Inelastic bouncing and optical trapping of cold atoms
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Chapter 5

A stochastic rainbow caustic observed in an inelastic atomic reflection

5.1 Introduction

Many techniques have been proposed and experimentally realized to reflect slow atoms. One of the methods is to make use of (near resonant) optical light fields which can create a spatially dependent potential. Of great importance was the proposal by Cook and Hill [9] to use an evanescent wave (EW) as a non-uniform laser field. In 1987 such an atomic mirror was experimentally realized by Balykin and co-workers [10], who observed specular reflection of an atomic beam from such an EW. One of the main features of such an EW is its small extension into the vacuum, typically the wavelength of the laser light. Therefore, the atoms spend only a short time in the optical field during the reflection, so that the number of scattered photons is small. In elastic mirrors, where the laser light is far from resonance, this reflection can be purely specular. When an atom scatters photons from the EW, it will gain momentum along the surface of reflection due to the absorbed photon recoil. We directly observed this lateral radiation pressure of $^{87}$Rb atoms reflecting from an EW [13]. In this experiment the laser was tuned to a closed transition.

The nature of the reflection changes dramatically when the laser is tuned to an open transition, so that photon scattering can change the internal state of the atom. When the hyperfine ground state of the atom changes during the reflection, the EW potential drops suddenly due to the increased detuning of the laser field. Thus the atom is slowed down by a high potential, and accelerated from the surface by a low potential. This process is called an elementary Sisyphus process [45], or evanescent-wave cooling [14, 15]. In this chapter we focus on the velocity distribution of the reflected atoms, which is found to be highly asymmetric. Moreover, this distribution contains a caustic at the lowest possible velocity. The spontaneous nature of the caustic makes it different from known caustics.
Caustics are ubiquitous phenomena in nature. Examples are the cusp-shaped patterns of light reflection on the inside of a coffee-cup and the patterns of bright lines observed on the bottom of a swimming pool (for a good review, see Berry [46]). The prime example of a caustic is the common rainbow, which can be understood in a ray-optics picture by considering how the scattering angle of a light ray depends on its impact parameter on a water droplet [47, 48] (see Fig. 5.1). Whereas the incident rays have smoothly distributed impact parameters, the outgoing rays pile up where the scattering angle has a local extremum. Such a divergence of the ray density, the caustic, appears at the rainbow angle. In atomic [49] and nuclear [50] scattering experiments analogous rainbow phenomena have also been observed. It is important to note that all of these examples are deterministic rainbows: every impact parameter results in a single outgoing angle. The distribution of impact parameters then results in a distribution of outgoing angles.

Previous experiments on the transverse velocity distribution of atoms bouncing elastically on corrugated mirrors [51, 52] also allow an interpretation in terms of caustics. However, those were of the usual kind, where the outgoing transverse velocity is a function of the position where the atom hits the mirror. The caustic then originates from atoms reflecting from inflection points on the mirror surface.

The examples of caustics that have been known so far have in common that the outgoing parameter (scattering angle) is a deterministic function of the incoming parameter (impact parameter). In this chapter we report on our observation of a new type of rain-

Figure 5.1: The optical rainbow

The optical rainbow is formed when (sun)light falls on spherical water droplets water with refractive index n, Fig. a. Consider a monochromatic ray of light falling on the droplet with an impact parameter b, defined as the displacement with respect to an axis through the center of the droplet. A part of the light is transmitted, with a change of direction described by Snell's law. Some of this light escapes the droplet after one internal reflection. The outgoing angle is determined by the impact parameter and the refractive index.

For $b=0$ the outgoing angle $\theta=0$. For increasing impact parameter the scattering angle increases to a maximum, and decreases again, see Fig.b. Because the droplet is uniformly illuminated, the distribution of outgoing angles is expected to be the greatest were the scattering angle varies most slowly with changes in the impact parameter. This divergence of the ray density, or "caustic", appears at the rainbow angle 42.5° (Fig.c). Above this angle the intensity is zero until the angle is approximately 50° where the secondary rainbow appears, which is the result of two internal reflections. The dark region in between these angles is called Alexander's dark band.

Each outgoing angle is associated with two possible impact parameters, with a different path length inside the droplet. These two paths interfere in the far field, producing a droplet-size dependent interference pattern, the supernumerary arcs. These are usually not visible since they are washed out due to spread in droplet size.
bow caustic existing by virtue of a stochastic process, which distributes a single-valued incident velocity over a range of bouncing velocities. To our knowledge, such stochastic caustics have not been observed before, and is therefore new in its kind.

5.2 Principle of the inelastic evanescent-wave mirror

When an atom is reflected by the EW, the dissipative part of the potential leads to photon scattering. Since the frequency of the EW-light is off resonance, two types of scattering can be distinguished \[53\]: Rayleigh, or elastic scattering, where the initial state equals the final state. The second type is called Raman, or inelastic scattering, where the initial state differs from the final state.

In the experiments presented here, we tune the EW laser field near an open transition to induce spontaneous Raman scattering, where the atom changes its hyperfine state. This process will also be referred to as "optical hyperfine pumping". To illustrate the basic process, we now consider a three-level atom, with an excited state \( F_e \) and two stable ground states \( F_g = 1, 2 \). These are the two hyperfine ground states in the case of \(^{87}\)Rb, separated in energy by the hyperfine splitting \( \Delta_{hfs} \) (Fig. 5.2a). We assume that the initial state of the atom is the \( F_g = 1 \) state. When it arrives at the surface with energy \( mv_f^2/2 \), it is slowed down and reflected by the EW potential \( U_i(z) \), see Fig. 5.2b, where the subscripts 1, 2 denote the hyperfine ground state. Using the approximation of unity Clebsch-Gordan coefficients, the optical potentials for the two ground states are written

\[ a) \quad A \text{ light ray with impact parameter } b \text{ leaves the droplet with an angle } \theta \text{ after one internal reflection.} \\
\text{b) The outgoing angle as a function of impact parameter.} \\
\text{c) The intensity as a function of outgoing angle. The divergence (caustic) appears at the so called rainbow angle.} \]
Figure 5.2: a) Relevant atomic levels of $^{87}\text{Rb}$. b) An atom enters the evanescent wave in its $F_g = 1$ state with initial velocity $v_i$. It is decelerated and spontaneously scatters a photon at a velocity $v_p$. After being pumped to $F_g = 2$ it accelerates and leaves the potential with asymptotic velocity $v_f$.

as:

$$U_1(z) = \frac{\hbar(I/I_s)\Gamma^2}{\delta_1} e^{-2\kappa z}$$  \hspace{1cm} (5.1)

$$U_2(z) = \frac{\hbar(I/I_s)\Gamma^2}{8(\delta_1 + \Delta_{hfs})} e^{-2\kappa z} \approx \frac{\delta_1}{\delta_1 + \Delta_{hfs}} U_1(z) = \beta U_1(z)$$  \hspace{1cm} (5.2)

where the detuning $\delta_1$ is with respect to the $|F_g = 1\rangle \rightarrow |F_e\rangle$ transition, and the intensity $I$ is the effective intensity at the surface as defined in Chapter 2. The parameter $\beta < 1$ describes the relative height of the two potentials. Spontaneous Raman scattering transfers the atom to the higher hyperfine ground state ($F_g = 2$). When transferred, the laser frequency is detuned from the atomic $|F_g = 2\rangle \rightarrow |F_e\rangle$ transition by approximately $\delta_1 + \Delta_{hfs}$. This increase of detuning results in a decrease of optical potential, i.e. $U_2(z) < U_1(z)$. Thus the atom is slowed down by a high potential, scatters a hyperfine changing photon, and is accelerated from the surface by a low potential. The energy of the reflected atoms is less than their initial energy, so they will bounce up less high than their initial position.
5.3 Measurements of the density profile of inelastically reflected atoms

When the atoms are dropped from the molasses they are in the upper hyperfine state, $F_g = 2$. Before undergoing an inelastic reflection, we prepared the atoms in the lowest hyperfine state, $F_g = 1$. This is done by laser light resonant with the open $F_g = 2 \rightarrow F_e = 1$ transition of the $D_2$ line. This optical pumping laser we call the depumper and was aligned such that it overlapped the guiding laser. The waist of this Gaussian beam was 0.7 mm with a total power of 50 $\mu$W, thus the transition is saturated. The depumper was applied for a few ms and started 28 ms after the atoms were released from the molasses, just before they arrive at the prism. To check the efficiency of the depumper, an absorption image was taken after the depumping pulse. Since the probe was resonant with the $F_g = 2$ state, no atoms were observed. When a repumper beam was applied just before the probe, all atoms reappeared.

We measured a slight difference in the vertical position of the atomic cloud when the depumper pulse was applied directly after releasing the atoms from the molasses. Due to the recoil of the absorbed photons, after 34 ms the atomic cloud has fallen roughly 0.2 mm less than without depumper pulse. No influence was found on the shape of the cloud, i.e. no heating of the cloud was measured.

When the cloud reaches the prism, the EW is switched on for typically 4 ms. The EW is only on during this time interval to suppress scattering of stray light which can exert radiation pressure on the atoms, and transfer them back to $F_g = 2$. Such stray light results from multiple reflections inside the prism and at the surfaces of the vacuum cell. After a variable delay $t_{probe}$, only those atoms that have been transferred to $F_g = 2$ are

![Figure 5.3: Time sequence of an inelastic reflection. After the atoms are released from the molasses, they fall toward the prism. Just before they arrive at the prism surface the depumper transfers the atoms to the $F_g = 1$ state. The evanescent wave reflects the atoms inelastically by pumping them back to $F_g = 2$. After time $t_{probe}$ the atomic cloud is imaged on the CCD camera.](image-url)
imaged on the CCD camera, where $t = 0$ is defined as the time that the center of the cloud reaches the mirror. This time-sequence is schematically depicted in Fig. 5.3.

A typical series with 2 ms time increments is shown in Fig. 5.4. Each image has been summed in a horizontal direction to obtain a density profile. The solid curve is the result of a calculation described below. The amplitude of the theoretical curve has been scaled such that the maximum optical density of the experimental curve coincides with the theoretical maximum value. This is the only fit parameter.

As expected, the atoms bounce up less high than their initial height. Furthermore, the spatial distribution shows another striking feature: it displays a high density peak at low $z$, and a long low-density tail extending upward. Note also that there is a time-focus: the density peak is sharpest when the slowest atoms reach their upper turning point. This density distribution is an immediate result of the scattering behaviour of the atoms during the reflection and will be explained in the next section.

### 5.4 Analysis: Stochastic Rainbow

The high density peak in the atomic density distribution of the inelastically reflected atoms is the direct result of a caustic, appearing at the lower edge of the velocity distribution.
The caustic can be understood by considering the atoms as point particles moving in the EW potential. This corresponds to the ray-optics limit for the optical rainbow. We consider an atom arriving at the surface in its $F_g = 1$ hyperfine ground state, with an initial downward velocity $v_i < 0$. The initial velocity is determined by the distance which the atom has fallen from the MOT to the surface, which is typically 34 cm/s corresponding to 5.8 mm. Its trajectory through phase space is determined by energy conservation:

$$U_i(0) \exp(-2\kappa z) + \frac{1}{2}mv^2 = \frac{1}{2}mv_i^2,$$

and is depicted in Fig. 5.5 by the thick line. The closest point to the surface is the motional turning point of the atom which is only a few hundred nm:

$$z_0 = (2K)^{-1} \ln(2U_1(0)/mv_i^2).$$

After being pumped to the $F_g = 2$ state at velocity $v_p$, the atom continues on a new trajectory determined by $U_2(z) = \beta U_1(z)$. The asymptotic velocity at which the atom leaves the surface is $v_f = \sqrt{v_p^2 + \beta(v_i^2 - v_p^2)}$.

To illustrate the formation of the caustic, possible trajectories starting at various positions in phase space are depicted as thin gray ($v_p < 0$) and black ($v_p > 0$) curves. For asymptotically large $z$ the density of curves represents the outgoing velocity distribution, showing the caustic where the trajectories pile up at the lowest possible velocity. The trajectories with the lowest outgoing velocities are those which originated near the turning point.

This distribution is similar to the angle distribution of the optical rainbow (5.1b). The density of outgoing trajectories diverges at the "rainbow velocity". Below this velocity the intensity is zero, similar to Alexander's dark band in the optical rainbow [47]. Above the rainbow velocity the intensity distribution decreases smoothly.

Despite the similarities, there is a crucial difference between a rainbow created by sunlight refracted by water droplets and our 'velocity caustic'. The appearance of a rainbow is due to a deterministic process, where the scattering angle is uniquely determined by the impact parameter. In our experiment the incoming velocity $v_i$ is nearly single-valued, and is distributed over a broad range of outgoing velocities by the stochastic process of spontaneous Raman scattering.

In view of the novel character of this caustic, the analogy with known examples can only be valid to some extent. For example, the position of the caustic is independent of the size parameter $\kappa$. Similarly the optical rainbow angle does not depend on the droplet size.

The limitation of the analogy becomes apparent through the parameter $\beta$ which determines the position of the caustic. This parameter characterizes the "degree of dissipation" and therefore has no analogy in the optical rainbow, or any other deterministic caustic. For example, the optical rainbow angle is determined by the index of refraction $n$ of water. However, the energy of the photons is not changed, and therefore $n$ cannot be compared to $\beta$. 

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Figure 5.5: Construction of the velocity caustic in terms of phase space trajectories. The velocity of the incident atoms is \( v_i = -0.34 \text{ m/s} \). The thick solid line shows the trajectory of the lower hyperfine state bouncing elastically. If the atom is pumped to the other hyperfine state it continues on a different trajectory. The thin lines represent possible outgoing paths, each starting at a different pumping velocity, depending on the position of Raman-transfer. The density of outgoing trajectories diverges, yielding a caustic in the velocity distribution. Shown in the upper curve are the total distribution (solid line), and the contribution of atoms moving toward (short dash) and away from (long dash) the surface while being transferred.
5.4.1 Quantitative analysis

In order to quantitatively analyze our experimental data, we write the optical hyperfine-pumping rate during the bounce as \( \Gamma'(z) = (1 - q)G_1(z)/\hbar\delta_1 \), where \( q = 0.72 \) is the branching ratio from the excited state to \( F_g = 1 \). We define \( \eta(v) \) as the survival probability for the atom to reach the velocity \( v \) without undergoing optical pumping. This function decreases monotonically in time as \( \dot{\eta} = -\Gamma'\eta \), which is reformulated as \( d\eta/dv = -\eta/v_c \), where \( v_c = 2\hbar\delta_1/(1 - q)m\Gamma \), with \( m \) the atomic mass. For \( |v| \leq |v_i| \), the solution is \( \eta(v) = \exp((-v - v_i)/v_c) \) (since the initial condition is \( \eta(v_i) = 1 \)). When the pumping process takes place at a certain velocity \( v = v_p \), the atom leaves the surface with velocity \( v_f = \sqrt{v_p^2(1 - \beta^2)} + \beta v_p^2 \) (Fig. 5.2). This results in a distribution of bouncing velocities, resulting from atoms which were pumped while moving toward \( (v_p^-) \) or away from \( (v_p^+) \) the surface:

\[
n(v_f) = \eta(v_p^-(v_f)) \frac{\partial v_p^-}{\partial v_f} + \eta(v_p^+(v_f)) \frac{\partial v_p^+}{\partial v_f}
\]

(5.3)

for \( \sqrt{\beta} |v_i| \leq v_f < |v_i| \). The result is plotted in Fig. 5.5. This distribution diverges at \( v_f = \sqrt{\beta}|v_i| \). Note again the strong resemblance with the optical rainbow distribution of outgoing angles, see Fig. 5.1c.

This velocity caustic originates from atoms which are pumped near the turning point. There are two reasons for scattering preferentially at this position. An atom spends a relatively long time at the turning point since its velocity is lowest there. In addition, the intensity of the EW, and thus the photon scattering rate is highest at the turning point. The divergence is an artefact of the ray-optics description. It disappears due to diffraction when the atoms are treated as matter waves.

To compare the experimentally obtained spatial distributions with the model, we first calculated the one-dimensional phase-space density \( \Phi(z, v) \). The spatial distribution is obtained by projecting \( \Phi \) on the \( z \)-axis. Initially the MOT is described by a normalized Gaussian, \( \Phi(z, v) \propto \exp(-(z - z_0)^2/2\sigma_z^2) \exp(-v^2/2\sigma_v^2) \) with \( z_0 = 5.8 \text{ mm}, \sigma_z = 0.5 \text{ mm} \) and \( \sigma_v = 2.8 \text{ cm/s} \) (Fig. 5.6a). The cloud falls due to gravity and expands due to thermal motion. When it arrives at the EW, its velocity distribution is nearly Gaussian centered around \( v = -34 \text{ cm/s} \), with a spread of \( \sigma_v = 0.9 \text{ cm/s} \). This decrease in velocity spread is due to the fact that it is the distribution at fixed position, not at fixed time. This is called kinematic compression. When the cloud is reflected, its velocity distribution is changed in the way described above. Then the phase space distribution evolves freely again except for gravity and we calculate the spatial distribution of inelastically bounced atoms after a time of flight \( t \) (Fig. 5.6). To illustrate the agreement with the experimental data, the result for a time \( t = 7 \text{ ms} \) is drawn as the thick line in Fig. 5.4.
Temporal evolution of phase space density (in units of M/h) along the z-direction. The MOT is described by a Gaussian distribution both in velocity and position. During the inelastic reflection, the velocity distribution is changed in the way described in the text. The atoms start with an upwards velocity, and move away from the surface. After some time the atoms turn around and fall back to the prism. Depicted are the distributions at $t = 9$ and $15$ ms after the bounce. The spatial distribution is obtained by projecting the distribution on the z-axis.
5.5 Energy dissipation: measurements of bounce height

By analyzing the shape of the atomic distribution, information is obtained about the scattering process, resulting in the caustic at the rainbow velocity. A typical example of inelastically reflected atoms is shown in Fig. 5.7a. It shows the density profile of atoms 14 ms after the reflection, when the peak reaches its turning point. It clearly does not rise to the initial MOT position (5.8 mm). The peak is the direct result of the caustic at the lowest possible velocity. Above the caustic a low density tail is present from atoms transferred moving towards or away from the surface. Below the caustic peak there are no atoms at all, similar to Alexander's dark band of the optical rainbow.

The height to which the caustic atoms bounce back up, is completely determined by the ratio of the optical potentials of both ground states, $\beta$, defined in Eq. 5.2. It does not depend on the intensity of the EW, or its decay length $\kappa^{-1}$. Fig. 5.7a shows three measurements of the reflected cloud for different steepness of the EW. Both the peak position and the shape of the cloud do not change. However, the number of reflected atoms does change: the duration of a reflection on a shallow potential is much longer compared with a steep potential. Therefore more atoms are pumped over to the $F_g = 2$

![Figure 5.7](image)
Figure 5.8: Measured height of the upper turning point as a fraction of the initial (MOT) height, for an s-polarized EW, near a) the D$_2$-line and b) the D$_1$-line. This represents $\beta$, the relative optical potential strength of the $F_g = 1$ and $F_g = 2$ ground states. The curves result from a calculation including all hyperfine levels, and have been averaged over the ground state magnetic sublevels.

The rainbow velocity is calculated as $\sqrt{\beta v_i}$, and determines the bouncing height for the density peak. Fig. 5.7b shows the atomic density profile at the upper turning point for three different detunings. The relative height of the atomic cloud with respect to the height of the MOT is a direct measure of the energy dissipation. For the highest detuning, the energy dissipation is the lowest, i.e. the atoms bounce up the highest.

To calculate the exact value of $\beta$ one has to calculate the light shifts $U_1$ and $U_2$ including all excited state hyperfine levels and their magnetic sublevels. We calculate $\beta$ by averaging over all magnetic states of the hyperfine ground state. Fig. 5.8 shows its dependency on the detuning for a s-polarized EW, detuned with respect to the $D_1$ and $D_2$ line. There is a small difference between $\beta$ for the two D-lines. This results from the different level structure in the excited state. When the frequency is tuned close to the $D_1$-line, $\beta$ is primarily determined by the $F_g = 1, 2 \rightarrow F_e = 2$ transitions. For the $D_2$-line the dominant transitions are the $F_g = 1 \rightarrow F_e = 2$ and the $F_g = 2 \rightarrow F_e = 3$ transition. The detuning for the $F_g = 2$ state is $\delta_2 = \delta_1 + \Delta_{hfs} - 87$ MHz, see Fig. 5.2. For very large detuning the 87 MHz can be neglected, but for $\delta_1 = 150 \Gamma$ the correction is 9%, and for $\delta_1 = 75 \Gamma$ it is almost 14%.

When the EW has a large detuning, the accuracy in determining this upper turning point is lower since the atomic density is lower. There are several reasons for this density
decrease: the upper turning point is reached at a later time so that the atomic cloud expands longer. In addition, a larger detuning causes less hyperfine pumping due to the reduced scattering rate of the EW (this will be discussed in Sec. 5.7). Finally, for larger detuning the potential is lower, i.e. the effective mirror size is small, and thus fewer atoms are reflected.

Since the relevant Clebsch-Gordan coefficients for the $D_1$ line are smaller compared to those of the $D_2$ line, the size of the atomic mirror is smaller for the former case. Therefore the number of reflected atoms is smaller, and when the detuning exceeds $170 \Gamma$ no atoms are detected.

5.6 Polarization dependence

As mentioned already in the previous section, the degree of dissipation, $\beta$, depends on the polarization of the EW. When the laser beam is s-polarized, the EW is linearly polarized as well, see Chapter 2. All relevant magnetic substates experience almost the same light shift induced by the EW. The time of flight distribution of the reflection in Fig. 5.9a shows the sharp peak, resulting from the caustic. The inset shows the optical potentials for both hyperfine ground states.

The situation is different when the laser beam creating the EW is p-polarized. Fig. 5.9b shows the density distribution of atoms reflecting from the same EW as Fig. 5.9a, but with orthogonal polarization. Clearly the density distribution is much broader and shows three distinct maxima corresponding to $\beta = 0.08$, $0.15$ and $0.23$. This behaviour can be understood from the optical potentials, also plotted in the inset of the figure. Because the EW is elliptically polarized, there is a large potential difference between different magnetic sublevels, due to different Clebsch-Gordan coefficients. Many different Raman transitions are possible with different initial and final states, each with another $\beta$. We have indicated the 15 caustic positions at $8.5$ ms after the reflection, by the vertical bars in Fig. 5.9b. The bar at $2.8$ mm corresponds to the position of elastically reflected atoms ($\beta = 1$). Although we calculate a series of possible transitions, only three are clearly visible but it is difficult to address a specific transition.

It should be noted that to distinguish the different $m_F$ transitions, the EW was only switched on for only $2$ ms. This short opening time of the EW also reduces the total number of reflected atoms, and therefore an optical guide was used, as described in Chapter 4.

5.7 Optical hyperfine pumping

As mentioned already in section 5.5, the total number of atoms that are optically pumped to the upper hyperfine state depends on two parameters of the EW: $\delta_1$ and $\kappa$. in this
Figure 5.9: Density distribution of atoms 8.5 ms after bouncing inelastically on an EW described by $\kappa^{-1} = 0.8 \lambda$, $\delta_1 = 136 \Gamma$ ($D_1$ line) and total power of 70 mW. a) The EW was created by a $s$-polarized laser beam. The inset shows the optical potential for the different hyperfine ground states. b) The EW was created by a $p$-polarized laser beam. The bars indicate the position of calculated heights at this moment, corresponding to different possible transitions between initial and final $m_F$-states.

To measure the fraction of transferred atoms, the experimental procedure is as follows: first an inelastic bounce is imaged using the timing as shown in Fig. 5.3. By integrating the atomic density distribution, we obtain the number of atoms transferred to the $F_g = 2$ state, $N_2$. Then the same procedure is repeated, with the addition that the repumper light is switched on, just before the probe pulse. This transfers all elastically reflected atoms to the $F_g = 2$ state, so that all atoms are imaged by the probe beam. The atomic density contains the total number of reflected atoms, $N_1 + N_2$. Fig. 5.10a shows three images taken in the latter fashion, for different detuning. For low detuning almost all atoms are transferred during the reflection, whereas for large detuning a substantial fraction of the atoms remain in the lower ground state. The result is presented in Fig. 5.10b where the fraction of transferred atoms is measured as a function of detuning.

The solid curve represents the result of the calculated transfer fraction. To obtain this value, we calculated the time dependent scattering rate as observed by the atoms, while being reflected. This can be written as

$$\gamma(t) = \gamma_0 \text{sech}^2 (\kappa v_{\perp} t)$$

\[ (5.4) \]
Figure 5.10: a) Absorption images of all atoms, both $F_g = 1$ and 2, taken 3 ms after reflection. The lower part of the cloud are the inelastically reflected atoms, whereas the upper part represents the elastic fraction. The dotted line indicates the prism surface. b) Probability for the atoms to be optically hyperfine pumped during the inelastic reflection. The solid line is the result of a rate equation calculation, the squares are the experimental measured values. The detuning is with respect to the $D_2$-line and the decay length is kept constant: $\kappa^{-1} = 0.5 \times 10^{-6} \text{ m}$.

where $\gamma_0 = \frac{mv^2}{2\hbar} \Gamma$ is the maximal scattering rate, reached at the motional turning point of the atoms. The duration of the bounce, $(\kappa v)^{-1}$, is typically 3 $\mu$s in our experiments. The fraction of transferred atoms is then calculated by solving the optical rate equations [53] (omitting the excited state population), interacting with this light pulse.

$$\dot{N}_2 = -\dot{N}_1 = (1 - q) N_1 \gamma(t)$$  \hspace{1cm} (5.5)

where $q = 0.72$ is the branching ratio to the $F_g = 1$ state, where we assume large detuning and equal population of all magnetic sublevels [16]. After being optically pumped from $F_g = 1$ to $F_g = 2$, the scattering rate decreases dramatically due to the increased detuning (a factor 565 to 20 as the detuning increases from $\delta_1/\Gamma = 50$ to 350). Therefore pumping back to the lower state is not taken into account.

We find that the calculated value is systematically lower than the experimental one, i.e. atoms have scattered more photons than expected. A possible reason could be diffusely scattered light from the surface of the prism, i.e. a fraction of the beam creating the EW is not reflected totally, and propagates into the vacuum. This can happen due to roughness of the prism surface. This has been observed in Ref. [54], where the number of
scattered photons is determined by measuring the broadening of an atomic cloud reflecting elastically on an EW.

Another possible reason for the large number of scattered photons could be that the EW light contains a resonant spectral background. Such a spectral background was investigated in Ref. [34], where the heating rate in a 'gravito-optical surface trap' decreased when the EW was spectrally filtered. Spectral background can occur due to amplified spontaneous emission in the tapered-amplifier chip. This type of chip we characterized in Ref. [35] and the resonant background was found to be smaller than 0.7 nW/Γ, which would increase the scatter rate by 4 photons per second. Photon scattering is thus negligible since the reflection takes only a few micro seconds. However, these values were measured under ideal conditions. It is conceivable that the tapered amplifier was not optimally aligned during the presented measurements, and the spectral background was higher. Finally, the number of transferred atoms may have been overestimated if a small amount of repumper light was present.

For the experimental data around $\delta_1 = 133 \Gamma$ the difference with theory is about 4 or 5 photons. This of the same order as found in previous experiments [29, 13], where we measured radiation pressure by the EW field. In the latter experiment we found an excess of 3 scattered photons. Similar experiments done by the Dalibard group [45] with cesium atoms showed roughly the same relative difference between theory and measurements. In this experiment the transfer was measured at much larger detuning (2 to 4 GHz), and the experimental values were roughly 2 times as large as predicted. The authors suspected this to be a consequence of light scattered from the prism due to surface irregularities.

Note that for increasing detuning the effective mirror size becomes smaller. When an atom is transferred too early in its approach towards the surface, the optical potential for the $F_g = 2$ state is too small to overcome the residual kinetic energy of the atom. The atom will hit the prism and be lost. As an example we calculated that this loss fraction is 33%, for EW with a detuning of 166 Γ, and 70 mW power. If this loss factor is taken into account, the theoretical curve is lowered. This further increases the difference between the theoretical expectation and the measured values.

5.8 Discussion of phase-space density

Considering the divergence of the distribution of outgoing velocities, it is interesting to calculate the maximum phase space density, $\Phi_{\text{max}}$, in the context of evanescent-wave cooling [14, 15, 45]. The initial one-dimensional phase space density (PSD) of a single atom is determined by the atomic density and temperature after molasses, Fig. 5.6 ($t = 0$). The PSD is represented in units of $M/h$, where $M$ and $h$ are the atomic mass and Planck’s constant respectively. After the reflection, the PSD is calculated for the $F_g = 2$ state of the atom. Due to the broad velocity distribution an unexpected result followed: although
energy is removed from the atoms, the pumping process leads to a decrease of phase-space density. After the reflection, $\Phi_{\text{max}}$ is roughly three times lower than the initial $10^{-4}$ of the MOT for our experimental conditions. This can be seen in Fig. 5.6. Note that due to Liouville’s theorem the maximum PSD does not change before and after the reflection since the atoms move in the conservative gravitational potential.

We compare $\Phi_{\text{max}}$ of the reflected atoms with that of the MOT, and define 

$$\alpha = \frac{\Phi_{\text{max}}}{\Phi_{\text{MOT}}}.$$ 

We find that the condition for $\alpha > 1$ is mainly determined by $\beta$ and $v_c$, both depending on the detuning, and the falling height of the atoms $h$. In Fig. 5.11, $\alpha$ is plotted as a function of $\delta_1$ and $h$.

For fixed falling height, the dependence of $\alpha$ on $\delta_1$ shows a maximum at the condition $2v_c = v_t$. In this case the pumping rate at the turning point reaches its maximum, i.e. the peak at the lower edge of the velocity distribution reaches its maximum possible value. In the limit of high $v_c \propto \delta_1$, the scattering rate at the turning point is low. When $v_c$ is too low the $F_g = 1$ state is too much depleted before it reaches the turning point. This also results in a low pumping rate.

Decreasing the fall height also increases $\alpha$ since this determines the velocity of the atoms arriving at the EW, hence the duration of the bounce. This means that in multiple bounce experiments, the first bounce reduces the falling height but leads to a decrease of $\Phi_{\text{max}}$. Only later bounces may lead to effective cooling of the atomic cloud, and an increase of $\Phi_{\text{max}}$. It should be noted that this calculation is only valid for a single inelastic bounce with an initial narrow distribution. After many bounces the spatial distribution will become barometric [15].

![Figure 5.11: Contourplot of the calculated ratio (α) of phase space density after and before the inelastic bounce. The thick line corresponds to α = 1. Other parameters where fixed: κ = 1.3 μm, MOT radius 0.3 mm, T= 30μK.](image)
5.9 Outlook: Interference phenomena

The comparison of the inelastic reflection with the optical rainbow gives rise to the intriguing question: can we observe the equivalent of supernumerary arcs in the atomic velocity distribution? Any interference effect in the velocity distribution should manifest itself in the time of flight density distribution.

The supernumerary arcs of the optical rainbow are due to interference of two rays with different impact parameter but the same outgoing angle. These two paths will interfere in the far field due to the wave character of light. The phase difference is due to the different path length inside the droplet. The outgoing angular distribution shows minima where the two paths interfere destructively, and maxima where they interfere constructively. The size of the droplet determines the position of these arcs. In nature supernumeraries are not frequently observed, due to the spread in droplet size.

The high-velocity tail of our reflected velocity distribution is also the sum of two contributions: of atoms moving toward and moving away from the surface when they

![Figure 5.12: a) Schematic picture of two different paths in phase space with equal final velocity, but with opposite transfer velocity. The grey area is proportional to the phase difference. The inset shows the influence of the photon recoil. b) The calculated final velocity distribution for a single atom for \( \kappa = 2 \times 10^6 \text{ m}^{-1} \), including interference (thin line). The thick line (which has an artificial offset) results from the same calculation but also includes the initial velocity spread of the atomic cloud, and shows less visible oscillations. The velocity distribution of the atoms falling on the EW is a Gaussian centered around \( v_z = -35 \text{ cm/s} \) with an r.m.s. width of 0.9 cm/s.](image)
are pumped to $F_g = 2$. Two trajectories with the same outgoing velocity, but with opposite pumping velocity $\pm v_p$ could interfere, resulting in oscillations in the velocity distribution. This is a nontrivial effect because it involves the spontaneous emission of a photon. Interference between possible paths during an atomic reflection producing Stückelberg oscillations have been observed in Ref. [55]. In this experiment different magnetic substates follow different paths through the EW. However, transitions between these states were stimulated Raman transitions and therefore coherent processes. In an inelastic reflection the transition is spontaneous, and therefore incoherent. The results of Ref. [56] are also interpreted as an interference phenomenon between spontaneous Raman processes, although it was not directly observed.

We calculated the accumulated phase difference $\Delta \phi$ of the two paths $z_F(v)$ (where the subscript denotes the hyperfine state) as seen in Fig. 5.12a. The grey area is proportional to the phase difference. The accumulated phase difference for an atom with final velocity $v_f$ is calculated as:

$$\Delta \phi = \frac{M}{\hbar} \int_{-v_p}^{v_p} (z_1(v) - z_2(v)) dv$$

(5.6)

were the pump velocity is given by $v_p = \sqrt{(v_f^2 - \beta v_t^2)/(1 - \beta)}$. Adding the two velocity distributions of atoms transferred toward and away from the prism including the phase difference, results in the thin curve in Fig. 5.12b for $\kappa = 2 \times 10^6$ m$^{-1}$. The fringe period decreases for higher outgoing velocities $v_f$. This is similar to the supernumerary arcs of the optical rainbow, where the angle distribution is described by the Airy function. In our case the fringe period increases for a steeper EW. For example, the first maximum in Fig. 5.12b would shift to 12.8 cm/s when $\kappa = 3 \times 10^6$ m$^{-1}$.

These oscillations in the velocity distribution should be observable in the time of flight density distribution. The velocity distribution shown above is for an atom arriving at the surface with a single valued velocity, scattering one photon. This picture however is incomplete and we will discuss some of the details of an experiment which can reveal this interference phenomenon.

(i) Kinematic properties of the falling atomic cloud- The main influence of the atom cloud falling on the EW is the spread in its velocity. Each velocity component produces a different outgoing velocity distribution. The initial velocity spread causes the fringes in the caustic distribution to smear out (Fig. 5.12b thick line), and for the largest velocities the fringes disappear. To increase the visibility of the oscillations, one can increase the fringe spacing by making the potential steeper.

(ii) Internal structure of the atoms- The internal structure of the atoms should be taken into account when they are not optically pumped to one magnetic substate. In our experiment all 3 magnetic states of $F_g = 1$ are occupied, and each of these states will reflect with different $\beta$ producing different density distributions as described in the previous section. This will obscure the interference pattern. To reduce possible transitions, the
atoms must be optically pumped to one single initial magnetic state. Experiments to accomplish such optical pumping are in progress.

(iii) Photon recoil - The recoil of the scattered photon will give the atom an extra velocity in a random direction. This is schematically shown in the inset of Fig. 5.12a. To end with a certain velocity, the corresponding transfer velocity is slightly shifted by the extra recoil represented by the horizontal arrow. The grey areas represent the “extra” phase due to the recoil. However, the extra phase due to the recoil is small when the transfer takes place around the turning point, and in this case has no effect on the interference pattern. When the atom scatters a photon far away from the surface, the slight velocity change will cause the atom to move a relatively long time on a different trajectory in its final electronic state, therefore obtaining more phase compared to the case when this recoil was not taken into account. These atoms suffer from this phase diffusion which destroys the interference.

(iv) Which way information vs phase information - The prediction of interference raises the question whether it is possible to obtain “which way” information by a measurement of the emitted photon. It seems that by measuring the time of emission, one can determine if the atom was going towards or away from the surface when it was transferred. For sufficiently low temperature this apparent possibility vanishes when atoms are described as wavepackets, subject to Heisenberg’s uncertainty relation $\Delta z \Delta v_z \geq \hbar/(2M)$, were $\hbar$ is Planck’s constant [58]. The shape of the wavepacket inside the EW depends on the initial shape of the disk, before the reflection. We distinguish two different initial conditions: small $\Delta z$ or small $\Delta v_z$.

For sufficiently narrow spread of incident velocity (low temperature), the wavepacket is stretched along the classical trajectory and only a limited fraction of its area extends into the area of phase space which corresponds with the phase difference. This wavefunction is able to “probe” the phase difference with high resolution. Note that it also covers two crossing points at once, see Fig. 5.13. In other words, the atom is not on its way towards or away from the surface, but it is both. Therefore one cannot retrieve which-way information from such an atom.

For initial narrow spread of position (high temperature), a large fraction of its area extends into the area corresponding to the phase difference and it is unable to resolve $2\pi$ phase difference. Indeed, the position of this wavepacket can be determined by measuring the timing of the emitted photon and prohibits interference.

The initial phase-space distribution thus determines whether it yields timing or phase information. The phase-space distribution of the falling atoms fixes the initial conditions of the atoms. For an initial cold cloud, the atoms are described by small velocity spread, and their wavefunctions are able to resolve the phase difference, yielding interference. For increasing temperature, the width in velocity of the wavepacket will be less extensive. The corresponding decreasing phase resolution results in loss of interference. In other words: a cold cloud yields interference and a hot cloud does not. The final result under typical
Figure 5.13: When the atomic wave function before reflection is approximated by a disk in phase space, it is elongated along the trajectory. The figure shows the wave packet around the turning point ($v_z = 0$) for two initial possibilities: the thick (thin) line was initially described by a small $\Delta v_z$ ($\Delta z$). The dotted and dashed curves represent the trajectories of a classical point particle in the $F_y = 1, 2$ state respectively. The coldest atom (thick line) has only small overlap with the grey area (the phase difference between the two trajectories) therefore providing high phase resolution.

Experimental conditions is shown in Fig. 5.12b, where the spatial and velocity properties of the falling cloud are taken into account. For decreasing temperature of the molasses the visibility of the fringes will increase.

5.10 Conclusions

We have investigated an inelastic evanescent-wave mirror for atoms by tuning the EW close to an open transition. Photon scattering by the EW leads to a change of hyperfine ground states. Since both states experience different optical potentials, we were able to extract energy from the atomic cloud. The density distribution of the reflected atoms is highly asymmetric and contains a divergence or caustic in the velocity distribution. The caustic is created by atoms which are transferred near the motional turning point inside the EW. This is a new type of caustic due its stochastic nature: the spontaneous Raman transition by scattering a photon from the EW. This effect can be compared to some extent with the optical rainbow, but it has a different physical origin.
We measured the caustic velocity for atoms reflecting from the EW as a function of detuning for both $D_1$ and $D_2$-line. Changing the decay length did not change the position of the caustic. We observed a spread in positions due to different possible transitions between magnetic sublevels when the polarization of the EW was elliptical. We measured the number of transferred atoms as a function of detuning, which was systematically higher than expected. We contributed this to optical pumping by stray light due to surface roughness.

The experimental data agree well with a model calculating the temporal evolution of the phase space density. These calculations showed that although energy is subtracted from the atoms, in general the phase space density decreases during an inelastic reflection due to the large velocity spread of the reflected atoms.

Finally we discussed the possibility of observing interference phenomena in an inelastic reflection. Every possible final velocity can be produced by transferring the atom at two positions in phase space: when the atom moves toward or away from the prism surface. This should result in an interference pattern in the velocity distribution of the reflected atoms. This a nontrivial type of interference since it is induced by a spontaneous process.