Chapter 2
Investment Models and Financing Constraints

Introduction

Under the Modigliani and Miller theorem (1958), that is if capital markets are perfect, a firm’s capital structure is irrelevant to its value. In this case internal and external funds are perfect substitutes and firm investment decisions are independent from its financing decisions. With imperfect capital markets, however, the costs of internal and external finance will diverge due to informational asymmetries, costly monitoring, contract enforcement and incentive problems, so that internal and external funds generally will not be perfect substitutes. Informational asymmetries lead to a link among net worth, the cost of external financing, and investment. Within the neo-classical investment model with financial frictions, an increase in net worth independent of changes in investment opportunities leads to greater investment for firms facing high information costs and has no effect on investment for firms facing negligible information costs. It follows that certain firms are expected to face financing constraints, in particular firms facing high information costs.

A major topic of interest in recent micro-econometric research on company investment has been to test for the possibility that investment spending is subject to significant financing constraints. This chapter presents a survey of this research and derives one particular model of firm investment that we use in chapters 3 and 4 to assess the impact of financial liberalisation on the financing constraints of firms. Section 2.1 reviews the literature on financing constraints. Section 2.2 presents the structural model of firm investment that we use to estimate the impact of financial liberalisation on financing constraints of firms. Section 2.3 describes the econometric techniques we employ to estimate our structural model of firm investment.

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1 Myers and Majluf (1984) present the informational asymmetry problems of equity financing, and Stiglitz and Weiss (1981) show that informational asymmetries may cause credit rationing in the loans market.
2 See Townsend (1979) for a model of costly state verification.
3 Jensen and Meckling (1976) show that in the presence of limited-liability debt the firm may have the incentive to opt for excessively risky investment projects that are value destroying.
2.1 Literature Review

One model that incorporates a relationship between investment and financial decisions is the hierarchy of finance model. This model assumes that internally generated finance for investment is available at lower cost than external finance due to asymmetric information and agency problems. In contrast to the standard neo-classical model of investment in which firms have access to unlimited sources of investment finance at an exogenously given cost, the hierarchy of finance model implies that investment and financial decisions are not generally independent. Investment spending of firms is sensitive to the availability of internal funds such as retained earnings, or more generally to financing constraints.

The hierarchy of finance model predicts that the Euler equation of the standard neo-classical investment model that describes the optimal path of investment, also known as the investment Euler equation, should hold across adjacent periods for a priori unconstrained firms but be violated for constrained firms. For this latter group of firms a wedge should exist between the marginal cost of investment today versus investing tomorrow. The hierarchy of finance model thus predicts that there are cross-sectional differences in effects of internal funds on firms' investment.

Following the work of Fazzari, Hubbard and Petersen (1988) a large body of literature has emerged to provide evidence of financing constraints (see the surveys by Schiantarelli (1995), Blundell, Bond and Meghir (1996), Hubbard (1998), and Bond and Van Reenen (1999)). This literature is based upon the hierarchy of finance model and assumes that the “premium” on external finance is an inverse function of a borrower’s net worth. A firm’s investment is defined to be financially constrained if a windfall increase in the supply of internal funds (i.e. a change which conveys no new information about the profitability of current investment) results in a higher level of investment spending. This has led to different a priori classifications of firms that have tried to distinguish financially constrained and not-constrained firms. From a theoretical point of view such sorting criteria should focus on a firm’s characteristics that are associated with information costs. A number of studies have grouped firms by dividend payouts⁴; other a

⁴ See Fazzari, Hubbard and Petersen (1988), and Hubbard, Kashyap and White (1995).
priori groupings of firms have focused on group affiliation, size and age, the presence of bond ratings, the degree of shareholder concentration, or the pattern of insider trading.

Such a priori classifications are usually assumed to be fixed over the sample period. This may be problematic if the characteristics of firms change over firms, or if the criteria used to split the sample are not external to the investment model. In addition, Lamont (1997) has shown that the finance costs of different parts of the same corporation can be interdependent, in such a way that a firm subsidiary’s investment is significantly affected by the cash flow of other subsidiaries within the same firm. Kaplan and Zingales (1997) question the usefulness of a priori groupings of firms. They divide the firms studied by Fazzari, Hubbard and Petersen (1988) into categories of “not financially constrained” to “financially constrained” based upon statements contained in annual reports, and find no support for the presence of financing constraints. The problem with their analysis is that it is difficult to make such classifications. Fazzari, Hubbard and Petersen (1996) note that the firm-years Kaplan and Zingales (1997) classify as most financially constrained are actually observations from years when firms are financially distressed. From a theoretical point of view, the Kaplan and Zingales (1997) critique is limited to the claim that differing cash flow sensitivities reveal different degrees of financing constraints under the alternative hypothesis that these types of firms are both subject to significant financing constraints.

Several empirical investment models have been derived from the investment Euler equation that can be used to test empirically the presence of financing constraints. Most studies on financing constraints since Fazzari, Hubbard and Petersen (1988) estimate a \( q \)-model of investment, pioneered by Tobin (1969) and extended to models of investment by Hayashi (1982). Financial variables such as cash flow are then added to the \( q \)-model of investment to pick up capital market imperfections. If markets are perfect, investment should depend on marginal \( q \) only. Marginal \( q \) is usually measured by average \( q \) (see Fazzari, Hubbard and Petersen (1988), Hayashi and Inoue (1991), and Blundell, Bond, Devereux and Schiantarelli (1992)). Hayashi (1982) has shown that only under certain

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6 See Devereux and Schiantarelli (1990).
7 See Whited (1992).
8 See Oliner and Rudebusch (1992).
strong assumptions, marginal $q$ equals average $q$. Also, using average $q$ as a measure for investment opportunities requires the assumption that capital markets are efficient.

For these reasons several researchers have departed from the strategy of using proxies for marginal $q$ and estimate the so-called Euler investment equation describing the firm's optimal investment directly (see Whited (1992), Bond and Meghir (1994), Hubbard and Kashyap (1992), Hubbard, Kashyap, and Whited (1995)). This is the so-called Euler model of investment introduced by Abel (1980). The advantages of the Euler approach are that it avoids the use of share price data and can relax the assumption of linear homogeneity of the net revenue function. The disadvantage of the Euler approach is that it relies on the period-by-period restriction derived from the firm's first-order conditions. An alternative approach, introduced by Abel and Blanchard (1986), forecasts the expected present value of the current and future profits generated by an incremental unit of fixed capital. This approach does not need to assume that the Euler investment equation holds across adjacent periods. Gilchrist and Himmelberg (1995, 1998) have extended this approach by using a vector autoregression forecasting framework to decompose the effect of cash flow on investment. The disadvantage of this approach is that it needs to assume a certain stochastic process on the variables of the investment model.

Empirical work has found that financial variables such as cash flow are important explanatory variables for investment. These findings are attributed to capital market imperfections that arise from informational asymmetries, costly monitoring, contract enforcement and incentive problems. Most studies of financing constraints focus on firms in one country. One of the few cross-country studies is by Bond, Elston, Mairesse and Mulkay (1997), who study firms' investment behaviour in Belgium, France, Germany, and the UK, and find that financial constraints on investment are more severe in the UK than in the three other countries. Mairesse, Hall and Mulkay (1999) study firms' investment behaviour in France and the US and find significant changes in the investment behaviour of French and US firms over the last twenty years. In related work, Demirgüç-Kunt and Maksimovic (1998) directly identify those firms that are financially constrained by estimating if the firm grows its sales at a rate that requires long-term external

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9 These assumptions are that the firm is a price-taker with constant returns to scale in both production and installation.
financing. They investigate how differences in legal and financial systems affect firms' use of external financing to fund growth, and show that in countries with efficient legal systems a greater proportion of firms use long-term financing.

Bond and Van Reenen (1999) point out that, although most of the results of the financing constraints literature appear to be consistent with priors on the extent of financing constraints for some types of firms in some periods, these tests could also be detecting other sources of misspecification in the underlying investment models. Furthermore, it should be noted that there are alternative models in corporate finance, such as Jensen's (1986) free cash flow theory, that could potentially account for the excess sensitivity of investment to cash flow and other financial variables. In the next section we will describe the $Q$ model and the Euler model of investment in more detail. In the following two chapters we use empirical specifications of these models to assess the presence of financing constraints.

### 2.2 Company Investment Models and Financing Constraints

In this section we derive the investment Euler equation from the firm's maximisation problem, which is to maximise the expected value of future dividends, subject to capital accumulation. Using the investment Euler equation as a starting point, we introduce two related models: the $Q$ model of investment and the Euler model of investment.

For simplicity we start by ignoring taxes and debt. The firm's value in the absence of taxes and debt can be derived from the capital market arbitrage condition

\[(1 + r_t)(V_t - D_t + N_t) = E_t[V_{t+1}],\]

where $r_t$ is the interest rate on riskless bonds between period $t$ and period $t+1$, $V_t$ is the market value of the firm's outstanding shares at the beginning of period $t$, $D_t$ is dividends paid out to shareholders in period $t$, $N_t$ is the value of new shares issued in period $t$, and $E_t[.]$ denotes the conditional expectation based on information available in

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10 In the free cash flow approach, managers have non-value maximising objectives and are subject to less effective monitoring when spending internal funds than external funds.
period $t$. For simplicity, we assume that the dividend received equals the amount of corporate earnings distributed by the firm. The model assumes that the marginal shareholder is risk neutral, so that the effects of risk on the firm’s required rate of return are not considered. The left hand side of (2.1) is the opportunity cost of holding shares in the firm. The right-hand side is the shareholder’s expected wealth at the start of the next period. Solving (2.1) forward for $V_t$ yields the firm’s value at time $t$

$$V_t = E_t [\sum_{j=0}^{\infty} \beta_{t,j} (D_{t,j} - N_{t,j})].$$

(2.2)

The $j$-period discount factor $\beta_{t,j} = \prod_{i=1}^{j} (1 + r_{t,i-1})^{-1}$ for $j \geq 1$, and $\beta_{t,1} = 1$. Since the model assumes that the marginal shareholder is risk neutral we have $r_t = t$. The sources and uses of funds relation for a firm that issues only equity can be written as

$$D_t = \Pi_t + N_t,$$

(2.3)

where $\Pi_t = \Pi(K_t, L_t, I_t)$ is the net revenue generated in period $t$ with $K_t$ the capital stock of the firm at time $t$, $L_t$ its vector of variable inputs, and $I_t$ its investment at time $t$. The capital stock accounting identity is given by

$$K_t = (1 - \delta)K_{t-1} + I_t,$$

(2.4)

where $\delta$ is the constant rate of economic depreciation. The firm’s objective is to maximise (2.2). We can write the firm’s objective function in dynamic programming form

$$V_t(K_{t-1}) = \max_{K_t, I_t} \{\Pi(K_t, L_t, I_t) + \beta_{t+1} E_t[V_{t+1}(K_{t+1})]\}.$$

(2.5)

To arrive at (2.5) we have assumed that the interest rate on default free bonds is given exogenously to the firm. Substituting (2.4) into (2.5) gives
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\[ V_t(K_{t-1}) = \max_{L_t} \{ \Pi((1 - \delta)K_{t-1} + I_t, L_t, I_t) + \beta_t' E_t[V_{t+1}((1 - \delta)K_{t-1} + I_t)] \} \]  \hspace{1cm} (2.6)

Differentiating (2.6) with respect to \( K_{t-1} \), the Euler equation characterising the optimal path of investment is then given by

\[ \lambda_t = (1 - \delta) \left( \frac{\partial \Pi}{\partial K} \right)_t + (1 - \delta) \beta_t' E_t[\lambda_{t+1}] \]  \hspace{1cm} (2.7)

where \( \lambda_t = \partial V_t / \partial K_{t-1} \) is the shadow value of capital at \( t-1 \). Differentiating (2.6) with respect to \( I_t \) gives the first-order condition for investment

\[ 0 = \left( \frac{\partial \Pi}{\partial K} \right)_t + \left( \frac{\partial \Pi}{\partial I} \right)_t + \beta_t' E_t[\lambda_{t+1}] \]  \hspace{1cm} (2.8)

From (2.7) and (2.8) we can obtain that

\[ (1 - \delta) \left( \frac{\partial \Pi}{\partial I} \right)_t + \lambda_t = 0 \]  \hspace{1cm} (2.9)

Equation (2.9) shows that the marginal benefit is equal to the marginal cost of additional units of capital.

Given (2.9) there are several ways of obtaining empirical investment models. Two widely used approaches are the \( Q \) model of investment and the Euler equation model of investment. The \( Q \) model imposes auxiliary assumptions that allow the unobservable shadow value of capital to be related to the observable ratio between the stock market’s valuation of the firm and its capital stock at replacement cost. In contrast, the Euler equation approach utilises equations (2.7) and (2.9) to eliminate the unobservable shadow value of capital. The Euler equation is an intertemporal condition relating current investment to last period’s investment and marginal product of capital. To obtain an empirical model of investment we specify the net revenue function as
\[ \Pi_t = p_t F(K_t, L_t) - p_t G(I_t, K_t) - w_t L_t - p_t^i I_t, \quad (2.10) \]

where \( F(K_t, L_t) \) is the firm's production function, \( G(I_t, K_t) \) is the convex adjustment-cost function of installing \( I_t \) units of capital, which is assumed to be homogeneous in investment and capital, \( p_t \) is the price of the firm's output, \( w_t \) is the vector of prices for the variable inputs, and \( p_t^i \) is the price of investment goods. To allow for imperfect competition we let \( p_t \) depend on output, with the price elasticity of demand \( \epsilon \) assumed constant (with \( \epsilon > 1 \)). Differentiating (2.10) with respect to \( K_t \) gives

\[ \frac{\partial \Pi}{\partial I_t} = -\alpha p_t \left[ \frac{\partial F}{\partial I_t} \right] - p_t^i, \quad (2.11) \]

and

\[ \frac{\partial \Pi}{\partial K_t} = \alpha p_t \left[ \frac{\partial F}{\partial K_t} - \left( \frac{\partial G}{\partial K_t} \right) \right], \quad (2.12) \]

where \( \alpha = 1 - (1/\epsilon) > 0 \).

### 2.2.1 No Debt and Taxes

The \( Q \) model of investment without taxes and debt can be derived as follows. Using equation (2.9) and (2.11) gives

\[ \left( \frac{\partial G}{\partial I_t} \right) = \left( \frac{\lambda_t}{p_t^i} \frac{1}{1 - \delta} - 1 \right) p_t^i \alpha p_t = (q_t - 1) \frac{p_t^i}{\alpha p_t}, \quad (2.13) \]

where \( \lambda_t / (1 - \delta) \) is the shadow value of one extra unit of capital in period \( t \), and \( q_t = \lambda_t / (1 - \delta) p_t^i \) is marginal \( Q \). Following Summers (1991), we specify
$G(I_t, K_t) = \frac{1}{2} bK_t [(I_t/K_t) - c]^2$ as a symmetric adjustment-cost function, which is strictly convex in investment, and homogeneous of degree one in investment and capital. In this case, equation (2.13) can be written as

$$\left( \frac{I_t}{K_t} \right) = c + \frac{1}{\alpha b} \left[ (q_t - 1) \frac{p_t^I}{p_t} \right].$$ (2.14)

In the above equation, $q_t$ is unobservable. We make the additional assumption that both the production function and the adjustment-cost function are homogeneous of degree one in their arguments. In addition, we assume that the stock market is strongly efficient. Under these conditions, Hayashi (1982) has shown that marginal $Q$ is related to the observable average $Q$ ratio, transforming equation (2.13) as follows

$$\left( \frac{I_t}{K_t} \right) = c + \frac{1}{\alpha b} \left[ \frac{V_t}{(1-\delta)p_t^I K_{t-1}} - 1 \right] \frac{p_t^I}{q_t} = \text{const.} + \beta Q_t,$$ (2.15)

where $V_t/(1-\delta)p_t^I K_{t-1}$ is the average $Q$ ratio. The numerator of average $Q$ measures the market valuation of the firm's fixed capital, whilst the denominator measures its replacement cost value in the presence of tax. Using the above, the empirical specification of the $Q$ model is

$$\left( \frac{I_t}{K_t} \right) = c + \beta Q_t + \nu_t,$$ (2.16)

where the error term $\nu_t$ can be interpreted as a shock to marginal adjustment costs.\(^\text{11}\) An implication of this ad hoc approach to introducing stochastic variation into the $Q$ model is that $Q_t$ should be an endogenous variable in the econometric model (2.16). Current

\(^{11}\) An alternative way of introducing stochastic variation into the $Q$ model is to assert that marginal $q$ contains measurement error. In view of both the assumptions required to measure marginal $q$ and the limitations of most publicly available datasets, the likelihood of significant measurement error in the $Q$ variable does indeed seem to be large.
shocks to adjustment costs will affect the current period’s net revenue ($\Pi_t$), and therefore the current value of the firm ($V_t$). This endogeneity will need to be taken into account when estimating the $Q$ model.

Clearly firms are not financially constrained in the $Q$ model of equation (2.13), where given current prices and interest rates, investment depends only on the current and expected future marginal revenue products of capital, as summarised in marginal $q$ through the shadow value of an additional unit of capital. However, firms’ investment may be financially constrained in the hierarchy of finance model of corporate finance, in which external sources of finance (for example, from new share issues or borrowing) are assumed to be more expensive than internal sources of finance (for example, from retained earnings). In its simplest form, that is without debt and taxes, the additional cost associated with the use of external finance can be introduced by an explicit transaction cost per unit of new shares issued. Together with non-negativity constraints on dividends and new share issues this leads to a hierarchy of finance model that has three distinct financial regimes. Retained earnings are the cheapest source of finance, so if the firm has sufficient earnings to finance its desired investment, it will issue no new shares. In this regime the basic model given by equation (2.13) described the firms investment.

If the firm does not have sufficient earnings to finance its desired investment, the non-negativity constraint on dividends is binding, and the shadow value of internal funds is strictly positive. In this case the firm has to decide whether or not to finance additional investment by using the more expensive external source of finance. If the investment projects that would be forgone by not issuing new shares are sufficiently profitable compared to the higher cost of external funds, the firm will choose to issue shares. If this is not the case, the firm will be in a financially constrained positions, in which both dividends and new share issues are zero. From the sources and uses of funds condition and the net revenue function (2.10), it follows that the level of investment expenditure is constrained to the level of cash flow $C_t$. Thus in this constrained regime, windfall changes in cash flow have a direct effect on the level of investment, holding marginal $q$ constant.\footnote{Allowing the firm to borrow will tend to weaken this sensitivity of investment to windfall fluctuations in cash flow, but will only eliminate it in the special case where debt acts as a perfect substitute for finance from retained earnings (Bond and Meghir, 1994).}
An alternative approach to obtain an empirical investment model is to combine (2.7) and (2.9) to eliminate $\lambda_i$. This is the so-called Euler approach. Provided that equation (2.9) holds not only in period $t$, but also for period $t+1$, we have

$$-(1-\delta)\beta_{i,t}E_i\left(\frac{\partial \Pi}{\partial I},_{i+1}\right) = -\left(\frac{\partial \Pi}{\partial I}\right)_i - \left(\frac{\partial \Pi}{\partial K}\right)_i. \quad (2.17)$$

If the firm’s net revenue function and the adjustment-cost function are assumed to take the same form as in the $Q$ model, and the firm operates under imperfect competition in the same way as in the $Q$ model before, then we have from (2.11)

$$\left(\frac{\partial \Pi}{\partial I}\right)_i = -b\alpha \phi_i \left(\frac{I}{K}\right)_i + b c \alpha \phi_i - p_i, \quad (2.18)$$

and from (2.12)

$$\left(\frac{\partial \Pi}{\partial K}\right)_i = \alpha \phi_i \left(\frac{Y}{K}\right)_i - \alpha \phi_i \left(\frac{\partial F}{\partial LK}\right)_i + b \alpha \phi_i \left(\frac{I}{K}\right)_i - b c \alpha \phi_i \left(\frac{I}{K}\right)_i. \quad (2.19)$$

where $Y_i = F_i - G_i$ denotes net output and $\alpha = 1 - (1/\epsilon) > 0$. Following Bond and Meghir (1994) we assume that the marginal products of variable factors $(\partial F/\partial L)$ can be replaced from the first-order conditions by $(w/\alpha p)$. The benefit of this assumption is that we do not have to specify a parametric form for the production function. Using the above, the empirical Euler equation under the null of no taxes and no financing constraints in (2.17) can be written as

$$\left(\frac{I}{K}\right)_{i+1} = c(1-\phi_{i+1}) + (1+c)\phi_{i+1} \left(\frac{I}{K}\right)_i - \phi_{i+1} \left(\frac{I}{K}\right)_i^2 - \frac{\phi_{i+1}}{b\alpha} \left(\frac{C}{K}\right)_i + \frac{\phi_{i+1}}{b\alpha} J, \quad (2.20)$$

$$+ \frac{\phi_{i+1}}{b(\epsilon - 1)} \left(\frac{Y}{K}\right)_i + \nu_{i+1},$$
where $\phi_{t+1} = (1 + \rho_{t+1})/(1 - \delta)$ with $\rho_{t+1} = (1 + r_{t+1} \gamma p_{t+1} / p_{t+1}) - 1$ being the real discount rate, $(C / K) = (p_{t}Y_{t} - w_{t}L_{t})/(p_{t}K_{t})$ is the ratio of real cash flow (operating profit) to the capital stock, and $J_{t} = (p_{t}^t / p_{t})[1 - p_{t+1}^t(1 - \delta)/[(1 + r_{t})p_{t}^t]]$ is the user cost of capital.

### 2.2.2 With Debt and Corporate Income Taxes

We now introduce debt and taxes on corporate income. For simplicity we ignore personal taxation on capital gains and dividend income. For a model that incorporates these taxes, see Bond and Meghir (1994). We incorporate financial frictions via the assumption that debt is the marginal source of external finance, and that risk-neutral debt holders demand an external finance premium, which is increasing in the amount borrowed due to agency costs. The idea is that highly leveraged firms have to pay an additional premium to compensate debt holders for increased costs due to information asymmetry problems.

The firm’s value with corporate taxes can again be derived from the capital market arbitrage condition in equation (2.1), where now $D_{t}$ is the dividend received from post-corporate tax earnings. Solving (2.1) forward for $V_{t}$ yields again the firm’s value at time $t$ in equation (2.2). The sources and uses of funds relation for a firm that issues both equity and debt can be written as

$$D_{t} = \Pi_{t} + N_{t} + B_{t} - (1 + (1 - \tau_{t})i_{t+1}B_{t+1}). \tag{2.21}$$

$\Pi_{t}$ is net revenue generated in period $t$, and $\Pi(K_{t}, I_{t}, I_{t})$ is the net revenue function, $i_{t+1}$ is the interest rate payable on bonds issued in period $t-1$, and $\tau_{t}$ is the rate of corporate income tax in period $t$, against which these interest payments are assumed to be deductible. The capital stock accounting identity is identical to the one used before and presented in equation 2.4.

The presence of debt implies that the firm can go bankrupt. Both the probability of bankruptcy and the interest rate charged by lenders will depend on the amount borrowed. In the event of bankruptcy we assume that ownership of the firm is transferred from the shareholders to the creditors, although the bankruptcy process in period $t$ may involve
dead-weight costs denoted by $X_t$. The value of the firm’s levered equity at time $t$ is then given by

$$V_t = \Pi_t + B_t + \beta_{t+1}(1-q_{t+1}^i)E_t[\Pi_{t+1}|nb_{t+1}] - E_t[(1+(1-\tau_{t+1})i_i)B_t],$$

(2.22)

where $q_{t+1}^i$ is the probability perceived in period $t$ that the firm will default on its bonds in period $s$, and $E_t[x|nb_s]$ is the conditional expectation of $x$, given that the firm does not default in period $s$. Bondholders will require the expected return from lending to equal the return available on riskless bonds. This gives the arbitrage condition for lenders

$$(1+i_t)B_t = (1-q_{t+1}^i)(1+i_s)B_t + q_{t+1}^iE_t[\Pi_{t+1}|b_{t+1}] - q_{t+1}^iX_{t+1},$$

(2.23)

where $E_t[x|b_s]$ is the expected value of $x$, conditional upon information available in period $t$, given that the firm does default on its bonds in period $s$. For simplicity we assume that tax on interest income is zero.

For debt rather than equity to be the firm’s marginal source of finance, we assume binding non-negativity constraints on dividend payments and new share issues, which implies that there is a shadow cost associated with raising new equity due to information asymmetry.\textsuperscript{13} The objective of the firm is to maximise firm value subject to the non-negativity constraints on dividend payments and new share issues, with associated Kuhn-Tucker multipliers denote by $\lambda^0_t$ and $\lambda^n_t$ respectively. Following Bond and Meghir (1994) we assume that both the bankruptcy probability $q_{t+1}^i$ and the interest rate $i_t$ depend on the amount borrowed $B_t$ and the size of the firm $K_t$ only through the ratio $(B_t/p^i_tK_t)$, where $p^i_t$ is the price of a unit of capital goods in period $t$. Moreover, we assume that that bankruptcy costs depend on $B_t$, but not on $K_t$, and that they are homogeneous of degree one in $B_t$.

\textsuperscript{13} Another way to introduce financial frictions is by limiting the amount of debt that the firm can raise at any point in time as in Whited (1992), Hubbard, Kashyap and Whited (1995), and Jaramillo, Schiantarelli and Weiss (1996).
Bond and Meghir (1994) show that the Euler equation characterising the optimal path for investment in the Euler investment model with debt and corporate taxes can be written as

\[-(1 - \delta) \beta r_i E_i \left[ (1 + \lambda_{0,i}^i) \left( \frac{\partial \Pi}{\partial t} \right)_{t+1} \right] = -(1 + \lambda_i^0) \left( \frac{\partial \Pi}{\partial K} \right)_i - (1 + \lambda_i^0) \left( \frac{\partial \Pi}{\partial K} \right)_i - \gamma_i \left( \frac{B_i^2}{p_i^i K_i^2} \right),\]

(2.24)

where \( \gamma_i = E_i \left[ \beta' r_i (1 + \lambda_{0,i}^i) \left( q'_{r,i} \left( \frac{X_{i+1}}{B_i} \right) + (q'_{r,i} - q'_{r,i}) \tau_{i+1} \right) \right] \),

and the derivatives of \( q'_{r,i} \) and \( i' \) with respect to the ratio \( (B_i / p_i^i K_i) \) are denoted by \( q'_{r,i} \geq 0 \) and \( i' \geq 0 \) respectively. Substituting (2.18) and (2.19) into (2.24), the empirical equation of the Euler investment model with debt and corporate taxes under the null of no financing constraints is

\[
\left( \frac{I}{K} \right)_{r+1} = c(1 - \phi_{r+1}) + (1 + c)\phi_{r+1} \left( \frac{I}{K} \right)_i - \phi_{r+1} \left( \frac{C}{K} \right)_i + \frac{\phi_{r+1}}{b c} J_i
\]

\[
+ \frac{\phi_{r+1}}{b (1 - \delta) a c} \left( \frac{Y}{K} \right)_i - \frac{(1 + r_i) \gamma_i}{b (1 - \delta) a c} \left( \frac{B_i}{K} \right)_i + \nu_{r,i}.
\]

(2.25)

For the \( Q \) model of investment, Hayashi (1982) has shown that, if the value of the firm is extended to include debt and corporate income tax is introduced, then tax-adjusted \( Q \) can be written as a function of the average \( Q \) ratio without debt, \( \frac{V_i}{(1 - \delta) p_i^i K_i} \), and the debt-to-replacement cost ratio, \( \frac{B_i}{(1 - \delta) p_i^i K_i} \), where \( V_i \) is the stock market valuation of the firm's equity, and \( B_i \) is the market value of the firm's debt and other net liabilities. Using the above, the empirical specification of the \( Q \) model of investment with corporate taxes and debt is
\[
\left( \frac{I}{K} \right)_t = c + \beta Q_t^a + v_t.
\]  

(2.26)

where \( Q_t^a \) is the tax-adjusted \( Q \), and \( v_t \) is interpreted as a shock to marginal adjustment costs.

2.3 Estimation Methodology

The \( Q \) model of investment states that under the null hypothesis of no financing constraints, investment should only depend on \( Q \). To test for the presence of financing constraints, it is common in the literature to add financial factors such as the cashflow-to-capital ratio to the \( Q \) model of investment. A positive coefficient of the financial variable suggests that the firm faces financing constraints. The problem with the cash flow measure is that it might be a good proxy for future investment opportunities as well. In the face of imperfect financial markets, the degree of leverage of the firm may deter the availability of external financing even after controlling for Tobin’s \( q \). In testing for the presence of financing constraints when using the \( Q \) model of investment it is thus common to add a proxy for the degree of leverage of the firm, such as the ratio of debt-to-capital.

The standard \( Q \) model assumes that current investment does not depend on lagged investment, which would be the case in a perfect world. However, in reality there may be a link between current and lagged investment, since firms often times make arrangements that are costly to cancel. Such a link can be incorporated into the \( Q \) model of investment by specifying \( G(I_t, K_t) = \frac{1}{2} bK_t [ (I/K)_t - \gamma (I/K)_{t-1} - c]^2 \) as adjustment-cost function, rather than the standard adjustment-cost function as proposed by Summers (1991).

The Euler investment model also incorporates both financial and leverage variables, and allows to test for strong persistence in investment to capital ratios by allowing current investment to depend on lagged investment. Let \( MPK_i \) indicate the marginal profitability of capital, that is, the investment opportunities of the firm, \( FIN_i \)
the internal financing available to the firm, and \( LEV_i \) the leverage of the firm. Here the subscript \( i \) refers to the company, and \( t \) refers to the time period. We will thus test for the presence of financing constraints by estimating the following extended empirical specification of models (2.25) and (2.26)

\[
\left( \frac{I}{K} \right)_{it} = c + \beta_1 \left( \frac{I}{K} \right)_{it-1} + \beta_2 MPK_{it} + \beta_3 FIN_{it} + \beta_4 LEV_{it} + d_t + f_t + \nu_{it},
\]

where we allow for time-specific effects \( d_t \) and firm-specific effects \( f_t \). Bond and Meghir (1994) show that the dynamics of the structural investment model imply that \( \beta_3 < 0 \), and \( \beta_4 < 0 \). If \( \beta_3 > 0 \), then the cash flow coefficient reflects financing constraints. The main discrepancies between the model in (2.27) and the Euler equation in (2.26) is that the Euler model of investment includes two additional explanatory variables, namely, the square of lagged investment-to-capital, \( \left( \frac{I}{K} \right)_{it-1}^2 \), and the user cost of capital. Both variables are neglected in our empirical work, since \( \phi \left( \frac{I}{K} \right)_{it-1}^2 \) is a small, second-order effect, and because the user cost of capital is difficult to measure.

The testable implications of equation (2.27) are as follows. In the absence of financial restrictions and agency problems, firm investment depends exclusively on the firm’s investment opportunities. However, to the extent that the firm faces constraints on external financing, its investment will be determined in part by its internal resources. Furthermore, in the face of imperfect financial markets, the degree of leverage of the firm may deter the availability of external financing. Therefore, we consider that a firm faces a better functioning financial system when, first, its investment is more responsive to changes in investment opportunities; second, investment is less determined by the internal resources; and, third, investment is less negatively affected by the firm’s leverage.

In essence, we have shown that the testable implications of the \( Q \) model and the Euler model of investment are not different, although they are based upon different assumptions. In fact, the specification in equation (2.27) is also similar to the one in
Gilchrist and Himmelberg (1998). The difference is that they assume that $MPK_a$ and $FIN_a$ follow a vector autoregressive process, and use a large number of variables to forecast the future marginal profitability of investment.

We will implement the model in (2.27) in the following chapters by taking different proxies for the variables $MPK_a$, $FIN_a$, and $LEV_a$. The investment opportunities $MPK_a$ are proxied by either the average $Q_a$ or the net sales-to-capital ratio $\left( \frac{S}{K} \right)_a$. The first measure of firm performance follows from the $Q$ model of investment. This measure is, however, problematic in countries with inefficient stock markets, and can only be used for listed firms. The sales-to-capital ratio is a proxy for the net output-to-capital ratio $\left( \frac{Y}{K} \right)_a$ in the Euler investment model. In fact, assuming a Cobb-Douglas production function, Gilchrist and Himmelberg (1998) show that the marginal profitability of fixed capital equals the ratio of net sales to capital (up to a scale parameter). This measure has the advantage that it can also be used for unlisted firms. We proxy the financial factors $FIN_a$ by either the cash-flow to capital ratio $\left( \frac{CF}{K} \right)_a$ or by the cash (and equivalents) to capital ratio $\left( \frac{\text{Cash}}{K} \right)_a$. The first variable has been the preferred measure to capture the availability of funds in the literature. Finally, we capture the degree of leverage of the firm $LEV_a$ by the square of the ratio of debt-to-capital $\left( \frac{B}{K} \right)_a^2$, because it is expected that agency costs are increasing in the level of debt. Related literature has generally included only long-term debt in the previous ratio, because agency costs are thought to be reflected most by long-term debt.\(^{14}\)

\(^{14}\) In Chapter 5 we also include short-term debt, because in the special case of Korea firms’ debt consists to a large extent of short-term debt, so that excluding short-term debt in the leverage ratio would understate the agency costs of debt to the firm.
2.4 Econometric Problems of Estimating Dynamic Investment Models

Dynamic investment models are likely to suffer from both endogeneity and heterogeneity problems. In the $Q$ model of investment the error term is a technology shock to the profit function. Since $Q$ may be a function of the technology shock it can be an endogenous variable. Hayashi and Inoue (1991) argue that a wide range of variables pertaining to the firm such as output and cash flow also depend on the technology shock, and are thus endogenous as well. The Euler investment model is likely to suffer from an endogeneity problem as well, since serial correlation in the error term\(^{15}\) would imply that the lagged investment ratio is an endogenous variable. The endogeneity of the explanatory variables calls for an instrumental variable procedure to obtain consistent estimates of the coefficients of interest. In addition, substantial differences across individuals in their investment behaviour may lead to a heterogeneity problem reflected by the presence of unobserved individual effects. Hsiao and Talmiscioglou (1997) argue that pooling data and grouping individuals according to certain \textit{a priori} criteria can also help overcome this heterogeneity problem.

In estimating the dynamic investment model (2.27) in the previous section, we will use Generalised Methods of Moments (GMM) estimators and panel data to control for potential unobserved individual effects, endogeneity of explanatory variables, and the use of lagged dependent variables. This section describes these GMM estimators for dynamic panel data models that have been introduced by Hansen (1982), Holtz-Eakin, Newey and Rosen (1988), Arellano and Bond (1991) and Arellano and Bover (1995). Consider the following generalisation of the model in (2.27)

$$y_u = \alpha y_{u-1} + \beta' x_u + \gamma' f_u + v_u,$$  \hspace{1cm} (2.28)

where

$$v_u = \eta_u + u_u.$$  \hspace{1cm} (2.29)

\(^{15}\) Since firms are likely to be subject to common shocks, the error terms are likely to be correlated across firms in each period. This suggests that identification of the Euler equation may be problematic without long time series of data for individual firms, since independence of the error terms across individuals is typically required for consistent estimation in panels when the number of time periods is fixed (Bond and Van Reenen, 1999).
and
\[ E(u_i | x_0, ..., x_T, \eta_i) = 0, \] (2.30)

where \( f_i \) is an observed individual effect and \( \eta_i \) is an unobserved individual effect. In this model, regardless of the existence of unobserved individual effects, unrestricted serial correlation in \( u_i \) implies that \( y_{i-1} \) is an endogenous variable. We allow for the possibility of simultaneous determination and reverse causality of the explanatory variables and the dependent variable by assuming weak exogeneity of the explanatory variables in the sense that they are assumed to be uncorrelated with future realisations of the error term.\(^{16}\)

If there are no strong unobserved individual effects we can apply a GMM estimator to equation (2.28) in levels. This estimator overcomes the potential problem of endogeneity of the explanatory variables and the use of lagged dependent variables. Under the assumption that the error term \( u_i \) is serially uncorrelated or, at least, follows a moving average process of finite order, and that future innovations of the dependent variable do not affect current values of the explanatory variables, the lagged variables \( (y_{i-2}, y_{i-3}, ..., y_{i}) \) and \( (x_{i-2}, x_{i-3}, ..., x_{i}) \) can be used as valid instruments in the GMM estimation. We call this the GMM level estimator.

In the presence of unobserved individual effects the GMM level estimator produces inconsistent estimates. An indication that unobserved individual effects are present is a persistent serial correlation of the residuals. To assess the validity of the GMM level estimates we use two specification tests suggested by Arellano and Bond (1991). The first is a Sargan test of over-identifying restrictions, which tests the validity of the instruments, and the second is a test of second-order serial correlation of the error term, which tests for the presence of unobserved individual effects.

If the GMM level estimates are not valid, we solve the estimation problem raised by the potential presence of unobserved individual effects by estimating the specific

\(^{16}\)In the setting of the investment model in (3.27) the assumption of weak exogeneity of the explanatory variables means that current explanatory variables may be affected by past and current investment-to-capital ratios, but not by future ones.
model in first-differences. If we remove the unobserved individual effect by first-differencing equation (2.28) we obtain

\[ \Delta y_a = \alpha \Delta y_{a-1} + \beta \Delta x_a + \Delta u_a. \]  

(2.31)

The use of instruments is again required because \( \Delta u_a \) is correlated with \( \Delta y_{a-1} \) by construction, and joint endogeneity of the explanatory variables might still be present. Under the assumptions that the error term \( u_a \) is not serially correlated and the explanatory variables are weakly exogenous, the following moment conditions apply to the lagged dependent variable and the set of explanatory variables:

\[
E(y_{a-s}, \Delta u_s) = 0 \quad \forall s \geq 2; t = 3, ..., T \quad (2.32)
\]

\[
E(x_{a-s}, \Delta u_s) = 0 \quad \forall s \geq 2; t = 3, ..., T . \quad (2.33)
\]

so that \( (y_{a-2}, y_{a-3}, ..., y_{a}) \) and \( (x_{a-2}, x_{a-3}, ..., x_{a}) \) are valid instruments. We refer to this estimator as the difference estimator. Arellano and Bond (1991) have shown that under the above assumptions the difference estimator is an efficient GMM estimator for the above model. To assess the validity of the assumptions of the difference estimator, we use again a Sargan test of over-identifying restrictions, which tests the validity of the instruments, and a test of second-order serial correlation of the error term, which tests whether the error term in the differenced model follows a first-order moving average process\(^\text{17}\).

Although the difference estimator solves the problem of the potential presence of unobserved individual effects, the estimator has some statistical shortcomings. Blundell and Bond (1997) show that when the dependent variable and the explanatory variables are persistent over time, lagged levels of these variables are weak instruments for the regression equation in differences. To overcome these statistical problems associated with the difference estimator one can use Arellano and Bover’s (1995) system estimator.

\(^{17}\) The use of endogenous variables dated \( t - 2 \) as instruments is only valid if \( u_a \) is serially uncorrelated, implying a first-order moving average error term in the differenced model.
The system estimator makes the additional assumption that the differences of the right-hand side variables are not correlated with the unobserved individual effects:

\[
E(y_u \eta_s) = E(y_u \eta_s) \quad \forall t, s, \\
E(x_u \eta_s) = E(x_u \eta_s) \quad \forall t, s.
\]  

(2.34)  

(2.35)

These assumptions may be justified on the grounds of stationarity. Arellano and Bover (1995) show that combining equations (2.32)-(2.33) and (2.34)-(2.35) gives the following additional moment restrictions:

\[
E(y_u \Delta y_{u-1}) = 0 \\
E(y_u \Delta x_{u-1}) = 0.
\]  

(2.36)  

(2.37)

Thus, valid instruments for the regression in levels are the lagged differences of the corresponding variables. Hence, we use \((y_{u-2}, y_{u-3}, \ldots, y_u)\) and \((x_{u-2}, x_{u-3}, \ldots, x_u)\) as instruments for the equations in first differences, and \(\Delta y_{u-1}\) with \(\Delta x_{u-1}\) as instruments for the equations in levels.

To assess the validity of the additional assumptions of the system estimator we consider two specification tests suggested by Arellano and Bond (1991), namely, a Difference Sargan test and a Hausman specification test. Both tests are designed to assess the validity of the additional instruments used in the levels equations of the system estimator. The Difference Sargan test statistic compares the Sargan statistic for the system estimator and the Sargan statistic for the corresponding first-differenced estimator. The difference Sargan test statistic is asymptotically distributed as \(\chi^2\) under the null hypothesis of validity of the instruments. The number of degrees of freedom of the difference Sargan test statistic is given by the number of additional restrictions in the system estimator, which equals the difference between the number of degrees of freedom

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18 Note that there may be correlation between the levels of the right-hand side variables and the unobserved individual effects.

19 Moment restrictions based on other lagged differences are redundant (see Arellano and Bover, 1995).

20 The instruments for the regression in differences are the same as before, that is, the lagged levels of the corresponding variables.
of the system estimator and that of the difference estimator. The Hausman statistics tests
the difference between the coefficients of the GMM system estimates and the
corresponding GMM first-differenced estimates, that is, the estimates without the
additional levels equations. The Hausman test statistic is a Wald test of the hypothesis
that the distance between the coefficients is zero, and the degrees of freedom is given by
the number of additional level equations.

We also introduce multiplicative dummies to assess differences across firms along
certain criteria. If we define $\Delta_u$ to be a firm-specific dummy variable, then introducing
this variable as a multiplicative dummy changes equation (2.28) as follows

$$
\begin{align*}
    y_u &= \alpha y_{u-1} + \beta' x_u + \delta' \Delta_u x_u + \gamma' f_u + v_u.
\end{align*}
$$

(2.38)

If the multiplicative dummy is an exogenous variable and $x_{u-2}$ is a valid instrument for
the endogenous variable $x_u$, then $\Delta_u x_{u-2}$ is a valid instrument for $\Delta_u x_u$. In estimating the
investment model in equation (2.27) we treat the weakly exogenous variables as
endogenous variables and potential multiplicative dummies as exogenous variables. If we
interact the weakly exogenous variables with the multiplicative dummies we use the
aforementioned appropriate transformations of these interacted variables as instruments.
Given the statistical problems and additional assumptions associated with the GMM
difference and system estimators, we will only use them in case the specification tests
reject the validity of the GMM level estimator.