Corporate Financial Risk Management

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Risk management and product markets

6.1 Introduction

We now have many theories that rationalize corporate risk management. However, in Chapter 4 we concluded that the existing theories are still too incomplete to adequately describe corporate risk management practices. We argue in this chapter that a study of the interaction between corporate risk management and competition on product markets may fill some of the gaps in the existing literature. Interesting questions emerge when considering such interaction. For example: Does a firm’s risk management strategy affect its strategic operating decisions (output, pricing, investment decisions)? Are there interactions between competitors’ risk management strategies? And are these strategies related to the structure on the product market?

The purpose of this chapter is to shed light on these questions. We focus on the impact of corporate risk management on a firm’s output decisions in a market with imperfect competition via the amount of internal wealth (financial slack). Risk management decisions today affect the availability of financial slack at future points in time and this may provide a link to the firm’s real decisions. Part of this mechanism has already been spelled out in Froot, Scharfstein and Stein (1993). In their study they suggested that their framework “...has implications for how companies’ hedging strategies should depend on both (1) the nature of product market competition, and (2) their competitor’s hedging strategies.” (Froot, Scharfstein and Stein, 1993, p. 1650). In this chapter we develop a model to study the interaction between risk management and product market decisions more closely.

Costly external financing or the inability to attract external financing may affect a firm’s strategic output decisions, especially in markets with imperfect competition. In this paper we show that risk management affects the strategic side effects of costly external financing and explore the interaction with respect to a firm’s optimal hedging and output choices.
We capture these issues in a two-stage game. In the first stage two firms simultaneously decide how much of their exposed internal wealth they will hedge. Then in the second stage the two firms simultaneously decide how much to produce. We therefore have a model with Cournot competition. Firms in our model however, need to finance the variable production costs. They can either do this with internal funds or - if available - attract external funds. External financing makes a firm a less aggressive competitor and therefore lowers both firms’ output choices. Risk management, in the first stage now also affects both firms’ expected profits; first through its impact on the expected costs of financing but also through its impact on both firms’ optimal output decisions. As a result, we find strategic interaction with respect to both firms’ hedging strategies. When both firms face serious constraints, such that external financing is difficult, we have an equilibrium where both firms hedge. However, in cases where financing constraints are less important, or where firms have more internal wealth, non-hedging is the equilibrium risk management strategy by both firms.

There are many real world anecdotes supporting the focus of our analysis. Companies sometimes motivate their hedging behavior by explicitly referring to the competitive consequences hedging may have on their product market strategies. Often this is related to the potential impact of corporate risk management on the firm’s market share. For example, in a recent roundtable conference on corporate risk management in the Journal of Applied Corporate Finance1, the treasurer of R.J. Reynolds Tobacco Company suggested that with a 100% hedging policy a firm can be just as exposed as when it does not hedge. Her argument was that if you are completely hedged and the exchange rate movement turns out to be favorable, while your competitor has left its exposure open, than your ability to gain market share has suffered.2

Belk and Glaum (1990, p. 7), found in an in-depth survey of risk management within UK multinationals that “... their decisions to hedge critically depend on what their competitors did (or what the companies thought their competitors did).” Also, in a detailed field study by Brown (2000) it appeared that the hedging strategies of a US manufacturing firm were affected by concerns related to the potential impact of hedging on its pricing strategy and market share. Moreover, the firm spent considerable resources in trying to generate information about both the main competitors’ exposure as well as their hedging strategies. Thus, it seems practitioners are well aware of this interaction between hedging and product market competition. The current

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2In an interview with a Dutch newspaper, the treasurer of KLM at that time, De Die, referred to the potential of loosing market share for the firm that did not hedge while its competitors hedged after an unfavorable change in the exchange rate (Het Financieele Dagblad, March 15, 1996, p. 15).
literature on corporate hedging, however, has ignored these effects and thus far provided no rationale for such (strategic) behavior. In this chapter we present such a rationale.

Our work is related to two strands in the literature. First, there is now a considerable literature that studies the impact of exchange rates on industry equilibrium and the firm's exposure to exchange rates in markets characterized by imperfect competition. Although this literature increases our understanding of a firm's economic exposure (especially in relation to market structure), it does not provide a rationale for such risk management in itself. Moreover, these papers assume that corporate hedging does not affect outcomes on product markets. The results of our analysis, however, suggest that corporate hedging under certain conditions may affect product market equilibrium.

Our work relates to a second strand of the literature that studies the interaction between financing and product market decisions. For example, Brander and Lewis (1986) argue that a benefit of debt financing is that it commits the firm to be more aggressive in the product market. Telser (1966) and Bolton and Scharfstein (1990) on the other hand, argue that debt may invite predatory strategies by competitors and therefore negatively affect firm value. These contributions generally consider the (strategic) interaction between capital structure and product market behavior but do not consider other financial decisions. In our model, hedging affects (the need of) costly external financing and reduces the volatility in internal funds. This not only reduces the expected funding costs, but also affects the outcome on the product market. As a result, hedging affects expected profits. Moreover, we find strategic interaction between two firms' risk management strategies.

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3 An exception is the study by Dumas (1994). He suggests that the passing through of exchange rate changes into prices may be influenced by the hedging decision of firms in the market. As a result, a firm's optimal hedge depends on its competitors' hedging decisions.

4 There may also be other explanations for this observation. For example, it might be the case that firms do not really know what to do when it comes to risk management. Then, it may make sense to look at your competitors in order to have something to go by.

5 Dornbusch (1987) and Froot and Klemperer (1989), for example, show that exchange rate changes do affect output prices when there is imperfect competition. Luehrman (1990), Von Ungern-Stemberg and Von Weizsacker (1990), and Marston (1996) all study a firm's currency-exposure in a market with imperfect competition. Adler (1994) incorporates a multi-period model of imperfect competition in a study of a firm's exposure.

6 An exception is Maksimovic (1990). He rationalizes a firm's decision to enter into a loan commitment contract with a bank from the impact that this loan commitment has on the outcome on the product market. The loan commitment reduces a firm's future marginal financing costs and therefore changes the firm's optimal output decisions. By entering into a loan commitment, a firm credibly commits to a more aggressive output strategy on the product market in the future.
We proceed as follows. In Section 6.2 we discuss some related literature that has focused on the interaction of product market competition and financial structure. Since, both this chapter and Chapter 7 focus on the impact of hedging on this interaction such a discussion is important. Section 6.3 presents the basic model. In Section 6.4 we derive our main results. Section 6.5 illustrates some key results with simple numerical examples. In section 6.6 we put our results in perspective and discuss their robustness. Section 6.7 summarizes and concludes.

6.2 Interaction of financial structure and product market competition

In this section we shortly review the literature on the interaction between financial structure and product market competition. Since hedging, both in Chapters 6 and 7, interferes with the interaction on financial structure and product markets such a survey helps to more closely define how these chapters will add to the literature. Maksimovic (1995) distinguishes four main approaches in this literature.

The first (and also earliest) approach discusses the role of financial structure in making credible implicit and explicit contracts with customers and rival firms. More debt decreases the firm’s ability to enter such (valuable) contracts. It increases the probability of default, thereby reducing the value of implicit contracts with customers and employees (Titman, 1984). Too much debt, a debt overhang, makes equity holders behave more opportunistically, thereby further reducing the value of such implicit contracts. An example of such behavior is that firms with a debt overhang may lower their product quality. Since customers anticipate this, they will try to protect themselves against such opportunistic behavior by paying less for the good. This will leave shareholders generally worse off thereby reducing the attractiveness of debt (Maksimovic, 1988 and Maksimovic and Titman, 1990). Corporate hedging may help preserve the value of implicit and explicit contracts with customers and employees and as such increase firm value (see Chapter 2 and Shapiro and Titman, 1992). The potential impact of hedging on a firm’s strategic behavior in product markets, however, is ignored and therefore does not produce any insight in the potential impact of hedging strategies on market share, nor does it provide a rationalization of the interest firms have in their competitors’ hedging strategies. In contrast, our work focuses particularly on these issues.

The second approach shows that the competitive environment of the firm is important in understanding firms’ strategies. These models are often called industry equilibrium models. Maksimovic and Zechner (1991) is a good example. It is widely known that more debt induces a firm’s shareholders to increase the riskiness of their activities. Maksimovic and Zechner (1991) show that considering the choices of competing firms in equilibrium is important in finding the product market strategy of a single firm that increases the firm’s risk. They consider an industry equilibrium model where a production strategy that for a single firm is risky (not taking industry
effects into consideration), may very well be the non-risky strategy when considering product market equilibrium. As a result, considering the choices of competing firms may cause some highly unlikely strategies - when seen from the standpoint of an isolated firm - being very rational. The optimal risk management decisions in industry equilibrium models have not been analyzed yet.

The third approach stresses the strategic commitment effects of debt when there is imperfect competition in the product market. The limited liability nature of equity induces shareholders to value income in the higher states of the world more than in the low states since most of the increase in wealth is then attributed to the firm’s debtholders (Brander and Lewis, 1986 and Maksimovic, 1988). As a result, firms with more debt commit to a more aggressive (higher) output strategy. Rival firms generally will respond with lower output. The existence of bankruptcy costs will induce more or less the same behavior (Brander and Lewis, 1988).

Finally, the fourth approach focuses on the relationship between the financial contract and product market strategies. More in particular, the central question in these papers is: “How does the existence of financial contracts which are motivated by agency problems open aggressive competition by rival firms?” Bolton and Scharfstein (1990) for example, show that an optimal financial contract that mitigates managerial incentive problems terminates funding when performance is poor. When taking the actions of rival firms into consideration this may not be the optimal financial contract since it will invite rival firms to prey on the firm. For example, rival firms may enter into aggressive product market strategies to make the financial constraints binding and induce exit of the firm. An important result is that, when taking product market strategies of other firms in consideration, financial contracts will be less sensitive to performance.

A recent contribution to this approach by Chevalier and Scharfstein (1996) has addressed the interaction between financial constraints and product markets in a more dynamical model, where building market share is important to generate future output. They show that, especially during recessions, liquidity-constrained firms will be more short-term oriented compared to less financially constrained competitors. As a result, they predict that more leveraged firms will be less inclined to build up market share and therefore will set higher prices compared to their less financially constrained rivals. Rival firms will also “respond” with higher prices, but since their focus is more on building market share they do not increase their prices too much. An empirical prediction from this literature is that if most firms in an industry are externally (internally) financed, the industry markup will be more counter-cyclical (pro-cyclical).

The third and the fourth approach in this literature have opposing empirical predictions with respect to the consequences of debt financing. In a Brander and Lewis type of model, more debt

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7 At least this will be the case in a Cournot model with strategic substitutes.

8 This was already pointed out by Telser (1966).

9 Because prices are strategic complements.
will induce a firm to be more aggressive (and its rivals less aggressive), while in a Bolton and Scharfstein model, more debt in a firm is expected to make rivals more aggressive.

Although the empirical evidence is still limited, it generally rejects the Brander and Lewis argument. Firms with more debt do not become more aggressive players in the product markets. Firms that have increased their leverage tend to reduce their output and increase prices. Rival firms tend to become more aggressive after such a recapitalization of a firm in the industry. Firms that entered a Leveraged Buy Out (LBO) reduce their investments (and capacity) after they have increased their leverage. Rival firms, however, increase their capacity and investments after such a recapitalization. Moreover, after a recapitalization more firms tend to enter the market (Chevalier, 1995 and Philips 1995). This suggests that a higher debt level invites more aggressive competition. Also Opler and Titman (1994) have shown that highly leveraged firms in distressed industries loose market share to their less levered competitors. Kovenock and Philips (1997) have further shown that recapitalizing firms in an industry also generally have the least productive plants and that capacity utilization increases after a recapitalization. The effects of increasing leverage on investment and plant closure in concentrated industries was significant; recapitalizing firms tend to cut back on investments and increase plant closure. Finally, Campbello (2000) recently found support for the Bolton and Scharfstein (1990) and Telser (1966) arguments. Debt has a negative impact on a firm's sales, especially when rival firms are relatively less levered during recessions. Furthermore, Campbello (2000) found that markups were more counter-cyclical the higher the amount of debt in an industry. This evidence supports Chevalier and Scharfstein (1996).

In this chapter, we will analyze whether there is a rationale for a firm to engage in risk management that is related to product markets. In our model risk management mitigates some of the effects of financial contracting on the product market. Our work therefore fits nicely with the literature we have just described.

6.3 The basic model

6.3.1 General outline and timing

Consider a risk neutral world where two firms labeled A and B compete for a homogeneous good on a product market characterized by imperfect (Cournot) competition.\(^{10}\) At date 1, both firms simultaneously choose the optimal production level and incur production costs. To finance these production costs, firms either rely on internal financing or (if possible) attract costly exter-

\(^{10}\)See Tirole (1988) for further discussion.
Firms decide how much to hedge

Uncertainty is resolved
Hedging contract is settled
Firms decide how much to produce
When necessary, firms attract external finance

Profits are realized
Firms repay their debt

FIGURE 6.1. Timing of the model

nal financing at date 1. The availability of internal wealth is uncertain since we assume that part of it is exposed to some risk factor.\textsuperscript{11} Firms can hedge this risk at date 0 with forward contracts.

Our analysis focuses on the relation between the output choice in the product market at date 1 and the firm's hedging decisions at date 0. Figure 6.1 sketches the sequence of events. We have a two-stage model where firms at date 0 simultaneously choose a hedge ($\phi^i$ where $i = A, B$), rationally anticipating the output decisions at date 1. At date 1, prior to the output decision, all uncertainty with respect to the risk factor is resolved and the hedging contract is settled. Given the level of internal wealth available and the costs of external financing, both firms simultaneously choose the level of output $q^i$. If necessary they attract additional (costly) external financing to pay the production costs. Finally, at date 2 firms receive income from sales, repay their debts (plus deadweight costs of external financing), and the model ends.

Firms in our model maximize total firm value.\textsuperscript{12} Below we provide a more detailed setup of our model. We first discuss the product market, then the financial markets, and finally the equilibrium concept relevant for our analysis.

\textsuperscript{11}From an investors' point of view (part of) this risk is diversifiable, which makes risk neutrality a somewhat innocent assumption.

\textsuperscript{12}In our model we do not consider uncertainty over costs or demand. Models with uncertainty over demand may give rise to a limited liability or bankruptcy effect of debt. Since shareholders do not take the bad outcomes into consideration they will enter into more aggressive strategies when financed with debt (see Brander and Lewis, 1986 and Showalter, 1996). Showalter (1996, 1999) further has shown that with uncertainty over costs, there is no strategic effect of debt. We do not want to focus on these strategic commitment effects here and therefore assume that at date 1, before the output decisions have been taken, all uncertainty is resolved. Alternatively, we can assume that firms in our model have no significant debt outstanding (equity financed).
6. Risk management and product markets

6.3.2 The product market

We describe competition on the product market with a simple Cournot model. More precisely, we consider a world where two firms labeled $A$ and $B$ compete for a homogeneous good. At date 1 both firms simultaneously choose quantities that maximize total firm value given the other firm's optimal production decision. We characterize demand on the product market by a simple linear inverse demand curve,

$$p(q^A, q^B) = a - b(q^A + q^B)$$

where $p$ is the equilibrium market clearing price of the good and $a$ and $b$ are constants both larger than or equal to zero. The output brought to the market by firm $A$ is denoted by $q^A$ and that of firm $B$ by $q^B$.\(^{13}\)

Both firms face total production costs characterized by

$$K^i(q^i) = c^i q^i$$

where $i$ denotes firm $A$ or $B$.

In our model these production costs play a central role. Production costs are made at date 1 and need to be financed either with internal wealth or externally.\(^{14}\)

6.3.3 The financial markets

The capital market

In our model both firms need to finance their production costs at date 1. These can be financed either from internal wealth ($w^i_1$) or with external funds. The firm's internal wealth available at date 1 is a function of some random marketable risk factor $\bar{\varepsilon}$ (e.g. an exchange rate) uniformly distributed over the support $[\underline{\varepsilon}, \bar{\varepsilon}]$, the firm's exposure $\theta^i$ to this risk factor, and some exogenous part (unexposed to the risk factor) $w^i_0$. More precisely, we describe the level of internal wealth of firm $i$ at date 1 as $w^i_1 = w^i_0 + \theta^i \bar{\varepsilon}$.\(^{15}\) This coincides with the case where the firm has receivables

\(^{13}\)We have chosen a relatively simple setting in which both firms produce and sell in a single currency. The exposure is exogenous; in our model a firm has internal wealth that is exposed to hedgeable price risk. Our motivation for this choice is twofold. First, exposures become complicated in an alternative setup where one firm, for example, is producing abroad especially with imperfect competition (see for example Luehrman, 1990 and Marston, 1996). Secondly, we want to stress the link between corporate risk management and financial contracting in this dissertation. For that purpose it is sufficient to have a rather simple specification of a firm's exposure.

\(^{14}\)An alternative interpretation of these production costs is as an investment in capacity. The investment in capacity (equal to $cq$) then determines how much the firm can produce in the second period.

\(^{15}\)In Chapter 2 (Section 2.4) we modelled the firm's exposure in a similar manner.
of which part is in foreign currency and therefore faces uncertainty with respect to the amount of internal funds available at date 1. We assume that firms have positive exposure to the risk factor so that high values of $\varepsilon$ correspond with high values of internal wealth.

Firms can also finance production costs externally. However, we assume that external financing is costly. There are a number of ways to model market frictions that make external financing costly. For example, Myers and Majluf (1984) show that if firms have private information they cannot credibly convey to the market, they will only be able to attract external financing at a discount. As a result, firms prefer to finance through internal means first. In our model we have decided not to model the costs of external financing explicitly. The major reason is that it would complicate the model considerably. Although it may add to our understanding we leave this for future work. We assume that external financing is costly and that the costs are an increasing function of the amount of external financing $C'(\varepsilon)$. More specifically, we consider two cases: first, we consider the case where firms face constant marginal financing costs ($C'(\varepsilon) = \alpha\varepsilon^4$); second, we consider the case where firms face convex external financing costs ($C'(\varepsilon) = 1/2\varepsilon^2$).

The first case holds for firms with good access to financial markets. The second case is representative for a firm with limited abilities to find external financing.\(^\text{16}\)

Since external financing is costly, in our model firms prefer internal financing. Only when internal wealth is exhausted, will firms switch to external financing. Therefore, the amount of external financing that firms attract is equal to $\varepsilon = \max\{0, c^i q^* - w^i\}$, where $q^*$ is the optimal date 1 output decision, $c^i$ the marginal production cost and $w^i$ the amount of internal wealth available at date 1 for firm $i$.

The uncertainty over the amount of internal wealth is resolved prior to the moment that both firms choose their output. Therefore, when both firms make their output decisions (at date 1) there is no uncertainty with respect to the amount of internal financing available to both firms.

The forward market

Both firms’ date 1 internal wealth is exposed to some uncertain risk factor $\varepsilon$. At date 0, both firms have the opportunity to simultaneously enter into hedging this risk. We consider linear hedging contracts only (forwards or futures). We will denote the size of the hedge contract entered by firm $i$ with $\phi^i$. This contract specifies the amount of foreign currency that has to be sold against the forward price $f$ at date 1. The payoff from a forward contract entered upon at date 0 is consequently $\phi^i Z_f$, where $Z_f (= f - \varepsilon)$ is the payoff from the forward contract at date 1. We assume that the expected value of the payoff from the forward contract at the entry date (date 0) is zero ($E(Z_f) = 0$), and thus $E(\varepsilon) = f$.

\(^{16}\)A convex cost function can be motivated by both the theory put forward in Myers and Majluf (1984) but also using Townsend’s (1979) theory of costly external financing. For further discussion see also Chapter 2.4.2 and Froot, Scharfstein and Stein (1993).
After the realization of the risk factor, at date 1, but before production decisions are made, both firms’ hedging strategies (and the amount of internal wealth) become common knowledge. We do not allow for the possibility where one firm learns about the hedging behavior of another firm before making its own hedging decision.

6.3.4 *Equilibrium concept*

In the case of a one-stage production game the concept of a Nash equilibrium would be sufficient. However, we have defined a two-stage sequential model, where firms decide simultaneously on hedging in the first period and then simultaneously on production. The hedging decisions (and thus the amount of internal wealth that both firms have, become common knowledge at date 1 prior to the production decisions. Since we assume that firms take the hedging decisions at date 0 in such a way that they fully anticipate the effect of their hedging decisions on the outcome of the product market, we need a more narrow equilibrium concept to rule out implausible behavior. We therefore will use the concept of subgame perfect equilibrium as defined by Selten (1977). To satisfy such an equilibrium, a strategy should satisfy that: (1) it is a Nash equilibrium for the entire game and (2) relevant action rules are a Nash equilibrium for every subgame.

An equilibrium is given by a combination of output and hedging decisions of each firm: \((q_i^A, \phi_i^A), (q_j^B, \phi_j^B)\) such that \((q_i^*, \phi_i^*)\) maximizes expected firm value given the optimal decisions of the other firm \((q_j^*, \phi_j^*)\) for \(i = A, B\) and \(j = A, B\) and \(i \neq j\).

In the next section we solve the model through backward induction. First, we derive optimal production decisions in the product market given each firm’s internal wealth and first-stage hedging decision. Then, moving backwards, we derive the firm’s optimal hedging decision.

Firms choose output in the second stage that maximize profits. The costs of external financing \(C^i(e)\) will affect the product market equilibrium. In the first stage firms choose the size of the hedge that maximizes expected profit. Hedging decisions (may) affect the cost of external financing and thus the outcome on the product market. We will explore under which conditions hedging is an equilibrium strategy and how such hedging decisions affect outcomes on the product market.

6.4 Interaction between corporate hedging and product markets

In this section, we solve the model backwards in two stages: first, we solve the equilibrium on the product market given both firms’ initial hedging strategy; second, we solve for the optimal first stage hedging strategy.
6.4 Interaction between corporate hedging and product markets

6.4.1 Second stage product market equilibrium

At date 1 both firms \((i = A, B)\) choose output that maximize profits

\[
\max_{q^i} \pi^i(q^i, q^j) = p(q^i, q^j)q^i - c_i q^i - C_i(c^i)
\]

taking the output decisions of the other \((j = A, B\) and \(i \neq j)\) as given.

Now, consider first the case where both firms are financially unconstrained and therefore do not need external financing. In that case \(C^i(c) = 0\). The first order conditions of the maximization problem are enough to derive the equilibrium outcomes if the second order conditions are satisfied.\(^{17}\) The first order conditions of this unconstrained problem are

\[
\begin{align*}
\frac{\partial \pi^A}{\partial q^A} &= p(q^A, q^B) + p_A(q^A, q^B)q^A - c^A = 0 \quad (6.1) \\
\frac{\partial \pi^B}{\partial q^B} &= p(q^A, q^B) + p_B(q^A, q^B)q^B - c^B = 0
\end{align*}
\]

where \(p_i\) denotes the partial derivative of the inverse demand curve with respect to \(q^i\). Using the two first order conditions we can derive the implicit reaction curves \(r^A\) and \(r^B\) for firms \(A\) and \(B\), respectively. Firm \(A\)'s reaction curve \((r^A)\) specifies the firm's optimal output decision as a function of firm \(B\)'s output decision. These reaction curves are respectively

\[
\begin{align*}
& r^A : q^A = \frac{1}{2} q^B + \frac{(a - c^A)}{2b} \\
& r^B : q^B = \frac{1}{2} q^A + \frac{(a - c^B)}{2b}
\end{align*}
\]

Figure 6.2 plots these two reaction curves. Note that both curves are downward sloping. Firm \(A\) will produce more as \(B\) produces less and vice versa. In the terminology of Bulow, Geanakoplos and Klemperer (1985) we therefore consider strategic substitutes.\(^{18}\) The point where the two reaction curves cross is the unique Nash equilibrium of this output game. Substituting the two reaction curves into each other gives the Nash equilibrium outputs

\[
\begin{align*}
& q^{A*} = \frac{(a - 2c^A + c^B)}{3b} \\
& q^{B*} = \frac{(a - 2c^B + c^A)}{3b} \\
\end{align*}
\]

Firm \(A\)'s optimal production decision depends on the demand function \((a)\) and the marginal

\(^{17}\)See appendix 6.1 that this is indeed the case.

\(^{18}\)Alternatively, with strategic complements the reaction curves are upward sloping. This implies that an increase in the output by one firm will also induce the competitor to increase its output.
costs of both itself and that of firm B. Note that $q^A*$ is negatively related to its own marginal costs ($c^A$) and positively related to its competitor’s marginal cost ($c^B$). This is the standard Cournot equilibrium when for both firms the amount of internal wealth available at date 1 ($w_1^i$) satisfies respectively $w_1^A \geq \frac{1}{3} (a - 2c^A + c^B) \frac{c^A}{b}$ and $w_1^B \geq \frac{1}{3} (a - 2c^B + c^A) \frac{c^B}{b}$.

Now consider the impact of external financing. With external financing the first order conditions have an additional term

\[
\frac{\partial \pi^A}{\partial q^A} = p(q^A, q^B) + p_A(q^A, q^B)q^A - c^A - C^A_e \frac{\partial c^A}{\partial q^A} = 0 \\
\frac{\partial \pi^B}{\partial q^B} = p(q^A, q^B) + p_B(q^A, q^B)q^B - c^B - C^B_e \frac{\partial c^B}{\partial q^B} = 0
\]

where $C^i_e$ the partial derivative of firm $i$’s cost of external financing with respect to $e$. Using that $\frac{\partial c^i}{\partial q^i} = c^i$ if $e > 0$, we can rewrite the two first order conditions and derive:

\[
q^A = -\frac{1}{2}q^B + (a - (1 + C^A_e)c^A)/2b
\]

\[
q^B = -\frac{1}{2}q^A + (a - (1 + C^B_e)c^B)/2b
\]

Notice that the last term is equal to zero when the firm does not need external financing in equilibrium. Then, $C(e) = 0$ and thus also $C_e = 0$. 

FIGURE 6.2. Reaction curves of firms $A$ and $B$ when both firms are financially unconstrained.
FIGURE 6.3. Reaction curves of firms A and B when firm A faces constant marginal financing costs at high levels of output.

Constant marginal financing costs

With constant marginal external financing costs \( C_e^A = \alpha^A \), the reaction curve shifts down when the firm needs external financing (see Figure 6.3). As a result a firm will produce less compared to if it had sufficient internal funds to finance the production costs.\(^{19}\)

If both firms need external financing in equilibrium the equilibrium output decisions are

\[
\begin{align*}
q^A &= \frac{a - 2c^A(1 + \alpha^A) + c^B(1 + \alpha^B)}{3b} \\
q^B &= \frac{a - 2c^B(1 + \alpha^B) + c^A(1 + \alpha^A)}{3b}
\end{align*}
\]

This is an equilibrium as long as \( w_i^1 < c_i^1(\frac{a - 2c^A(1 + \alpha^A) + c^B(1 + \alpha^B)}{3b}) \) for \( i = A, B \) and \( j = A, B \) and \( i \neq j \). Note that the impact of constant marginal financing costs is the same as an increase in both firms’ marginal production costs.

If one firm (say firm A) needs external financing while the other (firm B) does not, then the equilibrium output decisions are

\[
\begin{align*}
q^A &= \frac{a - 2c^A(1 + \alpha^A) + c^B}{3b} \\
q^B &= \frac{a - 2c^B + c^A(1 + \alpha^A)}{3b}
\end{align*}
\]

\(^{19}\)The impact of constant marginal costs of external financing on product market equilibrium is similar to an increase in the marginal cost of one of the producers.
This is an equilibrium as long as \( w_i^A < c^A \left( \frac{a-2c^A(1+\alpha^A)+c^B}{3b} \right) \) and \( w_i^B \geq c^B \left( \frac{a-2c^B+c^A(1+\alpha^A)}{3b} \right) \).

**Convex external financing costs**

In a similar manner, we can specify the optimal output decisions for firm \( A \) and \( B \) when these firms face convex external financing costs \( (C(e) = 1/2e^2) \). In this case, the marginal external financing costs are \( C'_e = e^i \). Substituting in the reaction curves give

\[
q^A = -\frac{1}{2}q^B + (a - (1 + e^A)c^A)/2b
\]

\[
q^B = -\frac{1}{2}q^A + (a - (1 + e^B)c^B)/2b
\]

(6.7)

Note that \( e^i = \max\{0, c^i q^i - (w_i^0 + \theta^i e + \phi^i Z_f)\} \), and thus is also a function of \( q^i \). To establish the new reaction curves we substitute this into equation 6.7. After a bit of rearranging we get the following reaction curves for firm \( A \) and \( B \) respectively:

\[
r^A : \quad q^A = -\frac{1}{2}q^B + (a - c_A)/2b \quad \text{for} \quad e^A = 0
\]

\[
q^A = -\frac{b}{2b + (c^A)^2} q^B + \frac{a-(1-w_0^A)c^A}{2b + (c^A)^2} \quad \text{for} \quad e^A > 0
\]

and for firm \( B \):

\[
r^B : \quad q^B = -\frac{1}{2}q^A + (a - c_B)/2b \quad \text{for} \quad e^B = 0
\]

\[
q^B = -\frac{b}{2b + (c^B)^2} q^A + \frac{a-(1-w_0^B)c^B}{2b + (c^B)^2} \quad \text{for} \quad e^B > 0
\]

where \( w_i^i = w_i^0 + \theta^i e + \phi^i Z_f \). Now obviously, output decisions respond more intensely compared to the case with constant marginal financing costs.

To illustrate these changes we have plotted the new reaction curves in Figure 6.4 for a specific set of parameters. The slopes of the reaction curves for firms that need external financing become steeper and the reaction curves shift inwards compared to the case where there is no such need of external financing. Notice that in our model external financing makes firms less rather than more aggressive (as in Brander and Lewis, 1986, and Showalter, 1996).

\[\text{In the case were firms are completely financially constrained, (have no opportunity to attract external funds), firms cannot produce more than available internal funds allow. Note that with the cost function of external financing used here, this is more or less what happens. It is possible to add a scaling factor} \eta \text{ to the cost function of external financing that is between 0 and 1 to reduce the impact of the cost function. However, this would further complicate our analysis.}\]
FIGURE 6.4. Reaction curves of firms A and B, when they face convex marginal financing costs at high levels of output.

If one firm (say firm A) is financially constrained in equilibrium while the other (firm B) is not, the optimal output decisions for firm A and firm B are respectively:

\[ q_A^* (\phi_A, \epsilon) = \frac{a + cB - 2cA + 2cA w_A^1}{3b + 2(cA)^2} \]  
\[ q_B^* (\phi_A, \epsilon) = \frac{ba - 2bcB + bcA - bcA w_A^1 + a(cA)^2 - cB(cA)^2}{(3b + 2(cA)^2)b} \]  

This is an equilibrium if \( w_A^1 < \frac{1}{3} \cdot \frac{cA + cB - 2cA}{b} \) and \( w_B^1 \geq \frac{cB ba - 2bcB + bcA - bcA w_A^1 + a(cA)^2 - cB(cA)^2}{(3b + 2(cA)^2)b} \).

Finally, if both firms are financially constrained in equilibrium, i.e. \( q_i^* > \frac{w_i}{c_i} \) for both \( i = A, B \), then the optimal output decisions are as follows:

\[ q_A^* (\phi_A, \phi_B, \epsilon) = \frac{ba + bcB - bcB w_A^1 + a(cB)^2 - 2cA b - cA(cB)^2 + 2cA w_A^1 b + cA w_A^1 (cB)^2}{3b^2 + 2b(cA)^2 + 2b(cB)^2 + (cA)^2(cB)^2} \]  
\[ q_B^* (\phi_A, \phi_B, \epsilon) = \frac{ba + bcA - bcA w_A^1 + a(cA)^2 - 2cB b - cB(cA)^2 + 2cB w_B^1 b + cB w_B^1 (cA)^2}{3b^2 + 2b(cA)^2 + 2b(cB)^2 + (cB)^2(cA)^2} \]  

where \( w_i^1 (\phi_i, \epsilon) = w_i^0 + \theta_i \epsilon + \phi_i Z_i \).

This is an equilibrium if \( w_i^1 < \frac{-c^2 w_i^0 b + c^2 b + ba - 2bc - c(c^2) + a(c^2)}{b(3b + 2c)^2} \) for \( i = A, B \) and \( j = A, B \) and \( i \neq j \).
6.4.2 The first stage hedging decision

In the above we have characterized the second stage output equilibrium decisions conditional on hedging decisions and the realization of the exchange rate. Note that with proportional financing costs, the output decisions are fixed once we know whether $A$ and/or $B$ needs external financing. With convex financing costs the output decisions are a more direct function of the hedge ratio and the realization of the exchange rate. We will now analyze the first stage hedging game.

Firms at date 0 choose hedge positions that maximize expected profits. They then anticipate future product market decisions contingent on the level of internal wealth. The hedging transaction itself has zero expected value. Therefore, to make hedging beneficial, it should contribute to expected profits in other ways. We argue that it does so via the costs of external financing and the consequences of these costs on the product market equilibrium. Our analysis of the first stage hedging decision will focus on this link.

To solve for the optimal first stage hedging decisions we will use the equilibrium outcome in the product market and then solve for the optimal hedge ratios. Hence, both firms choose a hedge that maximizes their expected profits

$$\max_{\phi'} E(\pi'(\phi', \phi^j, \epsilon)) \quad (6.10)$$

taking the optimal hedging decisions of the other firm as given.

If parameters are such that firms are financially unconstrained (and therefore do not need external financing) for any exchange rate, we can then show that:

**Proposition 14** If the financial constraint is not binding ($\zeta \geq \min\left\{ \frac{c^A q^A - w^A}{\theta^A}, \frac{c^B q^B - w^B}{\theta^B} \right\}$), for any exchange rate, there is an equilibrium where the firms have no incentive to hedge in the initial stage and both firms produce in the second stage the output as in a standard Cournot duopoly with:

$$q^{A*} = \frac{(a - 2c^A + c^B)}{3b} \quad (6.11)$$

$$q^{B*} = \frac{(a - 2c^B + c^A)}{3b}$$

**Proof.** If the minimum exchange rate $\zeta$ is equal to $\min\left\{ \frac{c^A q^A - w^A}{\theta^A}, \frac{c^B q^B - w^B}{\theta^B} \right\}$, neither firm is financially constrained and we are back in a simple (unconstrained) Cournot game. In Section 6.4.1 it is shown that the solution to this game satisfies $q^* = \frac{(a - 2c^i + c^j)}{3b}$ for $i = A, B$ and $j = A, B$ and $i \neq j$. ■

This first result serves as a benchmark. It defines an unconstrained equilibrium in the second stage where firms always have sufficient internal financing. Alternatively stated, firms have deep pockets. In such a case risk management decisions in the first period do not affect output decisions in the second period (there is no interaction between the two stages) and therefore risk management is irrelevant. Now consider the more interesting case where one of the two
firms, for at least some realizations of the exchange rate, need external financing to finance the marginal production costs. Below we first consider the case for constant marginal financing costs and then the case for convex financing costs.

Constant marginal financing costs

With constant marginal financing costs we can write the expected profits of firm A as

\[
E(\pi^A(\phi^A, \phi^B, \epsilon)) = \int_{\tilde{e}}^{\epsilon} (\pi^{A,I}(\phi^A, \epsilon)) f(\epsilon) d\epsilon + \int_{\epsilon}^{\tilde{\epsilon}} (\pi^{A,II}(\phi^A, \epsilon)) f(\epsilon) d\epsilon + \int_{\epsilon}^{\tilde{\epsilon}} (\pi^{A,IV}(\phi^A, \epsilon)) f(\epsilon) d\epsilon
\]

(6.12)
as long as \( \tilde{\epsilon} < \tilde{e} \), or as

\[
E(\pi^A(\phi^A, \phi^B, \epsilon)) = \int_{\epsilon}^{\tilde{\epsilon}} (\pi^{A,I}(\phi^A, \epsilon)) f(\epsilon) d\epsilon + \int_{\epsilon}^{\tilde{\epsilon}} (\pi^{A,III}(\phi^A, \epsilon)) f(\epsilon) d\epsilon + \int_{\epsilon}^{\tilde{\epsilon}} (\pi^{A,IV}(\phi^A, \epsilon)) f(\epsilon) d\epsilon
\]

(6.13)
if \( \tilde{\epsilon} < \tilde{e} \). Where \( \tilde{e} \) is defined as the exchange rate where firm A becomes unconstrained (does not need external financing) and \( \tilde{\epsilon} \) as the exchange rate where firm B does not need external financing to produce the equilibrium second stage output,

\[
\tilde{e} = \frac{c^A q^{A*} - w^A_0 - \phi^A f}{\theta^A - \phi^A}
\]

\[
\tilde{\epsilon} = \frac{c^B q^{B*} - w^B_0 - \phi^B f}{\theta^B - \phi^B}
\]

Note that \( \tilde{e} \) and \( \tilde{\epsilon} \) depend on the hedging strategies of firms A and B, respectively. Hence, if firm A becomes unconstrained first, then equation 6.14 describes firm A's expected profits. Conversely, if firm B becomes unconstrained first, then equation 6.15 describes firm A's expected profits.

The first term specifies the equilibrium profits of firm A where both firms are constrained. With a sufficiently low realization of the exchange rate (\( \epsilon \in [\tilde{e}, \tilde{\epsilon}] \)) both firms need external financing. In that case the equilibrium output decisions for firms A and B are given in equation 6.5. We denote these output decisions as \( q^{A*} \) and \( q^{B*} \), respectively. Note that the output decisions of both firms are constant in this interval. Hence,

\[
E(\pi^{A,I}(\epsilon, \phi^A)) = E[p(q^{A*}, q^{B*})q^{A*} - c^A q^{A*} - C^A(\epsilon^A)]
\]

where \( C^A(\epsilon^A) = \alpha^A(c^A q^{A*} - [w^A_0 + \theta^A \epsilon + \phi^{A*}(f - \epsilon)]) \)
The expected profits of A over this interval is a linear function in $\epsilon$ and $\phi^A$. When $\epsilon \in [\tilde{\epsilon}, \hat{\epsilon}]$ and $\tilde{\epsilon} < \hat{\epsilon}$, we have the case where firm B needs external financing while firm A does not. The output decisions of both A and B ($q^{AIII}$, $q^{BIII}$) can easily be derived from equation 6.6. The associated expected profits of firm A over this interval can be written as

$$E(\pi^{A,III}) = E[p(q^{AIII}, q^{BIII})q^{AIII} - c^Iq^{AIII} + (w_0^A + \theta^A\epsilon + \phi^A (f - \epsilon) - c^Aq^{AIII})]$$

However, when $\tilde{\epsilon} < \tilde{\epsilon}$ and $\epsilon \in [\tilde{\epsilon}, \hat{\epsilon}]$, so that firm A needs external financing while B does not, then the optimal output decisions are ($q^{AIII}$, $q^{BIII}$) as specified in equation 6.6. The expected profits of firm A over the specified interval are then equal to

$$E(\pi^{A,III}) = E[p(q^{AIII}, q^{BIII})q^{AIII} - c^Iq^{AIII} - C^A(\epsilon^A)]$$

where: $$C^A(\epsilon^A) = \alpha^A(c^Aq^{AIII} - [w_0^A + \theta^A\epsilon + \phi^A (f - \epsilon)])$$

Finally, the third term in both 6.14 and 6.15 considers the cases where both firms are unconstrained. Over the range where both firms do not need external financing ($\epsilon \in [\tilde{\epsilon}, \hat{\epsilon}]$, and $\tilde{\epsilon} < \hat{\epsilon}$ or $\epsilon \in [\tilde{\epsilon}, \hat{\epsilon}]$ when $\tilde{\epsilon} < \hat{\epsilon}$) both firms produce the unconstrained Cournot output decisions as given in equation 6.2 and the expected profits of firm A are equal to

$$E(\pi^{A,IV}) = E[p(q^{AIV}, q^{BIV})q^{AIV} - c^Iq^{AIV} + (w_0^A + \theta^A\epsilon + \phi^A (f - \epsilon) - c^Aq^{AIV})]$$

What adds to the profit in the unconstrained region is that the firm may still have leftovers; if a firm has not (completely) hedged it will realize a gain on the open position at these levels of the exchange rate.

We have now derived an expression for firm A’s expected profits. In a similar way we can derive the expression of the expected profits for firm B. To find the optimal hedge ratios we differentiate the expected profits of firms A and B with respect to their hedge ratios and set them equal to zero. This gives for firm $A$\(^{22}\)

$$\frac{dE[\pi^A(\phi^A, \phi^B, \epsilon)]}{d\phi^A} = \int_{\tilde{\epsilon}}^{\hat{\epsilon}} - (1 - \alpha^A)(f - \epsilon)f(\epsilon)d\epsilon + \int_{\tilde{\epsilon}}^{\hat{\epsilon}} \frac{d\pi^A}{d\phi^A} [\pi^{A,I}(\epsilon) - \pi^{A,II}(\epsilon)] + \int_{\tilde{\epsilon}}^{\hat{\epsilon}} (f - \epsilon)f(\epsilon)d\epsilon + \int_{\tilde{\epsilon}}^{\hat{\epsilon}} (f - \epsilon)f(\epsilon)d\epsilon = 0$$

\(^{21}\)These are $q^{A,II} = \frac{a - 2c^A + c^B(1 + \alpha^B)}{3b}$ and $q^{B,II} = \frac{a - 2c^B(1 + \alpha^B) + c^A}{3b}$, respectively.

\(^{22}\)We conjecture that in equilibrium firm A needs external financing at a lower exchange rate than firm B.
Since the expected value of the hedge contract is zero \( E(f - \epsilon) = 0 \), we can rewrite this as

\[
\frac{dE[\pi^A(\phi^A, \phi^B, \epsilon)]}{d\phi^A} = \int_\tilde{\epsilon}^{\tilde{\epsilon}} -\alpha^A(f - \epsilon)f(\epsilon)d\epsilon + \frac{d\tilde{\epsilon}}{d\phi^A}[\pi^{A,I}(\tilde{\epsilon}) - \pi^{A,II}(\tilde{\epsilon})] = 0 \quad (6.14)
\]

Hedging by firm \( i \) affects its expected profits through two channels. The first term refers to a change in the expected value of the financing costs while the second term represents changes through a shift in the exchange rate \( (\tilde{\epsilon} \text{ and } \tilde{\epsilon}) \) where the firm becomes constrained. The benefit obviously depends on how profitable the firm is under the scenario where both firms are constrained versus that where only one of the two firms is constrained. Hedging affects the expected profits through its impact on the likelihood that it is in a certain regime (with or without external financing) and through the impact on the costs of external financing.

Now consider first the case where firms are always financially constrained. Even after a positive draw of the exchange rate both firms need external finance. For this case we can derive the following proposition:

**Proposition 15** If both firms are financially constrained for every \( \epsilon \in [\epsilon, \tilde{\epsilon}] \) and both have constant marginal external financing costs, then the hedging decision is irrelevant.

**Proof.** This directly follows from equation 6.16. The only impact of hedging in this specific case is on the expected financing costs. Since these are a linear function of \( \epsilon \), hedging does not affect the expected profits. As a result, both firms' hedging decisions are irrelevant.

Also, if one firm is constrained for all \( \epsilon \in [\epsilon, \tilde{\epsilon}] \) while the other firm does not face costs of external financing (has deep pockets), then the hedging decision for the constrained firm is irrelevant.

Now consider the more general case where firms are financially constrained over certain intervals (realizations of the exchange rate and hedging strategies), while being unconstrained over others. In this case there is strategic interaction between both firms' hedging strategies. Hedging affects the regions where firms are constrained. This is important since both firm \( A \) and \( B \)'s hedging strategy determines to which regime \( A \) and \( B \) belong and therefore their expected profits. In Section 6.5, we will develop most of the intuition using numerical examples.

**Convex financing costs**

With convex financing costs we can write the expected profits of firm \( A \) as in equations 6.14 and 6.15. However, the expected profit functions over the different intervals are obviously not the same.

When both firms are financially constrained \( (\epsilon \in [\epsilon, \tilde{\epsilon}] \text{ and } \tilde{\epsilon} < \tilde{\epsilon}) \), equation 6.11 specifies the optimal output decisions (denoted by \( q^{A,I} \) and \( q^{B,I} \)). Note that these output decisions are not constant (as was the case with constant financing costs) but are a function of both firms' hedge
ratios and the realization of the exchange rate. As a result, the expected profits of firm A over this interval is also a function of the hedge ratios and the exchange rate \( E(\pi^{A,I}(\phi^A, \phi^B, \epsilon)) \). Hence,

\[
E(\pi^{A,I}(\phi^A, \phi^B, \epsilon)) = E[p(q^{AI*}, q^{BI*})q^{AI*} - c^Aq^{AI*} - C^A(e^A)]
\]

where \( C^A(e^A) = \frac{1}{2}(c^Aq^{AI*} - |w^A_0 + \theta^A\epsilon + \phi^A*(f - \epsilon)|)^2 \)

In the case where firm A is constrained, while B is not, equation 6.10 specifies the optimal output decisions. Note that the output decisions for both firms in this case depends only on firm A’s hedge ratio and the exchange rate, and hence on \( E(\pi^{A,III}(\phi^A, \epsilon)) \) and \( E(\pi^{B,III}(\phi^A, \epsilon)) \). Firm A’s expected profit over this interval is equal to

\[
E(\pi^{A,III}(\phi^A, \epsilon)) = E[p(q^{AI*}, q^{BI*})q^{AI*} - \epsilon^Aq^{AI*} - C^A(e^A)]
\]

where \( C^A(e^A) = \frac{1}{2}(c^Aq^{AI*} - |w^A_0 + \theta^A\epsilon + \phi^A*(f - \epsilon)|)^2 \)

For the case where B is constrained while firm A is not we can also infer from 6.10 the optimal output decisions in a similar way. The expected profits of firm A then depends on the hedge of firms A and B, \( E(\pi^{A,III}(\phi^A, \phi^B, \epsilon)) \), while the expected profits of firm B only depends on its own hedge ratio \( E(\pi^{B,III}(\phi^B, \epsilon)) \). Firm A’s expected profits are then equal to

\[
E(\pi^{A,II}(\phi^A, \phi^B, \epsilon)) = E[p(q^{AI*}, q^{BI*})q^{AI*} - \epsilon^Aq^{AI*} + \]

\( (w^A_0 + \theta^A\epsilon + \phi^A*(f - \epsilon) - c^Aq^{AI*})] \)

Finally, when both firms are unconstrained, then as with constant marginal financing costs both firms produce according to the unconstrained Cournot output decisions as given in equation 6.2 and the expected profits of firm A are equal to

\[
E(\pi^{A,IV}(\phi^A, \epsilon)) = E[p(q^{AI*}, q^{BI*})q^{AI*} - \epsilon^Aq^{AI*} + \]

\( (w^A_0 + \theta^A\epsilon + \phi^A*(f - \epsilon) - c^Aq^{AI*})] \)

Above we have specified the expected profits of firm A. In exactly the same way we can specify the expected profits for firm B. Consider now the impact of hedging.

If a firm is financially constrained over at least some intervals of the exchange rate, then there are two effects on the firms’ profits. First, the expected profits over the interval where a firm is constrained will be directly (negatively) affected by the expected costs of external financing. Since these costs are convex, less variation is preferred to more and thus, hedging
will reduce these expected costs. However, in our broader framework there is an additional effect; hedging decisions of both firms directly affect equilibrium output decisions as well as the expected profits via this channel. Moreover, they affect the exchange rate intervals \((\hat{c}, \tilde{c})\) that determines when each firm is (un)constrained.

Now, consider both firms’ optimal hedging strategies. Again we should differentiate the firm’s expected profit function with respect to its hedge size \(\phi^i\). This provides us with the reaction curves in hedging and in turn, the optimal hedge of firm \(A\) as their best response to firm \(B\)’s hedging strategy. Solving this gives us the equilibrium hedging strategies. However, since the equations become rather tedious, we only consider some specific cases below in order to develop some intuition as to what is going on.

We derive optimal hedge decisions first for two cases: 

1. for the case where parameters are such that for all \(\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]\) one firm (\(A\)) is constrained while the other (firm \(B\)) is not; and 
2. where parameters are such that for all \(\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]\) both firms are financially constrained.

**Proposition 16** For the case where one firm (\(A\)) is financially constrained while the other (\(B\)) is not for all \(\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]\), there is a unique subgame perfect equilibrium where the constrained firm fully hedges its exposure, while the unconstrained firm \(B\) has no incentive to hedge. The equilibrium is characterized as:

\[
\phi^A^* = 0^A
\]

\[
q^{A*} = \frac{a + c^B - 2c^A + 2c^A w^A_0 + 2c^A \theta^A f}{3b + 2(c^A)^2}
\]

\[
q^{B*} = \frac{ba - 2bc^B + bc^A - bc^A (w^A_0 + \theta^A f) + a(c^A)^2 - c^B(c^A)^2}{(3b + 2(c^A)^2) b}
\]  

**Proof.** The optimal second stage product market decisions as a function of \(w^*_i\) are given in equation 6.9. Substituting this into the expected profit function gives

\[
E(\pi^A) = \int_\underline{\epsilon}^\bar{\epsilon} \pi^A(\phi^A, \epsilon) f(\epsilon) d\epsilon
\]

To find the optimal hedging decision, now differentiate the expected profits function with respect to \(\phi^A\) and set it equal to zero.

---

23 This rationale of risk management has been established first in Froot, Scharfstein and Stein (1993), see also Section 2.2.4. Note also that in the case with constant marginal costs, there is no such effect.

24 We do not consider directly the more general case because as we will show later this becomes rather complex and encompasses the cases that we consider first.
This gives us

\[-b \sigma^2 \left(-\theta^A + \phi^A\right) \frac{4(c^A)^2 + 9b}{3b + 2(c^A)^2} = 0\]

Solving for \(\phi^A\) then shows that \(\phi^A* = \theta^A\).

Substituting this into equation 6.9 gives \((q^A*)\) as given in equation 6.13.

We can do the same for firm \(B\). However, \(E(\pi^B)\) does not depend on \(\phi^B\). As a result, the hedging decision is irrelevant. Substituting \(A\)'s optimal hedge strategy in 6.9 gives \((q^B*)\) as in 6.13.

Parameter conditions under which this is an equilibrium is: \(w_{0i}^A + \theta^i e + \phi^A*(f - \epsilon) < \frac{1}{3}c_i^A(\frac{a + c^B - 2c^A}{b})\) and \(w_{0i}^A + \theta^i e + \phi^i*(f - \epsilon) > c^B(\frac{2b\theta^i + b(c^A - c^B)\theta^i f + a(c^i)^2 - 2(c^i)b - (c^i)(c^j)^2}{(3b + 2(c^A)^2)b})\) for all \(\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]\).

If one firm (firm \(A\)) is always financially constrained for every \(\epsilon\), while the other (firm \(B\)) is not, then there is an equilibrium hedging strategy where firm \(A\) fully hedges its exposure. Firm \(B\) in this equilibrium obviously has no incentive to hedge. Note that as a result of the financial constraint, firm \(A\) in equilibrium will produce less output compared to the case where both firms are financially unconstrained while firm \(B\) produces more.

Now, consider the case where both firms are financially constrained for all \(\epsilon\). For this particular case we can show the following:

**Proposition 17** When both firms are financially constrained for all \(\epsilon \in [\underline{\epsilon}, \bar{\epsilon}]\), costly external financing gives rise to a unique subgame perfect equilibrium \(\{(q^{A*}, \phi^{A*}), (q^{B*}, \phi^{B*})\}\) where both firms fully hedge their exposure. The equilibrium is characterized as:

\[\phi^{i*} = \theta^i \quad \text{for} \quad i = A, B\]

\[q^{i*} = \frac{ba + b(c^i) - b(c^i)w_{0i} - b(c^i)\theta^i f + a(c^i)^2 - 2(c^i)b - (c^i)(c^j)^2 + 2(c^i)b\theta^i + (c^i)(c^j)^2\theta^i f}{3b^2 + 2b(c^i)^2 + (c^i)^2}\]

**Proof.** A sketch of the proof is given here (the full proof is in Appendix 6.2). First, we solve for the second period output decisions \((q^{A*}, q^{B*})\) and express them as a function of the date 0 hedging decisions (see equation 6.11).

Now substitute these equilibrium output decisions as a function of the hedge ratios and the exchange rate in the original expected profit function. Firms at date 0 choose the hedge that
maximizes expected profits. Therefore, determine the first order condition with respect to hedging

\[ \frac{\partial E(\pi)^i}{\partial \phi^i} = 0 \]

Using this we can establish the reaction curves in hedging (given in the Appendix 6.2 equation 4). Then substituting these reaction curves into each other and rewriting gives the optimal hedge decisions \( (\phi^A = \phi^B) \) for both firms. Finally, substitution the hedges in the original optimal output decisions gives us (after some rewriting) the equilibrium output decisions \( (q^A^*, q^B^*) \)

\[
q^A^* = \frac{ba + b(c^A) - b(c^B)w^B - b(c^B)\theta^B f + a(c^A)^2 - 2(c^A)b - (c^A)(c^B)^2 + 2(c^A)b_w^A + 2(c^A)b\theta^A f + (c^A)(c^B)^2w^A + (c^A)(c^B)^2\theta^A f}{3b^2 + 2b(c^A)^2 + 2b(c^B)^2 + (c^A)^2(c^B)^2} \\
q^B^* = \frac{ba + b(c^A) - b(c^A)w^A - b(c^A)\theta^A f + a(c^A)^2 - 2(c^B)b - (c^B)(c^A)^2 + 2(c^B)b_w^B + 2(c^B)b\theta^B f + (c^B)(c^A)^2w^B + (c^B)(c^A)^2\theta^B f}{3b^2 + 2b(c^A)^2 + 2b(c^B)^2 + (c^B)^2(c^A)^2} 
\]

An important implication of this proposition is that there is a unique hedging strategy for both firms. Both firms will enter at date 0 into a hedge equal to each firm's exposure. This directly follows from the concavity of the profit function in the output decisions. As a result, firms have an incentive to minimize the variation in output in order to maximize the expected profits. A full hedging strategy achieves this.

Another implication of this proposition is that firms' equilibrium hedging decisions are affected by competitive considerations if both firms are financially constrained. Both market conditions as well as the rival firms' hedging decisions determine the firm's own optimal hedging strategy. To demonstrate this, assume for example that firm A decides to hedge less \( (< \phi^A^*) \). This may occur, for example, if the firm is insufficiently solvent, and therefore cannot enter into the desired hedging strategy. Inspection of the reaction curve in equation (4) in the Appendix 6.2, then shows us that B's optimal hedging strategy also changes. With strategic substitutes (downward sloping reaction curves) firm B will hedge more the less A hedges. Our results therefore may explain firms' interest in rivals' hedging strategies. In the literature no rationalization for such behavior exists. The intuition behind this is the asymmetry in the firms' profits. Firms get hurt more when they cannot produce in low output states than in high output states; you gain more by producing when total output is low than that you gain when total output is
high. As a result, when both firms are always constrained, a firm will tend to overhedge if its competitor hedges less than its full exposure.\footnote{Finally, it is interesting to analyze the optimal hedging decisions if changes in the exchange rate can make: both firms constrained (for sufficiently low levels of the exchange rate); one firm constrained at intermediate levels of the exchange rate; or neither firm at high levels of the exchange rate. However, we have not yet come up with clear results with respect to these scenarios.}

Note further that the sign of the reaction curve in hedging is similar to that in the output game. This is a typical result (see Bulow et al. 1985). The first stage game inherits the sign from the second stage game. In the next section, we will discuss some numerical examples to further illustrate some of the intuition behind these models.

6.5 Numerical examples

To illustrate the ideas developed in this chapter, we consider some simple numerical examples. First we describe the setup of our basic example. Then, we consider the case where firms cannot use any external financing, followed by the case where firms face constant marginal financing costs. Finally, we consider an example with convex costs of external financing.

Basic setup

We consider a simple Cournot duopoly. Two firms ($A$ and $B$), face a linear inverse demand function in the second stage of our model,

$$ p = 100 - q^A - q^B $$

Both firms face the same cost function:

$$ K^i = 10q^i $$

for $i = A, B$.

In an unconstrained equilibrium, the problem for both firms in the second stage would be to choose $q^i$ that maximizes expected profits given the other firm's output decisions

$$ \max_{q^i} \pi^i = (100 - q^i - q^j)q^i - 10q^i $$

for $i = A, B$, $j = A, B$ and $i \neq j$. Solving this maximization problem gives the following two reaction curves:

$$ r^A : q^A_i = 45 - \frac{1}{2}q^B_i \quad (6.17) $$

$$ r^B : q^B_i = 45 - \frac{1}{2}q^A_i $$
There is a Cournot Nash equilibrium where both curves cross. The optimal production decisions are $q^A^* = 30$ and $q^B^* = 30$ respectively. The resulting equilibrium price is: $p^* = 40$. The expected profits of firms $A$ and $B$ in this case are $E(\pi^A^*) = E(\pi^B^*) = 900$.

Both firms’ initial wealth is exposed to foreign exchange rate uncertainty in the first stage. Suppose that the exchange rates in the first period can only take the values $\tilde{e} \in \{0.8, 1.2\}$, with equal probability. The expected value of the exchange rate at date 1 is equal to 1 ($E_0(\tilde{e}) = 1$) and thus the forward rate (at date 0) is also 1. The realization of the exchange rate at date 1, the firm’s exposure, and the unexposed part of internal wealth determine the amount of internal funds available. In the examples below we consider first the case (denoted with Case 1) where both firms can lock in the unconstrained equilibrium through hedging; that is, we assume $w_i^1 = 100 + 200e + \phi^i(f - e)$ for both firms. We then consider the case (denoted with Case 2) where a full hedging strategy does not ensure both firms the sufficient means with which to finance its production costs and we assume that the internal wealth for both firms is $w_i^1 = 80 + 200e + \phi^i(f - e)$. 

No external financing allowed

Consider first Case 1 where $w_i^1 = 100 + 200e + \phi^i(f - e)$ for $i = A, B$. Hence, both firms can, by entering into full hedging achieve the unconstrained product market equilibrium decisions irrespective of the realization of the exchange rate. In this case, both firms’ profits then equal 900.

What happens if both firms do not hedge ($\phi^A = \phi^B = 0$)? In that case, both firms will produce up to their constraint for the low realization of the exchange rate ($q^A = q^B = 26$) and the unconstrained Cournot outputs ($q^A = q^B = 30$) for the high exchange rate. The expected profits then are equal to 964.

Now consider the case where one firm (say firm $A$) fully hedges, while the other ($B$) does not hedge at all. As a result, firm $A$ produces $q^A = 30$, in both the low as well as the high exchange rate state. Firm $B$, however, has only limited resources ($w^B_i(\tilde{e}) = 260$) and therefore can only produce up to 26 in the low exchange rate state, while in the high exchange rate state it has sufficient means. However, it will then be optimal for firm $B$ to produce $q^B = 30$. The expected profits for $A$ and $B$ are $E(\pi^A) = 960$ and $E(\pi^B) = 912$, respectively. Table 6.1 shows the expected profits given each firm’s hedging strategy. As a result we may conclude that in equilibrium both firms will not hedge. Note that in this case both firms end up in the most profitable outcomes. Because firms do not hedge, they compete less aggressive in the low

---

$^26$Note that after a high realization of the exchange rate both firms have internal wealth equal to 40 after financing production costs.

$^27$We put the equilibrium outcomes in bold in the tables.
TABLE 6.1. Normal form of the first stage hedging game in pure strategies. Denotes firm A and B’s expected profits for each combination of hedging strategies in the case where firms cannot attract external finance for Case 1

<table>
<thead>
<tr>
<th></th>
<th>$\phi^B = 0$</th>
<th>$\phi^B = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^A = 0$</td>
<td>(964, 964)</td>
<td>(912, 960)</td>
</tr>
<tr>
<td>$\phi^A = 200$</td>
<td>(960, 912)</td>
<td>(900, 900)</td>
</tr>
</tbody>
</table>

TABLE 6.2. Normal form of the first stage hedging game in pure strategies. Denotes firm A and B’s expected profits for each combination of hedging strategies in the case where firms cannot attract external finance for Case 2

<table>
<thead>
<tr>
<th></th>
<th>$\phi^B = 0$</th>
<th>$\phi^B = 200$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^A = 0$</td>
<td>(964, 964)</td>
<td>(941.5, 966)</td>
</tr>
<tr>
<td>$\phi^A = 200$</td>
<td>(966, 941.5)</td>
<td>(952.952)</td>
</tr>
</tbody>
</table>

exchange rate state which causes expected profits to be higher compared to the case where both firms would have hedged.

Consider now Case 2 where hedging cannot bring the firm back to an unconstrained Cournot equilibrium (Case 2). We therefore assume that $w_i^1 = 80 + 200\epsilon + \phi^i(f - \epsilon)$ for $i = A, B$. Again we calculate the expected profits for each combination of strategies. The results are presented in Table 6.2. It is clear that considering pure strategies only, there is an equilibrium where both firms hedge. Note that this equilibrium however, does not produce the socially optimal equilibrium; a coordination problem prevents this.

Constant marginal financing costs

Assume that the marginal external financing costs for both firms ($\alpha^i$) are equal to 0.1. First, we consider the cases where $w_i^1 = 100 + 200\epsilon + \phi^i(f - \epsilon)$ and then we consider the case where $w_i^1 = 80 + 200\epsilon + \phi^i(f - \epsilon)$ for both firms.

Table 6.3 shows the expected profits for various hedging strategies of both firms. When we only consider pure strategies, there is (as in Case 1 with no external financing allowed) a subgame perfect equilibrium in pure strategies where both firms do not hedge. Comparing Table 6.3 with Table 6.1 (the case without external financing) we can see that the impact of risk management on expected profits is considerably less; with constant marginal financing costs there is less feedback on output decisions.

Again we do the same for Case 2 where $w_i^1 = 80 + 200\epsilon + \phi^i(f - \epsilon)$ for $i = A, B$. The results are presented in Table 6.4. We find that in this case not hedging by both firms is an equilibrium strategy. This contrasts the outcome of the game in Table 6.2.
Convex external financing costs

We first, consider Case 1 where \( w_i = 100 + 200\epsilon + \phi_i(f - \epsilon) \) for \( i = A, B \), but now assume that firms can attract external financing against an increasing marginal cost. Table 6.5 shows the results of this setup. Again, we have the result that not hedging by both firms is an equilibrium strategy.

Table 6.6, finally shows the results for Case 2 where \( w_i = 80 + 200\epsilon + \phi_i(f - \epsilon) \) for \( i = A, B \). Note that as in the case where firms could not attract external finance, we have an equilibrium where both firms hedge.

Next, we consider the case where firms are even more financially constrained and where hedging cannot lock in the unconstrained production outcome, but where firms have the opportunity to attract external financing. We assume that \( w_0^A = w_0^B = 0 \) and \( w_i = 200\epsilon + \phi_i(f - \epsilon) \) with \( \epsilon \in \{0.8, 1.2\} \) and external financing costs again equal to \( C(\epsilon) = 1/2\epsilon^2 \). Table 6.7 shows the expected profits given both firms’ anticipated optimal output and hedging strategies. (In Appendix 6.3 we provide the details behind the calculations of the expected profits and derive the unique equilibrium hedging strategies). It appears that full hedging dominates the non hedging strategy in terms of expected profits. If both firms completely hedge their exposures, the expected profits of both A and B are equal to 1000.6. If both do not hedge, the expected

<table>
<thead>
<tr>
<th>( \phi^A )</th>
<th>( \phi^B = 0 )</th>
<th>( \phi^B = 200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(962.9, 962.9)</td>
<td>(912.1, 959.1)</td>
</tr>
<tr>
<td>200</td>
<td>(959.1, 121.9)</td>
<td>(900, 900)</td>
</tr>
</tbody>
</table>

TABLE 6.5. Normal form of the first stage hedging game in pure strategies. Denotes firm A and firm B's expected profits for each combination of hedging strategies for the case where firms face convex external financing costs for Case 1
TABLE 6.6. Normal form of the first stage hedging game in pure strategies. Denotes firm A and firm B’s expected profits for each combination of hedging strategies for the case where firms face convex external financing costs for Case 2

<table>
<thead>
<tr>
<th>( \phi^A = 0 )</th>
<th>( \phi^B = 0 )</th>
<th>( \phi^A = 200 )</th>
<th>( \phi^B = 200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi^A = 0 )</td>
<td>( 962.7,962.7 )</td>
<td>( 988.7,892.3 )</td>
<td>( 900,900 )</td>
</tr>
<tr>
<td>( \phi^A = 200 )</td>
<td>( 988.7,892.3 )</td>
<td>( 900,900 )</td>
<td>( 900,900 )</td>
</tr>
</tbody>
</table>

TABLE 6.7. Expected profits with various risk management strategies when both firms are financially constrained and face convex costs of external finance

<table>
<thead>
<tr>
<th>( \phi^i = \theta^i, i = A, B )</th>
<th>( \phi^i = 0, i = A, B )</th>
<th>( \phi^A = \theta^A, \phi^B = 0 )</th>
<th>( \phi^A = 278, \phi^B = 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(\pi^A) )</td>
<td>1000.6</td>
<td>970.1</td>
<td>1000.7</td>
</tr>
<tr>
<td>( E(\pi^B) )</td>
<td>1000.6</td>
<td>970.1</td>
<td>985.2</td>
</tr>
</tbody>
</table>

profits are lower (970.1). If one firm hedges its complete exposure while the other does not, then the firm that hedges has a higher expected profit (1000.7) compared to the firm that did not hedge (985.2). Therefore, if we only consider the possibilities to enter into full hedging versus not hedging and consider the impact on the product market outcomes, we find that the hedging decisions affect expected profits and therefore that hedging dominates not hedging.

Finally, if \( B \) does not hedge, for example because it is insufficiently solvent such that banks do not want to enter into a derivatives contract, then it is optimal for firm \( A \) to increase its hedge to a level such that it is larger than its exposure. In the appendix 6.3 (equation 5) we derive the reaction curves in hedging. This learns that if \( B \) does not hedge \( \phi^B = 0 \), firm \( A \)'s optimal hedging strategy is to hedge more than its exposure namely \( \phi^A = 278 \). The expected profits if firm \( A \) hedges optimally and anticipates firm \( B \) is not hedging, for firms \( A \) and \( B \) are then 1004.3 and 991.2 respectively.

The intuition is as follows. After a high realization of the exchange rate the unhedged firm has the potential to increase its output to a level higher than that of a hedged firm. However, this increased output reduces the price. When the realization of the exchange rate is low the unhedged firm is more financially constrained than the hedged firm. Both firms in this state are however constrained. As a result, total supply is much lower than in the former state and thus the market price will be higher. Since the firm that has hedged (is least financially constrained) in this state will sell most, it will also benefit most in terms of profits from the higher market price. Thus, hedging is always an optimal strategy irrespective of the other firms' strategies; it pays to be less financially constrained than your competitor when total market supply is low (price is high).

This is an interesting observation. In terms of expected profitability, loosing market share in an oligopolistic market where total supply is high is less serious than losing market share in one where total supply is relatively low. Hedging enables the firm not to loose market share
6.6 Discussion

in the high value (low exchange rate, low total supply) range. Although this hedging comes with a cost in that the firm loses market share in the high exchange rate (but low value) range. However, the total impact of hedging on the expected profitability is positive.\textsuperscript{28}

6.5.1 Remarks

We can draw some interesting conclusions from the examples discussed in this section. First, we have shown that irrespective of the type of financing costs there is strategic interaction between two firm’s risk management strategies. Obviously, the more the impact of financing costs on output decisions the larger is the feed back effect and therefore this interaction. Second, the examples show that the game’s outcome depends on how financially constrained a firm is. More in particular, in the case where hedging makes both firms potentially unconstrained we find that not hedging by both firms is the equilibrium strategy. Whereas, for the case where hedging cannot make the firms financially unconstrained, hedging by both firms is the equilibrium strategy except for the case with constant marginal costs. Note that in the cases where hedging is the equilibrium strategy, not hedging would have produced higher profits. However, a coordination problem prevents firms from achieving higher profits. Our result therefore is merely the outcome of a prisoners’ dilemma.

6.6 Discussion

In this chapter we studied the interaction between risk management and product markets. In our model, the amount of internal wealth (wealth exposed to risk) provides the link between the hedging and output decisions. Such interaction is to be expected with firms that are severely financially constrained, face large exposures, and operate in a concentrated market. In this case, hedging decisions are not irrelevant and will affect outcomes in the product market. Moreover, there will be strategic interaction in the firms’ hedging strategies among competitors.

The model considered has some resemblance to that in Maksimovic (1990). He considers the opportunity to purchase a loan commitment from the perspective of imperfect product markets. The major insight of that paper was that a loan commitment (where the commitment rate was below the spot rate) reduced a firm’s marginal financing costs and, as a result, made the firm a more aggressive competitor in the product market (their reaction curve shifts outward). In

\textsuperscript{28}In the Appendix 6.4 we finally explore Case 2 in a setting where the exchange rate has a uniform distribution and a firm faces convex financing costs. We again find that hedging by both firms is an equilibrium. Moreover, we verify what the optimal response is of firm A given that firm B enters into full hedging. Although the insights developed there are not much different from those developed here we included them in the appendix to show the robustness of our results.
our model, corporate hedging reduces the need of costly external financing in certain states and therefore makes an externally financed firm also a more aggressive competitor.

Our model deviates in some important ways from Brander and Lewis (1986), Showalter (1996) and Glazer (1994). In these models debt financing creates an agency problem due to the limited liability nature of equity. Ex post, shareholders prefer to play their option to default and as a result, with uncertain demand (or uncertain costs in the case of Cournot competition)\(^{29}\), this will give an incentive to increase output in equilibrium. Hence, firms financed with debt strategically commit to be more aggressive in the product market because shareholders ignore the states where the firm defaults on its debt. In our model this strategic effect of external financing does not occur because all uncertainty is resolved at the time that the output decision has to be made. Moreover, our results are not driven by (the possibility of) bankruptcy. A second difference is that, unlike in the other papers, in our model the financing decision is directly linked to the production decision. Firms in our model need external financing in order to finance production. This creates a feedback effect from production to financing. Most other papers that address the interaction between capital structure and product market strategy do not have this.\(^{30}\)

What about the robustness of our results for alternative specifications of the model. We considered a duopoly model. In a model with more competitors strategic interaction will diminish. The more the potential impact of risk management on the firm’s financial constraint and the more concentrated the product market, the larger the benefits of hedging. Strategic interaction in a more concentrated industry increases. Hence, output decisions are more sensitive in a concentrated industry. In our model hedging affects these output decisions. As a result, strategic interaction in our model diminishes as the number of competitors increase.

When one considers heterogenous goods rather than homogenous goods, this will also reduce strategic interaction. Since firms do not react that strongly to each other, the impact of hedging also reduces in the case of heterogeneous goods along the same line of reasoning. Finally, it is interesting to consider what happens when we consider a more dynamic framework. In general, it is often suggested that as long as you have a finitely repeating game, the subgame perfect equilibrium strategy is simply the equilibrium strategy of a one shot game. The argument is supported by backward induction.\(^{31}\) However, it is not clear at the outset what this would imply for a firm’s optimal hedging strategy and the rationales developed in this chapter.

\(^{29}\)Showalter (1996) has shown that with price competition and cost uncertainty, there is no strategic commitment effect of debt.

\(^{30}\)An interesting exception is a recent paper by Povel and Raith (2000).

\(^{31}\)If there is a unique outcome in the last period, this is also the optimal strategy in the period before last, the one before that, etc. (see for example Varian, 1992).
6.7 Concluding Remarks

This chapter has shown that corporate hedging may affect product market competition and hence a firm’s real decisions. In particular, hedging may affect a firm’s future availability of funds. This may affect output decisions. We are well aware that this is only one possible way of how hedging may affect real decisions and product market competition. However, we feel that our explanation is powerful and has not received the attention it deserves.

Our work can be extended in several ways. First, we considered only symmetric cases. It is interesting to analyze the game where one firm has deep pockets while the other has not. Second, we have taken the amount of financial slack (internal wealth) as exogenous. In a more complete model, this should be made endogenous. Low levels of internal wealth have the advantage of reducing the "overinvestment problem" (Jensen, 1986). On the other hand it may increase the "underinvestment problem" (see e.g. Stulz, 1990). Bolton and Scharfstein (1990), moreover, have shown how competitors may limit the effectiveness of financial contracts to reduce the overinvestment problem, since these contracts invite predatory behavior by competitors.

Another aspect of our model which deserves attention is that hedging in our model only has competitive advantages. This is mainly due to the choice of a model: where hedging has no large costs. Firms in our model only get hurt after a low draw of the exchange rate (if they need costly external financing). Hedging reduces these expected costs. However, one may also argue that hedging may have considerable costs. Imagine, for example, that there is only one firm in the industry that hedges while the others do not. In this case competitors may decide to compete more aggressively after a positive draw of the exchange rate. Firms that did not hedge have gained from the positive realization while the firm that did hedge does not. If firms use this extra benefit to push that firm out of the market we find another important cost of hedging.

The choices of hedging strategies by firms in a duopoly therefore may result in one firm gaining financial strength vis-a-vis another firm. Realizing that firms cannot attract external financing easily, this may give the relatively strong firm an incentive to push its competitor out of the market. If changes in exchange rates are large enough to induce this type of predatory behavior, it also may give rise to strategic behavior in hedging. This also suggests a link to the literature on predation. These more dynamic links will be explored in the next chapter.
6.8 Appendix

6.8.1 Appendix 6.1

The first order conditions are

\[
\begin{align*}
\frac{\partial \pi^A}{\partial q^A} &= p(q^A, q^B) + p_A(q^A, q^B)q^A - c^A = 0 \\
\frac{\partial \pi^B}{\partial q^B} &= p(q^A, q^B) + p_B(q^A, q^B)q^B - c^B = 0
\end{align*}
\]

(1)

The second order conditions for a maximum are

\[
\begin{align*}
\frac{\partial^2 \pi^A}{\partial (q^A)^2} &= p_A(q^A, q^B) + p_{AA}(q^A, q^B)q^A + p_A(q^A, q^B) < 0 \\
\frac{\partial^2 \pi^B}{\partial (q^B)^2} &= p_B(q^A, q^B) + p_{BB}(q^A, q^B)q^B + p_B(q^A, q^B) < 0
\end{align*}
\]

with \( p = a - b(q^A + q^B) \), the s.o.c. for \( A \) and \( B \) is equal to: \(-2b\). We have assumed in the text that \( b \) is a positive constant. Thus the second order conditions for a maximum hold.

6.8.2 Appendix 6.2

The dynamic maximization problem is defined as follows:

\[
\max_{\phi, q^t} E(\pi^t) = E\{(a^* - b(q^t + q^t^0))q^t - c^t q^t - \frac{1}{2} e^2\}
\]

where \( e = c^t q^t^* - (w^0 + \theta^t \bar{e} + \phi^t Z_f) \). This should be solved backwards. We therefore first solve for the optimal production decisions in stage 2 \((q^A^*, q^B^*)\) and write these as functions of the hedging strategies at date 0, and the realization of the exchange rate at date 1.

Step one: derivation of optimal second stage production decisions

In equation 6.9, we have established the equilibrium output decisions as a function of the hedge ratios and the exchange rate. These were

\[
\begin{align*}
q^A^*(\phi^A, \phi^B, \epsilon) &= \frac{ba + bcB - bcBw^1_B + a(cB)^2 - 2c^A b - c^A(cB)^2 + 2c^A w^1_B + c^A w^1_B(cB)^2}{3b^2 + 2b(cB)^2 + 2b(cA)^2 + (cA)^2(cB)^2} \\
q^B^*(\phi^A, \phi^B, \epsilon) &= \frac{ba + bcA - bcA w^1_A + a(cA)^2 - 2c^B b - c^B(cA)^2 + 2c^B w^1_B + c^B w^1_B(cA)^2}{3b^2 + 2b(cA)^2 + 2b(cB)^2 + (cB)^2(cA)^2}
\end{align*}
\]

where \( w^1_i(\phi^t, \epsilon) = w^0_i + \theta^t \bar{e} + \phi^t Z_f \).
First stage decisions

To find the optimal hedging decisions, go back to the first stage and substitute the \((q^A, q^B)\) found in step one back into the original expected profit function

\[
E(\pi^I(\phi^I, \phi^J, \epsilon)) = E[p(q^I, q^J)q^I - c^Iq^I - C^I(e^I)]
\]

Then we take the first order condition with respect to the hedge ratio

\[
\frac{\partial E(\pi^I)}{\partial \phi^I} = 0
\]

After some rewriting and taking expectations (using the following properties \(E(Z_f) = E(f - \epsilon), f = E(\epsilon), E(Z_f) = 0, E[(Z_f)^2] = E[(f - \epsilon)^2] = \text{var}(\epsilon) = \sigma^2_\epsilon\) and \(E(Z_f\epsilon) = -\sigma^2_\epsilon\)), we can establish the following reaction curves in hedging

\[
\phi^A = -\frac{-4(c^A)^2b^2\theta^A - 2(c^A)b^2(c^B)\theta^B + 2(c^B)b^2(c^A)\phi^B - 6b(c^A)^2(c^B)^2\theta^A \\
-2(c^B)^4(c^A)^2\theta^A - 2(c^B)b(c^A)^3\theta^B + 2(c^B)b(c^A)^3\phi^B - (c^B)^3(c^A)^3\theta^B + \\
(c^B)^3(c^A)^3\phi^B - 2b(c^A)(c^B)^3\theta^B + 2b(c^A)(c^B)^3\phi^B - 9\theta^A b^3}{12\theta^A b^2(c^B)^2 - 4b\theta^A(c^B)^4}
\]

(4)

The reaction curve for \(\phi^B\) is symmetric, and thus:

\[
\phi^B = -\frac{-4(c^B)^2b^2\theta^B - 2(c^B)b^2(c^A)\theta^A + 2(c^B)b^2(c^A)\phi^A - 6b(c^A)^2(c^B)^2\theta^B \\
-2(c^A)^4(c^B)^2\theta^B - 2(c^A)b(c^B)^3\theta^A + 2(c^A)b(c^B)^3\phi^A - (c^B)^3(c^A)^3\theta^A + \\
(c^A)^3(c^B)^3\phi^A - 2b(c^B)(c^A)^3\theta^A + 2b(c^B)(c^A)^3\phi^A - 9\theta^B b^3}{12\theta^B b^2(c^A)^2 - 4b\theta^B(c^A)^4}
\]

Now substituting \(\phi^B\) into \(\phi^A\) and solving for \(\phi^A\) gives

\[
\phi^A = \theta^A
\]

Then given the symmetry in the problem the optimal hedge for \(B\) is

\[
\phi^B = \theta^B
\]

Hence, it is optimal for both firms in equilibrium to engage in a full hedging strategy.
Now substituting this back into equation 6.9 gives the optimal output decisions given the optimal hedge decisions

\[
q^{A*} = \frac{ba + b(c^B) - b(c^B)w_0^B - b(c^B)\theta^B f + a(c^A)^2 - 2(c^A)b - (c^A)(c^B)^2 + 2(c^A)bw_0^A + 2(c^A)b\theta^A f + (c^A)(c^B)^2w_0^A + (c^B)(c^A)^2\theta^A f}{3b^2 + 2b(c^A)^2 + 2b(c^B)^2 + (c^A)^2(c^B)^2}
\]

\[
q^{B*} = \frac{ba + b(c^A) - b(c^A)w_0^A - b(c^A)\theta^A f + a(c^A)^2 - 2(c^B)b - (c^B)(c^A)^2 + 2(c^B)bw_0^B + 2(c^B)b\theta^B f + (c^B)(c^A)^2w_0^B + (c^A)(c^B)^2\theta^B f}{3b^2 + 2b(c^A)^2 + 2b(c^B)^2 + (c^A)^2(c^B)^2}
\]

These are the optimal equilibrium output decisions.

6.8.3 Appendix 6.3

This appendix provides the more detailed calculations behind a numerical example given in Section 6.4. First consider the reaction curves in the product market. These are given in equations 6.7 and 6.8. Now substitute \( b = 1, a = 100, c^A = c^B = 10, w_0^A = w_0^B = 0 \) and \( \theta^A = \theta^B = 200 \) in these reaction curves to get

\[
q^A = -\frac{1}{102}q^B + \frac{90 - 10 + 10w_1^B}{102}
\]

\[
q^B = -\frac{1}{102}q^A + \frac{90 - 10 + 10w_1^A}{102}
\]

Substituting these into each other gives the equilibrium output decisions

\[
q^{A*} = -\frac{10}{10403} w_1^B + \frac{90}{103} + \frac{1020}{10403} w_1^A
\]

\[
q^{B*} = -\frac{10}{10403} w_1^A + \frac{90}{103} + \frac{1020}{10403} w_1^B
\]

Now we need to determine the optimal hedge ratios. We therefore substitute the \((q^{A*}, q^{B*})\) in the original expected profit functions \(E(\pi^A)\) and \(E(\pi^B)\), respectively. This gives us

\[
E(\pi^A) = E\left[ \frac{9270900}{1071509} w_1^A + \frac{413100}{10609} w_1^B + \frac{900}{1071509} w_1^B - \frac{110292809}{216444818} (w_1^A)^2 + \frac{53050200}{108222409} (w_1^B)^2 \right]
\]
Then differentiate with respect to $\phi^i$ in order to find the optimal hedges. That is,

$$\frac{\partial E(\pi^A)}{\partial \phi^A} = \frac{\partial E(\pi^A)}{\partial \pi^A} \frac{\partial E(\pi^A)}{\partial \phi^A} = 0$$

This gives us

$$\frac{\partial E(\pi^A)}{\partial \phi^A} = \left(\frac{9270900}{1071509} - 2\frac{110292809}{216444818} \pi^A + \frac{106110500}{108222409} \pi^B\right) Z_f = 0$$

$$\frac{\partial E(\pi^B)}{\partial \phi^B} = \left(\frac{9270900}{1071509} - 2\frac{110292809}{216444818} \pi^B + \frac{106110500}{108222409} \pi^A\right) Z_f = 0$$

Now substitute the following: $Z = f - \epsilon$, and $\pi^A = 200\epsilon + \phi^A(f - \epsilon)$ and $\pi^B = 200\epsilon + \phi^B(f - \epsilon)$. This gives us (after some rewriting)

$$E\left[-\frac{5}{216444818} \begin{pmatrix} -374544360 + 108636489 * (200\epsilon) + 108636489\phi^A(f - \epsilon) \\ 108636489\phi^A \epsilon - 42444200 * (200\epsilon) - 42444200\phi^B + 42444200\phi^B \epsilon \\ 42444200 * (200\epsilon) - 42444200\phi^B \sigma^2 \end{pmatrix} \right] (f - \epsilon)$$

Taking expectations we get

$$-\frac{5}{216444818} \left(-108636489 * (200\epsilon) + 108636489\phi^A \sigma^2 + 42444200 * (200\epsilon) - 42444200\phi^B \sigma^2\right) = 0$$

Solving for $\phi^A$ gives us the following reaction curves

$$\phi^A = 200 + \frac{42444200}{108636489}(200 - \phi^B)$$

$$\phi^B = 200 + \frac{42444200}{108636489}(200 - \phi^A)$$

Finally, substitute $\phi^B$ in $\phi^A$ and solve to get the optimal hedge ratio:

$$\phi^{A*} = \phi^{B*} = 200$$

Table 6.3 provides the details of the examples in Section 6.4.
We consider a slightly modified version of the numerical example of Case 2 with convex costs of external financing, where:

A.1 $\epsilon$ has a uniform distribution over the support $[0.5, 1.5]$;

A.2 internal wealth equals $w^I = 80 + 200\epsilon$.

Table 6.9 summarizes the results if we analyze the pure strategies. Again we find that hedging dominates the non-hedging strategy. Hence, there is a subgame perfect equilibrium where both firms hedge when we consider pure strategies (hedging or not hedging).

To improve our understanding we consider below what the best response is of firm $A$ to a full hedge by firm $B$. Given that $B$ fully hedges, $w^B = 280$ (irrespective the exchange rate). Now we can substitute this into equation 6.10 to derive the optimal output decision as a function of firm $A$’s hedging strategy ($\phi^A$) and the realization of the exchange rate ($\epsilon$). After simplification, this gives

$$q^A(\phi^A, \epsilon) = \frac{87890}{10403} + \frac{204000}{10403}\epsilon + \frac{1020}{10403} - \frac{1020}{10403}\phi^A\epsilon$$

$$q^B(\phi^A, \epsilon) = \frac{293890}{10403} - \frac{2000}{10403}\epsilon - \frac{10}{10403}\phi^A + \frac{10}{10403}\phi^A\epsilon$$

These are the equilibrium output decisions when both firms are constrained.

Substituting these output decisions in both firm’s profit functions gives firm $A$ and $B$’s profits as a function of $\epsilon$ and $\phi^A$ for the part where both firms are constrained. Hence:
6.8 Appendix

<table>
<thead>
<tr>
<th></th>
<th>$\phi^B = 0$</th>
<th>$\phi^B = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^A = 0$</td>
<td>(965.34; 965.43)</td>
<td>(935.7; 984.5)</td>
</tr>
<tr>
<td>$\phi^A = 1$</td>
<td>(984.5; 935.7)</td>
<td>(950.5; 950.5)</td>
</tr>
</tbody>
</table>

TABLE 6.9. Normal form of the hedging game in pure strategies with convex financing costs for case 2, however with exchange rate uniformly distributed

$$E(\pi^{A,cons}) = E\left\{(100 - \frac{87890}{10403} + \frac{204000}{10403} \epsilon + \frac{1020}{10403} \epsilon - \frac{1020}{10403} \phi^A \epsilon) - \left(\frac{293890}{10403} - \frac{2000}{10403} \epsilon - \frac{10}{10403} \phi^A + \frac{10}{10403} \phi^A \epsilon\right)\right\}$$

$$= E\left\{\frac{87890}{10403} + \frac{204000}{10403} \epsilon + \frac{1020}{10403} \epsilon - \frac{1020}{10403} \phi^A \epsilon - 0.5(10\left(\frac{87890}{10403} + \frac{204000}{10403} \epsilon + \frac{1020}{10403} \epsilon - \frac{1020}{10403} \phi^A \epsilon\right) - (80 + 200\epsilon + \phi^A(1 - \epsilon))^2\right\}$$

which, after simplification, gives us

$$E(\pi^{A,cons}) = E\{440.26 + 898.67\epsilon + 4.4934\phi^A - 8.3772\phi^A \epsilon - 388.39\epsilon^2 + 3.8839\phi^A \epsilon^2 - 9.7097 \times 10^{-3}(\phi^A)^2 + 1.9419 \times 10^{-2}(\phi^A)^2 \epsilon - 9.7097 \times 10^{-3}(\phi^A)^2 \epsilon^2\}$$

Alternatively, we can have the situation that if the exchange rate is sufficiently high and $A$ did not hedge too much after a favorable realization of the exchange rate, firm $A$ is unconstrained while firm $B$ still remains constrained. For that part the equilibrium output conditions are respectively

$$q^A = 30.985$$
$$q^B = 28.029$$

(6.19)

Note that the output decisions in this interval of the exchange rate do not depend on the exchange rate nor on the hedge size. Hence, for this interval the profits on the product market are fixed and $\pi^{A,uncon} = 960.0857$.

Now we still need to find the exchange rate where the firm is just unconstrained. Define this exchange rate as $\tilde{\epsilon}(\phi^A)$, the exchange rate where $q^{A,cons} = q^{A,uncon}(\tilde{\epsilon}(\phi^A), \phi^A)$. Solving this gives us

$$\tilde{\epsilon}(\phi^A) = \frac{1}{203} - \frac{46660 + 203\phi^A}{-200 + \phi^A}$$

(6.20)

We now have all the ingredients to write firm $A$'s expected profits as follows:
\[ E[\pi^A(\phi^A, \epsilon)] = \int_{\bar{\epsilon}}^{\bar{\epsilon}} \pi^{A, \text{cons}}(\phi^A, \epsilon) f(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\bar{\epsilon}} (\pi^{A, \text{uncons}} + 200(\epsilon - \bar{\epsilon}) + \phi^A(1 - \epsilon)) f(\epsilon) d\epsilon \]

(6.21)

The first term refers to the part of firm A’s profits, where both firms A and B are financially constrained. The second term refers to the part where firm A is financially unconstrained while B is financially constrained, plus the leftovers of the unhedged foreign exchange position. To find firm A’s optimal hedge ratio in response to firm B’s full hedging strategy we must maximize firm A’s expected profit function with respect to \( \phi^A \)

\[
\max_{\phi^A} \int_{\bar{\epsilon}}^{\bar{\epsilon}} \pi^{A, \text{cons}}(\phi^A, \epsilon) f(\epsilon) d\epsilon + \int_{\bar{\epsilon}(\phi^A)}^{\bar{\epsilon}} (\pi^{A, \text{uncons}} + 200(\epsilon - \bar{\epsilon}) + \phi^A(1 - \epsilon)) f(\epsilon) d\epsilon
\]

Take the first order condition:

\[
\frac{dE[\pi^A(\phi^A, \epsilon)]}{d\phi^A} = \int_{\bar{\epsilon}}^{\bar{\epsilon}} \frac{d\pi^{A, \text{cons}}(\phi^A, \epsilon)}{d\phi^A} f(\epsilon) d\epsilon + \pi^{A, \text{cons}}(\phi^A, \bar{\epsilon}) \frac{d\bar{\epsilon}(\phi^A)}{d\phi^A} + \int_{\bar{\epsilon}(\phi^A)}^{\bar{\epsilon}} \frac{d(\pi^{A, \text{uncons}} + 200(\epsilon - \bar{\epsilon}) + \phi^A(1 - \epsilon))}{d\phi^A} f(\epsilon) d\epsilon - [\pi^{A, \text{uncons}} + 200(\bar{\epsilon} - \bar{\epsilon}) + \phi^A(1 - \bar{\epsilon})] \frac{d\bar{\epsilon}(\phi^A)}{d\phi^A}
\]

(6.22)

Now differentiate in parts:

\[
\frac{dE(\pi^{A, \text{cons}})}{d\phi^A} = 4.4934 - 8.3772 \epsilon + 3.8839 \epsilon^2 - 1.9419 \times 10^{-2} \phi^A + 3.8838 \times 10^{-2} \phi^A \epsilon - 1.9419 \times 10^{-2} \phi^A \epsilon^2
\]

Now for the border (the exchange rate) where the firm is just constrained we have

\[
\bar{\epsilon}(\phi^A) = \frac{1}{203} - \frac{-46060 + 203\phi^A}{-200 + \phi^A}
\]

and its derivative

\[
\frac{d\bar{\epsilon}(\phi^A)}{d\phi^A} = \frac{6060}{203(-200 + \phi^A)^2}
\]

Finally, we need to differentiate the profit function of firm A for the part where firm A is unconstrained. Hence,
\[
\frac{d(\pi^{A,\text{uncons}} + (200(\epsilon - \bar{\epsilon}(\phi^A)) + \phi^A(1 - \epsilon))}{d\phi^A} = -\frac{200 \times 6060}{203(-200 + \phi^A)^2} + (1 - \epsilon)
\]

Now using the property that the firm A's constrained profit is evaluated at \(\bar{\epsilon}\) where it just becomes unconstrained is equal to the firm's unconstrained profit

\[
\pi^{A,\text{cons}}(\phi^A, \bar{\epsilon}) = \pi^{A,\text{uncons}}
\]

After some rewriting we have

\[
\frac{dE[\pi^{A}(\phi^A, \epsilon)]}{d\phi^A} = \int_{\bar{\epsilon}}^{\epsilon} \frac{d\pi^{A,\text{cons}}(\phi^A, \epsilon)}{d\phi^A} f(\epsilon)d\epsilon + \int_{\epsilon}^{\bar{\epsilon}} \frac{d(\pi^{A,\text{uncons}} + 200(\epsilon - \bar{\epsilon}) + \phi^A(1 - \epsilon))}{d\phi^A} f(\epsilon)d\epsilon - \phi^A(1 - \bar{\epsilon}) \frac{d\bar{\epsilon}(\phi^A)}{d\phi^A}
\]

Now, after substituting this gives

\[
1.1267 \times 10^{16} - 2.8508 \times 10^{16}(\phi^A) + 1.2576 \times 10^{14}(\phi^A)^2
-4.9261 \times 10^{-11} \frac{-1.215 \times 10^{10}(\phi^A)^3 + 1.6425 \times 10^7(\phi^A)^4}{(-200.0 + (\phi^A))^3} = 0
\]

Finally, solving this for \(\phi^A\) gives \(\phi^A = 0.396\) or \(\phi^A = 229.81\)

Substituting back into \(\bar{\epsilon}(\phi^A)\) shows that this \(\bar{\epsilon}(\phi^A) \notin [\epsilon, \bar{\epsilon}]\). As a result, the firm is financially constrained over the whole interval \([\epsilon, \bar{\epsilon}]\). Using Proposition 17, the optimal hedging strategy then is to fully hedge. Hence, firm A's optimal response to a full hedging strategy by B is also a full hedging strategy.

Hence, we get the same result for our more general case compared to that which we discussed in the numerical example in Section 6.4.
6. Risk management and product markets