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Chapter 5

Entropy statistics as a framework to analyse technological evolution

The previous chapters addressed product innovation in complex technological system in formal simulation models. In the coming chapters, product innovation is addressed empirically using data on technical characteristics and service characteristics of product technologies. The central issue of concern in this chapter holds whether statistical patterns in the evolution of product designs can be understood in the light of the formal models discussed hitherto.

The methodology that will be used is based on entropy statistics. Entropy statistics is applied to frequency distributions of product characteristics incorporated in the products offered in an industry. The changes that occur in the distribution of product designs describe the pattern in technological evolution over time. Entropy is a variety measure. The changes in entropy indicate patterns in the evolution of product variety. For example, an evolutionary pattern towards increased product standardisation is indicated by a fall in the entropy value over time. When one is able to describe the technological evolution in terms of the degree and nature of variety that occurs over time, one is able to test whether patterns in technological evolution corresponds to the predictions that can be derived from the product life-cycle model.

This chapter is organised as follows. The entropy methodology and how it can be applied to data on product characteristics is described in Section 5.1. The entropy methodology is applied to data on technical characteristics of aircraft and helicopter designs in Section 5.2. The following Chapter 6, Chapter 7, and Chapter 8 are introduced in Section 5.3.

5.1 ENTROPY STATISTICS

The NK-model of complex systems discussed in Chapter 2 and the generalised NK-model of product innovation in complex technological systems developed in Chapter 3 are both based on the concept of design space. A design space is an N-dimensional space containing all possible combinations between alleles of elements. For each element $n$ there exist $A_n$ alleles. Alleles of elements are coded starting from 0, 1 to $A_n-1$. Each product design can thus be coded as a string of alleles of elements. Each product is thus described by its design $s$ containing alleles $s_2...s_N$, and is part of a possibility set $S$, for which holds:

$$s \in S; s = s_1s_2...s_N; s_n \in \{0,1,...,A_n - 1\}$$

(5.1)

The total number of possible combinations between alleles of elements binds the total number of possible designs in the design space. The size of the design space $S$ is given by:
\[ S = A_1 \cdot A_2 \cdot \ldots \cdot A_N = \prod_{n=1}^{N} A_n \]  

(5.2)

Each design \( s \) is a multidimensional point in this design space. The set of products present in an industry at some point in time is described by the N-dimensional frequency distribution of designs incorporated in the products offered in an industry. Following Saviotti (1996), such a set of products will be called a "product population".

For example, consider an industry with a product population of a hundred products. Each product design is described in two dimensions that each contains two alleles, so there are four possible designs: 00, 01, 10, or 11. Forty-five of the hundred products are designed according to design 00, thirty products according to design 01, fifteen products according to design 10, and ten products according to design 11. As a percentage of the total number of products offered in the industry, design 00 covers 45 percent, design 01 covers 30 percent, design 10 covers 25 percent, and design 00 covers 10 percent. The N-dimensional (here, two-dimensional) frequency distribution is: \( p_{00} = 0.45, \ p_{01} = 0.30, \ p_{10} = 0.15, \ p_{11} = 0.10 \).

Over time, technological evolution in products can then be described as the change in the relative frequencies of designs in the population offered in an industry. For example, a product population may evolve from a distribution of product designs with many different designs with even frequencies (e.g., \( p_{00} = 0.25, \ p_{01} = 0.25, \ p_{10} = 0.25, \ p_{11} = 0.25 \)) into a distribution that is dominated by one or few designs with high frequencies (e.g., \( p_{00} = 0.97, \ p_{01} = 0.00, \ p_{10} = 0.00, \ p_{11} = 0.03 \)). The market evolves from a high product variety towards low product variety. The pattern of technological evolution can then be characterised by a progressive standardisation, which indicates the emergence of a dominant design (here, design 00).

Another example of a pattern of technological evolution is one in which there exist one or few designs with high frequencies at the start (e.g., \( p_{00} = 0.97, \ p_{01} = 0.00, \ p_{10} = 0.00, \ p_{11} = 0.03 \)) and that progressively proliferates towards a distribution of many different designs with even frequencies (e.g., \( p_{00} = 0.25, \ p_{01} = 0.25, \ p_{10} = 0.25, \ p_{11} = 0.25 \)). This pattern can be described as a pattern of progressive product differentiation in which new design variants are successfully introduced in the market.

The objective of this chapter is to develop an empirical methodology that can describe evolutionary patterns of technological developments in terms of the changes that occur in distributions of designs in product populations. The main argument made in this chapter holds that entropy statistics applied to frequency distributions of product designs can well describe the evolution of a technology in terms of the changes in the degree and nature of product variety. As an indicator of variety the entropy of the distribution of designs can indicate the extent to which a population is characterised by a "dominant design".

In the following, I thus analyse product populations as defined as sets of technological artefacts sharing the same design space. In this, the analysis that follows differs from market analyses that define product populations in terms of the degree of similarity of the services products provide (e.g. using data on the elasticity of substitution of consumers with regard to different products). The empirical analysis in this study, however, deals with the technological evolution of different artefact designs that are all represented in the same design space whatever the differences between different artefacts designs in terms of the level of service characteristics they provide.
5.1.1 Entropy as a measure of variety

Entropy measures the degree of randomness in a distribution. It is originally developed in thermodynamics where Boltzmann introduced the entropy concept in the context of randomly moving particles (see, e.g., Batten 1983; Prigogine and Stengers 1984; Dyke 1988). Each particle is described by its coordinates in what is called a particle’s state space. When many particles are randomly moving through state space, like particles of a gas in a box, the resulting spatial distribution of all particles in what is called “phase space” is flat. The flat distribution follows from the fact that at all times each particle has an equal probability to be present in any area in the box. The flat distribution in phase space at the macroscopic level indicates maximum randomness in the behaviour of particles in state space at the microscopic level. This distribution is characterised by maximum entropy.

When particles behave in a non-random way, some areas in the box are filled with more particles than other areas, and the resulting distribution at the macroscopic level is skewed. In that case, the entropy of the distribution is lower compared to the case in which all particles move randomly. In the extreme case when all particles cluster in one area, entropy is lowest.

When one translates the entropy concept to agents that search for product designs by combining alleles of different elements, an individual designer “moves around” in design space by experimenting with different combinations of alleles analogous to particles moving around in a state space. A population of designers (firms) moving around in a state space, i.e. design space, makes up a frequency distribution of designs in phase space.

In this case, maximum entropy corresponds to the case in which all designs occur at the same frequency. Such a complete flat distribution occurs when designers would randomly “move around” in design space and randomly choose the alleles of elements. In this hypothetical case, any product design has an equal probability to occur and the product population would be one with even frequencies of all designs. This hypothetical case refers to a situation in which designers have not learnt anything about the properties of different designs and simply choose a design at random analogous to randomly moving particles in a box.  

A skewed distribution occurs when few of the possible designs are dominating the product population. In that case, the frequency of some designs is high, while the frequency of most designs is low or zero. Such a distribution has a low entropy value indicating that product variety is low. In this case, designers have not chosen a design at random, but have somehow learned what designs are to be preferred, for example, by applying a search strategy in design space or a search strategy in function space as described in Chapter 2 and Chapter 3.

The entropy measure thus indicates the degree of variety of product designs in a product population. To describe a product population as a frequency distribution of designs, let each

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101 In the biological context, maximum entropy refers to a distribution of genotypes in which each possible genotype has the same frequency in the population. Again, this distribution is hypothetical as it reflects the end of a long series of mutations that lead to a more and more flat distribution of genotypes, while selective pressure is absent (Shpak and Wagner 2000: 32).

102 Savioitii (1988b: 91) proposed the use entropy statistics to measure product variety using data of product characteristics. Frenken et al. (1999b, 2000) and Van Herpen and Pieters (2000) actually applied the entropy measure as a measure of product variety using data on product characteristics. Many other applications of entropy in social science have been explored. In economics, entropy is best known as a measure of industrial concentration as an alternative to the Herfindahl-index (Theil 1967; Zadjenweber 1972; Krehm 1977). Other applications of entropy statistics include analyses of inequality (Theil 1967) and of input-output tables (Theil 1967; Batten 1983). In sociology, entropy statistics has been applied to racial distributions at different levels of aggregation (Theil 1972). In innovation studies, entropy has been used as a measure of variety in patents and R&D expenditure of different technologies (Grupp 1990; Kodama 1990; Carpenter and Temple 1996). In scientometrics, Leydesdorff (1991, 1992, 1995) introduced entropy statistics in analyses of word and citation distributions.
product be a string of \( N \) alleles in an \( N \)-dimensional design space.\(^{103}\) Each dimension is labelled here as \( X_n (n=1,\ldots,N) \) with a total of \( A_n \) alleles. Since the first allele of an element is labelled as “0”, the second allele as “1”, \( \textit{et cetera} \), I label the alleles of the first dimension by \( i=0,1,\ldots,(A_1-1) \), for the second dimension \( j=0,1,\ldots,(A_2-1) \), \( \textit{et cetera} \). The frequencies at which a design with alleles \( ij...w \) occurs is denoted by \( p_{ij...w} \) (where index \( w \) stands for the allele of the last \( N \)th element). For example, when twenty percent of all products in a product population are designed according to string “032001”, I have \( p_{032001} = 0.20 \).

The entropy of an \( N \)-dimensional distribution is given by (Theil 1972: 116, 155):

\[
H (X_1, X_2, \ldots, X_N) = - \sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} \cdots \sum_{w=0}^{A_N-1} p_{ij...w} \cdot \log_2 (p_{ij...w}) \quad (5.3)^{104}
\]

Throughout our analyses, entropy is measured in bits using logarithm two. The entropy measure is an indicator of dispersion in a distribution. The higher the entropy of a distribution, the larger its dispersion. In the context of frequency distributions of designs described in terms of alleles, entropy can be considered as an indicator of product variety.\(^{105}\)

Entropy is zero when all products present in the population are designed in exactly the same way, \( \text{i.e.} \), when all products share the same alleles for all elements. This design has frequency one and all other designs have zero frequency, thus:

\[
H_{\text{min}} (X_1, X_2, \ldots, X_N) = -1 \cdot \log_2 (1) = 0 \quad (5.4)
\]

Entropy is positive otherwise. The larger the entropy value, the larger the product variety in a frequency distribution of designs. The maximum entropy is limited by the size of design space \( S \). When all \( S \) possible combinations of alleles have an equal frequency, one obtains for all designs \( p_{ij...w} = 1/S \). The entropy of this distribution equals:

\[
H_{\text{max}} (X_1, X_2, \ldots, X_N) = - S \cdot \left( \frac{1}{S} \right) \cdot \log_2 \left( \frac{1}{S} \right) = - \log_2 \left( \frac{1}{S} \right) = \log_2 (S) \quad (5.5)
\]

This value is the maximum possible entropy value for a distribution of products designs with a design space of \( S \) possible designs (Theil 1972: 6).\(^{106}\)

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\(^{103}\) Note that dimensions in design space can be taken in a broader sense than in the sense of components. Any technical dimension along which designs can differ, can be taken into account in design space. For example, the number of engines can also taken as a dimension with allele 0 = “one engine”, 1 = “two engines”, \( \textit{et cetera} \).

\(^{104}\) \( 0 \cdot \log_2 0 = 0 \).

\(^{105}\) In a study on changes in the degree of standardisation of nuclear power stations in the United States and France, David and Rothwell (1994b) calculate Herfindahl indexes to indicate standardization for each subsystem. Herfindahl measures can be used for indicating standardisation in the same way as the uni-dimensional entropy measure (formula 5.6). Their study differs from the analysis below in that they summed the uni-dimensional Herfindahl index using a “performance-weights” for each subsystem, whereas the present study uses multi-dimensional entropy (formula 5.3). The performance-weights were calculated from the estimated likelihoods of a plant experiencing a forced outage due to an identified malfunction in the sub-system in question.

\(^{106}\) Since maximum entropy equals the logarithm of the size of the design space, it holds that for a linear increase in the size of the design space, maximum entropy increases logarithmically. This property reflects that the higher the existing maximum possible variety in a population, the smaller the marginal increase in maximum entropy caused by the addition of an extra possibility in the state space (\( \text{i.e.} \) design space). This property of the entropy formula reflects the idea that variety is not a linear function of the number of possible states. The addition of a new possible design always increases maximum possible variety, but decreasingly so.
For N-dimensional distributions, one can also measure the entropy for each dimension separately on the basis of the marginal frequencies. The uni-dimensional or marginal entropy values indicate the variety in a product population with respect to one design dimension only, and is given by:

\[
H(X_i) = - \sum_{j=0}^{A_i-1} p_{i..} \cdot \log_2 (p_{i..}) \tag{5.6.1}
\]

\[
H(X_2) = - \sum_{j=0}^{A_2-1} p_{.j..} \cdot \log_2 (p_{.j..}) \tag{5.6.2}
\]

\[
H(X_N) = - \sum_{w=0}^{A_N-1} p_{...w} \cdot \log_2 (p_{...w}) \tag{5.6.3}
\]

Box 5-1 gives examples of frequency distributions of product designs with minimum, intermediate and maximum entropy for a design space with two elements and two alleles \((N=2, A_n=2)\). The first distribution is completely dominated by one design with relative frequency equal to one. This distribution of designs is characterised by minimum entropy and minimum product variety. The second distribution is characterised by a skewed distribution in which all possible designs occur, but at different frequencies. This distribution is characterised by intermediate entropy and intermediate variety. In the third distribution, all four possible designs have equal frequency. This distribution of designs is characterised by maximum entropy and maximum product variety.

Summarising, entropy indicates the variety of product designs in a product population. When all products are designed according to one and the same design, entropy equals zero. When all possible product designs occur at even frequencies in a product population, entropy is maximum equalling \(\log_2 (S)\). Any other distribution of designs in a product population has an entropy value between zero and \(\log_2 (S)\).

### 5.1.2 Mutual information as a measure of dependence

The measure that indicates the degree of dependence between different dimensions of a frequency distribution is the measure of mutual information \(T\). The N-dimensional mutual information is given by (Theil 1972: 126, 155):

\[
T(X_1, X_2, ..., X_N) = \sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} ... \sum_{w=0}^{A_N-1} p_{i...w} \cdot \log_2 \left( \frac{P_{i...w}}{P_{i..} \cdot P_{.j..} \cdot ... \cdot P_{...w}} \right) \tag{5.7}
\]

in bits.
Case 1: Minimum entropy

Frequencies: \( p_{00} = 1.0, \ p_{01} = 0.0, \ p_{10} = 0.0, \ p_{11} = 0.0 \)
Marginal frequencies: \( p_0 = 1.0, \ p_1 = 0.0, \ p_0 = 1.0, \ p_1 = 0 \)

\[ H(X_1, X_2) = -l\{ l \cdot \log_2 (1) \} = 0 \text{ bits} \]
\[ H(X_1) = -l\{ l \cdot \log_2 (1) \} = 0 \text{ bits} \]
\[ H(X_2) = -l\{ l \cdot \log_2 (1) \} = 0 \text{ bits} \]

Case 2: Intermediate entropy

Frequencies: \( p_{00} = 0.45, \ p_{01} = 0.30, \ p_{10} = 0.15, \ p_{11} = 0.10 \)
Marginal frequencies: \( p_0 = 0.75, \ p_1 = 0.25, \ p_0 = 0.60, \ p_1 = 0.40 \)

\[ H(X_1, X_2) = -l\{ 0.45 \cdot \log_2 (0.45) \} - l\{ 0.3 \cdot \log_2 (0.3) \} - l\{ 0.15 \cdot \log_2 (0.15) \} - l\{ 0.1 \cdot \log_2 (0.1) \} = 1.78 \text{ bits} \]
\[ H(X_1) = -l\{ 0.75 \cdot \log_2 (0.75) \} - l\{ 0.25 \cdot \log_2 (0.25) \} = 0.81 \text{ bits} \]
\[ H(X_2) = -l\{ 0.6 \cdot \log_2 (0.6) \} - l\{ 0.4 \cdot \log_2 (0.4) \} = 0.97 \text{ bits} \]

Case 3: Maximum entropy

Frequencies: \( p_{00} = 0.25, \ p_{01} = 0.25, \ p_{10} = 0.25, \ p_{11} = 0.25 \)
Marginal frequencies: \( p_0 = 0.50, \ p_1 = 0.50, \ p_0 = 0.50, \ p_1 = 0.50 \)

\[ H(X_1, X_2) = -4 \cdot l\{ 0.25 \cdot \log_2 (0.25) \} = 2.00 \text{ bits} \]
\[ H(X_1) = -2 \cdot l\{ 0.5 \cdot \log_2 (0.5) \} = 1.00 \text{ bits} \]
\[ H(X_2) = -2 \cdot l\{ 0.5 \cdot \log_2 (0.5) \} = 1.00 \text{ bits} \]

Box 5-1: Entropy values of three example distributions

The mutual information value \( T \) only applies to multi-dimensional frequency distributions as it is a dependence measure. The measure indicates the extent to which alleles along different dimensions are co-occurring in products. The mutual information value equals zero when there is exist no dependence between any of the dimensions. In that case, the joint frequency of alleles of elements \( p_{ij...w} \) corresponds exactly to the frequency that could be expected from the product of the marginal frequencies \( (p_i \cdot p_j \cdot ... \cdot p_w) \). When the product of marginal frequencies does not correspond to the joint frequency, there is dependence between dimensions and \( T \) takes on a positive value. The less the product of marginal frequencies corresponds to the joint frequency, the higher the value \( T \).

The mutual information measure directly relates to the concept of entropy as the mutual information of a distribution can be derived from the multi-dimensional and marginal entropy values. It has been shown that the mutual information of a multi-dimensional distribution equals the sum of the marginal entropy values minus the multi-dimensional entropy value (Sahal 1979: 129; Theil and Fiebig 1994: 12), i.e.:

\[ T (X_1, X_2, ..., X_N) = \left( \sum_{n=1}^{N} H(X_n) \right) - H (X_1, X_2, ..., X_N) \]  \hspace{1cm} (5.8)
From this equation, it is readily derived that the mutual information is always zero when the multi-dimensional entropy is zero. As derived in equation (5.4), multi-dimensional entropy is zero when one design occurs with frequency one implying that the alleles incorporated in this design also occur with frequency one. The sum of marginal entropy values equals zero, implying that mutual information equals zero:

\[
T (X_1, X_2, ..., X_N) = \left( \sum_{n=1}^{N} H(X_n) \right) - H(X_1, X_2, ..., X_N)
\]

\[
T (X_1, X_2, ..., X_N) = \left( \sum_{n=1}^{N} -1 \cdot \log_2 (1) \right) = -1 \log_2 (1) = 0 + 0 = 0
\]

Put differently, when there is no variation along dimensions, there cannot be dependence between dimensions.

It can also be derived that the mutual information is always zero when the multi-dimensional entropy is maximum. As derived in equation (5.5), entropy is maximum when all possible designs in design space have an equal frequency 1/S. In that case, the alleles along each dimension also have an equal frequency with marginal frequencies equal to 1/A_n. Mutual information becomes:

\[
T (X_1, X_2, ..., X_N) = \left( \sum_{n=1}^{N} H(X_n) \right) - H(X_1, X_2, ..., X_N)
\]

\[
T (X_1, X_2, ..., X_N) = \left( \sum_{n=1}^{N} \log_2 (A_n) \right) - \log_2 (S)
\]

\[
T (X_1, X_2, ..., X_N) = \log_2 (A_1 \cdot ... \cdot A_n) - \log_2 (S)
\]

\[
T (X_1, X_2, ..., X_N) = \log_2 (S) - \log_2 (S) = 0
\]

In the following, I will apply the mutual information measure not only to the N-dimensional frequency distribution, but also to each of the two-dimensional frequency distributions that can be derived from the N-dimensional frequency distribution. The number of T-values of two-dimensional frequency distributions is bounded by the number of pairs of design dimensions.\(^{107}\) For example, when N=3, one can compute three two-dimensional mutual information values (see also, Frenken 2000: 263-264, included in the study as Chapter 8):

\(^{107}\) The number of two-dimensional distributions that can be derived from an N-dimensional distribution equals \(\frac{N^2 - N}{2}\).
The T-values for two-dimensional distributions indicate the dependence between two design dimensions and are informative with regard to the importance of epistatic relations. A high mutual information between two dimensions suggests that an important epistatic relation exists between the two dimensions, since designers offer predominantly alleles in particular combinations. These combinations reflect complementarities between alleles. For example, when gasoline engines would exclusively be used in metal cars and electric engines in plastic cars, mutual information between the engine and the material dimensions would be high.\(^{108}\)

To illustrate the N-dimensional and two-dimensional mutual information measure, consider the example in Box 5-2 of two frequency distributions with equal entropy but different mutual information. In the first population mutual information is zero while in the second population mutual information is high. The six mutual information values for the two-dimensional distributions indicate along which dimensions dependence is present. The example shows that strong dependence exists between the first and second dimension, the first and third dimension, and the second and third dimension. Allele “0” in the first dimension \((i=0)\), allele “0” in the second dimension \((j=0)\) and allele “0” in the third dimension \((k=0)\) always co-occur. And, allele “1” in the first dimension \((i=1)\), allele “1” in the second dimension \((j=1)\) and allele “1” in the third dimension \((k=1)\) always co-occur.

As mutual information measures the extent in which alleles along dimensions are co-occurring in products, it is a measure that indicates clusters of alleles. In the example of the first frequency distribution in Box 5-2, there are no multiple clusters as all designs are derived from a common core 000#.\(^{109}\) The variation is limited to the third and fourth dimension, and this variation exactly corresponds to what is expected from the marginal frequencies. This distribution is an example of a dominant design that can be characterised as design 000# since these alleles always occur in products. The other designs are derived from this dominant design by mutations in the third element and the fourth element.

By contrast, in the example of the second frequency distribution in Box 5-2, all designs are either derived from either cluster 000# or from cluster 111#. Within each cluster, variations are limited to mutations in the fourth dimension. In this case, one can speak of two dominant designs, one characterised by 000# and one by 111#. This clustering of designs is indicated by the high value for mutual information of this distribution.

\(^{108}\) Two-dimensional mutual information measurements have also been used in other contexts to analyse the dependence structure between pairs of elements in a complex system, for example in a model of cellular automata (Langton 1990) and in a model of auto-catalytic networks (Ulanowicz 1996).

\(^{109}\) Symbol # stands again for any allele as in Chapter 2.
Case 1: low mutual information

Distribution: $p_{0000} = 0.45$, $p_{0001} = 0.30$, $p_{0010} = 0.15$, $p_{0101} = 0.10$

$$H(X_1,X_2,X_3,X_4) = -(0.45 \cdot \log_2 (0.45)) - (0.3 \cdot \log_2 (0.3)) - (0.15 \cdot \log_2 (0.15)) - (0.1 \cdot \log_2 (0.1)) = 1.78 \text{ bits}$$

Case 2: high mutual information

Distribution: $p_{0000} = 0.45$, $p_{0001} = 0.30$, $p_{1110} = 0.15$, $p_{1111} = 0.1$

$$H(X_1,X_2,X_3,X_4) = -(0.45 \cdot \log_2 (0.45)) - (0.3 \cdot \log_2 (0.3)) - (0.15 \cdot \log_2 (0.15)) - (0.1 \cdot \log_2 (0.1)) = 1.78 \text{ bits}$$

Box 5-2: mutual information values of two example distributions with same entropy value

How does the mutual information measure relate to the model of product innovation in complex systems as developed in Chapter 3 and Chapter 4? Following the model, the existence of clusters of alleles indicates product differentiation resulting from epistatic relations between the functioning of alleles. Clusters of designs are expected to reflect local optima in design space as the co-occurrence of alleles indicates complementarity between dimensions. In this specific example, complementarities *casu quo* epistatic relations are limited to combinations of alleles in the first, second, and third dimension, while the fourth dimension shows no dependence with other dimensions.

Summarising, entropy indicates the degree of product variety in a product population and mutual information indicates the nature of product variety. A low mutual information value indicates that the variety in product designs is predominantly derived from one cluster of alleles as clustering between alleles is low ("a dominant design"). A high value indicates
that the variety in product designs is predominantly derived from different cores as different clusters of alleles are indicated. The existence of clusters reflects the existence of epistatic relations between elements that render different combinations of alleles optimal for different user groups.

5.1.3 Average disaggregated entropy

The entropy can be calculated of frequency distributions at any level of aggregation. Disaggregation of a distribution into several sub-distributions using a grouping variable allows one to measure the extent to which entropy disappears at the group-level. In the context of the technological evolution of a product population, a particular relevant grouping variable is the firm that offers the product on a market. This grouping allows one to address questions related to specialisation patterns of forms along different technological trajectories.

This disaggregation of the entropy of a distribution into entropy values of several sub-populations informs one about the degree of specialisation. The extreme degree of specialisation occurs when each firm would offer products that are based on one firm-specific design and each firm would use a different design. The entropy of each sub-population of products at each firm level would then be zero and the entropy of the whole product population would be positive. This example reflects a situation in which each firm is perfectly specialised in one design.

To measure the entropy of a distribution of designs that is incorporated in products offered by a single firm \( b \), one obtains for the entropy value of the distribution at the level of the firm \( b \) (Theil 1972):

\[
H_b (X_1, X_2, \ldots, X_N) = -\sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} \cdots \sum_{w=0}^{A_N-1} p_{ij...w} \cdot \log_2 \left( \frac{p_{ij...w}}{p_{-b}} \right)
\]

(5.9)

where \( p_{ij...w} \) stands for the relative frequency of products with design \( p_{ij...w} \) offered by firm \( b \) in the whole product population, and \( p_{-b} \) stands for the relative frequency of all products offered by firm \( b \). The value for \( p_{-b} \) can be derived from the frequencies \( p_{ij...w} \) as follows:

\[
p_{-b} = \sum_{i=0}^{A_1-1} \sum_{j=0}^{A_2-1} \cdots \sum_{w=0}^{A_N-1} p_{ij...w}
\]

(5.10)

When there is a total of \( B \) firms in an industry \( (b=1, \ldots, B) \), the average entropy of designs at the level of \( B \) firms is derived by the weighted sum of entropy values at the firm level. The weights are based on the firms' relative share of products in the total population of products (Theil 1972: 19). The average firm entropy is then given by:

\[
\bar{H} (X_1, X_2, \ldots, X_N) = \sum_{b=1}^{B} p_{-b} \cdot H_b (X_1, X_2, \ldots, X_N)
\]

(5.11)

in bits where \( p_{-b} \) stands for the relative number of products offered by firm \( b \) in the total product population offered at the level of the industry.
The difference between the total industry entropy and the average firm entropy informs one about the degree of specialisation among firms within an industry. Two extreme cases can be distinguished. There is a maximum specialisation when each firm offers only one variety of product designs, and there is the case of minimum specialisation when each firm offers the same variety of product designs. All other cases lie in between the two extreme cases of maximum and minimum specialisation.

In the case of maximum specialisation when each firm would offer products that are all based on one design, then the entropy of distributions of designs at each firm level $H_b$ would equal zero for all firms. Consequently, average firm entropy would equal zero and technological specialisation would be maximum.

Reversibly, in the case of minimum specialisation when each firm would offer the same variety of product designs, the frequencies of designs at the firm level would exactly correspond to the frequencies at the industry level. Then, the entropy of distributions of designs at each firm level $H_b$ would equal the industry entropy $H$. Consequently, the average firm entropy equals the industry entropy. This situation is one of total absence of technological specialisation among firms since all firms offer the same distribution of designs as the distribution of designs at the industry level.

The average entropy at the firm level can thus be considered an inverse measure of the degree of specialisation of firms. The lower the average entropy at the firm level the higher the degree of specialisation among firms, and the higher the average entropy at the firm the lower the degree of specialisation among firms.\(^{110}\)

It can be shown that the average entropy of firms cannot exceed the total entropy of the product population as a whole (Theil 1972: 65). And, since the average entropy of firms has a minimum possible value of zero, one can express the average entropy at the firm level as a percentage of total entropy value of the product population at the industry level. This relative measure normalises the average firm entropy for changes in the total entropy value as the maximum possible average entropy at the firm level is bounded by the entropy at the industry level.

In Box 5-3, two examples are given of distributions of designs at firm levels for different degrees of specialisation. In the first case, specialisation is low as all firms offer a distribution of products designs that is very similar to the overall distribution at the industry level. The low degree of specialisation is indicated by the high value for average firm entropy as a percentage of total industry entropy (over 98 percent). Thus, all firms approximately offer the same distribution of designs as the distribution at the industry level.

In the second case, specialisation is high, as the distributions of designs at the firm levels differ considerably from the distribution of designs at the industry level. All firms offer products that are based on only one or two specific product designs. The high degree of specialisation is indicated by the low value for average firm entropy as a percentage of total industry entropy (below 22 percent). Thus, firms are highly specialised in offering products with specific designs.

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\(^{110}\) In a similar way, Theil (1972) used the disaggregation measure to measure to what extent racial variety in a city, as expressed by the entropy value of a racial distribution, disappears when one disaggregates at the level of districts. The lower the average entropy at the district level, the less racial variety at the level of districts. In the same way, Theil (1972) measured to what extent average entropy is falling when entropy at the district level is disaggregated at the school level.
Consider an industry in which a hundred products are offered with a frequency distribution of product designs: 
\[ p_{00} = 0.4, \ p_{01} = 0.3, \ p_{10} = 0.2, \ p_{11} = 0.1. \] Entropy equals: \( H(X_1, X_2) = 1.85 \) bits. There are four firms present in the industry. Firm 1 offers 40 products and the other firms offer 20 products each.

**Case 1: low degree of specialisation**

Firm 1 offers 13 products “00”, 15 products “01”, 8 products “10” and 4 products “11”:

\[
H_1(X_1, X_2) = 0.13/0.4 \log_2 (0.13/0.4) + 0.15/0.4 \log_2 (0.15/0.4) + 0.08/0.4 \log_2 (0.08/0.4) + 0.04/0.4 \log_2 (0.04/0.4) = 1.85 \text{ bits}
\]

Firm 2 offers 9 products “00”, 5 products “01”, 4 products “10” and 2 products “11”

\[
H_2(X_1, X_2) = 0.09/0.2 \log_2 (0.09/0.2) + 0.05/0.2 \log_2 (0.05/0.2) + 0.04/0.2 \log_2 (0.04/0.2) + 0.02/0.2 \log_2 (0.02/0.2) = 1.81 \text{ bits}
\]

Firm 3 offers 9 products “00”, 5 products “01”, 4 products “10” and 2 products “11”

\[
H_3(X_1, X_2) = 0.09/0.2 \log_2 (0.09/0.2) + 0.05/0.2 \log_2 (0.05/0.2) + 0.04/0.2 \log_2 (0.04/0.2) + 0.02/0.2 \log_2 (0.02/0.2) = 1.81 \text{ bits}
\]

Firm 4 offers 9 products “00”, 5 products “01”, 4 products “10” and 2 products “11”

\[
H_4(X_1, X_2) = 0.09/0.2 \log_2 (0.09/0.2) + 0.05/0.2 \log_2 (0.05/0.2) + 0.04/0.2 \log_2 (0.04/0.2) + 0.02/0.2 \log_2 (0.02/0.2) = 1.81 \text{ bits}
\]

\[
\overline{H}(X_1, X_2) = (0.4 \cdot 1.85) + (0.2 \cdot 1.81) + (0.2 \cdot 1.81) + (0.2 \cdot 1.81) = 1.83 \text{ bits}
\]

\[
\frac{\overline{H}(X_1, X_2)}{H(X_1, X_2)} \cdot 100\% = \frac{1.83}{1.85} \cdot 100\% = 98.9\%
\]

**Case 2: high degree of specialisation**

Firm 1 offers solely 40 products “00”

\[
H_1(X_1, X_2) = 0.4/0.4 \log_2 (0.4/0.4) + 0 + 0 + 0 = 0 \text{ bits}
\]

Firm 2 offers 10 products “01” and 10 products “11”

\[
H_2(X_1, X_2) = 0 + 0.1/0.2 \log_2 (0.1/0.2) + 0 + 0.1/0.2 \log_2 (0.1/0.2) = 1.00 \text{ bit}
\]

Firm 3 offers solely 20 products “01”

\[
H_3(X_1, X_2) = 0 + 0.2/0.2 \log_2 (0.2/0.2) + 0 + 0 = 0 \text{ bits}
\]

Firm 4 offers 10 products “10” and 10 products “11”

\[
H_4(X_1, X_2) = 0 + 0 + 0.1/0.2 \log_2 (0.1/0.2) + 0.1/0.2 \log_2 (0.1/0.2) = 1.00 \text{ bit}
\]

\[
\overline{H}(X_1, X_2) = (0.4 \cdot 0) + (0.2 \cdot 1.00) + (0.2 \cdot 0) + (0.2 \cdot 1.00) = 0.40 \text{ bits}
\]

\[
\frac{\overline{H}(X_1, X_2)}{H(X_1, X_2)} \cdot 100\% = \frac{0.40}{1.85} \cdot 100\% = 21.6\%
\]

Box 5-3: Two cases of average entropy of the same distribution at the industry level
5.1.4 Relating changes in mutual information with changes in average entropy

As discussed in the previous section, a dominant design is indicated when the entropy of a distribution falls and the mutual information falls. Only when the fall in entropy is observed jointly with a fall in mutual information, can the fall in entropy be attributed to the emergence of one dominant cluster of alleles present in the product population. In that case, the remaining product variety is based on variations of a dominant design, instead on variations in multiple clusters of designs. In the previous section, it was also shown how one can indicate the degree of specialisation by means of disaggregation of entropy at the firm level. The question now becomes what changes one expects in the degree of specialisation when a dominant design emerges at the industry level?

From the discussion of economies of scope in Chapter 4, it is expected that a fall in mutual information is related to a fall in the degree of specialisation among firms. When all firms follow a dominant design, they are expected to come up with the similar varieties of the dominant design as to exploit economies of scope. By contrast, when mutual information is rising, product variety is increasingly based on the existence of multiple clusters of designs. In that case, economies of scope arise only from variations within each cluster, and firms are expected to specialise their product offering on one cluster of designs that are based on one knowledge base.

This hypothesis can be further illustrated by going back to the example in Box 5-2 of two product populations with the same entropy value, but a different mutual information value. In the case of the first frequency distribution in Box 5-2, mutual information is absent. Since all designs are based on a single dominant 00## design economies of scope is expected to be realised in the production of all four designs, since they are based on a single set of core alleles 00##. Therefore, a firm that offers products of all four designs is expected to incur less cost per design than a firm specialised on offering products that are based on a single design, i.e.:

\[ C(0000,0001,0010,0011) < C(0000) + C(0001) + C(0010) + C(0011) \]

For the second frequency distribution in Box 5-2, it is unlikely that economies of scope can be realised in the production of all four designs, since the four designs are not based on the same core alleles. Variation exists in all four dimensions. However, this does not mean that one should expect four different firms to specialise in one of the four different designs. As explained above, one can distinguish in this example between two “dominant designs”, one based on 000# and one based on 111#. Economies of scope are then expected for the production of the two pairs of designs that are based on one of the two knowledge bases, i.e.:

\[ C(0000,0001) < C(0000) + C(0001) \]

and,

\[ C(1110,1111) < C(1110) + C(1111) \]

From this discussion, one can conclude that specialisation is expected to increase when the number of design families increases. And, since mutual information indicates the degree in which a product population is clustered in design families, one expects an increase (decrease)
in mutual information to go hand in hand with an increase (decrease) in the degree of specialisation among firms.

5.2 **Empirical Analysis of Aircraft and Helicopter Evolution**

The entropy measure, the mutual information measure, and the average firm entropy measure provide one with three measurements to describe the evolution of a distribution of designs in an industry. As it is argued in this section, the three measures will prove sufficient to characterise the product life-cycle of a product technology in terms of changes in the degree and nature of product variety. To illustrate the entropy methodology, it is applied in this chapter to data on technical characteristics on the alleles incorporated in product models of aircraft and helicopters. The measurements show the pattern in product variety over time and indicate whether a dominant design emerged. The application to two technologies does not allow me to draw general conclusions from the results, but only specific conclusions regarding the industries in question. When applied to data on a large number of other technologies, the entropy methodology can be used to test the validity of the product life-cycle model in general. Such a larger research program is further discussed in Section 9.5 in this study.

5.2.1 Data

The data concern the alleles of six technical characteristics incorporated in aircraft designs introduced in the period 1913-1984 and the alleles of five technical characteristics incorporated in helicopter designs introduced in the period 1940-1983. The technical characteristics of aircraft designs concern engine type, number of engines, wing type, number of wings, number of tails, and number of booms. The technical characteristics of helicopter designs concern engine type, number of engines, number of blades, number of shafts, and number of rotors per shaft. The dimensions of technical characteristics and alleles are listed in Table 5-1 for the data on aircraft and in Table 5-2 for the data on helicopters. The data of these dimensions have been compiled for a sample of 731 aircraft models and a sample of 144 helicopter models. The data have been compiled from Jane’s (1978, 1989) encyclopaedia on aviation, which are known to be among the most comprehensive encyclopaedia of aviation.

Apart from the data on technical characteristics, the data on service characteristics such as speed and range are also available. These data are used in empirical analyses in the following *Chapter 6*, in which the trends in variety in technical characteristics is related to trends in variety in service characteristics. The analyses in *Chapter 6* thus relate the technological evolution in design space to the dynamics of product differentiation in function space. In this chapter, I concentrate on the evolution of variety in technological characteristics space only.

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111 Part of the data used in this chapter has been collected in a previous project funded by the ESRC (Saviotti and Bowman 1984; Saviotti and Trickett 1992; Saviotti 1996).

112 The data on the six technical characteristics of aircraft and the five technical characteristics of helicopters are the same as those used in Frenken *et al.* (1999b), which is included in this thesis as *Chapter 6*. 

150
AIRCRAFT

Number of observations: 731
Time span: 1913-1984

Sources:

Dimensions and alleles:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>Engine type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 = 5$</td>
<td>0 piston-propeller, 1 turboprop, 2 jet, 3 turbofan, 4 rocket</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_2$</th>
<th>Number of engines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2 = 7$</td>
<td>0 one, 1 two, 2 three, 3 four, 4 six, 5 eight, 6 twelve</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_3$</th>
<th>Wing type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3 = 4$</td>
<td>0 straight, 1 delta, 2 swept, 3 variable swept</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_4$</th>
<th>Number of wings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4 = 3$</td>
<td>0 monoplane, 1 biplane, 2 triplane</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_5$</th>
<th>Number of tails</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_5 = 2$</td>
<td>0 one, 1 two</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_6$</th>
<th>Number of booms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_6 = 3$</td>
<td>0 one, 1 two, 2 three</td>
</tr>
</tbody>
</table>

Table 5-1: Dimensions and alleles for data on aircraft and helicopter designs
HELIICOPTERS

Number of observations: 144
Time span: 1940-1983

Sources:


Dimensions and alleles:

<table>
<thead>
<tr>
<th>$X_1$</th>
<th>Engine type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 = 5$</td>
<td>0 piston, 1 piston turbo, 2 ramjet, 3 gas generator, 4 turboshift</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_2$</th>
<th>Number of engines</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_2 = 3$</td>
<td>0 one, 1 two, 2 three</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_3$</th>
<th>Number of blades</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_3 = 7$</td>
<td>0 two, 1 three, 2 four, 3 five, 4 six, 5 seven, 7 eight</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_4$</th>
<th>Number of shafts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_4 = 2$</td>
<td>0 one, 1 two</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$X_5$</th>
<th>Number of rotors per shaft</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_5 = 2$</td>
<td>0 one, 1 two</td>
</tr>
</tbody>
</table>

Table 5-2: Dimensions and alleles for data on aircraft and helicopter designs
The dimensions of technical characteristics that are taken into account in the data on aircraft and helicopters obviously cover only a sub-set of technical characteristics that can be considered relevant in the evolution of aircraft and helicopter technology. For example, regarding aircraft, the type of material, the type of landing-gear, and the type of nose could not be used in the following analyses for reasons of data availability. Though these characteristics and other are relevant to the understanding of aircraft history, these were only available for very few of the 731 aircraft models for which the six included characteristics were available. Similarly, technical characteristics other than the five dimensions included in the database on helicopter models, were only available for few of the 144 helicopter models, and could therefore not be used in the following analyses. The selection of technical characteristics has thus been based on an attempt to optimise the trade-off between the size of the sample and the number of characteristics.

Each product model has been coded as a string of alleles. For each dimension $n$, alleles are coded starting from 0 up to $A_n$. The size of the design space of possible aircraft designs that is covered by the data equals $5 \cdot 7 \cdot 4 \cdot 3 \cdot 2 \cdot 3 = 2520$ designs, and the size of the design space of possible helicopter designs is covered by the data equals $5 \cdot 3 \cdot 7 \cdot 2 \cdot 2 = 420$ designs.

The distributions of product designs that are used to measure entropy and mutual information at particular moments in time, are not the yearly distributions of product designs. A yearly distribution is too small a time-span as aircraft and helicopters designs are typically products that remain on offer for many years after their introduction. I used moving ten-year distributions, but the entropy calculations for five-year and fifteen year distributions yielded the same trends as in the results based on ten-year distributions discussed below.

The results in the figures below are shown per year, where each year stands for a ten-year period. A year corresponds to the first year of a ten-year period. Thus, the year 1913 stands for the distribution of designs introduced between 1913 and 1922, the year 1914 stands for the entropy of the distribution of product designs introduced between 1914 and 1923, et cetera.

5.2.3 Results on entropy and mutual information

Entropy values and mutual information values were computed using the distributions of product designs in all products introduced during a ten-year period. The results on entropy and mutual information for aircraft are given in Figure 5-1-1 and for helicopters in Figure 5-1-2. The trends are quite different for the two different technologies, with entropy and mutual information rising for aircraft and, after an initial rise, falling for helicopters. The result show that product variety in designs in aircraft has been steadily risen up to the 1950s, while product variety in product designs in helicopters has been falling after an initial period of rising variety.

Starting with analysing the patterns of technological evolution of helicopters, one may conclude that the results on entropy and mutual information on helicopter distributions indicate the emergence of a dominant design. The entropy has fallen from the 1955 onwards showing that product variety in product designs has fallen after an initial rise. The value for mutual information peaked earlier in 1949 and also shows a declining trend hereafter. The two results point to the emergence of a dominant design in the early fifties.
Figure 5-1-1: Entropy (upper curve) and mutual information (lower curve) for aircraft

Figure 5-1-2: Entropy (upper curve) and mutual information (lower curve) for helicopters
Figure 5-2-1: Two-dimensional mutual information values for aircraft

Figure 5-2-2: Two-dimensional mutual information values for helicopters
The falling trends in entropy and in mutual information for helicopters indicate that the product variety and dependence between alleles has fallen over time. Thus, the variety that remained was increasingly based on small variations around a dominant design as indicated by the falling mutual information. The most frequent design after the fifties, which can be considered the dominant design, is the two-engine turboshift design with four blades and one shaft and one rotor per shaft (string “41200”). Variations have been primarily based on variations in the number of blades and the number of engines. This result confirms the historical study of Bilstein (1996: 91) who identified the twin-turboshift Kaman-model introduced in 1954 as the “pioneering” design.

The results on entropy and mutual information for aircraft are quite different from the patterns for helicopters and the aircraft results do not support the dominant design thesis. Entropy increased in the early decades and decreased only slightly in the thirties. Hereafter, entropy increased again rapidly to level off in the late-fifties. Mutual information increased slightly in the early history and decreased slightly in the thirties. Hereafter, mutual information increased rapidly and steadily to level off in the sixties. The results suggest that only in the thirties some indication can be found of a dominant design evident from a slight and temporary fall in entropy and mutual information.

The period of the thirties indeed corresponds to the age of the Douglas DC3 introduced in 1936 with design “010000”, which is commonly considered as the dominant design in aircraft history (Miller and Sawers 1968; Nelson and Winter 1977; Constant 1980). During this period the design “010000” has a high frequency reflecting that many other firms developed products based on this design.

However, as indicated by the rising trends in entropy and mutual information from the forties onwards, the results clearly point out that the alleged dominance of the DC-design did not last long. Product variety as indicated by entropy and the degree of clustering as indicated by mutual information rose over the whole period from the forties to the sixties. These results reflect that from the forties onwards no one design family cluster, but several design families gradually emerged. These different design families came into existence with the successful development of three new engine types: turboprops, jet engines, and turbofan engines.

To understand the nature of the clustering that has taken place, the values for the two-dimensional mutual information are particularly informative. These values are given in Figure 5-2-1 for aircraft for all fifteen pairs of dimensions, and in Figure 5-2-2 for helicopters for all ten pairs of dimensions. From the figure for aircraft, it is clear that the rise in mutual information in the post-war period is primarily related to rising mutual information between engine type and wing type, between engine type and number of engines, and between number of engines and wing type. The values for these three pairs of design dimensions have increased very rapidly. By contrast, the two-dimensional mutual information values for helicopters show decreasing trends. Only one pair of dimensions (dimension one and dimension three) shows high values over the whole period reflecting complementarities between number of engines and number of rotors. This relationship points to the common use of more rotors when more engines are incorporated in a helicopter design to carry the higher weight.

As noted, the results for the two-dimensional mutual information values for aircraft are more pronounced. The emergence of several design families can now be related to important epistatic relations. The results indicate that clustering concentrated in the dimensions of engine type, number of engines, and wing type. The functionality of a particular engine type depends heavily on the complementarity with the type of wings used and with the number of engines used. And, the functionality of the number of engine used...
depends heavily on the wing type used in an aircraft. The local optima in fitness landscapes are thus primarily characterised by different alleles for “core elements” engine type, wing type, and the number of engines. Along these dimensions specific combinations between alleles are dominant.

One can speak of a branching pattern of different design families that gradually occur as a process of “speciation” into different families occupying different local optima reflecting different niches (Saviotti 1996; Levinthal 1998). For example, turbofan engines are complemented by swept wings to optimise speed and take-off weight in transport operations, while jet engines are complemented by delta wings with smaller wing span to optimise speed and manoeuvrability for fighter operations. And, the less powerful piston propeller and turbopropeller are complemented by conventional straight wings and used for low-speed transport operations.

1. Dominant design: one- and two-engine piston propeller monoplanes with straight wings, one tail and one boom
   Strings: predominantly 000000 and 010000
   Niche: small-sized, short-range, low-payload, low-speed aircraft for business market and trainers

2. Dominant design: two-engine turboprop monoplanes with straight wings, one tail and one or two booms
   Strings: predominantly 110000 and 110001
   Niche: medium-sized, short-range, low-payload, low-speed aircraft for cargo and passengers

3. Dominant design: one-engine and two-engine jet monoplanes with delta wings one tail and one or two booms
   Strings: predominantly 201000, 211000, 201001, and 211001
   Niche: small-sized, medium-range, low-payload, supersonic aircraft for fighter market

4. Dominant design: two-, three and four-engine turbofans with swept wings, one tail and one booms
   Strings: predominantly 333000, 313000, and 323000
   Niche: large-sized, long-range, large-payload, high-speed aircraft for cargo and passenger aircraft

Source: Frenken et al. (2000: 238)

Box 5-4: Description of four design families in aircraft technology

In total, four design families can be distinguished that refer to typical combinations in which alleles co-occur as listed in Box 5-4.
5.2.4 Results on the number of firms participating in an industry

The number of firms participating in an industry is measured by counting the number of firms that have introduced at least one new product in a ten-year period. Thus, the same time window is used as in the analyses of entropy and mutual information. In Figure 5-3-1 and Figure 5-3-2, the counts of the number of firms in each ten-year period are given for the aircraft and the helicopter industries, respectively.

The results show that in both industries the industrial dynamics are characterised by an initial period of around 25 years in which the number of firms have risen, and a period hereafter in which the number of firms has been falling. The peak in the number of aircraft firms occurred in the mid-thirties at 85 firms and the peak in the number of helicopter firms occurred in the late-fifties at 31 firms. In both industries “shake-out” phenomena can thus be indicated. Note that the fall in aircraft firms after the thirties was interrupted during the period 1945-1960 during which the number of firms slightly increased again. It is during this period that many new varieties were introduced offering many opportunities for new firms notably in the business aircraft segment. However, the overall trend from the thirties onward has been a declining one with the number of firms decreasing from 85 in the period 1933-1942 to 32 firms in the period 1974-1983.

The patterns of industrial dynamics found for aircraft and helicopters correspond to the patterns found in many other industries, in which an initial period of several decades is followed by a more or less rapid drop in the number of firms within an industry (Utterback and Suarez 1993; Klepper 1997). As discussed in Section 1.2 and Section 4.1, many authors have related the drop in the number of firms to the emergence of a dominant design. In the history of aircraft, the peak in the number of firms occurs indeed during a period in which according to historians (Constant 1980; Vincenti 1990; Bilstein 1996) a dominant design emerged. This design was based on the Douglas DC-3, an all-metal, two-piston-propeller-engine monoplane that was introduced in 1936. About this time, the number of aircraft firms peaked at around 85 firms. Similarly, the number of helicopter firms peaked around the mid-fifties only few years after the time a dominant design was indicated in the previous section.

Thus, in both industries the number of firms in the industry has first risen and fallen hereafter. The peaks in the number of firms in the history of the aircraft and helicopter industries correspond to the time at which alleged dominant design emerged. However, the subsequent evolution of the two technologies hereafter was completely different. Whereas helicopter variety declined after the emergence of the alleged dominant design, aircraft variety started to increase again. Thus, although the industrial dynamics in both industries are similar for what concerns the trends in the number of firms, the two patterns in technological evolution as expressed in product variety are opposite. This result indicates that a peak in the number of firms need not always point to the emergence of a dominant design. A shake-out in the aircraft industry has not prevented product variety to increase in later years. The prediction of the product life-cycle model that the peak in the number of firms coincides with the peak in product variety only holds for the evolution of helicopter technology.

113 The data do not cover the last fifteen years, but indications exist that the number of firms in both industries continued to fall during this period (Jane’s 1998). In both European and American aviation industry many companies have merged. Also note that the results are based on a sample. The real number of firms has been higher over the whole period, but the trend is likely to be as indicated by the results.

114 Interestingly, after the peak in the number of aircraft firms in the thirties, the number of aircraft firms increased slightly again during the fifties, but steadily declined hereafter. This temporary rise can be related to the introduction of a number of new engine technologies at the time, as it will be explained below.
5.2.5 Entropy disaggregation at the firm level

Average entropy values at the firm level were computed using the distributions ten-year of product designs as in the previous analyses. The results are given in Figure 5-4-1 for aircraft and Figure 5-4-2 for helicopters. The figures show the entropy in product designs at the industry level in the upper curve (corresponding to the upper curves in Figure 5-1-1 and Figure 5-1-2) and the average firm entropy in the lower curve. Figures 5-5-1 and figure 5-5-2 show the same results, but now in the form of average firm entropy as a percentage of the total industry entropy. The main observation from the results is again that different patterns are found for the two industries. As it is argued below, these differences can be related to the different results for entropy and mutual information for the two technologies found in the previous analyses.

Starting with the analysis of the helicopter industry, the results show that average entropy at the firm level as a percentage of industry entropy remained at low level, but has risen spectacularly during the last 20 years (1964-1983). At this point in the history of the helicopter industry, the number of firms had suddenly dropped from around 30 to around 15 firms. Thus, a smaller number of firms each produced a larger variety of designs and an increasingly more similar variety of product designs as variety at the industry level fell. This result is in line with the thesis discussed in Section 5.1.4 that the degree of specialisation between firms falls once a dominant design emerges and a shake-out occurs. Surviving firms, on average, shifted from offering few different designs to offering a large variety of designs based on a common core through which economies of scope can be realised.

In the aircraft industry, the reverse trend occurred. The entropy at the firm level as a percentage of industry entropy first increased to almost 35 percent, but decreased from the thirties to a value below 15 percent in the mid-forties. In the post-war period, this percentage remained rather low fluctuating around 20 percent. This result indicates that on average the firms in the aircraft industry have specialised in offering fewer varieties during the period after the thirties.

How can one understand the increasing specialisation trend among aircraft firms after the thirties? From the high values for mutual information in Section 5.2.3 indicating the co-existence of several design families, it can be suggested that as several design families emerged, firms tend to specialise in one design family. This explanation is indeed in line with the earlier discussion on the benefits of specialising in a single knowledge base to exploit economies of scope. With the number of design families rising over time aircraft firms increasingly specialised while helicopter firms de-specialised when one dominant design family emerged.115

115 Descriptive statistics of the post-war data show indeed a large number of firms that specialised in one design family. Piper and Cessna specialised in one- and two-engine piston propeller aircraft. These models were primarily used in the business and trainer market. Antonov, Beechcraft, and De Havilland Canadair specialised in turboprop aircraft with predominantly two engines used for cargo and passenger transport. Convair, Daussault, and Sukhoi specialised in fighter aircraft with delta- or variable swept wing and one or two jet engines. And, Ilyushin, Boeing, Lockheed, Douglas, Tupolev, McDonnell Douglas specialised in two- or four-engine turbofan aircraft with swept wings. These large aircraft models are used for mass-transport of cargo and passengers.
Figure 5-3-1: Number of firms in the aircraft industry

Figure 5-3-2: Number of firms active in the helicopter industry
Summarising, the results on the helicopter industry confirm the product life-cycle model in that a dominant design emerged, a shake-out occurred, and firms de-specialised. The history of the helicopter industry can be considered as a “paradigmatic” example of the product life-cycle model. The development of the aircraft industry, however, did not match the predictions of the product life-cycle model. Only for the period up till the mid-forties, a pattern of rising and falling product variety and number of firms was found. After this period, product variety increased further with the development of several design families: a piston propeller family, a turbopropeller family, a jet family, and a turbofan family. With the development of several design families, aircraft firms generally specialised in one out of the several design families.

Though the analysis of only two industries cannot serve as an empirical test of the product life-cycle, the result on the history of aircraft designs cannot be considered as confirming evidence. As in some of the studies on the product life-cycle reviewed in Section 4.1, the analysis of the history of aircraft technology also shows that the pattern in product innovation does not necessarily follow the logic of the product life-cycle model. The growing variety in aircraft design leading to four design families suggest that product innovation has not dropped significantly in the course of history. Around the time of the introduction of the alleged dominant design the Douglas DC-3, product variety only temporarily stabilised. In the post-war period, the entropy analysis showed a rapid increase in product variety suggesting a high activity in product innovation.

To account for evolutionary patterns in product variety, the product life-cycle theory may turn out too limited a framework. In particular, the product life-cycle model rules out the possibility of the emergence of multiple co-existing dominant designs. As in the example of the aircraft industry, different designs can be dominant in different market segments or “niches”. A framework that encompasses this possibility has been developed in a study by Frenken et al. (1999b), which is integrally included in this study as Chapter 6. In this study, four industries have been analysed. Apart from the aircraft and helicopter industries, the motorcycle and microcomputer industries have been analysed. From this analysis, building blocks are formulated for a more general theory of dominant designs and product life-cycles.

The analysis of data on aircraft and helicopter design in this chapter was meant to show how the entropy methodology can be applied and how one can interpret the results. To test the product life-cycle using the entropy methodology, many other technologies need to be analysed in this way. Such an extensive research program requires careful data collection of product characteristics of many technologies covering as many years as possible, an issue on which I reflect in more detail in Section 9.5.1.

5.3 INTRODUCTION TO CHAPTER 6, CHAPTER 7 AND CHAPTER 8

The following chapters, Chapter 6, Chapter 7, and Chapter 8, concern three other empirical studies of technological development. The studies address the dynamics of innovation in the light of the product life-cycle model and in the light of related concepts of lock-in, technological paradigm, and national systems of innovation. The measurements are primarily based on entropy statistics, the methodology of which has been described in this chapter.

116 This pattern in technological evolution has also been found by Levinthal (1998) in his analysis of wireless telephony and by Almeida (1999) in his analysis of electric motors.
Figure 5-4-1: Entropy (upper curve) and average entropy (lower curve) for aircraft

Figure 5-4-2: Entropy (upper curve) and average entropy (lower curve) for helicopters
Figure 5-5-1: Average firm entropy as a percentage of industry entropy for aircraft

Figure 5-5-2: Average firm entropy as a percentage of industry entropy for helicopters
All three chapters have been published earlier as journal articles and have been included integrally in this study as Chapter 6, Chapter 7, and Chapter 8. To link the articles to the previous arguments made in this study, this section shortly describes the three chapters and shows how the empirical analyses addresses questions raised in previous chapters.

5.3.1 Variety and niche creation

The first study in Chapter 6 concerns a study on product variety in the aircraft, helicopter, motorcycle and microcomputer industries, which has been published as: Koen Frenken, Pier Paolo Saviotti, Michel Trommetter (1999) ‘Variety and niche creation in aircraft, helicopters, motorcycles and microcomputers’, Research Policy 28, pp. 469-488. The study in Chapter 6 extends the analysis in Chapter 5 in two ways. First, the analysis concerns four instead of two technologies. Second, variety is measured not only using entropy statistics but also using a recent variety measure developed by Weitzman (1992). The study in Chapter 6 however does not include analyses of mutual information, average firm entropy, and the number of firms as in Chapter 5.

In the study in Chapter 6 the entropy measure and the Weitzman measure are used to map the changes in the product variety during the evolution of the four technologies. The entropy measure is used in same way as done in Chapter 5 but to different distributions of product designs. In the study in Chapter 6, we did not apply the entropy measure on distributions covering a particular period of years (“clock time”), but on distributions that each contained a particular number of product releases (“event time”). Distributions with a same number of events were chosen to render the results comparable with the results of the second variety measure – called the Weitzman measure (Weitzman 1992, 1993) – which can be best applied to a given number of observations. It should be noted that the results on entropy in Chapter 6 based on event time show the same trends for aircraft and helicopters as in Chapter 5, which are based on clock time rendering the results quite robust.

The Weitzman measure is complementary to the entropy measure. The Weitzman measure applies to matrices of pair-wise distances between two product designs. Two types of pair-wise distances have been computed. The first distance applies to technical characteristics and is called Hamming distance. This measure equals the number of alleles that two product designs do not share.\footnote{Hamming distance is also explained in Section 4.3. See also Kauffman (1993: 199).} Hamming distance equals zero when designs do not differ in any dimension, and Hamming distance equals $N$ when designs differ in all dimensions. The second distance measure is applied to service characteristics, and is an Euclidean measure. This distance measure maps the difference between pairs of designs in continuous service characteristics space. Following the generalised model in Chapter 3, the Euclidean distance measure thus indicates the difference between two products in terms of the level of services they provide as measured by bundles of service characteristics.

By means of measuring product variety in design space as expressed in the values of technical characteristics (the alleles) as well as in function space as expressed in the values of service characteristics (the fitness levels), we are able to link changes in design to product differentiation. Two types of differentiation can be distinguished. First, vertical differentiation in service characteristics can take place when product innovation leads to designs with higher quality sold at a higher price and designs with lower quality sold at a lower price. Second, horizontal differentiation can take place when different products are
adapted to preferences of different user groups that apply different weights to service characteristics. As discussed in more detail in Chapter 3, both types of product innovations are expected to increase the product variety as measured in Hamming space of technical characteristics (the elements of products) and Euclidean space of service characteristics (the functions of products).

The variety analysis is done using data on four product technologies: aircraft, helicopter, motorcycle, and microcomputer data. For aircraft, helicopter and motorcycle products, the variety patterns could be well explained by an evolutionary model that relates technological variety to the scope for niche creation. The scope for niche creation for a technology is generally increasing over time due to product innovations that enlarge the high end of the market for a product (faster, bigger, safer, et cetera) as well as product innovations that enlarge the low-end of the market (slower, smaller, less safe, et cetera). For example, the values for the Weitzman measure applied to service characteristics of aircraft shows that variety in function space has increased considerably in the course of the history of aircraft. As an illustration, I show in Figure 5-6-1 and Figure 5-6-2 two trees that have been constructed from the Weitzman algorithm as applied in Chapter 6. The first tree covers the very first period of 1913-1916 and the second tree covers the very last period of 1979-1984. These trees show the increase in product differentiation in aircraft over time in various dimensions of service characteristics, including power, speed, length, and range, as measured by the values on the y-axis.

The scope for niche creation can be limited by the existence of inter-technological competition by other technologies that already occupy a region in service characteristics space at lower prices. This explains why product variety in helicopters decreased, whereas in aircraft and motorcycle technology new niches have been created. The presence of aircraft technology affected the range of product differentiation that could take place in helicopters as niche creation in the high end of the helicopter market has been restricted by the presence of competitive aircraft designs at the low end of the aircraft market.

For microcomputers the relation between technological variety and the scope for niche creation did not hold. Both variety in design space and variety in function space have been declining over the period studied, while the range of services that can be offered technically has been steadily expanding. This result suggests that particular factors have to be taken into account to understand the evolution of product variety in microcomputers. First, the rate of decline in costs of elements per unit speed has been fallen extremely fast in computing compared to other technologies. The resulting fast increase of the value-for-money of new microcomputer models rendered low-end computers quickly obsolete. In other words, niche creation in low ends of service characteristics is not feasible.

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118 These figures also have been included in Frenken et al. (2000: 235-236). Note that in this study, the periods corresponding to each family tree have erroneously been transposed by the editor.
119 One may argue that, when helicopter is considered as a separate industry distinct from aircraft, small aircraft should also be considered a separate industry distinct from large aircraft. In that case, variety analyses and industrial dynamics should cover separate market segments instead of all aircraft models. However, the methodological choice to distinguish between helicopters and aircraft has been based on the difference in design space of helicopters and aircraft, while the different types of aircraft can all be represented in the same design space.
120 A comparison has been made between cars and microcomputers if technical change in automobiles had progressed at the rate of technical change in microcomputers in the last few decades. Brock and Colander (2000: 88): “an auto that performed at the level of a Ferrari would cost about 10 cents today!” This rapid technical change is also reflected in Moore’s Law, which still holds today, stating that computing speed doubles each 18 months (MacKenzie 1992; Van Lente 1993; Brock and Colander 2000: 89).
Figure 5.6-1: Evolutionary tree of period 1, 1913-1916, computed using the Weitzman measure (Frenken et al. 1999a; Frenken et al. 2000)
<table>
<thead>
<tr>
<th>firm</th>
<th>model code string</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake</td>
<td>LA-4-200 PA-28</td>
</tr>
<tr>
<td>Piper</td>
<td>Y-121</td>
</tr>
<tr>
<td>Harbin</td>
<td>BAE Jetstream FMA (commuter)</td>
</tr>
<tr>
<td>BAE</td>
<td>1A-63</td>
</tr>
<tr>
<td>Aermacch</td>
<td>Gullstream MB-339K</td>
</tr>
<tr>
<td>Gulfstream</td>
<td>A1 Rockwell 6'146-200</td>
</tr>
<tr>
<td>Amer.</td>
<td>30000</td>
</tr>
<tr>
<td>31000</td>
<td>20000</td>
</tr>
<tr>
<td>110000</td>
<td>11000</td>
</tr>
<tr>
<td>Dornier</td>
<td>Shorts 128-2</td>
</tr>
<tr>
<td>Shorts</td>
<td>360</td>
</tr>
<tr>
<td>Embraer</td>
<td>BAE Jetsream 3 F-16XL</td>
</tr>
<tr>
<td>Boeing</td>
<td>120</td>
</tr>
<tr>
<td>Boeing</td>
<td>33200</td>
</tr>
<tr>
<td>31000</td>
<td>31000</td>
</tr>
<tr>
<td>31000</td>
<td>31000</td>
</tr>
</tbody>
</table>

**service characteristics:**

| engine power     | 149 | 149 | 746 | 1342 | 1400 | 1780 | 9120 | 2960 | 3580 | 566 | 1750 | 2237 | 1152 | 10000 | 17108 | 32000 | 38320 |
| wing span        | 11.58 | 10.67 | 17.23 | 15.85 | 9.69 | 11.05 | 23.72 | 15.37 | 30.63 | 15.55 | 22.81 | 19.78 | 26.34 | 10.43 | 12 | 37.95 | 47.57 |
| max. take-off weight | 1220 | 1247 | 5500 | 6600 | 4650 | 6150 | 30935 | 10886 | 22090 | 3842 | 11793 | 9600 | 40597 | 21772 | 16100 | 104325 | 136080 |
| max. speed       | 241 | 282 | 282 | 488 | 740 | 907 | 916 | 916 | 474 | 325 | 393 | 500 | 778 | 2156 | 2500 | 863 | 863 |
| range             | 1327 | 1334 | 1280 | 1167 | 1500 | 1222 | 7495 | 5106 | 1545 | 642 | 1697 | 2907 | 2473 | 4630 | 3700 | 4800 | 5749 |

Figure 5-6-2: Evolutionary tree of period 43, 1979-1984, computed using the Weitzman measure (Frenken et al. 1999a; Frenken et al. 2000)
The second factor that is rather specific to microcomputers concerns to the rate of increasing returns to adoption of standards. In particular, compatibility requirements between hardware and software and network externalities among users cause a high rate of increasing returns to adoption of particular computer designs. These factors explain why design dimensions such as the type of operation system, the type of disk drive, and type of screen have standardised very quickly.

5.3.2 Scaling trajectories

Chapter 7 deals with scaling in successive product designs along technological trajectories in civil aircraft, an issue which has been discussed in Section 1.3 of this study. The study has been published as: Koen Frenken, Loet Leydesdorff (2000) ‘Scaling trajectories in civil aircraft (1913-1997)’, Research Policy 29, pp. 331-348.

In this study, a measure is proposed to indicate scaling of designs along technological trajectories over time. This measure can be applied to series of individual products of a particular firm as well as to series of distributions of products at the level of the industry as a whole. The basic idea is to measure the change in the ratio between characteristics in a series of product designs. Perfect scaling can then be defined as increases or decreases in the levels of characteristics without changes in the ratios between characteristics. The degree of change in ratios is measured using the formula of the expected information content in an \textit{a priori} distribution of ratios between characteristics of one design compared to an \textit{a posteriori} distribution in ratios between characteristics of another design. The value of the expected information content indicates to what extent two designs differ in terms of the distribution of ratios between characteristics.

The data used in this study are the same aircraft data as the ones used in Chapter 5 and Chapter 6 but only concern the civil aircraft models. The civil aircraft data have been extended for the period of 1985-1997 using Jane’s (1998). A reason to focus on civil aircraft is that many scholars have described the history of civil aircraft in great detail, so one can confront the results of the analysis with historical notions.

The results for aircraft show two cycles of low degree of scaling followed by a high degree of scaling. These results confirm the historical studies and earlier empirical studies that have described two successive technological paradigms in civil aircraft, one based on piston propeller engine technology and one based on turbofan engine technology (Miller and Sawers 1968; Constant 1980; Mowery and Rosenberg 1982). The first paradigm started with the success of the Douglas DC-3 introduced in 1936, which was subsequently scaled in various versions up to the late fifties. The second paradigm started with the success of the Boeing 707 introduced in 1957, which was also scaled up to the late nineties. During each of the two product life-cycles, an initial period of a low degree of scaling is followed by a period of a high degree of scaling. The latter period reflects the paradigmatic stage of technological development, and the former period reflects the pre-paradigmatic stage of development. Note that the conclusion of two successive technological paradigms in civil aircraft is not incompatible with the conclusion that after the World War II no single paradigm gained dominance at the level of the aircraft industry as a whole. The former conclusion refers to technological development in one market segment only, while the latter covers all segments including civil aircraft, business aircraft, fighters, bombers, cargo, and trainers.
The scaling measure is applied both at the firm level and the industry level. By means of mapping dynamics at firm levels and industry levels, we attempted to relate micro-level strategies of particular firms (Douglas, Boeing) to each other and to the macro-level developments at the world level of the industry. In this way, we show how Boeing has overtaken the leading position of Douglas through radical innovation. This type of analyses informs one about the success of different firm strategies in product innovation at different stages of the product life-cycle.

5.3.3 A complexity approach to innovation networks

Chapter 8 concerns the last study reported in this study. This study contains a theoretical part based on Kauffman’s NK-model and an empirical analysis using entropy statistics. The study deals with international specialisation patterns in technology-market-country combinations in the world aircraft industry. This study has been published as: Koen Frenken (2000) ‘A complexity approach to innovation networks. The case of the aircraft industry (1909-1997)’, Research Policy 29, pp. 257-272.

In this study, it is shown how the original NK-model of complex systems as described in Chapter 2 can also be used to model complementarities between competencies of actors in an “innovation network”. The actors that are distinguished in the development of a product innovation are producers that contribute with technological knowledge, users that contribute with knowledge on product requirements, and national governments that contribute with facilitating infrastructure and financial services. Using this model, one can understand specialisation of producers in particular technologies, in particular markets, and in particular countries as a consequence of interrelated competencies between producers, consumers and governmental bodies. Once joint strategies prove successful, networks tend to become interlocked leading countries to become specialised in particular technology-market combinations (cf. Walker 2000).

The international specialisation patterns that have emerged are measured by mutual information values of the three-dimensional distributions of technologies, markets and countries, and by the mutual information values of the two-dimensional distributions of technologies and markets, technologies and countries, and markets and countries. Thus, the complementarities between actors can be mapped by the mutual information measures in the same way as the complementarities between alleles of elements of a complex artefact have been computed in Chapter 5.

The results show that the degree of specialisation has risen substantially in the post-war period. Countries increasingly specialised in the development of product innovations using a specific technology for a specific market segment. Regarding Vernon’s (1966) international product life-cycle model holding that technologies in their mature stage shift to low-age countries, the study adds an important nuance to Vernon’s model. Low-wage countries indeed typically entered the aircraft industry by using a mature technology, but some high-wage countries also remained active in product innovation in mature technologies. This can be understood by complementarities that can arise in innovation networks in high-

121 Collaboration between producers, users, and governments is also central to Lundvall’s (1988) concept of national innovation systems and David’s (1994b) discussion of path dependence and institutional lock-in.

122 This type of network is not the only type of innovation network. Another common type of innovation network is a network of producers collaborating in the development of one technology. This type of innovation network is discussed in Section 9.4.4, in which issues for further research are discussed.
wage countries developing new products based on mature technology for specific user groups (e.g., American firms that developed piston-propeller aircraft primarily for American business market).

The study also goes into the more recent development of transnational collaborations between firms within consortia (e.g., Airbus). In the light of the NK-model developed earlier in the article, this institutional development is understood as a means for firms to escape their historical specialisation patterns through alignment with partners that have built up different competencies in other technologies or markets. The study ends with reflections on the importance of transnational innovation networks in the development of new technologies by means of recombinations of national competencies.