Chapter 4

Coordination of Speculation

4.1 Introduction

The recent financial crises in Asia not only show that countries are interdependent, but also that speculators select one country at a time for their attack, given that there are several candidates that more or less face the same kind of problems. Since individual speculators rarely have sufficient resources to attack a fixed exchange rate successfully by themselves, coordination among speculators is essential. As coordination takes time, the actual date of the attack depends on the dynamic process which governs the evolution of coordination. Following this line of reasoning, we try to give an answer to the question raised by Obstfeld and Rogoff (1995, p.86): 'the speculative attack on the British pound in September 1992 would certainly have succeeded had it occurred in August - so why did speculators wait?' Our answer emphasizes that just before the EMS crisis several currencies were vulnerable to a speculative attack. Hence, speculators had to coordinate so that they aimed at the same currency. As this coordination takes time, the exchange rate is not seriously attacked as early as the state of fundamentals might suggest. Instead, the parity is only abandoned as soon as a focal point has emerged.

In this chapter we study the consequences of adding coordination problems among currency speculators to Krugman's seminal (1979) model of balance of payments crises and, particularly, its linear variant developed by Flood and Garber (1984).\(^1\)\(^2\) These models show that one reason for a balance of payments crisis lies in the fact that a fixed exchange rate regime will collapse when policymakers pursue an expansionary domestic credit policy relative to the anchor country in the presence of perfect capital mobility. In the presence

\(^1\)From now on denoted by KFG.

\(^2\)Flood and Garber also consider a stochastic discrete-time variant of Krugman's model which gives rise to a forward premium.
of sufficiently liquid speculators, the KFG model in effect predicts that a fixed exchange rate can only survive if the rate is equal to the perfectly observed equilibrium rate. The implications of this result are at odds with the following five stylized facts of balance of payments crises.

First, the currency is usually overvalued prior to the occurrence of an attack.\textsuperscript{3} Second, an exchange rate jump occurs at the moment of a successful speculative attack, instead of the continuous transition from fixed to flexible rates as predicted by KFG.\textsuperscript{4} Third, speculators make profits, at least as a group. Indeed, a major distinction between flexible and fixed exchange rates is that speculators play a zero sum game against each other under the first arrangement (one speculator’s profits are another speculator’s losses), whereas they are able to make profits under the second arrangement. These profits are made at the expense of the central bank which suffers capital losses on its foreign exchange positions. Of course, profits for the group of speculators are related to currency overvaluation before the crisis and devaluation at the moment of the successful attack.

A fourth stylized fact of recent speculative attacks is contagion: the occurrence of several, subsequent, speculative attacks on different currencies, with the result that a successful attack on one currency increases the probability of a successful attack on another currency. As KFG use a two-country model, in which one currency is fixed to an anchor currency, their set-up is unable to shed any light on this issue of contagion. We present a multi-country extension of the KFG framework in which contagion arises naturally. In essence, the reason that several, subsequent, attacks occur is that all countries in our model have weak fundamentals. Contagion arises because a successful attack in one country both increases the speculative funds and decreases the number of options that are available to speculators. As a consequence, successful coordination among speculators occurs faster if there have been more successful attacks in the recent past and this implies that subsequent speculative attacks occur with ever shorter time intervals in-between.

A fifth, and for our purpose last, stylized fact of speculative attacks that deserves an explanation is that reserves often show large fluctuations before the fall of a fixed exchange rate regime. In KFG, reserves decrease monotonically before the attack. In our model, these fluctuations in reserves before the attack are the consequence of speculators communicating with each other by means of trading. They do this by taking alternating short positions in the currency they prefer to attack. These short positions in turn give

\textsuperscript{3}See, for example, Dornbusch and Werner (1994) and Sachs et al. (1996a) on the Mexican peso-crisis; Dornbusch et al. (1995) on the experience of several countries with currency crises and collapses; and Helpman et al. (1994).

\textsuperscript{4}See also Obstfeld and Rogoff (1995) on these two aspects of speculative attacks. Obstfeld (1995) discusses the experience of several countries with fixed and flexible exchange rates in general.
rise to the fluctuations in reserves before the attack.

In this chapter we will explain these five stylized facts by analyzing the coordination problems among speculators that arise in a multi-country extension of the KFG model in which speculators are only able to communicate imperfectly with each other by means of trading. First, in Section 4.2, we discuss the discrete time KFG model. Then, in Section 4.3, we analyze the dynamics of coordination when two identical countries fix their currencies vis-à-vis a third anchor currency. Section 4.4 considers the general case for $m$ identical countries that fix their currencies vis-à-vis some anchor currency and presents the qualitative implications of our framework for contagion. Section 4.5 concludes and gives some suggestions for possible extensions and alternatives. The Appendix contains a specific illustration of the dynamics of coordination when communication between speculators is formalized by signals being drawn from a Beta distribution.

### 4.2 The Discrete Time KFG Model

In this section we review KFG’s model in which there is one country that fixes its exchange rate against some second anchor country. We write their model in discrete time which allows a more tractable and natural analysis of the incentives of speculators:

\[
\frac{M_t}{P_t} = a_o - a_1i_t \quad a_1 > 0 \tag{4.1}
\]

\[
M_t = R_t + D_t \tag{4.2}
\]

\[
\Delta D_t = D_t - D_{t-1} = \mu \quad \mu > 0 \tag{4.3}
\]

\[
P_t = P_t^*S_t \tag{4.4}
\]

\[
i_t = i_t^* + \frac{S_{t+1} - S_t}{S_t} \tag{4.5}
\]

Equation (4.1) gives the money market equilibrium condition, where $M_t$, $P_t$ and $i_t$ are the home country’s domestic money stock, price level, and interest rate in period $t$, respectively. Equation (4.2) shows that the domestic money stock equals the country’s
international reserves ($R_t$) plus domestic credit ($D_t$). Equation (4.3) states that domestic credit grows by a constant amount $\mu$. Equations (4.4) and (4.5) impose purchasing-power parity and uncovered interest-rate parity, respectively. The exchange rate, $S_t$, is expressed as the number of units of home currency per unit of foreign currency. An asterisk attached to a variable indicates ‘foreign’. The last term on the right hand side of (4.5) shows that the expected equals the actual rate of exchange rate change; i.e. the assumption of perfect foresight is imposed.

At $t = 0$ reserves are equal to $R_0$. As long as the exchange rate is fixed at $\bar{S}$, reserves adjust to maintain money market equilibrium: $\Delta R_t = -\mu$. If we denote the combined funds of speculators by $\theta = R_0$, the moment when reserves have reached the critical level is equal to time $t = 0$. The central bank defends the fixed exchange rate until it has run out of reserves. From that point on, the exchange rate floats freely forever. Substituting (4.4) and (4.5) in (4.1) gives:

$$M_t = \gamma S_t - \delta (S_{t+1} - S_t),$$

where $\gamma = (a_0 P^* - a_1 P^*t)$, which is assumed to be positive, and $\delta = a_1 P^*$. After the attack, money market equilibrium requires that equation (4.6) holds. Moreover, after the attack the stock of reserves is zero, so that $M_t = D_t$ and $\Delta M_t = \Delta D_t = \mu$. Using the trial solution $S_t = \lambda_0 + \lambda_1 M_t$, we arrive at the following expression for the ‘shadow floating’ exchange rate:

$$S^f_t = \frac{\delta \mu}{\gamma^2} + \frac{1}{\gamma} M_t$$

We assume $S^f_t = \bar{S}$. What will be the optimal moment to attack the currency if speculators cooperate? On the one hand, they would be willing to wait forever as the difference between the shadow floating rate and the fixed exchange rate becomes larger and larger. On the other hand, however, reserves keep on falling gradually until there are no reserves left. At $t = 0$ reserves are equal to $R_0$. In each period, reserves decrease by the amount $\mu$ and, when reserves are equal to zero at some $t = g$, the exchange rate will float, even though there has not been a speculative attack. The moment where reserves are exhausted without an attack is given by $R_0 - g \mu = 0$, i.e. when $g = \frac{R_0}{\mu}$. Hence, speculators are not able to make any profit when they delay their attack until $t = g$.

The optimal moment to attack the fixed exchange rate regime if speculators cooperate and form a coalition is obtained by the maximum value of aggregate profits.\footnote{Profits of the coalition are divided equally among the identical speculators.}
4.2. The Discrete Time KFG Model

\[ \pi_t = \left( S_t^F - \bar{S} \right) R_t \]  \hspace{1cm} (4.8)

As \( \left( S_t^F - \bar{S} \right) = \frac{\mu}{\gamma} \) and \( R_t = R_0 - t\mu \), we have:

\[ \Delta \pi_t = \frac{\mu}{\gamma} \left[ R_0 + (1 - 2t) \mu \right] \]  \hspace{1cm} (4.9)

The term in brackets gets smaller and smaller as time proceeds. Profits, \( \pi_t \), over time for the coalition are shown in Figure 4.1. \( R_t \) has a slope of \( -\mu \) and \( S_t^F \) of \( \frac{\mu}{\gamma} \).\(^6\) Thus, aggregate profits for the speculators are maximized when the attack takes place at \( t = t^* \), where:

\[ t^* = \frac{R_0 + \mu}{2\mu}, \]  \hspace{1cm} (4.10)

which follows from setting \( \Delta \pi_t = 0 \) in eq. (4.9).

\(^6\)We have drawn the figure as if the model is in continuous time.
However, in general, attacking at \( t^* \) may be in the interest of the group of speculators, but no individual speculator has an incentive to wait this long if a credible commitment is not possible. This is because in the KFG model, speculators are assumed to be liquid in the sense that the combined speculative funds of all the speculators planning to participate in the attack are larger than the amount of reserves held by the central bank. As a consequence, there will be competition for reserves among speculators. This competition sets in motion a process of backward induction such that the attack will already occur at \( t = 0 \). As can be seen in Figure 4.1, then profits for speculators are equal to zero as the exchange rate does not jump. Moreover, the domestic currency has been undervalued, and reserves fall monotonically, preceding the attack.

These results are clearly at odds with the stylized facts of balance of payments crises as mentioned in Section 4.1. This contrast follows from the fact that in the KFG model a fixed exchange rate regime can only be supported as long as the fixed rate is equal to the equilibrium exchange rate. The reasons for this are that the equilibrium exchange rate is determined by the economies' fundamentals, these fundamentals are perfectly observed by liquid speculators, and there is a unique fixed exchange rate value that corresponds with the fundamental equilibrium rate. As a consequence, as soon as a country adopts an expansionary domestic credit policy, the fixed exchange rate regime will collapse.

### 4.3 Coordination in the KFG Model

In this section we focus our attention on coordination problems among speculators that arise in a multi-country extension of KFG.\(^7\) In this way, we show that waiting by speculators is the inevitable consequence of the fact that successful coordination, accomplished through imperfect communication, takes time. It does not mean that speculators remain passive in the run-up to a speculative attack. They are taking short positions in weak currencies as soon as a misalignment occurs. However, it takes time before a sufficient number of speculators have taken short positions in the same currency for the fixed exchange rate regime to collapse.

We adapt the KFG model by assuming that there are two identical countries that fix

\(^7\)The issue of coordination in a multi-country speculative attack setting is also studied by Bui et al. (1996, 1998). However, they focus on coordination between policymakers. In a recent paper, Morris and Shin (1998) analyze coordination and costs of speculative trading in a two-country model with self-fulfilling currency attacks. They demonstrate that these models do have a unique equilibrium when speculators face uncertainty about the fundamentals. Obstfeld (1996) discusses coordination problems among two speculators, also in a model of self-fulfilling currency attacks.
their exchange rates vis-à-vis a third country, the anchor country. The situation of each of these two 'pegging' countries is described by the model in Section 4.2. In particular, from \( t = 0 \) onwards both currencies are vulnerable to an attack in the sense that their currencies are overvalued. We assume that every speculator is small; i.e. has a limited amount of speculative funds. The type of coordination that arises is such that speculators need each other. In other words, a sufficient fraction of the total number of speculators has to focus attention on the same currency, as this is the only way to gather sufficient resources to buy up all the central bank’s reserves. The total amount of speculative funds of all speculators is again equal to \( \theta \).

We assume a continuum of speculators normalized on \([0, 1]\) and that initial beliefs at \( t = 0 \) are such that a fraction \( \rho_0(1) \) of total speculative funds available to all speculators opts for currency 1, and a fraction \( \rho_0(2) = 1 - \rho_0(1) \) of the speculators opts for currency 2. Without loss of generality we assume that \( \rho_0(1) > \rho_0(2) \). Opting for a currency means that a speculator takes a costless short position in that currency.

The total value of all the short positions taken against a currency at the beginning of a period implies a corresponding drop in the reserves of that country. However, if the total value of these short positions proves to be insufficient for the exchange rate to collapse, these positions are cancelled at the end of the period, and the stock of reserves returns to the level it would have had in the absence of speculation. Thus, reserves change from one period to the next because of monetary expansion with \( -\mu \) and, additionally, ‘inside’ each period because of speculation. The latter change is equal to \( -\rho_t(i)\theta \) for \( i = 1, 2 \). The accounting equation for reserves of country \( i \) then becomes, with \( R_0 \) denoting the stock of reserves at the beginning of time \( t = 0 \):

\[
R_t(i) = R_0 - t\mu - \rho_t(i)\theta
\]  

Equation (4.11) implies that \( \Delta R_t(i) = -\mu - \Delta \rho_t(i)\theta \). Thus, if speculative pressure proves insufficient for a successful attack, the short positions \( \rho_t(i)\theta \) are cancelled and we move to period \( t + 1 \) inheriting a stock of reserves equal to \( R_0 - t\mu \) from the previous period.\(^8\)

At the beginning of each period speculators reconsider in which of the two currencies they want to take a new position. Since they know that they have to coordinate in order for the attack to be successful, they decide to take a short position in that currency in which most speculators took a short position during the previous period, as reflected in that country’s temporary drop in reserves. Thus, during the process of coordination,

\(^8\)Notice that the identity in (4.11) is identical to the change in reserves in the previous section except that there \( \rho_t(i)\theta = 0 \) until reserves are exhausted completely in one stroke.
speculators communicate by means of trading.

However, there are several reasons why, in practice, it is difficult for speculators to determine which part of the reserve fluctuations can be attributed to speculative positions taken by other speculators. The stock of reserves also changes if there is a current account imbalance or if the country obtains some new funding, for example from an international financial institution, other central banks, the international capital market, or a state enterprise. Moreover, speculative pressure may not show up in reserve changes because the foreign assets of a commercial bank, rather than the official reserves, are used to finance speculative outflows. Moreover, the value of reserves will fluctuate because periodically they are reassessed and since the rate of return on reserves is added. In all these cases, communication between speculators through reserve changes is far from perfect, as it becomes hard to discover which currency other speculators attacked in the previous period.

Of course, the analysis of coordination depends crucially on the dynamic process that is assumed to govern the selection among alternatives.\(^9\) However, whatever the procedure chosen, coordination usually takes time. The coordination process starts as soon as a currency becomes misaligned and is completed when a sufficient number of speculators have selected the same currency. Then we say that a focal point has emerged. We are interested in the factors that determine when such a focal point has emerged, as this moment indicates a successful attack and, therefore, determines the size of the jump in the exchange rate and the resulting profit for each speculator participating in the attack. First of all, we have to determine the required value of short positions in order for an attack to be successful. Then we should determine the actual value of short positions per currency over time.

We again assume that the total value of financial funds of all speculators, \(\theta\), is such that the required fraction of these funds, \(\rho_0(i)\), needed for a successful attack against currency \(i = 1, 2\) at \(t = 0\), is equal to 1. In other words, at \(t = 0\) an attack can only be successful if all speculators coordinate their activities on the same currency. Thus, as in the KFG model discussed in the previous section, \(\theta = \theta_0\) denotes the critical level of reserves. In general, a successful attack occurs as soon as:

\[
\rho_i(1) \geq \rho_i^*(1)
\]  

4.3. Coordination in the KFG Model

The required fraction of speculators for a successful attack declines over time as central bank reserves are falling in both peripheral countries from one period to the next due to the expansionary domestic credit policy according to:

$$\rho_t^1(1) = \rho_0^1(1) - \frac{t\mu}{R_0} = 1 - \frac{t\mu}{R_0} \quad (4.13)$$

Thus, at a time $t$ for which the inequality in (4.12) holds, the fraction of financial funds of speculators used to take short positions in currency 1 implies such a large drop in the reserves of country 1 that the fixed exchange rate regime collapses. We are left with determining $\rho_t^1(1)$.

As was mentioned above, communication among speculators occurs on the basis of trading in currencies. More specifically, we assume that each speculator receives an idiosyncratic, noisy signal at each time $t$, about these fluctuations in reserves during the previous period for both countries. These signals are assumed to be independently and identically distributed and refer to the fraction of speculators that took a short position in each currency in the previous period. Since they are searching for the currency that attracted most speculation in the previous period, it is $\rho_{t-1}(1) - \rho_{t-1}(2)$ that matters. Thus, the signal received by speculator $je [0, 1]$ at time $t$ is defined as:

$$z_{jt} = \rho_{t-1}(1) - \rho_{t-1}(2) + \varepsilon_{jt}, \quad (4.14)$$

where $\varepsilon_{jt}$ is an idiosyncratic noise term with cumulative distribution function $F$ and corresponding probability density function $f$.

We impose the behavioral assumption that speculators who receive a positive signal opt for a short-position against currency 1. After all, in this case, the information obtained suggests that the fraction of speculators that attacked the first currency is larger than the fraction that attacked the second currency. Then, the fraction of speculators receiving a positive signal is equal to $1 - F (\rho_{t-1}(2) - \rho_{t-1}(1))$ where $F$ is monotonically non-decreasing given our behavioral assumption. As a consequence, for currency 1 at time $t$, we have:

$$\rho_t^1(1) = 1 - F (\rho_{t-1}(2) - \rho_{t-1}(1)) \quad (4.15)$$

This is the equation for the dynamics of coordination. We make the following assumption on the shape of the density function:

\footnote{Caplin and Leahy (1994) also discuss a model, applied to industry investment, in which individuals gather or receive private information, which they convey to others through changes in their behavior.}
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A1 the domain of $f$ is included in $[-1,1]$ (i.e. $\forall t, \epsilon_{jt}[-1,1]$) and $f$ is symmetric bell-shaped.

Given this assumption, we have the following result:

R1 $\rho_t(1)$ converges towards 1 if $\rho_0(1) > \frac{1}{2}$; $\rho_t(1)$ converges towards 0 if $\rho_0(1) < \frac{1}{2}$; for $\rho_0(1) = \frac{1}{2}$ we have the (unstable) equilibrium $\rho_t(1) = \frac{1}{2}$.

Proof. Given symmetry of $F$ we have $\rho_t(1) = F(2\rho_{t-1}(1) - 1)$ and, without loss of generality, we can consider the case where $\rho_0(1) > \frac{1}{2}$. It suffices to show that $\rho_t(1)$ will exceed any value $k$, satisfying $\rho_0(1) < k < 1$, within finite time. $F$ is non-decreasing and under the other assumptions the derivative $F' = f$ is non-increasing (since $f$ is bell-shaped) on $[\frac{1}{2},1]$, which implies that $F$ is concave on $[\frac{1}{2},1]$. The only fixed points in $[\frac{1}{2},1]$ being $\frac{1}{2}$ and 1, the increments $F(2\rho_t(1) - 1) - \rho_t(1)$ are positive and bounded away from 0 on $[\rho_0(1),k]$. This implies that $\rho_{t+1}(1) - \rho_t(1)$ for $\rho_0(1) < \rho_t(1) < k$, is bounded from below by a positive constant $a = \min(F(2\rho_t(1) - 1) - \rho_t(1), F(2k - 1) - k)$, so that $\rho_t(1)$ will exceed $k$ in no more than $(k - \rho_0(1))/a$ time steps. \[\blacksquare\]

A graphical characterization of the dynamics of coordination is shown in Figure 4.2. According as speculators receive more noisy signals, communication among them is more difficult, and the dynamics of coordination become slower. This is illustrated in Figure 4.2 by the curve labelled $h$ (high noise) relative to the curve labelled $l$ (low noise) and is summarized in Result 2:

R2 The number of time steps required to exceed a given level $k$, satisfying $\rho_0(1) < k < 1$, starting from a given initial value $\rho_0(1) > \frac{1}{2}$, is non-increasing with decreasing noise scale.

Proof. Consider the sequence $\rho^*_t(1)$ upon a multiplication of $\epsilon_{jt}$ by a noise scale factor $s$, governed by the dynamics of $F^*(x) = F(x/s)$ for $0 < s < 1$. A lower value of $s$ implies less noisy signals. Since $F$ is non-decreasing, $F^*(2x - 1) \geq F(2x - 1)$. This implies $\rho^*_{t+1}(1) \geq \rho_{t+1}(1)$ whenever $\rho^*_t(1) \geq \rho_t(1) > \frac{1}{2}$. For $\rho^*_0(1) = \rho_0(1)$ it follows by induction that $\rho^*_t(1) \geq \rho_t(1)$. \[\blacksquare\]

Given our assumption $\rho_0(1) > \rho_0(2)$, so that $\rho_0(1) > \frac{1}{2}$, it follows that the fraction of the total value of financial funds of all speculators used in short positions against
currency 1 increases over time. In other words, country 1 gradually becomes a focal point for speculators. In the Appendix we present an illustration of the dynamics of coordination for the special case in which the signals are drawn from a Beta distribution.

This immediately implies that speculative pressure against currency 2 decreases over time. For that reason policymakers in country 2 may think that their currency has withstood speculative pressure even though their currency is becoming increasingly overvalued. This follows from the fact that speculators recognize that they have to coordinate their activities on one country, which implies that the other country, temporarily as we will see in the next section, becomes less vulnerable to such an attack.

Substituting equations (4.15) and (4.13) in (4.12) gives us an expression for the conditions that determine the moment of successful coordination; i.e. the moment when a focal point has emerged:

\[
F \left( 2\rho_{t-1}(1) - 1 \right) \geq 1 - \frac{t\mu}{R_0}
\]  

(4.16)

Following Result 1 the lhs increases in t, while the rhs decreases in t. At \( t = 0 \) the requirement for successful coordination is equal to \( \rho_0(1) = 1 \). In other words, we obtain the results of KFG, where the attack takes place at \( t = 0 \), in the special case that all speculators select the same currency at the start of the coordination game. Of course, if there is only one ‘pegging’ country, as in KFG, the condition \( \rho_0(1) = 1 \) is trivially satisfied.

In general, we see that the time it takes before a focal point has emerged increases if initial differences in the volume of short positions taken in the two currencies are small;
i.e. when $\rho_0(1)$ is close to $\frac{1}{2}$. In that case, it is hard for speculators to determine which country was selected by most speculators in the previous round, given the amount of noise in communication. Also, the emergence of a focal point takes more time if the level of domestic credit growth is smaller relative to the initial amount of reserves, since then the required fraction of speculators for a successful attack decreases less fast.

Following Result 2, as communication gets more noisy, it takes longer before successful coordination occurs, as it becomes harder to infer which currency other speculators have attacked. For $\rho_0(1) < 1$, successful coordination can be achieved already in the second round if communication is almost perfect. Then all speculators observe at $t = 1$ that most short positions were taken in currency 1 at $t = 0$.

Although the KFG model has a unique equilibrium where the attack takes place at the moment when a country with a fixed exchange rate regime adopts an expansionary domestic credit policy, our extension of KFG to two symmetric countries and liquidity-constrained speculators contains multiple-equilibria. In the spirit of Obstfeld (1996), if a sufficient number of speculators coordinate their trading against the same currency, the fixed exchange rate regime collapses. If not, the fixed parity remains in place.

Denoting the moment when a focal point has emerged by $t = \bar{t}$, we know how long speculators have to wait until their attack is successful and therefore we know:

1. the duration of overvaluation of the nominal exchange rate: $\bar{t}$ periods;

2. the degree of overvaluation, which is also the jump in the exchange rate, and the width of the gap between the fixed rate and the rate at which the currency floats without being attacked successfully: $S^F_i - \bar{S} = \frac{i\mu}{\beta}$;

3. aggregate profit for the speculators participating in the attack: $\theta \rho_\epsilon(1) \frac{i\mu}{\beta}$;

4. the central bank’s loss of reserves due to the attack equals, from (4.11) with $R_1(1) = 0$, $\rho_\epsilon(1) \theta = R_0 - \bar{t} \mu$; and,

5. the fluctuations in reserves, inside a period due to speculation, in reserves in each country, $-\mu - \rho_\epsilon(i) \cdot \theta$ for $i = 1,2$, respectively.

4.4 The General Case and Contagion

In this section we will briefly argue that all our qualitative results for $m = 2$ carry over to the general case, in which $m > 2$ identical countries fix their currency vis-à-vis the currency of an anchor country. We rank the $m$ countries according to their vulnerability
to an attack, as perceived by speculators on the basis of their initial beliefs. Without loss of generality, we rank them in decreasing order of vulnerability from 1, ..., m. In other words, the fraction of speculators that opt for country \( i = 1, ..., m \), on the basis of initial beliefs in period \( t = 0 \), is, in decreasing order, \( \rho_0(1), \rho_0(2), ..., \rho_0(m) \). This gives rise to a drop in reserves in \( t = 0 \) equal to \(-\mu - \rho_0(1) \cdot \theta, ..., -\mu - \rho_0(m) \cdot \theta\). Again, as in the previous section, speculators do not perfectly observe these variations in reserve levels due to speculation inside a period. They receive a noisy signal, in each period \( t \), about these temporary fluctuations in reserves for each country \( i = 1, ..., m \).

If, for example, the difference in previous period trade against currency 1, respectively \( m \), is large then \( m \) is no longer a candidate for taking a short position in its currency. In that case, despite the noise involved in the communication among speculators, no one will take a short position in currency \( m \) anymore and speculative pressure against this currency will cease. Policymakers in this country observe that speculative pressure against its currency decreases and may even become zero, which may give them the impression that trust by financial markets in their currency has been restored.

Since 1 is the most preferred alternative in \( t = 0 \) and because of the continuum of speculators, we can be sure that the number of speculators that opt for this alternative will increase over time so that \( \rho_t(1) \) will approach unity. As the noise is constant, while the difference in previous period trade against the currencies widens, more and more speculators will take a short position in currency 1.

Since the required fraction of speculators for a successful attack, \( \rho_t^*(1) \), is the same as discussed in the previous section, we can conclude that the selection of a focal point in the case where \( m \) countries peg their exchange rate depends on the same factors which governed the process of coordination in the two-country case. The only difference is that successful coordination takes longer to achieve as more currencies are ready for an attack. Because it takes longer for a focal point to emerge, the duration of overvaluation will be longer, the eventual devaluation larger, and the loss of reserves in the attack smaller.

After speculators have selected a focal point and, consequently, attacked the fixed exchange rate regime of country 1 successfully, nothing prevents them from coordinating their speculative activities on the remaining \((m - 1)\) currencies. In our model, subsequent speculative attacks on different currencies arise because all countries have weak fundamentals and are vulnerable to a speculative attack. Nevertheless, the speculative attack on currency 1 in the first round facilitates a subsequent speculative attack on the other currencies because the process of coordination occurs faster, implying faster selection of a new focal point. The reasons are threefold: (1) The profits made by speculators in the first attack increase their financial funds, implying a faster coordination process; (2) After
the collapse of the first currency, the amount of reserves of the other countries, $R_1$, is much smaller than the amount of reserves, $R_0$, that the central bank of the first country started with; and (3) The number of strategies has decreased.

For these reasons, our analysis predicts that subsequent speculative attacks take place faster and faster. As a consequence, the decrease in speculative pressure against all currencies, except the first during the process of coordination on country 1, is only temporary and policymakers should not feel comfortable about this decrease in pressure from financial markets. As soon as currency 1 has been attacked, speculative pressure returns to the other currencies with a vengeance. Successful coordination on one country implies a faster emergence of a new focal point and this contagion implies that subsequent speculative attacks occur at increasingly shorter time intervals.\footnote{It may even be the case that the reserves of the countries which remain vulnerable to an attack have decreased, or speculative funds have increased, so much that several countries can be attacked successfully at the same point in time.}

As all countries are identical, our set-up would predict that currencies attacked later will lose less reserves in the attack, although both the devaluation and the degree of overvaluation of the nominal exchange rate will be larger. Also the duration of overvaluation will be longer. However, coordination in the first round may have taken so much time that some of the $m$ countries have already solved their problems. Speculators are then too late to attack those currencies.

Consequently, our analysis also suggests that, even though countries are identical, some may be hit by an attack, while others have enough time to reverse their policies and avoid a currency crisis. This implies that it is possible to follow expansionary domestic credit policies under fixed exchange rates temporarily, as long as other peripheral countries do the same. For, as in KFG, the critical level of reserves is reached at $t = 0$ in our approach as well. However, since the combined speculative funds are divided among all the overvalued currencies, the actual level of reserves at which a successful attack takes place can be much lower compared to the case of a unilateral expansionary credit policy as in KFG. Then, the policymaker has to weigh the benefits of temporary discretionary policy against the expected costs of a speculative attack if his country is selected by speculators.

### 4.5 Concluding Remarks

Assuming that each speculator has a limited amount of resources for speculation allowed us to analyze the selection of a focal point. For a successful attack a sufficient number...
of speculators have to select the same currency. Speculators communicate, imperfectly, with each other by trading in a currency. In our approach the short positions taken by speculators are reversed periodically. This occurs in all countries with overvalued currencies. Speculators not only take these short positions to provoke the collapse of the exchange rate, but these positions are also the medium through which they communicate with each other. As soon as the total value of these short positions is sufficient to buy up all the remaining reserves of a country, the currency is devalued. speculators make profits, and subsequently their attention will be directed to the next overvalued currency for an attack. The policymakers involved may be surprised, since speculative pressure on their currency had decreased recently.

This explanation of why speculators have to wait before their attack is successful clarifies how and when market expectations coordinate on a particular set of expectations. When one strategy is ultimately selected by a number of players sufficient for a successful attack, we say that a focal point has emerged. In our case, the initial beliefs of the speculators are the factor that determines which currency will eventually become the focal point.

Our analysis produces the following general conclusions:

1. The emergence of a focal point takes more time when, at the start of the coordination game, there are only slight differences in the ‘popularity’ of different countries, when the number of currencies vulnerable to an attack increases, when communication between speculators is noisier, and when domestic credit creation relative to the initial amount of reserves is smaller.

2. It takes time before a sufficient number of speculators take short positions in the same currency for the fixed exchange rate regime to collapse. Then, referring back to the quotation cited in the introduction, a currency may be successfully attacked in September, although the state of fundamentals allowed an attack in August.

3. The duration and extent of overvaluation of the currency before the attack, as well as the size of the devaluation when the attack is launched, depend positively on how long it will take for speculators to select a focal point.

4. The loss of reserves of the central bank in the speculative attack depends negatively on the time it takes for speculators to coordinate their actions.

5. Reserves of the countries under attack fluctuate. These fluctuations are the consequence of the short positions taken by speculators which allow them to communicate
indirectly. Speculative pressure increases over time on the currency being selected as a focal point and decreases temporarily on the currencies of the other countries involved in the coordination game.

6. A successful attack on one currency implies faster successful coordination of speculative activities on the remaining overvalued currencies, since speculative funds have increased, the options for speculators are reduced, and reserve levels of the remaining countries have decreased. Subsequent speculative attacks on different countries therefore occur faster and faster. Countries which become later focal points will lose less reserves in the final attack, experience a longer period of overvaluation of the currency prior to the attack, and undergo a larger devaluation at the moment of the attack.

Coordination takes time when communication between speculators is imperfect, and during this time currencies become overvalued despite perfectly observed weak fundamentals. It is even possible that coordination takes too long, i.e. by the time it happens policymakers have already reversed their policy of excessive domestic money creation. The exchange rate may not be attacked successfully in this case, even though there was an interval where speculators could make profits by attacking the currency. In effect, this suggests that, following expansionary domestic credit policies in the presence of fixed exchange rates and perfect capital mobility is possible for some time as long as other peripheral countries do the same and speculators are liquidity constrained.

As mentioned, KFG consider the occurrence of a speculative attack as the rational response of speculators to weak fundamentals. Alternative explanations based on multiple equilibria and self-fulfilling expectations can be found in Obstfeld (1986, 1994), although Krugman (1997b) too argues that a model which stresses the importance of self-fulfilling crises must be founded on weak fundamentals in one way or another. More specifically, Obstfeld (1994) argues that if speculative currency crises are a manifestation of possible multiple equilibria, an obvious barrier to understanding them is the lack of any convincing account of how and when market expectations coordinate on a particular self-fulfilling set of expectations.

In this chapter we shed some light on this issue of coordination. Although we discussed our coordination process in the KFG framework, the analysis can easily be extended to situations in which several countries are vulnerable to a speculative attack for other reasons than weak fundamentals caused by an expansionary domestic credit policy. For example, many recent studies argue that a major cause of the recent financial crises in Asia had to do with an excessive build-up of short-term obligations denominated in foreign
currency (see, for example, Chang and Velasco (1998b, 1999)). Our study of the selection process by speculators could also be fruitfully applied to a situation in which several countries have weak fundamentals because they all face liquidity problems.

Several extensions of our approach present themselves. For example, how does our analysis change when speculators command different amounts of speculative funds? More specifically, does the presence of a 'big' speculator, who can act as a leader, facilitate coordination? Our emphasis on coordination problems among speculators can also shed some light on other issues. For example, how does the analysis change when some of the $m$ countries can expect support from an international lender (of last resort)?

Additionally, some authors have pointed out that the presence of a lender of last resort has an adverse effect on the incentives of policymakers. Then the probability that such a country has to deal with a speculative attack will increase. Besides this moral hazard effect, the presence of a lender of last resort in our framework points to a second channel which may increase the probability that a balance of payments crisis will occur, i.e. the country with international support becomes slightly different from the other countries and may therefore become an easy focal point for speculators. Further research on coordination problems among speculators seems fruitful to understand many, still puzzling, characteristics of speculative attacks in practice.

### 4.6 Appendix

In this Appendix we will give a specific illustration of the dynamics of coordination and make the case that more imperfect communication hampers coordination, thus delaying the moment of a successful attack. As was mentioned above, communication among speculators occurs on the basis of trading in currencies. More specifically, we assume that each speculator receives an idiosyncratic, noisy signal at each time $t$, about these fluctuations in reserves during the previous period for both countries. These signals are assumed to be identically and independently drawn from a distribution that satisfies Assumption 1 in the text.

For the signal distribution at each time $t$, we assume the following Beta distribution on $[-1,1]$:

$$f_t(x) = \frac{\Gamma (\alpha_t + \beta_t)}{2\Gamma (\alpha_t) \Gamma (\beta_t)} \left( \frac{1 + x}{2} \right)^{\alpha_t-1} \left( \frac{1 - x}{2} \right)^{\beta_t-1}, \quad (A4.1)$$

\[12\]Indeed, the need to coordinate a speculative attack may be one of the reasons for the existence of hedge funds which assemble pools of private capital to engage in highly leveraged position-taking.
where the constant 2 is added for normalization and $\alpha_t, \beta_t > 0$. The parameters $\alpha_t$ and $\beta_t$ are chosen such that the mean signal is equal to $\rho_t(1) - \rho_t(2) = 2\rho_t(1) - 1$; i.e. signals are distributed around the true difference in the previous period temporary reserve loss.

Since the expectation of the distribution is equal to $\frac{(\alpha_t - \beta_t)}{(\alpha_t + \beta_t)}$, this gives the condition,

$$\frac{(\alpha_t - \beta_t)}{(\alpha_t + \beta_t)} = 2\rho_t(1) - 1 \quad (A4.2)$$

which implies $\frac{\alpha_t}{\beta_t} = \frac{\rho_t(1)}{\rho_t(2)}$. For $\rho_t(1) > \rho_t(2)$, we set:

$$\beta_t = c, \quad \alpha_t = c \frac{\rho_t(1)}{\rho_t(2)} \quad (A4.3)$$

which ensures that the signals are distributed in the required way in the simplest way.\(^{13}\)

Given $\rho_0(1) > \frac{1}{2}$, the new fraction of speculators taking short positions in currency 1 at $t + 1$ follows from the probability of getting a positive signal on $\rho_t(1) - \rho_t(2)$. This is given by $0 \int f_t(x)dx$. For $c = 1$ this reads:

$$\rho_{t+1}(1) = 1 - 2^{-\alpha_t} \quad (A4.4)$$

Upon substitution of $\alpha_t = c \frac{\rho_t(1)}{\rho_t(2)} = \frac{\rho_t(1)}{1 - \rho_t(1)}$, one obtains.

$$\rho_t(1) = 1 - 2^{-\rho_t(1)} \quad (A4.5)$$

As a consequence, for $\rho_0(1) > \frac{1}{2}$, $\rho_t(1)$ converges to unity. For $\rho_0(1) < \frac{1}{2}$, $\rho_t(1)$ converges to 0. For $\rho_0(1) = \rho_0(2)$, we have the (unstable) equilibrium $\rho_t(1) = \rho_t(2) = \frac{1}{2}$.

Given our assumption $\rho_0(1) > \rho_0(2)$, so that $\rho_0(1) > \frac{1}{2}$, and the continuum of speculators, it follows that the fraction of the total value of financial funds of all speculators used in short positions against currency 1 increases over time. In other words, country 1 gradually becomes a focal point for speculators.

What is the effect of varying the variance of our Beta distribution on $\rho_{t+1}(1)$? The variance is given by:

$$\frac{4\alpha_t \beta_t}{(\alpha_t + \beta_t + 1)(\alpha_t + \beta_t)^2} = \frac{4\rho_t(1)\rho_t(2)^2}{c^2(c + c\rho_t(2) + 1)(\rho_t(2) + 1)^2}, \quad (A4.6)$$

which shows that given $\rho_t(1)$ and $\rho_t(2)$, the variance of the signal is a decreasing function of $c$. Figure 4.3 shows an example of the probability density function at $t = 1$ of the signals for $\rho_t(1)/\rho_t(2) = 1.5$ and for $c = 2$.

\(^{13}\)For $\rho_t(1) \leq \rho_t(2)$, this would require setting $\alpha_t = c$ and $\beta_t = c \rho_t(2)/\rho_t(1)$. 
A decrease in the noise with which speculators communicate reduces the time that it takes before speculators coordinate. A higher value of $c$, which decreases the variance of the signal, implies faster dynamics of $\rho_t(1)$ as indicated in Figure 4.2 by the curve labelled $l$ compared to high variance which decreases the speed of coordination as illustrated by the curve labelled $h$. In the extreme case of perfect communication between speculators, which arises when $c$ goes to infinity, there is instantaneous coordination after one period, at least as long as $\rho_1(1) \neq \frac{1}{2}$.