X-ray waveguiding studies of ordering phenomena in confined fluids
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Appendix A

On the asymmetry of the angular intensity distribution

We discuss the asymmetry that the angular intensity distribution \( I(\theta_t, \theta_e) \) may display around the diagonal \( \theta_t = \theta_e \). Consider, for example, the contour plots shown in Figs. 5.3, 6.4 and 6.5. These intensity distributions were all obtained for waveguiding geometries in which the refractive-index profile of the waveguide is asymmetric with respect to \( z = L/2 \). In the following we derive an expression for \( I(\theta_t, \theta_e) \) which clearly shows that this is at the origin of the observed asymmetry.

Let us consider a waveguide of total length \( L \) which is divided into three sections (see Fig. A.1a). We assume that these sections each have a \( z \)-independent refractive-index profile, i.e. \( n_1 = n_1(x) \) and \( n_2 = n_2(x) \). In order to obtain an expression for \( I(\theta_t, \theta_e) \), we first derive an expression for the field amplitude at the exit of the waveguide. According to Eq. (2.15), the field at \( z = L_1 \) is given by

\[
\Psi(x, L_1) = \sum_{m=0}^{\infty} c_m^{(1)} \phi_m^{(1)}(x) e^{-i\beta_m^{(1)} L_1}, \tag{A.1}
\]

with

\[
c_m^{(1)} = \frac{1}{P_m^{(1)}} \int_{-\infty}^{\infty} \phi_t(x) \phi_m^{(1)}(x) dx, \tag{A.2}
\]

where

\[
P_m^{(1)} = \int_{-\infty}^{\infty} \left| \phi_m^{(1)}(x) \right|^2 dx \tag{A.3}
\]

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is the power in mode \(m\). Here, \(\phi_i(x) = \phi_i(\theta_i, x)\) denotes the incident wave field and the superscripts \((1)\) refer to section 1. In Eq. (A.2) we assumed, for ease of notation and without loss of generality, that the mode profiles \(\phi_m^{(1)}(x)\) are real-valued functions. The upper limit of the summation in Eq. (A.1) is set to infinity so as to indicate the contribution of the radiation modes. The field amplitude at the end of the second section is given by:

\[
\Psi(x, L_1 + L_2) = \sum_{m=0}^{\infty} c_m^{(2)} \phi_m^{(2)}(x) e^{-i\beta_m^{(2)} L_2},
\]  

(A.4)

with

\[
c_m^{(2)} = \frac{1}{P_m^{(2)}} \int_{-\infty}^{\infty} \Psi(x, L_1) \phi_m^{(2)}(x) dx.
\]  

(A.5)

Substituting Eq. (A.1) into Eq. (A.5), we can rewrite Eq. (A.4) as:

\[
\Psi(x, L_1 + L_2) = \sum_{m=0}^{\infty} \sum_{k=0}^{\infty} c_k^{(1)} e^{-i\beta_k^{(1)} L_1} C_{km}^{(1,2)} \phi_m^{(2)}(x) e^{-i\beta_m^{(2)} L_2},
\]  

(A.6)

with

\[
C_{km}^{(p,q)} = \int_{-\infty}^{\infty} \phi_k^{(p)}(x') \phi_m^{(q)}(x') dx'.
\]  

(A.7)

By repeating the above steps for the third section, we finally obtain the field amplitude at the exit of the waveguide:

\[
\Psi(x, L) = \sum_{l,m,k=0}^{\infty} c_k^{(1)} e^{-i\beta_k^{(1)} L_1} C_{km}^{(1,2)} e^{-i\beta_m^{(2)} L_2} C_{ml}^{(2,1)} \phi_l^{(1)}(x) e^{-i\beta_l^{(1)} L_3}.
\]  

(A.8)

The intensity of the exiting wave field measured in the far-field at an angle \(\theta_e\) is given by Eq. (2.52):

\[
I(\theta_i, \theta_e) = 4 \left| \left( \frac{i}{R\lambda} \right)^{1/2} e^{-i\kappa R} \int_{-\infty}^{\infty} \Psi(\theta_i, x, L) \phi_e(\theta_e, x) dx \right|^2.
\]  

(A.9)

On the right-hand side of Eq. (A.9), we added \(\theta_i\) and \(\theta_e\) in order to indicate the implicit dependence of \(\Psi(x, L)\) and \(\phi_e(x)\) on these parameters. Making use of time-reversal symmetry of the incoming and outgoing waves, i.e.,

\[
\phi_e(\theta_e, x) = \phi_i(\theta_e, x),
\]  

(A.10)
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we rewrite Eq. (A.9) as:

\[ I(\theta_i, \theta_e) = 4 \left( \frac{i}{R\lambda} \right)^{1/2} e^{-ik_0R} \int_{-\infty}^{\infty} \Psi(\theta_i, x, L) \phi_i(\theta_e, x) dx \right|^2 \quad (A.11) \]

By inserting Eq. (A.8) into Eq. (A.11) and performing the integration, we obtain

\[ I(\theta_i, \theta_e) = \frac{4}{R\lambda} \sum_{l,m,k=0}^{\infty} c_k^{(1)}(\theta_i) e^{-i\delta_k^{(1)} L_1} C_{km}^{(1,2)} e^{-i\delta_k^{(2)} L_2} C_{ml}^{(2,1)} e^{-i\delta_k^{(1)} L_3} c_l^{(1)}(\theta_e) \right|^2 \quad (A.12) \]

By interchanging \( \theta_i \) and \( \theta_e \), we obtain

\[ I(\theta_e, \theta_i) = \frac{4}{R\lambda} \sum_{l,m,k=0}^{\infty} c_k^{(1)}(\theta_i) e^{-i\delta_k^{(1)} L_3} C_{km}^{(1,2)} e^{-i\delta_k^{(2)} L_2} C_{ml}^{(2,1)} e^{-i\delta_k^{(1)} L_1} c_l^{(1)}(\theta_e) \right|^2 \quad (A.13) \]

If the sections are distributed asymmetrically with respect to \( z = L/2 \), i.e., \( L_1 \neq L_3 \), then Eqs. (A.12) and (A.13) are not equivalent (see Fig. A.1). Hence, the intensity distribution \( I(\theta_i, \theta_e) \) is then asymmetric with respect to the diagonal line \( \theta_i = \theta_e \).