Corporate Financial Risk Management

Ligterink, J.E.

Publication date
2001

5

Risk management and security design

5.1 Introduction

In Part I we surveyed the literature that rationalizes corporate risk management. The general conclusion from this literature is that risk management may lead to efficiency gains by mitigating - at the firm level - some of the detrimental consequences of market imperfections. This literature however ignores one of the most apparent characteristics of risk management, the transfer of firm-specific risk to outsiders. Firms use a variety of alternatives to transfer risk. For example, if a firm issues a security (like debt or equity), enters into a forward contract or purchases an option, it transfers risk. Moreover, firms that purchase derivatives are likely to affect the risk characteristics of their initial funding sources like debt and equity. This raises questions with respect to the firm's use of derivatives and its effect on the design of funding instruments like debt and equity in the (optimal) allocation of risk. Especially interesting is the question how risk management may improve the firm's ability to design securities if at all. The purpose of this chapter is to analyze the potential role of risk management and security design in a model where risk sharing is important.

Both corporate risk management and the design of a firm's securities (like debt and equity) are important in allocating risks. In a complete financial market such (re)allocation of risk would not affect investors' utilities and therefore would be irrelevant.[1] Therefore, in order to analyze the dual role of corporate risk management and security design in (re)allocating risks it is important to consider incomplete asset markets as a starting point. In such markets, firms can improve risk sharing opportunities in the economy by offering securities that complete the market. Markets may, however, remain incomplete when there are significant costs associated with

[1] For an extensive discussion on this see Chapter 2.

security design. Firms might then try to exploit differences in risk preferences among investors by issuing multiple types of securities and market these to those clienteles that value them most (Allen and Gale, 1988).[2] More specifically, the incompleteness of financial markets may induce firms to design financial structures with extreme securities. Each security within such a financial structure pays the firm's full income in a specific state and zero in all other states. The primary reason firms gain from such a security design is that by splitting the cash flows over separate securities they facilitate more optimal risk sharing in the economy.

Building on the insights developed in Allen and Gale (1988, 1994) we argue that corporate risk management may have a positive role in security design. We thereto characterize risk management as a way to change the distribution of a firm's income *among* different states of the world. Security design on the other hand concerns the allocation of the firm's income streams over different securities.[3] We then disentangle the separate contributions of risk management and security design in risk sharing. If both risk management and security design decisions affect the allocation of risk, how do they interact?

To study this we develop a one period (two dates) model as in Allen and Gale (1988), in which firms maximize the proceeds from issuing securities to investors with different degrees of risk aversion. At the initial date, firms issue securities, i.e. claims on future cash flows or output. Investors purchase securities to smooth uncertain future consumption. The costs of security design (together with short sale constraints) make the financial market incomplete. We extend this framework by allowing firms to enter into risk management (hedging). Hedging transactions offer firms the possibility to reduce the variability of their cash flows and partially adjust the allocation implied by the securities. In effect, hedging offers the firm fine-tuning possibilities which in the end enables it to increase the proceeds of the initial sale of financial securities. If there are important restrictions to investors' use of derivatives, corporate risk management may enhance the firm's possibilities to design securities that exploit differences in investors' preferences (and therefore increase the proceeds from security design). Hedging, by shifting income from low valued states to those where investors value income relatively higher, increases the firm's initial proceeds from issuing securities as compared to the Allen and Gale (1988) model which excludes hedging.

Subsequently, we introduce marketing costs in our framework (as in Madan and Soubra, 1991). We assume that approaching investors is costly and that if securities are too narrowly designed for a small group of investors, a firm may face considerable costs of marketing the securities. We model this idea by assuming that a firm that approaches an investor with a very specific security (for example an Allen and Gale type of extreme security) may not succeed in selling the security. Running such a risk of having an unsold inventory of securities, the firm

[2] This only holds as long as there are important restrictions on short selling for investors.

[3] Since we want to focus on risk sharing, we ignore other characteristics like those related to control.

has another reason (apart from exploiting differences in risk preferences among investors) to structure and price its securities; to reduce its marketing costs. Corporate hedging enables the firm to smooth its date 1 output and issue securities at date 0 that are more attractive to a larger group of investors. We argue that risk management enables the firm to issue more generic securities. A typical example is debt, a contract which pays out the same income over a wide range of (non-default) states. The benefits of issuing more generic rather than very specific (extreme) securities lie in a reduction of marketing costs. More generic securities attract a broader investment base and as such increase the firm's net proceeds from issuing securities in an incomplete asset market (Madan and Soubra, 1991).

We think that our approach, although preliminary, offers an important perspective on the rationale for corporate risk management. Corporations try to minimize the use of equity and take on debt instead. They do this by, among other things, transferring marketable risks via derivatives rather than through the firm's equity. The case of United Grain Growers, presented in the introduction of this dissertation, is a good example of such a transaction. Theories to date (see Chapter 2 for an overview) offer various explanations for such behavior. These approaches, however, do not consider issues related to risk sharing. If risk sharing is important in the design of securities, risk management may contribute to firm value because it enables the firm to optimize on the design of securities. The approach taken in this chapter explores this idea.[4] The objective of this chapter is not to develop a full fledged model of security design with endogenous corporate risk management. Instead, the main purpose is to provide a perspective on the way risk management may affect the firm's risk sharing through its more basic securities and how risk management and security design might interact. The focus is on the benefits of risk sharing in the economy. Therefore the perspective developed in this chapter is broader as taken elsewhere in this dissertation.

The role of securities markets in the allocation of risk goes back to the seminal works of Arrow (1964) and Debreu (1959). These contributions focussed primarily on complete financial asset markets. Later work by Radner (1972) and Diamond (1967) considered incomplete financial markets but the primary focus was on the firm's real decisions. More recent contributions in the security design literature (especially Allen and Gale, 1988, 1991, 1994; and Madan and Soubra, 1991) explicitly identify improvements in risk sharing possibilities in incomplete asset

[4] An important characteristic of the security design literature in corporate finance is that the optimality of financial contracts has been developed from first principles. Securities, like debt and equity are not taken for granted but are endogenous in these models. For example in the context of Allen and Gale (1988), the question is how optimal securities would look like if risk sharing in incomplete markets is important for corporations.

markets as the motivation for security design.[5] Our work is directly related to that literature, but adds risk management to the spectrum of financial decisions.

Section 5.2 and 5.3 provide the basic framework and present two illustrative examples that will be used throughout the chapter. We illustrate the general idea of Allen and Gale (1988) and that of Madan and Soubra (1991), that firms may gain from security design if the firm's securities facilitate optimal risk sharing. In Section 5.4 we include corporate hedging in both settings and study to what extent corporate hedging affects optimal security design. Section 5.5 discusses some limitations of our approach. Section 5.6 considers possible extensions. Finally, section 5.7 concludes.

5.2 A risk sharing security design framework

One of the primary functions of financial markets is the allocation of risk between firms and investors. Cash flow rights attached to securities traded in financial markets enable investors to take or reduce risks. For the suppliers of securities in financial markets (firms) it is beneficial to sell securities to those investors that require the lowest risk premium. A firm therefore has an incentive to design securities that are in line with the risk preferences of investors. A recent strand of the security design literature considers the allocation of risk of central importance in the design of financial securities (especially Allen and Gale, 1988, 1994; and Madan and Soubra, 1991).

In this section and in Section 5.3 we take a closer look at risk sharing as a motivation for security design. In two illustrative examples we discuss the major insights developed in the studies by Allen and Gale (1988) and Madan and Soubra (1991). We will then extend these examples to analyze the potential benefits of risk management in Section 5.4.

The model

Consider a simplified economy with a countable but infinite number of firms and an equal number of investors.[6] There are two dates, date 0 and date 1 with a single consumption good at each date.[7] There is uncertainty with respect to the state of the world that prevails at date 1.

[5]Alternative explanations of security design will be discussed in Section 5.5.

[6]In other words, we assume that there is one firm per investor. The fact that we have an infinite number of investors and firms, ensures that the equilibrium (if one exists) will be competitive.

[7]The consumption good will act as numeraire. This assumption is crucial for the results. The introduction of money or multiple goods, creates indeterminacy of equilibrium allocation in incomplete markets. If securities pay off in units of account, the value of these units in each of the states is indeterminate. In incomplete financial markets this nominal indeterminacy translates into real indeterminacy. For a discussion of this issue see Pesendorfer (1995) and references therein.

Uncertainty is reflected in a finite set of states of the world $s \in S$ at date 1, where $S = \{1, 2\}$. Initially (at date 0) nobody has information about which state will be realized at date 1. The prior probability π_s, that one of the states $s \in S$ occurs, is common knowledge. At date 1 all agents observe the prevailing state of the world.

At date 0 firms jointly make decisions on their production technology and issue claims against the state dependent output to be realized at date 1. Assume that the owners of the firm are risk neutral and only care about current (date 0) consumption and therefore maximize the proceeds from issuing securities. In this setting, we first analyze which securities firms should optimally issue in equilibrium.

To complete the framework we make the following two assumptions:

- costly security design makes financial markets incomplete;
- no short sales allowed.

Arrow (1964) and Debreu (1959) show that with complete financial markets investors can achieve complete risk sharing. In such a world security design (from a risk sharing perspective) cannot increase the investor's utility. However, to complete the market requires at least as many (linearly independent) securities as there are states in the world.[8] Since issuing multiple securities by firms is assumed to be costly, markets are (initially) incomplete.

The second assumption (no short sales) guarantees that investors cannot expand the supply of a security. If a firm issues multiple securities by splitting its cash flows to benefit from differences in risk preferences among investors and investors are allowed to short the security, the supply of the security can be enlarged to infinity. In that case, the firm cannot capture the costs from security design and only issues one composite security (unlevered equity) in equilibrium. When short selling is not allowed, the firm will be able to capture (part of the) monopoly rents from security design.[9]

Investors

Investors have time-additive von Neumann-Morgenstern utility functions and differ in their degree of risk aversion with respect to date 1 consumption. Investors' income (endowments) at the initial date have been normalized to zero.[10] Since at $t = 0$ investors do not know what state of the world will occur at $t = 1$, they have an incentive to purchase securities in order to smooth

[8] With dynamic trading the number of securities is actually lower than the number of states to complete the market.

[9] The main results hold when there is limited short sales. For an extensive discussion see Allen and Gale (1994).

[10] But we assume that investors face no constraint in purchasing the securities issued by the firm.

consumption over states. The only reason for investors to buy the firm's securities therefore is risk sharing.

There are two types of investors in the economy. One group (denoted by a) is risk averse with respect to date 1 consumption and one group is risk neutral (denoted with type n). Differences in preferences between investors are reflected in their respective utility functions:

$$U_i(c_0, c_1) = c_0 + u_i(c_1(s)) \tag{5.1}$$

where $i \in \{a, n\}$ and where $c_1(s)$ denotes the consumption at date 1 in state s. Observe that by definition all investor types are risk neutral with respect to their consumption at date 0. For the risk neutral investor, $u_n(c_1(s))$ (the utility at date 1 in state s) is linearly increasing in consumption and therefore $u_n'(c_1(s))$ is positive and constant. For the risk averse investor: $u_a'(c_1(s) > 0$ and $u_a''(c_1(s) < 0$.

Both investor types maximize their expected utility

$$EU_i(c_0, c_1) = c_0 + \pi_1 u_i(c_1(1)) + \pi_2 u_i(c_1(2)) \tag{5.2}$$

Investors can purchase securities (i.e. claims on future consumption) at $t = 0$ by giving up current consumption in exchange for future consumption. A security is defined as a claim issued by the firm on its future (date 1) state contingent cash flow stream.[11] These securities enable investors to smooth consumption over different states of the world at date 1. The preferences of investors determine security prices. For example, the maximum price that investor a wants to pay for a security that trades off one unit of current consumption in exchange for one unit of future consumption in state 1, depends on his marginal rate of substitution:

$$\frac{\partial EU_a(c_0, c_1)/\partial c_1(1)}{\partial EU_a(c_0, c_1)/\partial c_0} \tag{5.3}$$

It follows directly from equation 5.1 that for both investor types a and n the marginal utility of consumption c_0 is equal to 1. Using this and equation 5.3, we can now express the marginal rate of substitution of one unit of consumption in state 1 at date 1 in terms of the date 0 consumption good as

$$p_{a1} = \pi_1 \frac{\partial u_a(c_1(1))}{\partial c_1(1)} \tag{5.4}$$

This is an expression for the maximum (state) price that an investor of type a is willing to pay for a security that provides one unit of consumption good at date 1 in state 1. The state price of a type a investor for consumption in state 2 is thus equal to

[11] We therefore abstract from other relevant characteristics of a security such as control rights.

$$p_{a2} = \pi_2 \frac{\partial u_a(c_1(2))}{\partial c_1(2)} \tag{5.5}$$

Along these lines we can now derive the state prices for the risk neutral investor

$$\begin{aligned} p_{n1} &= \pi_1 \frac{\partial u_n(c_1(1))}{\partial c_1(1)} \\ p_{n2} &= \pi_2 \frac{\partial u_n(c_1(2))}{\partial c_1(2)} \end{aligned} \tag{5.6}$$

Note however, that for risk neutral investors $\frac{\partial u_n(c_1(s))}{\partial c_1(s)}$ is a constant and independent of consumption at date 1.

We have now established a convenient framework to price securities. These prices are consistent with investors' risk preferences. We will use this pricing framework to study the firm's optimal security design decisions.

Firms

Firms design and issue securities at date 0. Securities are claims on the firm's state-dependent output realized at date 1. The vector y_s describes a firm's future production in the different states of the world. Thus, in our framework, the security design is nothing more than writing a rule for partitioning the cash flow stream over a set of securities.[12] Firms can only issue securities with non-negative payoffs (securities with limited liability). Denote the number of k different types of securities issued under a financial structure with F_k. The payoffs of each security issued by the firm is given by the vector r^j. All payoffs of the securities of a firm should add up to the state dependent output y_s ($\sum_{j=1}^{k} r^j = y_s$). Firms design claims on their future (uncertain) output that maximize proceeds.

Issuing securities is costly. Suppose that issue costs are positively related to the number of securities issued by the firm. For example, the costs to create a financial structure with one financial security is denoted by $C(1)$, the costs of issuing two types of securities equals $C(2)$, with three types of securities the costs are $C(3)$, etc. We then assume that $C(3) > C(2) > C(1)$. In order to make sure that no trivial solution exists where there are no firms operating, we assume that $C(1)$ is normalized to 0.

Now assume that a firm issues a security that entitles the investor to an amount of the consumption good in each state s equal to $r(s)$, where $r(s)$ is the vector that describes the securities' payoff in each state. What will be the proceeds if the firm offers such a security to an investor of type a? From equations 5.5 and 5.6, we can derive the maximum price the investor of type a

[12] The payoff of the securities and the prices are in terms of the consumption good.

is willing to pay for a marginal amount of such a security. This equals

$$v_a = \sum_{s \in S} p_{as} \times r(s) \tag{5.7}$$

and depends on the marginal valuation of one unit of date 1 consumption in state s (in terms of the date 0 consumption good). In a similar way, we can find the maximum price that a risk neutral investor would pay for such a security.

The firms' problem then can be restated as the partitioning of its state dependent income stream over its securities such that the sum of the market values of the securities is maximized;

$$v^j = \max_i \sum_s p_{is} r^j(s). \tag{5.8}$$

Equilibrium conditions

An equilibrium is defined as the choice of a set of securities issued by the firm, a pricing system for each security and investors' portfolios with the following equilibrium conditions:

1. firms maximize the current value of consumption;
2. investors maximize their expected utilities;
3. markets clear.

It can be shown that an equilibrium exists under the assumptions presented earlier.[13] In such an equilibrium firms design securities that correspond with investors' risk preferences. These securities maximize the firm's date 0 revenues and therefore satisfy condition 1. The securities issued by the firm offer investors the ability to share risks at prices which maximize the investors' expected utility of wealth. With the set of prices derived in equilibrium markets will clear.

To explain the intuition behind the model more carefully we will use a numerical example. We first describe the investors' problem, then the firm's decision problem, and determine the equilibrium conditions. We then derive the optimal securities issued in equilibrium given the costs of security design. In Section 5.4 we will extend this example with the possibility of risk management by the firm.

[13]The equilibrium is constrained efficient; that is a central planner faced with the same transaction costs as in the market cannot make anyone better off without reducing someone else's utility. For an extensive discussion of efficiency and the proof of the existence of such equilibria see Allen and Gale (1988,1994).

Numerical example 1a

Investors

We consider two types of investors with the following utility functions:

$$\begin{aligned} u_a(c_1(s)) &= 2\ln(1+c_1(s)) \\ u_n(c_1(s)) &= 0.8c_1(s) \end{aligned}$$

Investors maximize their expected utility

$$E(U_i(c_0, c_1(s))) = c_0 + \sum_{s\in\{1,2\}} \pi_s u_i(c_0, c_1(s))$$

Assume that the probability π_s that state s occurs is $\frac{1}{2}$ for each state $s \in \{1,2\}$. Since the securities of a single firm form a negligible part of any investor's portfolio, in equilibrium investors value individual securities based on the marginal contribution of each security to their expected utility of wealth (see equation 5.7). We therefore need to calculate the state prices.

For every state and type of investor, we can identify the state prices (marginal utility of one unit of the consumption good at date 1 in state s). For example, the investor of type a, values a marginal amount of the security that pays 1 unit of the consumption good in state 1 as

$$p_{a1} = \frac{\partial EU_a/\partial c_1(1)}{\partial EU_a/\partial c_0} = \pi_1 \frac{2}{1+c_1(1)}$$

Similarly, we can calculate p_{a2} $(= \pi_2 \frac{2}{1+c_1(2)})$, p_{n1} $(= 0.4)$ and p_{n2} $(= 0.4)$.

Firms

Assume for expositional reasons that there is only one type of firm in the economy. If the firm operates, its output vector is $y_s = (4,1)$, and $y_s = (0,0)$ otherwise. Since the firm's owners only care about current consumption, the firm's problem is to create securities at date 0 (claims on the state dependent date 1 output y_s) that maximize the current proceeds of these securities.

Assume that initially there is an equilibrium where all firms have a financial structure F_1 with one composite security only. The payoff of such a security resembles that of unlevered equity. The payoffs of this security are then equal to: $r^1 = (4,1)$. Appendix 5.1 derives the state prices in such an equilibrium (see also Table 5.1).

Since the costs of security design $C(1)$ have been normalized to 0, it is clearly optimal for the firm in our example to operate and issue securities. The market value of a firm issuing a financial structure with only one security equals 2.

Can a firm do better, that is increase its proceeds by issuing more than one security in this setting? As we will see in the next subsection, this is indeed the case if the costs of issuing such a financial structure are not too high.

Investor type i	**Demand for unlevered equity**	**p_{is}**	
		State *1*	**State *2***
a	0.5	1/3	2/3
n	0.5	0.4	0.4

TABLE 5.1. Equilibrium with one composite security

Allen and Gale's optimal (extreme) securities

Consider the optimal securities for one specific firm if all other firms issue unlevered equity. Table 5.1 shows that the risk neutral investors value income in state 1 higher than the risk averse investors ($p_{n1} > p_{a1}$) and that risk averse investors value income in state 2 higher than risk neutral investors. As a result, a firm maximizes its proceeds by selling the complete state 1 output to the risk neutral investors and the complete state 2 output to the risk averse investors. We refer to these as extreme securities.

In our example it is thus optimal for a firm to issue a financial structure F_2 with two securities: one that pays off the complete state 1 output and zero in the state 2, i.e. $r^1 = (4, 0)$, and another that only pays off in state 2 (i.e. $r^2 = (0, 1)$). Subsequently, the firm should sell these two claims against a price that matches the investors' highest state valuations. Using equation 5.8, we can derive that it is optimal to sell the securities with the following payoffs:

- $r^1 = (4, 0)$, to the risk neutral investors against price $v^1 = 1.6$;[14]
- $r^2 = (0, 1)$, to the risk averse investors against price $v^2 = 2/3$.

The market value of a firm issuing these extreme securities where all other firms issue only one security (unlevered equity) is $1.6 + 2/3 = 2.26667$. Note that the issue proceeds are higher than compared to the case where the firm issues only one composite security.

Whether or not firms will enter in the process depends on the costs of security design (that is the cost of issuing two rather than one security by the firm). If these costs are lower than 0.26667 (that is the difference in the value of a firm with only one security, and that of a firm with two extreme securities) more firms will have an incentive to issue extreme securities in equilibrium.

The costs of security design therefore drive the equilibrium outcome. To illustrate this, in Appendix 5.2, we examine the equilibrium when the costs of issuing two securities $C(2)$ equals 0.1. In this equilibrium, a fraction equal to 0.41667 of the firms will issue only one security (unlevered equity with payoff $(4, 1)$). This security will be sold to the risk averse investors against a price equal to 2. The remaining fraction of the firms issue two securities $\{(4, 0), (0, 1)\}$

[14]Prices v are according to equation 5.8. Here, $v^1 = \sum_s p_{is} r(s) = 0.4 * 4 + 0.4 * 0 = 1.6$

Investor type	p_{is}	
	State 1	**State 2**
a	0.3749	0.5
n	0.4	0.4

TABLE 5.2. Equilibrium with transaction costs C(2)=0.1.

and face the costs $C(2)$. Both firms' market values (net of costs) equal 2. Table 5.2 summarizes this equilibrium.

The example illustrates the main findings of Allen and Gale (1988). In an incomplete asset market firms will issue multiple claims against its state dependent output in equilibrium. The costs of security design determine the number of securities firms will issue in equilibrium. Firms either issue one composite security or they issue a set of securities where each pays all income or nothing at all in that particular state. In the new equilibrium, risk averse investors have gained; state prices have become more equal. The benefits of security design in this competitive setting will go to the investors and not to the firms.

5.3 Placement frictions or marketing costs

One objection that has been raised against the Allen and Gale framework is the way the costs of security design are being treated. As described in the previous section, these costs depend on the number of securities that a firm issues, but not on the particular structure of the security sold to the market. This suggests that a very complex security can be sold to the market at the same costs as a standard contract. Several authors (Ross, 1989; Madan and Soubra, 1991 and Pesendorfer, 1995) have stressed that there are important differences in the costs of issuing securities, dependent on the structure of the security, and emphasize that these costs - labeled as marketing costs - may play an important role in financial innovation and security design.

Below, we will especially focus on the argument provided by Madan and Soubra (1991) who argue that marketing securities is not costless *and* that these costs depend on the type of securities being issued. The more specific a security, the harder it will be to find potential customers and therefore, the higher the marketing costs associated with issuing that security. Marketing costs in Madan and Soubra (1991) are either modeled as the costs of unsold inventory (of securities) or as the costs of approaching multiple investors.

In our numerical example below we will use a relatively simple way of modelling marketing costs. We assume that the firm can offer a security only once to an investor. If it sets its price too high, it will not be able to sell the security. Hence, we model the marketing costs as unsold inventory. Although this creates rather high marketing costs, the level of these costs is not that important for our analysis.[15]

If we explicitly include these marketing costs in our framework, extreme securities are rarely optimal. An extreme security is structured and priced according to the risk preferences of a very specific group of investors (those that value the payoffs of the security highest). This is unlikely to be a large and easily identifiable group so that marketing costs of these securities will typically be high. The higher the marketing costs, the lower the net proceeds from these securities for the firm. Taking these marketing costs into consideration, firms will generally issue securities that appeal to a larger set of investors. Although this will reduce the gross proceeds of the issue, it will also reduce the marketing costs and in the end maximize the firm's net revenues from issuing securities. An important result of this analysis therefore is that securities that are more in line with the average preferences of investors will maximize the net proceeds of security design. These securities typically include more debt like instruments and leveraged equity instead of the optimal extreme securities identified by Allen and Gale (1988).

In addition to the way costs are treated, there are two other important differences in the approach taken by Allen and Gale (1988) and Madan and Soubra (1991). First, in Allen and Gale the analysis of the firm's security design problem is captured in a general equilibrium framework. Madan and Soubra, in order to stress the role of marketing costs, choose to consider a partial equilibrium framework.[16] A second difference is that Madan and Soubra consider the security design problem of a financial intermediary rather than that of a firm.[17]

Example 2a

To illustrate how marketing costs affect security design we develop a second example that is slightly different from the one we presented before.[18] We consider an economy where investors' marginal state valuations are as in Table 5.3. We assume that the proportion of investors of type a (risk averse) is equal to 0.6 and the proportion of the investors of type n (risk neutral) is 0.4. Operating firms generate a state dependent income at date 1 equal to $y_s = (4, 1)$. In our analysis we focus on the security design problem of a single firm.

[15]An alternative way of modeling marketing costs is with a small transaction cost per approach. In our more general model in the appendix we also discuss this approach. Since hedging in our model is costless, we do not have a trade-off between marketing costs and hedging. As a result we have chosen for the simplest approach modelling the marketing costs as unsold inventory.

[16]They therefore do not consider the effects on state prices, nor do they consider issues related to optimal risk sharing in the economy.

[17]We will discuss these differences in Section 5.5.

[18]This new example is needed to rule out the trivial case where issuing one composite security (unlevered equity) is optimal. In order to keep the structure of the example as simple as possible we changed the individual state valuations from those in the former example.

Investor type	p_{is}	
	State 1	**State 2**
a	1/3	2/3
n	0.5	0.5

TABLE 5.3. State prices of investors.

Consider first the case where the firm issues only one security, unlevered equity with a payoff equal to $r^1 = (4, 1)$. What would be the optimal pricing strategy for the firm? In an Allen and Gale framework the firm sells its security to the investors that most value the security or

$$v^j = \max_i \sum_s p_{is} r^j(s) \tag{5.9}$$

Hence, using Table 5.3, the firm would sell the security to the risk neutral investors at a price of 2.5 (=0.5 $\times$ 4 + 0.5 $\times$ 1). However, since the firm does not know which investor type it approaches, and since the proportion of risk neutral investors is 0.4, the expected proceeds are only equal to 1 (2.5$\times$0.4=1). It is obvious that the firm can increase the expected proceeds in the presence of these marketing costs by reducing its price to a level that also attracts the risk averse investors. The maximum price risk averse investors are willing to pay for unlevered equity according to the marginal state valuations is equal to 2 $(1/3 \times 4 + 2/3 \times 1 = 2)$. Because risk neutral investors will also purchase the security at this price, the expected proceeds for the firm at this price are 2.

Now consider the Allen and Gale extreme securities. We have shown that in the absence of marketing costs, extreme securities that either pay the full output or zero maximize the gross issue proceeds for the firm. However, as we already suggested, such extreme securities are not optimal in the presence of marketing costs. We will illustrate this for our next example. Since the risk neutral investors value income in state 1 relatively higher than risk averse investors, it is optimal to sell all state 1 income to the risk neutral investors. The proceeds of a security that pays $(4, 0)$ and is sold to the risk neutral investors when there are no marketing costs is equal to 2. With marketing costs, the expected proceeds of that similarly priced security is much lower, namely 0.8 $(0.4 \times 2 = 0.8)$. Can a firm do better? Yes, by reducing the price to a level at which it also attracts risk averse investors. To attract the risk averse investors, the price should be reduced to $4/3$ $(1/3 \times 4 = 4/3)$. In that case both the risk neutral and the risk averse investors will purchase the security and the expected proceeds increase to $4/3$.

With respect to the second security $(0, 1)$, it is clear that in a world without marketing costs this security would be sold to the risk averse investors at a price of $2/3$. However, with marketing costs, it is optimal to sell the security at a lower price that also attracts risk neutral investors. This occurs at a price equal to 0.5. Total proceeds in the presence of marketing costs are 1

$5/6$ $(4/3 + 0.5 = 1\ 5/6)$. In this example where marketing costs are incorporated, selling one security (unlevered equity) dominates issuing extreme securities.

Consider now the financial structure with debt (a security that pays the same income over the two states $(1, 1)$) and levered equity (a security that pays 3 in state 1 and zero in the other state $(3, 0)$). In the absence of marketing costs, these securities would always be dominated by the extreme securities. However, this is not the case when we incorporate marketing costs. A firm that issues such securities can increase its value to 2 in a world with marketing costs by selling the two securities $r^1 = (1, 1)$ and $r^2 = (3, 0)$ against a price $v^1 = v^2 = 1$. Observe that both securities are priced such that all investors purchase the securities.[19]

There are two important differences between a world with or without marketing costs. First, it is no longer optimal for the firm to sell the securities at the highest possible price. Such a security would be too specific and thus have very high marketing costs. In our simple example, due to unsold inventory the expected net proceeds would be much lower when selling the security at the highest possible price. Second, in the presence of marketing costs extreme securities are no longer optimal. The two securities we identified at the end of the example dominate the extreme securities. By making some securities attractive for more investor types, the firm reduces its marketing costs and therefore increases the expected net issue proceeds.

We have restricted the notion of marketing costs to unsold inventory in an example where firms could only approach investors once. Madan and Soubra (1991) consider also other possible cases. For example, they consider a case where firms can approach investors several times but each approach has a cost. This, however, reflects the same idea, less attractive securities are associated with higher marketing costs.

In our simple setting, we also considered only two types of investors. The basic idea however, becomes even stronger when we consider many different types of investors (e.g. a continuum of types). According to Allen and Gale, a firm with income in two states should split this income stream into two securities with each security paying either all state income, or nothing (if the costs of security design are low enough). These securities should then be sold to the investors that value the security most. Since with a continuum of investors the firm will have to market the optimal extreme securities to two out of the continuum of investor types, it becomes a hazardous task. Allowing for marketing costs (e.g. costs of unsold inventory or, more realistically, the cost of approaching investors) subsequently makes it very unlikely that a firm will issue such extreme securities. Firms will choose their securities such that they appeal to a larger group of investor types.

[19]This is not always the case but depends on the distribution of investor types and differences in their marginal state valuations. In our example with the specific parameters it can easily be verified that this is the optimal pricing strategy for this financial structure.

5.4 Corporate hedging and security design

5.4.1 Introduction

In this section, we consider the interactions between corporate risk management and security design. Building on the framework we developed in the previous sections, we analyze the effect(s) of hedging on the firm's security design problem.

For this purpose we first need to define and characterize corporate risk management. If a firm enters into a forward contract (for hedging purposes) it transfers income from high to low income states.[20] This is an important characteristic; with derivatives, a firm can transfer income *among* states. The security design problem as we posed it in the former sections was to partition a firm's state dependent income stream over one or more securities. This suggests that security design and risk management play a different role in the allocation of risk.[21]

We now illustrate if and why it may be optimal from a security design point of view to reallocate part of the firm's risk to financial markets via hedging. Since an important part of the variation in the firm's state dependent income is generally caused by other (non-hedgeable) risks, it is rather unlikely that a firm can hedge all variation in state income away. In order to capture this, we assume (initially) that the firm can only enter into one forward contract. We define the hedging contract as a contract with a payoff which requires one unit of the date one consumption good in the high state or pays off one consumption good in the low state ($h = (-1, 1)$).[22] As usual, the use of forward contracts does not require an up-front payment.

With respect to the use and pricing of forward contracts, we make some very strong assumptions.[23] We first assume that the forward contract is priced at its expected value (or alternatively that it is held by risk neutral investors). We furthermore restrict the use of the forward contracts to corporations; we do not allow investors to enter into such forward contracts on their own account. Finally, we assume that firms can only enter into long positions. These are obviously rather strict assumptions. We defend them on two grounds. First, investors' use of derivatives is rather limited if compared to corporate use. Second, we are interested in studying the interaction

[20] We focus on hedging in this chapter and ignore the possibility of speculation.

[21] Note that a change in production technology is an alternative way to shift income from one state to another. We assume however that this is not efficient. For example, one may argue that a firm cannot shift income across states without loosing some of that income.

[22] Remember that we have a model with only one consumption good. As a result, it is a bit hard to think of a forward. However, considering multiple goods and money creates new problems in itself; the outcome may become indeterminate (see footnote 8). We therefore choose to model this in a very simplistic way.

[23] We will defend these assumptions in Section 5.5 more extensively. There we will also discuss the robustness of our results with respect to these assumptions.

between security design and risk management at the firm level. If firms have motivation to enter into derivatives that are not shared by investors (and do not harm the investors), this artificial assumption is not so disturbing. We will discuss the robustness of our findings with respect to these assumptions after our analysis.

The key insight that we develop in this section is that a firm's hedging strategy actually may support its security design. We first show how firms can increase the gross proceeds from securities by combining a proper hedge strategy with security design. The benefits of such a risk management strategy, however, directly follow from restrictions we imposed on investors' direct use of forward contracts. Hence, the basic idea developed here will only hold if investors' access to derivatives markets is indeed seriously restricted.

We then continue our analysis in a world with marketing costs. We find that hedging is a particularly effective tool to optimize the design of securities. Moreover, since investors do not have an incentive to use these forwards in a related way the constraints that we imposed are easier to accept. Below we develop these insights extending the two numerical examples we put forth in Sections 5.2 and 5.3. In Appendix 5.3, we discuss a slightly more general model to illustrate the contributions of risk management to security design.

5.4.2 The benefits of hedging: higher revenues from security design

We first study the link between corporate risk management and security design in an Allen and Gale framework where firms (but not investors) have the possibility to enter into a forward contract that changes the firm's income. We argue that in this framework a firm can increase the proceeds from its securities with an appropriate risk management strategy. We will illustrate this using example 1a. We then study the same problem in a setting with marketing costs of security design an extension of example 2a.

Example 1b: hedging and security design

Consider a single firm's security design and hedging decisions when (in equilibrium) all other firms issue one security. We have shown in example 1a (Section 5.2) that in an Allen and Gale framework it was optimal for a firm to issue extreme securities if the costs of security design were not too high.[24] These securities $(4, 0)$ and $(0, 1)$ could be sold at prices of, 1.6 and $2/3$ respectively. The market value of a firm issuing such securities therefore was equal to $1.6 + 2/3 = 2.2667$.

Now assume that the firm engages in risk management through the use of forward contracts. Corporate hedging enables the firm to transfer one unit of income from the state in which it has the highest income to the one that has the lowest. Does this make sense? This depends on the marginal utility of consumption of the investors in these respective states. If the marginal utility

[24] We refer here to costs associated with the number of securities being issued, not marketing costs.

of consumption in the state where the firm receives one additional unit of income is higher than that where it loses one additional unit of income, then hedging can be value-increasing.

In the context of our example, entering into a forward contract shifts income from state 1 to state 2. The firm's income stream before entering a forward contract is equal to $y_s = (4, 1)$. With a forward contract $h = (-1, 1)$ the firm's income stream y_s^H equals $(3, 2)$. Now it is straightforward to see that hedging in combination with issuing extreme securities improves the firm's market value in comparison to not hedging.

Table 5.1 shows that the state prices in state 2 is 0.4 for the risk neutral investors and $2/3$ for the risk averse investor. For state 1 these are respectively 0.4 and $1/3$. The firm can increase its market value if it is able to shift income to the state with the highest marginal utility of consumption (and thus the highest price). More precisely, if in our equilibrium only one firm enters into hedging, the firm can increase its value by hedging and then issue a financial structure with the extreme securities with payoffs of $r^1 = (3, 0)$ and $r^2 = (0, 2)$, respectively. The firm should sell and price these securities as follows:

- $r^1 = (3, 0)$, sell to risk neutral investors at a price equal to 1.2 ($v^1 = 3 \times 0.4 = 1.2$);
- $r^2 = (0, 2)$, sell to risk averse investor at a price equal to $4/3$ ($v^2 = 2 \times \frac{2}{3} = 4/3$).

Note that the total market value of a firm issuing these extreme securities ($1.2 + 4/3 = 2.5333$) exceeds the market value of a firm that cannot hedge (whose maximum is 2.26667 see Section 5.2). Hedging provides the firm with an additional tool for risk sharing; it enables the firm to move output *across* states, while the firm's initial security design can only distribute the output in one particular state over a number of securities. This additional feature of hedging over security design increases the potential revenues from securities even further.

Note however, that in order to achieve the benefits described above, risk averse investors' marginal state valuations must be negatively correlated with the firm's output. Only then will, reducing risk be beneficial. If the opposite case holds, risk averse investors value the unhedged security higher than a hedged security. In our example, hedging puts security payoffs more in line with the marginal state valuations of investors by taking one unit from the unpreferred state and adding it to the preferred state.

The motivation for corporate risk management however heavily depends on the assumption that access to forward markets is restricted to firms. If this is not the case, investors have a strong incentive to enter such contracts themselves. This would change their marginal utility of consumption in a particular state and thus reduce the potential benefit of corporate risk management substantially. Moreover, it may reduce the benefits of security design compared to the case where there are no forward markets.

5.4.3 *Hedging and security design with marketing costs*

In Section 5.3. we argued that marketing costs have a considerable impact on security design. Taking these costs into account implies that security design should not only focus on receiving the highest price but also on increasing the attractiveness of these securities for a sufficient mass of investors. In this section we will show that corporate risk management (hedging) enables the firm to make securities more attractive for investors and as a consequence, reduces the marketing costs associated with a financial structure.

We will illustrate this potential role of risk management by extending example 2 with the possibility to hedge as defined before.[25]

Example 2b: hedging to reduce marketing costs

Consider the setup in example 2a, Section 5.3. All firms generate an income $y_s = (4, 1)$ and the state valuations (marginal utility of consumption in the respective states) for the different types of investors are as given in Table 5.3. We established that the firm was indifferent between issuing one composite security and a financial structure with two securities $(F_2 : \{(1, 1), (3, 0)\})$. Both strategies generate the same market value net of marketing costs and dominate the financial structure with extreme securities. Assume that firms have the ability to hedge by entering into a forward contract as described in the introduction to this section $(h = (-1, 1))$. As a result of such a hedging transaction, the firm's output available for security holders becomes: $y_s^H = (3, 2)$. Does such a hedge increase the proceeds from security design net of marketing costs?

The hedged output, equal to y_s^H $(= (3, 2))$, enables the firm to issue more debt $(2, 2)$ and a new security with payoff $(1, 0)$. Both the risk averse and risk neutral investors value the debt-like instrument at the same price, 2. This is an especially attractive feature; to increase the expected revenues from security sales, the firm does not have to lower the price to attract more investors. Therefore, it prevents mispricing and reduces marketing costs. Given the parameters we have chosen, it is optimal to distribute as much output as possible via this security. Hedging helps to transfer output over states and issue more debt. The second security has the residual payoff $(1, 0)$. Although it is preferred (valued higher) by the risk neutral investors, it is priced according to the preferences of the risk averse investor at 1/3 $(=\frac{1}{3} \times 1)$ in order to reduce marketing costs and increase the expected value of the net proceeds. Sold against this price the residual security with payoff $(1, 0)$ attracts all investors. The total proceeds of a firm combining such hedging and security design strategies increases from 2 to 2 $1/3$.[26]

---

[25]For a slightly more general model of the contribution of risk management in an Allen and Gale (1988) and Madan and Soubra (1991) model see Appendix 5.3.

[26]If we would allow the firm to use more forward contracts, the optimum could be realized with 1.5 hedge contract. Firm value then increases to 2.5 and marketing costs reduce to 0.

To sum up, in our example hedging reduces marketing costs by increasing the payoffs to generic securities. For these securities the mispricing necessary to attract a broader investor base is less severe and thus the possibility of hedging increases the firm's market value net of marketing costs.

5.4.4 Discussion

We have characterized hedging and security design as performing two separate roles in risk sharing. Hedging transfers the output of a firm between states. The security design decisions concern the distribution of a given output stream over securities issued by a firm. We have established that hedging may contribute to security design in two ways. First, by shifting a firm's output to states valued higher by investors hedging increases the price of the firm's securities. However, it is important to stress that the value of hedging in this first case depends crucially on the assumption that investors cannot enter into such hedge contracts themselves. Although there may be several reasons why the access of investors to forwards and futures is limited (size of derivatives contracts is too large or creditworthiness of investors is too little), the assumption seems rather restrictive.

The second way hedging is beneficial to the firm's security design is by making the securities more attractive to a broader investment base thereby reducing the marketing costs associated with issuing these securities. It is important to stress that in this case investors do not have an incentive to mimic the firm's risk management since only firms carry marketing costs. Hence, here the assumption that access to risk management is restricted to firms seems less restrictive.[27] Therefore, we think that marketing costs may be important in explaining corporate risk management in relation to security design.

A characteristic of corporate risk management is that it is a more frequently used decision. Firms tend to purchase derivatives (often short-term) to a much greater extent than that they restructure or issue new securities. Although security design is sometimes combined with risk management (think of foreign currency denominated debt to hedge a firm's exposure) risk management is generally separate from the security design decision.

Another characteristic of corporate risk management is that it focuses on very specific (market) risks rather than general risk. An important reason may be that market risks can be transferred to the financial market in a relatively efficient way. It is efficient because there are very little contracting problems and as a result, the transfer of risk occurs at a relatively low price.

[27]However, it is still important to note that these forward contracts do not complete the market.

5.5 Limitations

The framework and the examples used in the previous sections were stylized. The only purpose we had in mind was to examine the potential role of risk management in a framework where risk sharing was important for the firm's security design. In this section, we will discuss the rather strict assumptions that were made and the impact they may have had on our results. To this end, we distinguish two sets of assumptions. First, those related to the framework we have chosen. Second, we consider those assumptions that are specific to our analysis. How restrictive are these assumptions and more importantly, how will they affect our results?

5.5.1 Framework specific limitations

We start with a discussion of the limitations of the basic framework we used. The repackaging of the firm's cash flows in line with investors' preferences is a central element in the analyses of both Allen and Gale (1988) and Madan and Soubra (1991). In our examples it was the firm that designed and issued the securities. One may also argue that a financial intermediary (e.g. an investment bank) could do this more efficiently. For example, in the Allen and Gale framework a financial intermediary could buy the securities of all firms, use these as collateral, and issue a multitude of securities against them. The financial intermediary would then market these new securities to those investors that value them most.[28] Moreover, the financial intermediaries might develop expertise in this area by doing it for a large set of firms and therefore economize on costs. Alternatively, it could use the cash flow rights of many firms in developing securities rather than those of one firm, thereby spreading risks.

There are some important reasons why we prefer to focus on firms designing the securities rather than financial intermediaries. First, it is unlikely that outsiders (financial intermediaries) and insiders (firms) share the same information with respect to the payoff of securities. If they do not share this information it becomes much harder for outsiders to achieve the same benefits.

Moreover such asymmetric information may create well-known adverse selection and moral hazard problems that are of great importance in financial contracting. An important part of the security design literature rationalizes financial contracts from this perspective. For example, Townsend (1979) shows that debt contracts are optimal for firms seeking external financing. In the presence of moral hazard, i.e. when firms have an incentive to be dishonest about cash flow realization, investors need to incur costs to verify the outcome ex post. With a debt-like contract these verification costs are minimized because, only in states where the firm cannot fulfill its

[28] This actually is the setup of Madan and Soubra (1991) and Pesendorfer (1995).

obligations, is verification required.[29] Other contracts (like equity) require verification over a wider range of cash flow realizations and therefore are more costly.

Nachman and Noe (1994), Boot and Thakor (1993) and Rahi (1996) study the security design decision when there is ex ante asymmetric information about the quality of the firm. It is well-known from the study by Myers and Majluf (1984) that asymmetric information causes mispricing and that such mispricing is most severe for equity. In Nachmann and Noe (1994) the analysis focuses on the role of debt as an optimal financing device for a firm with private information. Because debt contracts are relatively information insensitive, mispricing of a security issued by a firm with some private information is minimized with a debt contract. The authors derive conditions under which there is a pooling equilibrium where all firms issue debt.

Boot and Thakor (1993) also consider the financing problem of a firm when there is asymmetric information with respect to its value. They take, a now more or less, standard approach in finance where price and trading volume are determined by the interplay of traders (as in Kyle, 1985). In their model they have informed traders (those that pay to obtain information about the true fundamental value of the firm) and so-called liquidity traders (those that trade for other reasons, e.g. liquidity reasons). There is a competitive market maker to set prices and clear the market. The paper therefore combines the financing problem of a firm under asymmetric information with market microstructure elements. The key result in their paper is that undervalued (or high quality) firms have an incentive to design two securities rather than one composite security. One of these securities is completely information insensitive (debt) while the other is extremely information sensitive. Issuing these information sensitive securities stimulates information production (the reward for information production is higher with these securities). Overvalued (or low quality) firms mimic the undervalued firms since otherwise they will be screened out in equilibrium. Hence, there will be a pooling equilibrium where firms issue two securities.

Rahi (1996) combines in his analysis an ex ante asymmetric information problem as in Boot and Thakor (1993) and Nachmann and Noe (1994) with differences in risk preferences of investors. In order to be able to capture the benefits of risk sharing, the firm's securities should be designed, if at all possible, in such a way that it mitigates the negative effects of private information. This suggests a trade-off between the insurance gains from flotation on a stock market and the speculative gains resulting from private information.[30]

---

[29]Not only are verification costs minimized; the agency costs of not telling the truth under a debt contract are also minimized.

[30]In order to share risk with private information, the securities should be designed in such a way that this adverse selection problem is mitigated as much as possible. Rahi (1996) shows that the optimal security in such a world is equity (with unlimited liability). In equilibrium securities reveal all private information. See also Demanque and Laroque (1995).

Most of the papers described above consider the allocation of cash flow rights. A separate literature focuses on allocation of control rights as a motivation for security design (see Harris and Raviv, 1992, for a survey of this literature).

When one considers these rationales for security design, the comparative advantage of financial institutions in (re)designing financial securities becomes less obvious. Taking this into account, we have chosen to focus on the firm's security design problem rather than that of a financial intermediary. We are interested in the effect of the firm's hedging decision on the securities issued by the firm and think that in many cases the firm is in an optimal position to do the repackaging of claims on future cash flows.

A second common assumption that is typical for the literature we build on is that short sales are not allowed. Although one may argue that short selling securities is costly and not very common and therefore that the assumption of "no short sales" is not unreasonable, derivatives, like options and futures, enable investors to achieve short positions in a relatively cheap way. It is therefore interesting to consider the implications when we allow for (limited) short sales.

The short sale constraint in our model provides firms with a monopoly in security design. Monopoly rents are important because otherwise the firm has no incentive to design new securities, especially if this is costly. Allowing for unlimited short sales breaks this monopoly. By shorting securities investors can enlarge the supply of new securities developed by the firm but do not face the same costs as firms do to create such securities. Therefore short selling by investors will drive the monopoly rents of firms from security design to zero and takes away the firms' incentive to develop new securities. Firms will then only issue a composite security (unlevered equity) that pays off the firm's total income in each particular state (in equilibrium).[31]

The effect of *limited* short sales on a firm's security design problem is not completely clear. It depends on both the costs of security design by firms and on that of short selling by investors. For example, assume that investors face fixed costs of short selling. In that case investors can always enter into arbitrage by short selling. As long as their position is large enough, the above mentioned profits from arbitrage will make up for the fixed costs of short selling. Given the firm's costs of security design there will be no equilibrium with security design.

For an equilibrium with security design, investors should face increasing marginal costs of short selling. Consider the costs of short selling a convex increasing function, and the costs of security design for the firm sufficiently low. Then, we can have an equilibrium where the firm innovates and captures some of the benefits, and the investors by short selling expand the supply of the security limited by the increasing costs it faces.[32] If the costs of short selling are relatively

---

[31]Note that in such an equilibrium investors may be worse off; their risk sharing possibilities are reduced compared to equilibria with security design.

[32]For a more detailed discussion of the importance of the short sale constraint see Allen and Gale (1988, 1991, 1994).

low for investors and the costs of security design for firms are relatively high, the same situation will prevail as when short sales are allowed; there will be no security design in equilibrium.

5.5.2 *Model specific limitations*

Aside from the limitations following from the framework we have chosen, we can identify limitations that are due to the specific assumptions we have made in our analysis, especially those involved with respect to the forward transactions. First, we assumed that only firms could purchase hedge contracts. This is obviously far from reality. Forwards and futures contracts are among the most liquid financial securities. Although they require sufficient credit worthiness, it is hard to believe that investors have no access to these markets. Limiting the use of futures contracts to investors in relation to security design is more realistic when we allow for information frictions. Firms generally have private information with respect to hedgeable price risks. There are many reasons why firms may prefer not to disclose this information. For example, disclosure may not be desirable for strategic reasons, or it may simply be too costly. This makes investors' potential use of futures contracts for risk sharing purposes rather imprecise and puts firms in a much better position to hedge these risks. We therefore argue that restricting forwards and futures to firms for risk sharing purposes is not such an unrealistic approximation after all.

Note that if risk averse investors had full information about the state dependent payoffs of all securities and they could purchase forwards, they would have bought them most certainly in example 1b. This would however reduce the opportunities for security design. The opening of a futures market for all investors then would have the same impact as allowing for short sales; it may result in equilibria where there will be no security design. However, if in a world with marketing costs investors could purchase the same derivatives as firms, it is unlikely that the investors will copy the firm's decisions.

The second strong assumption in our example is with respect to the pricing of the forward contract. We assumed that forwards are priced against their expected values (and therefore implicitly assumed that the counterparty of these contracts was risk neutral). With a risk averse counterparty, firms should pay (or receive) a premium. What is actually needed is a model where the pricing of these hedging instruments is endogenous. This however is far from trivial, and several problems arise if one wants to model this in an incomplete financial market. First, it is well known that the introduction of a new class of assets may change the equilibrium decisions considerably in an incomplete asset market. For example, we have already argued that with a futures market there may very well be equilibria where there is no security design by corporations. Due to the presence of futures markets, firms will not issue multiple securities but only one composite security. This may even reduce investors' utilities such that with futures markets they are subsequently worse off. But the inclusion of a new set of assets not only changes the set of securities issued in equilibrium, it will also affect the market prices of the available securities. Generally it is assumed that derivatives are redundant and priced by arbitrage. However, this seems to ignore some externalities of the introduction of these derivatives. For example, Back (1993), considers the introduction of an option on a stock in a model that is more or less similar to Boot and Thakor (1993). He then shows that under certain conditions the introduction of an option on the stock causes the volatility of the underlying asset to be stochastic. "*The change in the price process reflects a change in the information transmitted to by volume and prices when the option is traded.*" The intuition is as follows. The informational content of a buy order of a call option is substantially different from that of the purchase of the underlying security. This implies that after the introduction of an option a richer class of signals will be received by the market.[33] The analysis by Back is important since it shows that the introduction of a derivative may have repercussions on the underlying price process. With respect to the case of options on stocks, this implies that options are not redundant anymore which may have important drawbacks for the pricing. The analysis of these issues in incomplete markets is still in its infancy.

For our simple analysis it is important to realize that the inclusion of derivatives in a model with security design may have externalities in a market where investors have private information. These externalities are not trivial and therefore force us to be modest about the robustness of our model.

A third assumption in our example is that we assumed hedging to be costless. Apart from a risk premium that may be included in the forward price, costs of hedging are a larger bid ask spread (that often increases with the maturity of the hedge) compared to spot transactions; a reduction in credit line (entering forwards binds some of the internal wealth); and additional costs for setting up a risk management department. Although hedging costs generally tend to be relatively small it is obvious that these costs make hedging an increasingly less attractive device of risk sharing; firms will hedge only if the benefits (higher proceeds from securities) are higher than the costs of hedging. Only if the costs of hedging are convex in the number of hedging contracts may we find some interior solution; firms do hedge to the point where the marginal costs of hedging is outweighed by the potential marginal benefit a firm can capture through improved security design.

5.6 Extensions

In Section 5.4 we studied risk management in a framework of security design. One of the characteristics of hedging that we mentioned (but not fully explored) there was about the timing

[33] In order to avoid having fully revealing equilibria, it is important to have so-called liquidity traders in both markets and the trades in these markets to be imperfectly correlated. This is an assumption in his model.

of hedging vis-a-vis the security design decision. We argued that firms hedge more frequently than that they issue securities and that hedging in general is relatively short-term. Although we ignored this feature in our simple one-period model, we think that some additional insights can be gained when we consider the timing of hedging versus that of issuing securities.

Securities cannot be changed without (high) costs. Hedging is generally a relatively cheap way to adapt the payoffs of a firm's securities. But why should it be necessary to change the securities' payoffs? One reason might be the resolution of uncertainty. In our example where risk sharing was the main motivation for security design, the original securities may after some time have a risk profile that do not perfectly match the preferences of security holders. It may then very well be in the interest of the security holders that the firm hedges to prevent undesired outcomes, especially if the investors are relatively sensitive with respect to which state exactly occurs. A change in the securities (recontracting with the investors) is costly. However, if hedging strategies may substitute for these decisions it may be a worthwhile and relatively cheap instrument.

For example, consider a starting firm that issues securities to finance its activities but has a lot of uncertainty. After some time, uncertainty becomes more predictable; for example, the firm becomes an important exporter to the US and therefore faces an exchange rate risk with respect to the US dollar. The firm's exposure with respect to the US dollar at the time the firm's securities were designed was unknown. After resolution of uncertainty the firms' hedging decisions can be directed to bring the securities' payoffs more in line with investors' initial expectations or current preferences. As an alternative the firm could have changed its financial structure. However, this seems a very costly alternative.

The timing also adds to the precision and the efficiency of the hedge. If hedging transactions occur too early, measurement errors may bring the firm into inefficient hedges (over- or underhedges). This may even be worse than not hedging.[34]

Finally, in our analysis we considered only cases where all states were identifiable. At date 0 all agents knew the firm's output in those states and investors' preferences were defined over these states. These are rather strong assumptions. In reality, we may have the case where there is a much finer partition of state output realizations. There may even be uncertainty over the exact distribution of output over these states or about the states themselves (in the sense that they are unidentifiable at date 0). What consequences might this have? Does this make the separation of the hedging and security design decision more likely? To answer these questions one needs a model that takes into consideration the sequential nature of risk management and security design.

[34] Dumas (1994) also has a similar rationalization why firms generally tend to hedge relatively late.

5.7 Conclusion

In this chapter we focused on risk sharing. More specifically, the basic idea we analyzed was how the firm's risk management affects or interacts with the design of securities. We identified two possible rationales for risk management, both related to security design. In a model of Allen and Gale (1988) we first showed that hedging may lead to higher proceeds from security design. With hedging firms can transfer income between states. This enables firms to develop securities that are better in line with investors' preferences such that they can raise the price of their securities. The restrictions under which this holds however are particularly strong.

We subsequently argued that a second benefit of corporate risk management is the reduction of marketing costs of securities. Risk management can be used to design payoff structures that attract a broader investor base. This reduces the marketing costs of security design considerably and increases the net proceeds for the firm. We feel this to be an interesting but preliminary result. Moreover, it is in line with the observation that firms use risk management to lower their equity base and issue more generic financing instruments (like debt).

The economic literature on security design is still in its infancy and generally very abstract. However, we think that it pays to continue research along this line. Current theories tend to neglect risk sharing issues and the interaction between a firm's security design and its risk management. In addition, including information frictions to the model, but allowing for risk management may increase our understanding of corporate financing practices.

There are some additional motivations why investors may prefer to hedge. For example, hedging may be beneficial since it increases the transparency of a firm's security. By taking out some of the noise that is hedgeable, a security's payoff more closely mirrors the performance of a firm.

The timing of the hedge in relation to security design is also interesting. In considering this issue, a more dynamic model is required. Progress in this direction may increase our understanding of the role of hedging in developing optimal securities (from a risk sharing point of view).

5.8 Appendix

5.8.1 Appendix 5.1

This appendix derives the characteristics of an equilibrium in which firms issue one composite security (example 1a).

With the marginal state valuations derived in the text and the initial security, we can derive the maximum value that an investor of type i is willing to pay for a security that pays $r^j(s)$ at date 1 in terms of consumption goods at date 0.

If all firms issue only one composite security, unlevered equity, with state dependent payoff at date 1 equal to $r^1(s) = y_s = (4, 1)$, and if we normalize the costs of such a financial structure to 0 for the firm, the proceeds of such a security depends on to whom it is to be sold. To maximize the proceeds the firm should sell it against the highest possible price, that is to the investors that value the security most (has the highest marginal utility of consumption). The state prices for the risk neutral investors p_{ns} is equal to 0.4 for every state. Given the endowments of the risk averse investors, if the utility of consumption of the risk averse investors is larger than 0.4 for every state, then risk averse investors will hold all securities. However, as a starting point consider the case where both investor types hold the security with payoff $r^1(s) = (4, 1)$ in equilibrium. This can only happen if the market price of that security reflects risk neutral investors' valuations of the payoffs of that security. Otherwise, the risk neutral investors will never hold the security. Hence, the price of the security in equilibrium, given that risk neutral investors will hold some of that security, should be equal to

$$v^1 = \sum_s 0.4 \times r^1(s) = 2$$

What can we say about the risk averse investors? They hold the securities in equilibrium as well. Both investor types can only hold the security if it has a price equal to 2. Given this market price, what proportion of the securities will the risk averse investors hold? A risk averse investor's demand for the security denoted by α, directly follows from $v^1 = 2 = \sum_s \frac{1}{2}\frac{2}{1+\alpha * r_s} r^1_s$. Substituting gives that the risk averse investors will hold half of the supply of the securities while risk neutral investors will hold the other half. Using this we can calculate the equilibrium state prices (see Table 5.1). Note that the risk averse investor's state prices (p_{as}) in Table 5.1 equals $\frac{1}{2}\frac{2}{1+\alpha * r_s}$, where αr_s is the total payoff from securities that the risk averse investors receive and consume in a particular state.

5.8.2 Appendix 5.2

This appendix derives the equilibrium where the costs of security design are $C(2) = 0.1$.

In equilibrium, there will be firms that issue one security and have zero costs of security design and other firms that issue the extreme securities as specified above. Both types of firms will have the same market value. The market value of the firms that issue one security remains 2. To have an equilibrium where some firms issue one security and others the two extreme securities, requires that the market value of the firm with two securities (after costs) is also 2. The firm's securities market value before costs therefore is $2 + 0.1 = 2.1$.

Since r^1 is sold to the risk neutral investors at a price equal to 1.6, the price of the firm's second security will be pushed to 0.5 to compensate the firms for the costs of security design. With these figures, we can calculate the proportions of firms issuing one security and of risk averse investors that are holders in the one security firms to reach an equilibrium. These are

0.41667 and 1 respectively.[35] Furthermore, we can calculate the marginal utility of consumption of the risk averse investors in both states (p_{as}). Table 5.2 summarizes the equilibrium when the costs ($C(2)$) are equal to 0.1.

The market value of firms that issue one security $(4, 1)$ is equal to that of the firms that issue the two extreme securities $\{(4,0), (0,1)\}$ and incur the costs of security design. Both will have a market value equal to 2. The extreme security with payoff in the first state $r^1 = (4, 0)$ will be sold completely to the risk neutral investors and have a market value equal to 1.6. The other extreme security with payoff $r^2 = (0, 1)$ will trade at a price equal to 0.5 and will be sold to the risk averse investors. The market value of the firm issuing the two extreme securities and selling these to the investors that value the security most is thus $1.6+0.5 = 2.1$. Net of transaction costs, this equals the market value of a firm issuing one security. Note that in equilibrium, risk averse investors have gained in the sense that compared to Table 5.1, the difference in the marginal utility of consumption between the different states has become smaller. The benefits of security design in this competitive setting will fall to the investors and not to the firms.

5.8.3 Appendix 5.3

In this appendix we provide a slightly more formal model of hedging in a risk sharing framework, first in an Allen and Gale type of analysis and then in a model with marketing costs as in Madan and Soubra (1991).

Allen and Gale

In Allen and Gale, firms choose a financial structure that maximizes the security proceeds (market value given a financial structure $MV(F_k)$) net of the costs of issuing multiple (k) securities ($C(k)$) or

$$\max_{F_k} MV(F_k) - C(k)$$

where F_k is the set of k securities issued by the firm. Hence, $F_k = \{r^j, ..., r^k\}$, such that:

$$\sum_{j=1}^{k} r^j = y_s$$

[35]Denote b as the proportion of firms that issue one security and α as the risk averse demand for the unlevered equity. Then the values given in the text have been calculated by solving the following two equations simultaneously: $2 = \sum_s \pi_s * \frac{2}{1+\alpha b r_s^1 + (1-b) r_s^2} r_s^1$ and $0.5 = \sum_s \pi_s * \frac{2}{1+\alpha b r_s^1 + (1-b) r_s^2} r_s^2$. We thereby restrict α and b to assume values between 0 and 1.

where y_s is the output vector of the firm, and r^j is the payoff vector of the j-th security. To maximize this, firms should issue securities against the highest price.

Investors choose securities that maximize their expected utility. (Investors have time additive Von Neumann Morgenstern utility functions). The investors' state valuations p_{is} therefore are equal to:

$$p_{is} = \frac{\partial EU_i(c_0, c_1)/\partial c_1(s)}{\partial EU_i(c_0, c_1)/\partial c_0}$$

To maximize the proceeds from security design, firms should design and sell these securities at the highest price. The value of security j is

$$v^j = \arg\max_i \sum_{s \in S} p_{is} r^j$$

and the market value of the firm with a financial structure F_k then is equal to

$$MV(F_k) = \sum_{j=1}^{k} v^j$$

Finally, we need a specification of the costs of security design. In Allen and Gale these costs ($C(k)$) are a step function of the number of securities (k) issued by the firm.

How would hedging contribute to increase firm value? Note first that hedging expands the opportunity for the firm to issue a larger set of securities. Since

$$\sum_{j=1}^{k} r^j = y_s + \phi Z_f$$

where Z_f is the payoff of one hedge contract over the different states $s \in S$, the payoff of ϕ hedge contracts is equal to ϕZ_f and the price of the forward contract satisfies the following condition: $E(Z_f) = 0$.

Given this enlarged opportunity to issue securities, the firm's maximization problem becomes

$$\max_{F_k, \phi} MV(F_k) - C(k)$$

Madan and Soubra

In Madan and Soubra, firms maximize the proceeds from securities (market value given a financial structure $MV(F_k)$) net of issuing costs of securities. However, Madan and Soubra consider marketing costs of issuing securities that not only depend on the number of securities (k) but also on the types of securities that the firm issues: ($C(F_k)$). As a result a firm's maximization problem turns into

$$\max_{F_k} MV(F_k) - C(F_k)$$

where F_k is again the set of k securities issued by the firm. Also, $F_k = \{r^j, ..., r^k\}$, with

$$\sum_{j=1}^{k} r^j = y_s$$

where y_s is the output vector of the firm as before.

The maximum price that investor i is willing to pay for the j-th security depends on the investors' state valuations p_{is}. When investors choose securities that maximize the investor's expected utility and investors have time-additive Von Neumann Morgenstern utility functions) these are again equal to

$$p_{is} = \frac{\partial EU_i(c_0, c_1)/\partial c_1(s)}{\partial EU_i(c_0, c_1)/\partial c_0}$$

and thus,

$$v^j(i) = \sum_{s \in S} p_{is} r^j$$

The market value of the firm with a financial structure F_k is equal to

$$MV(F_k) = \sum_{j=1}^{k} v^j$$

In Madan and Soubra, the marketing costs of a financial structure also depends on the price against which the claims are being issued. The higher the price of the security, the harder it is to sell these securities and thus also the higher the marketing costs. Marketing costs can be modelled in two ways.

Consider first the costs if the firm can approach investors only once. If the price is too high the security remains unsold. In that case the marketing costs of a financial structure F_k is equal to

$$C(F_k) = \sum_{j=1}^{k} v^j G(v^j)$$

where v^j is the price against which the j-th security is being offered and $G(v^j)$ is the proportion of potential buyers where $v^j(i) < v^j$.

Alternatively, we can model the marketing costs as some small fixed transaction cost c per approach. If we again define $G(v^j)$ as the proportion of potential buyers where $v^j(i) < v^j$, then the expected total marketing cost will be equal to

$$C(F_k) = \sum_{j=1}^{k} \frac{c}{1 - G(v^j)}$$

Hedging again expands the opportunity for the firm to issue a larger set of securities. Since

$$\sum_{j=1}^{k} r^j = y_s + \phi Z_f$$

With the opportunity to hedge firms maximize

$$\max_{F_k, \phi} MV(F_k) - C(F_k)$$

Now the firm will not only use hedging to increase the value of the claims but may also use hedging to reduce the marketing costs of the financial securities.