T_c \). At the Ising spin phase transition, long-range spin correlations occur, that might influence the Time Warp dynamics. We call this critical behavior of the first kind the physical critical behavior. The tendency to be correlated can be measured using the correlation function

\[ C(r) = \langle s_0 s_r \rangle, \]

where \( s_r \) is a spin that is located \( r \) lattice sites away from \( s_0 \). Some results for the correlation function are shown in Fig. 6.3, which shows \( C(r) \) at several different temperatures. The important feature in the figure is not the average value of \( C(r) \), but rather the amount that \( C(r) \) increases above this average value as \( r \) becomes small. At low temperatures, \( C(r) \) increases slightly at short distances, however the enhancement is very small. The correlation function at \( T = T_c \approx 2.27 \) differs from the low temperature behavior in two ways. First, the relative alignment at short distances is much larger than the value at large \( r \). Second, the correlations are now long-range as \( C(r) \) approaches the \( r \to \infty \) limit very slowly as \( r \) is increased. As the temperature is increased further to temperatures above \( T_c \), the correlations become smaller in magnitude and again extend over only a few lattice spacings.

![Figure 6.3: Ising spin correlation functions at several temperatures.](image)

The critical behavior of the second kind is inferred from the dynamical behavior of the Time Warp protocol. We expect that in the low temperature Ising regime the Time Warp dynamics reaches a self-organized critical state, which we call computational critical behavior.
The average rollback length and rollback length distribution is studied at different temperatures in order to determine the influence of the Ising spin phase transition on the Time Warp protocol. It is expected that around the Ising spin phase transition, the long-range spin correlations increase the average rollback sizes. It is well known that these long-range correlations result in moving islands of actively flipping spins, located in a sea of inactive spins. This separation of activity can trigger very large rollbacks whenever an active island moves over a processor boundary.

We are interested in rollback length distributions in order to do a first order check of SOC in Time Warp dynamics. Remember that for SOC systems it is well known that many observables scale as power-laws.

### 6.3 A First Indication of Self-Organized Criticality in Time Warp

A series of experiments are executed using the APSIS parallel simulation environment (Chapter 3) on the Distributed ASCI Supercomputer (DAS), see Section 5.5 for a description of the DAS supercomputer. The Ising spin simulation model used in the experiments is described in Section 5.4. We experiment with two different grid decompositions for the parallel simulation of the Ising dynamics on a \( L \times L \) square lattice: a one-dimensional “slice” decomposition and a two-dimensional “box” decomposition. Both decompositions are constructed to assure optimal load balance. For all parameter instances of the simulation experiments we measure the average rollback length and rollback length distributions.

For the first series of experiments in this section we have fixed the lattice size to \( L = 220 \), the number of processors to \( P = 12 \), and the virtual time window (VTW) to 3000, using a “sliced” 1D decomposition. In Fig. 6.4 the average rollback lengths are shown for the temperature range [0.1–2.7]. Ideally, rollback avalanches are measured instantaneously over the system, that is over all processors used by the parallel simulation. However, instantaneous measurement requires freezing the forward simulation, and allowing only rollbacks to occur. This would fundamentally alter the dynamics of the Time Warp protocol, and is therefore unacceptably intrusive. Hence, each processor records the local rollbacks for analysis. The rollback lengths are averaged over time for all processors. The results of three different runs are depicted in the figure. Close to the Ising phase transition (\( T_\text{c} \approx 2.27 \) for infinite lattices), the expected peak in the average rollback length can be observed. From this figure, three different regimes can be identified: the **physical sub-critical phase** (\(< T_\text{c} \)) , the **physical critical phase** (\( \approx T_\text{c} \)) and the **physical super-critical phase** (\( > T_\text{c} \)).

The different phases influence the rollback length distributions. In the physical sub-critical temperature regime [0.1–1.4], power-law scaling is found (see Fig. 6.5), i.e., the Time Warp dynamics appear to be in a *computational critical regime*. As the temperature approaches the physical super-critical regime
a transition to exponential scaling can be observed (see Fig. 6.6). Close to the critical temperature, length distributions with “fat tails” (power-law distributions with exponential cutoff) develop due to the emergence of long-range spin correlations.

The scaling exponent $\alpha$ in the physical sub-critical regime seems to be uni-

Figure 6.4: Average rollback length for different temperatures. For each temperature, the results of 3 experiments are shown. Using the simulation parameters $L = 220$, $P = 12$, and $VTW = 3000$, and a 1D decomposition.

Figure 6.5: Rollback distribution for temperatures in the range 0.1–1.5, fitted exponent has value $-1.21 \pm 0.01$. Using the parameters $L = 220$, $P = 12$, and $VTW = 3000$, and a 1D decomposition.
Figure 6.6: Rollback distribution for temperatures in the range 1.5–2.7. Using the parameters \( L = 220, P = 12, \) and \( VTW = 3000, \) and a 1D decomposition.

versal for all temperatures in this regime. From the experimental data a power-law with exponent \( \alpha = -1.21 \) is fitted (linear fit of the logarithmic values). Because the rollback length distribution obeys power-law scaling, we conjecture that the rollback dynamics are in a SOC regime.

The fact that the distributions in Fig. 6.5 begin to deviate from a power-law

Figure 6.7: Average rollback length for different temperatures. For each temperature, the results of 3 experiments are shown. Using the parameters \( L = 220, P = 12, \) and \( VTW = 3000, \) and a 2D decomposition.
at large rollback lengths is a finite size effect, which is further investigated in Section 6.4. At small rollback lengths, the curve deviates from a straight line because discreteness effects of the rollback lengths come into play. As the rollback cascades approach the size of what is assumed to be the size of a real-world system's component particle, in our study an event, it becomes impossible for the fractal pattern to repeat at this scale (Bak et al. 1988; Brunk 2000; Frette et al. 1996).

For the 2D decomposition we repeat the same set of experiments as for the 1D case, again with $L = 220$ and $P = 12$. As for the 1D case, we observe a peak in the average rollback lengths around $T_c$ (see Fig. 6.7). Again a transition from power-law scaling to exponential scaling is observed (see Figs. 6.8 and 6.9). In the physical sub-critical regime we find $\alpha = -1.25$, slightly larger than in the 1D case. This tendency is in accordance with the expectation that slightly shorter distances between processor partitions enable a faster propagation of cascading rollbacks.

In the next series of experiments, different parameters are varied. Note that two different processes intervene: the Ising simulation process and the Time Warp process. An important parameter for both processes is the lattice size. Due to finite size effects, increasing the lattice sizes causes the Ising spin phase transition point $T_c$ to shift. For the Time Warp process, the probability to select a boundary cell decreases for increasing lattice sizes. If the number of processors is increased and the lattice size is kept fixed, the probability to select a boundary cell increases. Therefore we experiment with different numbers of processors. The virtual time window is a very important parameter that determines the maximum length of the rollbacks. This parameter is studied in

![Figure 6.8](image-url)

Figure 6.8: Rollback distribution for temperatures in the range 0.1–1.5, fitted exponent has value $-1.25 \pm 0.02$. Using the parameters $L = 220$, $P = 12$, and $VTW = 3000$, and a 2D decomposition.
the last series of experiments.

### 6.4 Finite-Size Scaling Effects

#### 6.4.1 Influence of lattice size

Because the peak in the average rollback appears close to $T_c$, we expect that this peak is related to the long-range correlations of the Ising phase transition. To support this hypothesis we have conducted a number of experiments with increasing lattice sizes in order to study the presence of finite size effects. For finite lattices, the critical temperature $T_c$ shifts with increasing lattice sizes.

Due to limited computer and time resources we did not extract any critical exponents from the generated data; a more detailed study of this phenomenon is therefore necessary. To simulate an Ising spin system on one specific $T \approx T_c$, more than 2 days (see Fig. 6.11) of computing time on 12 Pentium II (at 200 MHz) nodes is needed; this gives an indication on the total amount of computer time required to run these experiments. Figure 6.10 shows the results for varying lattice size experiments (using $L = \{110, 220, 440, 880\}$ and $P = 12$). A shift towards $T_c$ is observed for increasing lattice sizes.

Furthermore we find that for all lattice sizes in the SOC regime $\alpha \approx -1.21$ for 1D decompositions and $\alpha \approx -1.25$ for 2D decompositions. For 2D decomposition the rollback distributions are shown for $T = 1.0$ in Fig. 6.12.
6.4 Finite-Size Scaling Effects

Figure 6.10: Average rollback lengths for $L = \{110, 220, 440, 880\}$ for varying $T$ using the parameters $P = 12$, $VTW = 3000$, and a 2D decomposition.

Figure 6.11: Run times for $L = \{110, 220, 440, 880\}$ for varying $T$ using the parameters $P = 12$, $VTW = 3000$, and a 2D decomposition.

6.4.2 Varying the Number of Processors

To study the influence of the number of processors on the rollback length distribution in the SOC or computational critical regime, a series of experiments with $P = \{4, 8, 12, 24\}$ using a 2D decomposition has been performed. The lattice size has been fixed to $L = 220$. The rollback distributions of Ising spin simulations at $T = 1.0$ are shown in Fig. 6.13. Similar results are seen for
other temperatures $T$ in the SOC regime. The results indicate that for increasing $P$ the rollback length distributions converge. Again, the scaling exponent $\alpha$ is not influenced by increasing $P$.

The average rollback lengths for different processors in the range $T = [0.1 - 2.7]$ are shown in Fig. 6.15. For 8, 12 and 24 processors, again, a peak around $T_c$ can be distinguished. For $P = 4$, this peak is not present. Apparently, the critical Ising spin dynamics does not reduce the performance of the Time Warp
6.4 Finite-Size Scaling Effects

Figure 6.14: Rollback distributions for \( P = \{4, 8, 12, 24\} \) at \( T = 2.7 \) using the parameters \( L = 220, VTW = 3000 \), and a 2D decomposition. Note that the horizontal axis has a linear scale.

protocol if only 4 processors are used.

In the low temperature regime the average rollback length increases with the number of processors. From Fig. 6.13 we observe that the rollback length cutoff size increases with \( P \). Therefore it is expected that the average rollback lengths must increase with \( P \) in the low \( T \) regime (see Fig. 6.15).

This is not valid anymore in the high \( T \) regime, where the rollback length distributions approximately collapse (see Fig. 6.14) to the same exponentially decreasing distribution. As a direct consequence, the average rollback lengths will collapse (see Fig. 6.15). In the high \( T \) regime the rollback lengths are not influenced by \( P \) as in the low \( T \) regime. The processors are synchronized frequently in this regime, due to a high acceptance ratio of flipped spins. Therefore, there is hardly any real time to build large simulation time differences between the processors, resulting in only small rollback lengths.

It is interesting to compare the runtimes for different \( P \) in the same temperature range. The results are presented in Fig. 6.16. From the figure one can derive that in the low \( T \) regime, the runtime scales down if more processors are used. This is also valid for the high temperature regime. Around \( T_c \), we find non-trivial scaling of the parallel runtime for different number of processors. Obviously, using only 4 processors gives the best result, which could be expected from the significantly lower average rollback length in this regime. Using 12 processors gives the worst results in this case.

For the high and low temperature regimes \( P = 24 \) gives the best performance results. Even though, in the low \( T \) regime, the average rollback length is maximal for \( P = 24 \), the extra overhead is beneficially applied to efficiently exploit the parallelism present in the simulation.
Figure 6.15: Average rollback lengths for $P = \{4, 8, 12, 24\}$ for varying $T$ using the parameters $L = 220$, $VTW = 3000$, and a 2D decomposition.

Figure 6.16: Run times for $P = \{4, 8, 12, 24\}$ for varying $T$ using the parameters $L = 220$, $VTW = 3000$, and a 2D decomposition.

Although the average rollback lengths in the low $T$ regime are much larger than the average rollback lengths in the high $T$ regime, the execution times are comparable. This is a result of the frequency of rollback events. In the low $T$ regime this frequency is much lower, due to the reduced acceptance probability of spin flips. It seems that, the average rollback lengths and the rollback frequency are balanced to approximately similar execution times for the low and high $T$ regimes.
6.4.3 Different Virtual Time Window Sizes

An important parameter of the Time Warp protocol is the so-called virtual time window (VTW). This parameter controls the asynchronicity of the simulation. It specifies the maximum difference between the local virtual time and the global virtual time (the minimum of all local virtual times). It is expected that this parameter greatly influences the rollback dynamics. For the experiments presented in this section we have varied the VTW parameter while keeping all other parameters fixed \((P = 12\) and \(L = 220\)).

In Fig. 6.17 the rollback distributions are depicted for \(T = 1.0\) for experiments with VTW parameters in the range \([750, 6000]\). Obviously, a small virtual time window decreases the maximum rollback length.

![Figure 6.17: Rollback distributions for VTW = \(\{750, 1500, 2000, 2250, 2500, 2750, 3000, 6000\}\) at \(T = 1.0\) using the parameters \(L = 220\), \(P = 12\), and a 1D decomposition.](image)

In Fig. 6.18 the rollback distributions for VTWs around 3000 are shown. There is a transition from \(VTW = 2750\) to \(VTW = 3000\). The VTW values \(\{3000, 3250, 3500\}\) produce similar rollback length distributions, while \(VTW = 4000\) deviates.

From Fig. 6.19 it can be observed that the peak of the average rollback length shifts and broadens for increasing VTW. This effect is caused by the Ising dynamics. Large virtual time windows effectively result in a more pronounced influence of the finite sub-lattices (decomposed over the 12 processors). Due to the increased asynchronicity for larger time windows the sub-lattices are effectively loosely coupled and act more like individual Ising spin lattices. It is a well-known fact in Ising spin simulations that decreasing the lattice size results in a broadening and shifting of the spin correlation peak around \(T_c\).

The average rollback lengths roughly decrease for decreasing virtual time
Figure 6.18: Rollback distributions for $VTW = \{2750, 3000, 3250, 3500, 4000\}$ at $T = 1.0$ using the parameters $L = 220$, $P = 12$, and a 1D decomposition.

Figure 6.19: Average rollback length for $VTW = \{750, 1500, 2000, 2250, 2500, 2750, 3000, 3500, 4000, 6000\}$ for varying $T$ using the parameters $L = 220$, $P = 12$, and a 1D decomposition.

window size (see Fig. 6.19). This is a result of the Time Warp dynamics. Small virtual time windows only allow for a small build up of local virtual time differences.

For smaller VTW values the average rollback lengths are comparable over the entire temperature range. For these values it can be concluded that the Time Warp dynamics are not constrained by the details of the Ising dynamics.
6.5 Summary and Discussion

The increased synchronization frequency disables the build up of large time differences.

Somewhere there is a crossover point where increasing the maximum rollback lengths (by increasing VTW) does not improve the progress in simulation time due to the increased protocol overhead. For this specific simulation instance it seems that VTW = 2750 is optimal for the low and high temperature regime (see Fig. 6.20). For the regime around $T_c$ it is almost optimal. Note that VTW = 3000 is comparable to VTW = 2750 in the low and high T regimes, while around $T_c$, VTW = 3000 produces significantly higher execution times. Hence the run times are highly susceptible for VTW around $T_c$, as a consequence of the critical Ising dynamics.

In contrast to the disappearance of the average rollback length peak in Fig. 6.19 for increasing VTW, a peak remains in the runtime curves. This can be explained from reduced rollback frequencies in lower temperature regimes. The increase of the runtimes around $T_c$ with increasing VTW can be explained from the increased average rollback lengths (see Fig. 6.19) and the fat tail in the rollback size distribution around $T_c$ for large virtual time windows (data not shown).

6.5 Summary and Discussion

In this chapter we have intensively studied the dynamical behavior of the Time Warp protocol for parallel discrete event simulations. As a simulation case we considered a model that supports tuning of the correlation length, namely the
Ising spin model, which is basically a cellular automata model. A property of the Time Warp protocol is the appearance of so-called rollbacks whenever a causality error occurs. This rollback mechanism can trigger a cascade of events that need to be undone. The local rollbacks are recorded by the simulation process, as the instantaneous global cascaded rollbacks cannot be administered without unacceptable intrusion into the Time Warp dynamics. It is known that so called slowly-driven, interaction-dominated threshold (SDIDT) systems can exhibit power laws without any apparent tuning. The specific feature of these dynamical systems is called self-organized criticality (SOC).

From the inset in Fig. 6.16 we can see that the optimistic simulation of the Ising spin system scales linearly with the number of processors for low temperatures $T = [0 - 1.5]$. However, it is found that the Ising spin phase transition influences the rollback behavior, and consequently the runtime. Around the critical temperature (physical critical behavior) $T_c$, the average rollback lengths increase dramatically, as well as the simulation runtimes, due to long-range spin correlations. The non-trivial scaling in runtimes around the critical temperature shows in Fig. 6.16 that the best performance is obtained with only 4 processors. For physical sub- and super-critical temperatures the simulation runtimes approximately coincide.

For the rollback dynamics three different phases can be distinguished: physical sub-critical, physical critical, and physical super-critical rollback length scaling behavior. In the sub-critical regime the scaling behavior appears to behave like a power-law, with exponents independent of the temperature. In this regime we conjecture that computational critical (SOC) behavior appears. Around the critical phase large rollback lengths become more abundant due the long-range spin correlations. Here the computational complexity and the physical complexity are entangled and contribute both to the runtime and rollback behavior in a non-linear way. In the physical super-critical phase a negative exponential distribution of the rollback lengths is observed.

Obviously a lot of work remains to be done in the study of physical- and computational critical behavior in Time Warp. The results presented in this chapter are, to our knowledge, the first series of experiments that have ever been conducted to study the influence and the appearance of critical behavior in Time Warp. The entanglement of the computational and physical complexity, and their non-trivial contribution to the runtime behavior might have consequences for other optimistic simulations.