Robust photometric invariance in machine color vision
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Chapter 5

Robust Photometric Invariant Segmentation of Multispectral Images *

Abstract

Our aim is to detect homogeneously colored regions invariant to surface orientation change, illumination intensity, shadows and highlights in multispectral images, robust against sensor noise. Therefore, different polar angle representations of a spectrum are examined for invariance to these photometric effects using the dichromatic reflection model. Based on the CCD-camera sensitivity, a theoretical expression of the certainty associated with the polar angular value under the influence of noise is obtained. The expression is employed by the segmentation technique to become robust against sensor noise. We verify empirically that uniformly colored regions are detected by the proposed method invariant to surface orientation change, shadows and highlights.

5.1 Introduction

Multispectral imaging has received a great deal of attention recently. Spectral imaging is used, for example, in remote sensing, computer vision, and industrial applications. Spectral information has become an important quality factor in many industrial processes because of its high accuracy [8]. Spectral images can be obtained, for example, by a CCD-camera with narrow-band interference filters [10]. Tominaga [20] [21] describes two generations of a multichannel vision system based on the use of a CCD-camera and six color filters to reconstruct the surface spectral reflectance and illuminant spectral power distribution. Baronti et al. [2] uses a multispectral imaging

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system for the non-invasive analysis of works of art. Haneishi et al. [7] uses five color filters for archiving spectral images of art works.

In the past, various color image segmentation methods have been proposed which do not account for the image formation process [5] [14]. Drawback of these color segmentation methods is that the values of the color features, on which the segmentation method relies, depend on the geometry of the object, the viewpoint of the camera and on the illumination conditions. As a consequence, the obtained segmentation results may be affected negatively by shadows, shading and highlights. In contrast, image processing methods that account for the process of image formation are studied since the mid-1980s, see for instance the work collected by Wolff, Shafer and Healey [22]. Here, Shafer [18] presented the dichromatic reflection model, a physical model of reflection which states that two distinct types of reflection - surface and body reflection - occur, and that each type can be decomposed into a relative spectral distribution and a geometric scale factor. Klinker et al. [12] developed a color segmentation algorithm based on the Shafer’s model. The method is based on evaluating characteristic shapes of clusters in red-green-blue (RGB) space followed by segmentation independent of the object’s geometry, illumination and highlights. To achieve robust image segmentation, however, surface patches of objects in view must have a rather broad distribution of surface normals which may not hold for objects in general.

Healey [9] proposed a method to segment images on the basis of normalized color. However, Kender [11] showed that normalized color and hue colors are singular at some RGB values and unstable at many others. For instance, the essential singularity of normalized coordinates is at black ($R = G = B = 0$), whereas for the hue the singularity is at the achromatic axis ($R = G = B$). As a consequence, both color spaces become unstable near the singularity where a small perturbation of RGB value might cause a large jump in the transformed values. Traditionally, these effects are either ignored or suppressed by an ad hoc thresholding of the transformed values. For example, Ohta [16] rejects normalized color values if the sum of RGB is less than 30, and rejects hue values if the saturation times the intensity is less than 9. Healey [9] rejects all sensor measurements that fall within the sphere of radius $4\sigma$ centered at the origin in RGB-space.

A more elegant strategy to deal with unstable color values would be to generate the reliability of a transformed color together with the output and to incorporate the reliability in the image processing method. An early effort in this direction is the work of Burns and Berns [4]. The authors analyze the error propagation from a measured color signal transformed into the CIE $L^*a^*b^*$ color space. In this way, an indication is obtained of how the color space transform influences the mean, variance and covariance of the colors under the influence of noise. Shafarenko, Petrou and Kittler [17] use an adaptive filter for noise reduction in the CIE $L^*u^*v^*$ space prior to 3-D color histogram construction. The filter width depends on the covariance matrix of the noise distribution in the CIE $L^*u^*v^*$ space.

Our aim is to detect homogeneously colored regions invariant to surface orientation change, illumination intensity, shadows and highlights in multispectral images, robust against sensor noise. The contribution of this paper is two-fold:
1. We propose two polar coordinate models to encode spectra and analyze these models for invariance for photometric effects using the dichromatic reflection model.

2. We propose computational models for estimating the effect of sensor noise on the stability of the polar coordinates. The expression of the reliability of the polar coordinate representation is incorporated in the segmentation method.

The segmentation method is derived from the physics-based dichromatic reflection model. This makes it possible to characterize what kinds of images are likely to be segmented successfully by our algorithm. The model describes the reflection of materials which are optically inhomogeneous. In that case the light interacts with a medium that causes the bigger component of surface matter, as well as with particles of a colorant to produce scattering and coloration. Many common materials have these characteristics, including most paints, varnishes, paper, ceramics, and plastics. Therefore, we anticipate that images of objects of these materials are properly segmented by the proposed method. As a consequence, the proposed method is unsuited for homogeneous materials.

The paper is organized as follows: In section 5.2, the dichromatic reflection reflection model and a camera model are described in detail. In section 5.3, distance measures are proposed and examined for photometric invariance. In section 5.4 the effect of sensor noise is propagated through the polar angle representations of spectra. The photometric invariant region detection methods are described in section 5.5. In section 5.6, the theoretical estimated uncertainty in polar angular representation is compared empirically to the real uncertainty. Experiments are carried out to evaluate the invariance to photometric effects of the proposed segmentation methods. A discussion is given in section 5.7.

5.2 Reflection and Camera Models

In this section, we discuss a camera and image formation model. Based on the models, we examine clusters shapes drawn by uniformly colored objects in multispectral color space.

5.2.1 Multispectral Imaging

Over the past few years, new imaging sensors have become available commercially. Optical Insights Ltd. introduced the “MultiSpec Imager”. The device is mounted to a monochrome CCD-camera and contains four different color filters. Four different filtered two-dimensional images are then projected onto the monochrome CCD grid. Further, Spectral Imaging Ltd. introduced the Inspector V7 spectrograph. A spectrograph transforms the monochrome CCD-camera to a line scanner: One axis displays the spatial information, whereas along the other axis the visible wavelength range is displayed. In this paper, the Jain CV-M300 camera is used with 576 pixels
along the optical axis. We use the Inspector V7 spectrograph with shortest observable wavelength of 410 nm and longest wavelength of 705 nm. Setting the wavelength interval to 5 nm, the number of spectral samples obtained is 59. The pixels at position \((x, \lambda)\) of image \(h\) can therefore be averaged in spectral direction by a uniform filter. Let \(K' = \text{round}(576/59)\). If \(K'\) is odd, then the size of the filter \(K = K'\), else \(K = K' - 1\). The averaged spectral image \(h'\) is

\[
h'(x, \lambda) = \frac{1}{K} \sum_{i=y_{\lambda}+[K/2]}^{y_{\lambda}-[K/2]} h(x, \lambda_i) \tag{5.1}
\]

### 5.2.2 Camera and Image Formation Models

We use a linear camera model to describe the relation between input signal \(h\) at position \((x, \lambda)\) and the output signal \(c\) as

\[
c(x, \lambda_i) = \gamma_i h(x, \lambda_i) + d(x) \tag{5.2}
\]

where \(d(x)\) denotes the dark current independent of the wavelength and \(\gamma_i\) denotes the camera gain for the \(i\)th color channel. For the moment, we ignore the dark current for notational simplicity. For the same reason, the notation for position is left out of the equations. The camera gain may further be refined as consisting of two terms

\[
\gamma_i = \gamma_e \cdot \gamma_{w,i} \tag{5.3}
\]

where \(\gamma_e\) denotes the electronic gain and \(\gamma_{w,i}\) denotes the white-balancing gain.

For inhomogeneous, dielectric materials, the measured input signal \(h\) of (5.2) is described by the dichromatic reflection model. Consider an image of an infinitesimal surface patch. Using \(N\) narrow-band filters with spectral sensitivities given by \(f_i(\lambda)\), the measured sensor values are given as:

\[
h(\lambda_i) = m_b(\vec{n}, \vec{s}) \int_{\lambda} f_n(\lambda)E(\lambda)c_b(\lambda)d\lambda + m_s(\vec{n}, \vec{s}, \vec{v}) \int_{\lambda} f_n(\lambda)E(\lambda)c_s(\lambda)d\lambda \tag{5.4}
\]

for \(h(\lambda_i)\) denoting the sensor response for the the \(i\)th wavelength. \(E(\lambda)\) denotes the spectral power distribution of the incident light, \(\vec{n}\) denotes the surface patch normal, \(\vec{s}\) is the direction of the illumination source, and \(\vec{v}\) is the direction of the viewer. \(\int_{\lambda} f_n(\lambda)E(\lambda)c_b(\lambda)d\lambda\) and \(\int_{\lambda} f_n(\lambda)E(\lambda)c_s(\lambda)d\lambda\) are constants depending on the surface albedo \(c_b(\lambda)\) and Fresnel reflectance \(c_s(\lambda)\). Geometric terms \(m_b\) and \(m_s\) denote the geometric dependencies on \(\vec{n}, \vec{s}, \vec{v}\). In this paper, we make no specific assumptions for functions \(m_b\) and \(m_s\) but simply require that

\[
0 \leq m_b(\vec{n}, \vec{s}), m_s(\vec{n}, \vec{s}, \vec{v}) \leq 1 \tag{5.5}
\]

It is important to observe that (5.5) captures one of the attractive features of the dichromatic reflection model: the model is far more general than typical models used in computer vision and computer graphics, and includes most such models as special cases. For instance, a possible instantiation of \(m_b\) is Lambert's cosine law (then \(m_s = 0\)). Similarly, a possible instantiation of both \(m_b\) and \(m_s\) is Phong's model for specular reflection [3].
5.2.3 White-Balancing

According to (5.4), a matte, white reference standard with constant spectral response can be described taking \( c_b(\lambda) = 1 \) and \( m_b(\vec{n}, \vec{s}) = 1 \). Furthermore, assume the camera is not white-balanced so \( \gamma_{w,i} = 1 \), say, for all color channels \( i \). The measured sensor values are obtained substituting the body reflection of (5.4) in (5.2) as

\[
w(\lambda_i) = \gamma \int \lambda f_i(\lambda)E(\lambda) d\lambda
\]

for \( w \) denoting the sensor response for the white matte reference standard. The gain parameter \( \gamma_{w,i} \) of (5.3) is adjusted, either by the white-balancing procedure of the CCD-camera or else manually, as

\[
\gamma_{w,i} = \frac{1}{w(\lambda_i)}
\]

The output of a white-balanced camera system is now described by substitution of (5.4) in (5.2) as

\[
c(\lambda_i) = \frac{m_b(\vec{n}, \vec{s}) \int \lambda f_i(\lambda)E(\lambda)c_b(\lambda_i) d\lambda}{\int \lambda f_i(\lambda)E(\lambda, \lambda) d\lambda} + \frac{m_s(\vec{n}, \vec{s}, \vec{\nu}) \int \lambda f_i(\lambda)E(\lambda)c_s(\lambda_i) d\lambda}{\int \lambda f_i(\lambda)E(\lambda) d\lambda}
\]

Under the assumption of neutral interface reflection where the spectral distribution of the reflected light is independent of the wavelength \( c_s(\lambda_i) = c_s \), the specular term of (5.8) rewrites to

\[
\frac{m_s(\vec{n}, \vec{s}, \vec{\nu}) \int \lambda f_n(\lambda)E(\lambda)c_s(\lambda_i) d\lambda}{\int \lambda f_n(\lambda)E(\lambda) d\lambda} = m_s(\vec{n}, \vec{s}, \vec{\nu})c_s
\]

making the surface reflection term of (5.4) independent of the spectral distribution of the light source. Due to the white-balancing operation and the neutral interface reflection assumption, the color channels \( c(\lambda_i) \) produce equal output when a achromatic object is imaged. From (5.9) also follows that the direction of specularities coincides with the direction of the achromatic axis.

Assume that the filter \( f_i(\lambda) \) is a narrow band filter modeled as a unit impulse that is shifted over \( i \) wavelengths: The transmission at \( \lambda_i = \delta \) and zero elsewhere. Under neutral interface reflection, (5.8) rewrites to

\[
c(\lambda_i) = m_b(\vec{n}, \vec{s})c_b(\lambda_i) + m_s(\vec{n}, \vec{s}, \vec{\nu})c_s
\]

making the body reflection term of (5.4) independent of the spectral distribution of the light source. In vector notation, a spectrum is denoted as

\[
\vec{c} = m_b(\vec{n}, \vec{s})\vec{c}_b + m_s(\vec{n}, \vec{s}, \vec{\nu})\vec{c}_s
\]

The vectors \( \vec{n}, \vec{s}, \vec{\nu} \) are three-dimensional. The vectors \( \vec{c}, \vec{c}_b, \vec{c}_s \) are \( N \)-dimensional, with \( N \) the number of samples taken in the wavelength range.
Figure 5.1: The half-ray \( \vec{B} \) is drawn by a matte surface in RGB-space. Similarly, the triangularly-shaped plane defined by \( \vec{B} \) and \( \vec{S} \) is drawn by a shiny surface in RGB.

5.2.4 Cluster Shapes in Sensor Space

Consider the camera output for a white-balanced RGB camera described in (5.10). For a matte surface, the observed color depends on \( c_b(\lambda_i) \). The observed colors of a matte, dull surface can therefore be represented by the color cluster vector \( \vec{B} \), see figure (5.1), where the direction of \( \vec{B} \) is based on \( c_b(\lambda_i) \).

The intensity of the color depends on the geometrical term \( m_b(\vec{n}, \vec{s}) \). A uniformly colored surface may give rise to a broad variance of sensor values due to the varying circumstances induced by the image-forming process such as a change in object orientation, illumination intensity and position. Therefore, the extent of \( \vec{B} \) is based on \( m_b(\vec{n}, \vec{s}) \). Since no negative sensor values occur, a precise description of the cluster \( \vec{B} \) is the notion of a half-ray.

For a shiny surface under neutral interface reflection, the observed color depends on both terms \( c_b(\lambda_i) \) represented by color cluster vector \( \vec{B} \) and unit vector \( \vec{c}_s \) corresponding to the vector \( \vec{S} \), see figure (5.1). The intensity of \( \vec{S} \) is affected by the surface geometry and viewing direction. According to (5.4), for a given point on a shiny surface, the contribution of the body reflection \( \vec{B} \) component and specular reflection component \( \vec{S} \) add up. Hence, the observed colors of the shiny surface must be in a plane. The triangularly-shaped plane is spanned by vectors \( \vec{B} \) and \( \vec{S} \). The cluster shapes derived this way for a RGB camera also hold for a multispectral camera.
5.3 Photometric Invariant Distance Measures

In the previous section it was shown that uniformly colored matte objects draw half-rays in multispectral color space due to changes in the surface orientation, illumination intensity and shading. Due to specularities, shiny objects draw triangularly-shaped planes in multispectral space. For photometric invariant region detection, we aim to model the shape of the clusters as either a half-ray or a triangularly-shaped plane. A measured spectrum is then assigned to one of these clusters based on the minimal distance of the spectrum to the clusters. In this section, we therefore investigate different distance measures.

5.3.1 Combined Angle and Magnitude Distance

Androutsos et al. [1] recently proposed a combination distance measure which is composed of an angle and magnitude component:

\[
\delta(x_i, x_j) = 1 - \left[ 1 - \frac{2}{\pi} \arccos \left( \frac{\vec{x}_i \cdot \vec{x}_j}{\|\vec{x}_i\| \|\vec{x}_j\|} \right) \right] \left[ 1 - \frac{\|\vec{x}_i - \vec{x}_j\|}{\sqrt{3} \cdot 255^2} \right]
\]

(5.12)

where \(\vec{x}_i\) and \(\vec{x}_j\) are three-dimensional color vectors. Consider two RGB color spectra belonging to matte objects which can be described by the dichromatic reflection model (5.10). The spectra are denoted \(\vec{P}\) and \(\vec{Q}\). Our aim is to analyse the photometric invariance of the distance measure of (5.12) between \(\vec{P}\) and \(\vec{Q}\). Assuming arbitrary values for the geometry, \(m_b(n, \beta) = \alpha\), the spectrum \(\vec{P}\) is abbreviated to

\[
\vec{P} = \alpha[ c_p(R) \ c_p(G) \ c_p(B) ]^T
\]

(5.13)

Similarly, spectrum \(\vec{Q}\) is denoted as

\[
\vec{Q} = \beta[ c_Q(R) \ c_Q(G) \ c_Q(B) ]^T
\]

(5.14)

Substitution of \(\vec{P}\) and \(\vec{Q}\) into the angle component of (5.12) gives

\[
1 - \frac{2}{\pi} \arccos \left( \frac{c_p(R)c_Q(R) + c_p(G)c_Q(G) + c_p(B)c_Q(B)}{\sqrt{c_p^2(R) + c_p^2(G) + c_p^2(B)} \sqrt{c_Q^2(R) + c_Q^2(G) + c_Q^2(B)}} \right)
\]

(5.15)

independent of the geometry terms \(\alpha\) and \(\beta\). However, multiplication with the magnitude component gives

\[
1 - \frac{\left[ (\alpha c_p(R) - \beta c_Q(R))^2 + (\alpha c_p(G) - \beta c_Q(G))^2 + (\alpha c_p(B) - \beta c_Q(B))^2 \right]^{1/2}}{\sqrt{3} \cdot 255^2}
\]

(5.16)

dependent of the geometry terms \(\alpha\) and \(\beta\). As a result of multiplication, the combination distance measure is unfit for photometric invariant segmentation (which the authors did not claim anyway).
Chapter 5. Robust Photometric Invariant Segmentation of Multispectral Images

5.3.2 Distance from a Point to a Line

Let \( \vec{P} \) denote a \( N \)-dimensional vector. For a matte surface, the spectrum \( \vec{P} \) for a constant value of \( m_b(\vec{n}, \vec{s}) = k \) is described as

\[
\vec{c} = k \vec{c}_P
\]  

(5.17)

Let \( \vec{l} \) be a line passing through the origin and through a vector \( \vec{R} \) denoted as

\[
\vec{l} = \alpha \vec{R}
\]  

(5.18)

where the variable \( \alpha \) can be thought of as corresponding to the geometry function \( m_b(\vec{n}, \vec{s}) \) thus

\[
\vec{l} = m_b(\vec{n}, \vec{s}) \vec{c}_R
\]  

(5.19)

Let \( \vec{Q} \) be a point on \( \vec{l} \) such that \( \vec{P} - \vec{Q} \perp \vec{e} \). \( \vec{Q} \) is found as

\[
\vec{Q} = \vec{e} \left( \frac{\vec{P} \cdot \vec{e}}{\vec{e} \cdot \vec{e}} \right)
\]  

(5.20)

The shortest distance \( d(\vec{P}, \vec{l}) \) is the Euclidean distance between \( \vec{P} \) and \( \vec{Q} \):

\[
d(\vec{P}, \vec{l}) = d(\vec{P}, \vec{Q})
\]  

(5.21)

For the analysis of photometric invariance of distance measure (5.21), the point \( \vec{Q} \) is to be obtained. Substitution of the expressions (5.17) and (5.18) into (5.20) yields

\[
\vec{Q} = \vec{c}_R \left( \frac{k_b \vec{c}_P \cdot \vec{c}_R}{\vec{c}_R \cdot \vec{c}_R} \right)
\]  

(5.22)

giving \( \vec{Q} \) independent of the geometry term \( \alpha = m_b(\vec{n}, \vec{s}) \) of the line \( \vec{l} \). We have thus established the important property that the distance between \( \vec{P} \) and \( \vec{l} \) is independent of the geometry of the object whose spectra draw the line \( \vec{l} \) in the multispectral space. As a consequence, the distance from spectrum \( \vec{P} \) to a line \( \vec{l} \) is photometric invariant for the geometry of the object. The obtained result does not depend on independence of the spectral distribution of the illumination obtained by (5.9). Therefore, similar arguments hold for cameras that are not white-balanced, as well as for broad-band cameras.

5.3.3 Distance between Chromaticity Polar Angles

Spectra can be transformed into polar coordinates. To define polar coordinates descriptors, the origin \( O \) and a positive horizontal axis are fixed. Then each \( N \)-dimensional point \( \vec{P} \) can be located by assigning to it polar coordinates \((\rho, \vec{\theta})\) where the one-dimensional term \( \rho \) gives the distance from \( O \) to \( \vec{P} \) and the \((N-1)\)-dimensional term \( \vec{\theta} \) gives the angles from the initial axis to \( \vec{P} \).
A spectrum is transformed to polar coordinate representation as
\[ \rho_i = |\vec{c}| \] (5.23)
\[ \theta_c(\lambda_i) = \arctan\left( \frac{c(\lambda_i)}{c(\lambda_N)} \right), \quad 1 \leq i \leq N - 1 \] (5.24)
where \( \rho_i \) encodes the intensity of the spectrum and \( \theta_c(\lambda_i) \) the chromaticity of the spectrum. \( \theta_c(\lambda_i) \) takes on values in the range \( 0 \leq \theta_c \leq \frac{\pi}{2} \).

For the analysis of photometric invariance of the chromaticity angular representation of spectra, substitution of (5.10) as \( c(\lambda_i) = m_b(\vec{n}, \vec{s})c_b(\lambda_i) \) in (5.24) gives
\[ \theta_c(\lambda_i) = \arctan\left( \frac{c_b(\lambda_i)}{c_b(\lambda_N)} \right) \] (5.25)
independent of geometry term \( m_b(\vec{n}, \vec{s}) \).

The quadratic distance, \( e \), between any two \( M \)-dimensional vectors of angles \( \vec{\theta}_1 \) and \( \vec{\theta}_2 \) is defined as follows:
\[ e^2(\vec{\theta}_1, \vec{\theta}_2) = \sum_{i=1}^{M} (\Delta(\theta_{1i}, \theta_{2i}))^2, \quad 0 \leq \theta_{1i}, \theta_{2i} < 2\pi \] (5.26)
Here, \( \theta_{1i} \) denotes the \( i \)th of \( M \) angles for the first vector. The distance \( \Delta(\theta_i, \theta_j) \) takes values in the interval \([0, 2]\) and is defined as follows:
\[ \Delta(\theta_i, \theta_j) = [(\cos(\theta_i) - \cos(\theta_j))^2 + (\sin(\theta_i) - \sin(\theta_j))^2]^{1/2} \] (5.27)
The angular difference \( \Delta \) is indeed a distance because it satisfies the following metric criteria:
- \( \Delta(\theta_i, \theta_j) \geq 0 \) for all \( \theta_i \) and \( \theta_j \)
- \( \Delta(\theta_i, \theta_j) = 0 \) if and only if \( \theta_i = \theta_j \)
- \( \Delta(\theta_i, \theta_j) = \Delta(\theta_j, \theta_i) \) for all \( \theta_i \) and \( \theta_j \)
- \( \Delta(\theta_i, \theta_j) + \Delta(\theta_j, \theta_k) \geq \Delta(\theta_i, \theta_k) \) for all \( \theta_i, \theta_j, \) and \( \theta_k \)

The proof of the first three conditions is trivial. To see the triangular inequality, consider two angles \( \theta_i, \theta_j \). Define
\[ \vec{\theta}_i = \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}^T \] (5.28)
and define \( \vec{\theta}_j \) in similar fashion. Since \( \Delta(\theta_i, \theta_j) = d(\vec{\theta}_i, \vec{\theta}_j) \) where \( d \) denotes the well-known Euclidean distance, the triangular inequality is proven.

Since chromaticity polar angles are independent of the geometry of the object, as was shown in (5.25), the distance between two chromaticity angles is photometric invariant as well.
5.3.4 Distance from a Point to a Plane

The shape drawn by spectra belonging to a shiny surface is bounded by two vectors \( \vec{S} \) and \( \vec{B} \) in \( N \)-dimensional space. For a white-balanced camera, vector \( \vec{S} \) corresponds to the achromatic axis. Vector \( \vec{B} \) corresponds to the surface albedo. Both vectors emanate from the origin. To achieve photometric invariant segmentation results, we examine the distance from a spectrum \( \vec{P} \) to the plane \( \vec{v} \) spanned by these vectors.

We discuss the case for \( N = 3 \) in RGB-space. In the RGB color space, \( \vec{v} \) can be described by the normal equation of the form \( \vec{v} = \vec{Q}_1 \vec{R} - \vec{Q}_2 \vec{G} + \vec{Q}_3 \vec{B} = 0 \) as \( \vec{v} \) passes through the origin. Here, the three-dimensional vector \( \vec{Q} \) is obtained as \( \vec{Q} = \vec{B} \times \vec{S} \) where \( \times \) denotes the outer-product. The distance of \( \vec{P} \) to \( \vec{v} \) is given by

\[
d(P, v) = \frac{\vec{Q} \cdot \vec{P}}{\sqrt{\vec{Q} \cdot \vec{Q}}} \quad (5.29)
\]

To examine the photometric invariance of (5.29), consider a shiny surface recorded with a white-balanced camera. The spectrum \( \vec{P} \) recorded at the surface is described by (5.10). Assuming constant values \( m_b(\vec{n}, \vec{s}) = k_b \) and \( m_s(\vec{n}, \vec{s}, \vec{v}) = k_s \), the spectrum \( \vec{P} \) for a RGB-camera is

\[
\vec{P} = k_b [ c_P(R) \ c_P(G) \ c_P(B) ]^T + k_s [ 1 \ 1 \ 1 ]^T \quad (5.30)
\]

The plane \( \vec{v} \) is defined by vectors \( \vec{S} \) and \( \vec{B} \), see figure (5.1). Assuming variable values \( m_b(\vec{n}, \vec{s}) = \beta \) and \( m_s(\vec{n}, \vec{s}, \vec{v}) = \sigma \), vector \( \vec{B} \) is described as

\[
\vec{B} = \beta [ c_B(R) \ c_B(G) \ c_B(B) ]^T \quad (5.31)
\]

Similarly the vector \( \vec{S} \) is described as

\[
\vec{S} = \sigma [ 1 \ 1 \ 1 ]^T \quad (5.32)
\]

The plane \( \vec{v} \) is determined by the outer-product \( \vec{Q} = \vec{B} \times \vec{S} \). It follows that

\[
\vec{Q} = \beta \sigma [ (c_B(G) - c_B(B)) \ (c_B(B) - c_B(G)) \ (c_B(B) - c_B(R)) ]^T \quad (5.33)
\]
giving the parameters for the normal equation of \( \vec{v} \). Substitution of (5.30) and (5.33) into (5.29) gives:

\[
d(P, v) = \left| \frac{(k_b c_P(R) + k_s)(c_B(G) - c_B(B)) + (k_b c_P(G) + k_s)(c_B(B) - c_B(G))}{\sqrt{B_G - B_B}^2 + (B_B - B_G)^2 + (B_R - B_G)^2}} \right| + \left| \frac{(k_b c_P(B) + k_s)(c_B(R) - c_B(G))}{\sqrt{B_G - B_B}^2 + (B_B - B_G)^2 + (B_R - B_G)^2}} \right| \quad (5.34)
\]

independent of the geometry term \( \beta = m_b(\vec{n}, \vec{s}) \) and specular term \( \sigma = m_s(\vec{n}, \vec{s}, \vec{v}) \) of the plane \( \vec{v} \). We have thus established the property that the distance between \( \vec{P} \)
and υ is independent of the geometry of the object whose spectra draw the plane υ in RGB-space. This is an important observation as the distance from spectrum \( \bar{P} \) to a plane υ is shown to be photometric invariant for the geometry and specularities of the object.

A drawback of the approach is the following: the plane υ is bounded on one side by the achromatic axis \( S \), see figure (5.1). Using the distance from a point to an unbounded plane therefore has the undesired effect that complementary colors, e.g. blue and yellow map on a single plane. Opponent colors will therefore be assigned to the same color cluster. To overcome the problem, we propose to compute the distance between hue polar angles instead.

### 5.3.5 Distance between Hue Polar Angles

Consider a \( N \)-dimensional spectrum \( \bar{c} \) defined by (5.10) transformed to polar coordinate representation as

\[
\rho_s = 1 - \min\{c(\lambda_1), \ldots, c(\lambda_N)\} \quad (5.35)
\]

\[
\theta_h = \alpha[\ c(\lambda_i) - [1 - \rho_s], \ \phi(i, N) \ ] \quad (5.36)
\]

where \( \theta_h \) takes on values in the range \( 0 \leq \theta_h < 2\pi \) and where

\[
\phi(i, N) = \frac{i - 1}{N - 1} \cdot \frac{4}{3}\pi \quad (5.37)
\]

and

\[
\alpha(w_i, \theta_i) = \text{arctan} \left( \frac{\sum_{i=1}^{N} w_i \sin(\theta_i)}{\sum_{i=1}^{N} w_i \cos(\theta_i)} \right), \quad (5.38)
\]

The function \( \phi \) takes on values in the range \( 0 \leq \phi(i, N) \leq \frac{4}{3}\pi \). The function \( \alpha \) takes on values in the range \( 0 \leq \alpha < 2\pi \). The function denotes the weighted average of a series of \( N \) angles \( \theta_i \) with corresponding weight \( w_i \). The average is computed by decomposing the angular value into a horizontal and vertical component. The saturation of the spectrum is encoded by \( \rho_s \). The angle \( \theta_h \) can be thought of as the hue obtained directly from multispectral data. The function \( \phi(i, N) \) assigns a hue-angle to the \( i \)-th of \( N \) spectral samples. The range from \( 0 \cdots 4/3\pi \) is reserved for the colors ranging from red through green through blue, so that the range from \( 4/3\pi \cdots 2\pi \) represents the purplish colors. The choice of \( 4/3\pi \) is somewhat arbitrary but can be defended taking the hue computation into account based on conventional red-green-blue colors where a similar division is employed. For example, consider the following equation for the hue [13]:

\[
\theta = \text{arctan} \left( \frac{\sqrt{3}(G - B)}{(R - G) + (R - B)} \right) \quad (5.39)
\]

Equation (5.37) assigns hue-angle \( \theta_h = 0 \) to the red channel, hue-angle \( \theta_h = 2/3\pi \) to the green channel, and \( \theta_h = 4/3\pi \) to the blue channel. Let \( \rho_s = 1 - \min\{R, G, B\} \),
then the weights of (5.38) are defined as \( w_1 = R - \rho_s \) for the red channel, \( w_2 = G - \rho_s \) for the green channel, and \( w_3 = B - \rho_s \) for the blue channel. Substitution of these results into (5.36) gives

\[
\theta = \arctan \left( \frac{(R - \rho_s) \sin(0) + (G - \rho_s) \sin(2/3\pi) + (B - \rho_s) \sin(4/3\pi)}{(R - \rho_s) \cos(0) + (G - \rho_s) \cos(2/3\pi) + (B - \rho_s) \cos(4/3\pi)} \right)
\]

\[
= \arctan \left( \frac{\frac{1}{2}\sqrt{3}G - \frac{1}{2}\sqrt{3}B}{R - \frac{1}{2}G - \frac{1}{2}B} \right)
\]

identical to (5.39).

The polar coordinates are illustrated in figure (5.2). The hue polar angle \( \theta_h \) is invariant to the geometry and specularities: For a multispectral camera with narrowband filters, consider substitution of (5.10) in the term \( c(\lambda_i) - [1 - \rho_s] \) of (5.36) as

\[
c(\lambda_i) - [1 - \rho_s(x)] = mb(\vec{n}, \vec{s}) [c_b(\lambda_i) - 1 + c_b(\lambda_p)]
\]

independent of specularity term \( m_s(\vec{n}, \vec{s}, \vec{\tau}) \). Moreover, the hue polar angle is independent of shadows and geometry as the substitution of (5.41) in (5.36) gives

\[
\theta_h = \arctan \left( \frac{\sum_{i=1}^{N} [c_b(\lambda_i) - c_b(\lambda_p)] \sin[\phi(i, N)]}{\sum_{i=1}^{N} [c_b(\lambda_i) - c_b(\lambda_p)] \cos[\phi(i, N)]} \right)
\]

independent of geometric term \( m_b(\vec{n}, \vec{s}) \). Similar arguments hold for the white-balanced RGB-camera.

The distance between two hue polar angles \( \theta_{h,i}, \theta_{h,j} \) is computed as \( \Delta(\theta_{h,i}, \theta_{h,j}) \) where \( \Delta \) is defined in (5.27). Because the hue polar angle is independent of the geometry of the object and independent of shadows and specularities, the distance between two hue polar angles is therefore photometric invariant as well.

### 5.3.6 Conclusion

Concluding the results obtained so far, consider table (5.1). The distance from a spectrum to a line, \( d(P, \vec{\ell}) \), and the distance between chromaticity angles, \( \Delta(\theta_{c,i}, \theta_{c,j}) \), are both photometrically invariant for the geometry of objects. Similarly, the distance from a spectrum to a plane \( d(P, \vec{v}) \), and the distance between hue-angles, \( \Delta(\theta_{h,i}, \theta_{h,j}) \), are both photometrically invariant to the geometry and the highlights. However, the drawback of the computation of the distance from spectrum to a plane is that opponent colors map onto the same plane. Therefore, the angular representation of spectra is adopted in the remainder of the paper. In the next section, the effect of sensor noise on the angular representations is investigated.

### 5.4 Error Propagation

In general, the result of a measurement of a quantity \( u \) is properly stated as

\[
\hat{u} = u_c \pm \sigma_u
\]
5.4. Error Propagation

Figure 5.2: Polar coordinate representation of a spectrum depicted as a Euclidean map. $\rho_s$ encodes the saturation of the spectrum and $\theta_h$ encodes the hue. The hue polar angle forms a half plane emanating from the origin in multispectral space. The hue range from $0 \cdots 4/3\pi$ is reserved for the colors ranging from red (700 nm) through green (550 nm) to blue (400 nm). The range from $4/3\pi \cdots 2\pi$ (dashed part of the hue circle) represents purplish colors.

where $u_e$ is the best estimate for the quantity $u$ and $\sigma_u$ is the uncertainty or error in the measurement of $u$. Suppose that $u, \cdots, w$ are measured with corresponding uncertainties $\sigma_u, \cdots, \sigma_w$, and the measured values are used to compute the function $q(u, \cdots, w)$. If the uncertainties in $u, \cdots, w$ are independent, random and small, then
Table 5.1: Taxonomy of photometric invariant distance measures. The first column: \( \delta(\vec{x}_i, \vec{x}_j) \) refers to the combination measure proposed by Androutsos et al. \( d(\vec{P}, \vec{L}) \) denotes distance from a spectrum \( \vec{P} \) to a line \( \vec{L} \), \( \Delta(\theta_{c,i}, \theta_{c,j}) \) denotes distance between two chromaticity angular representations, \( d(\vec{P}, v) \) denotes distance from a spectrum \( \vec{P} \) to a plane \( v \), and \( \Delta(\theta_{h,i}, \theta_{h,j}) \) denotes distance between two hue-angular representations. The second column denotes the invariance to the geometry of an object. The third column denotes the invariance to highlights reflected by an object. The fourth column denotes whether spectra with opponent colors (e.g. yellow and blue) map onto the same plane.

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Distance measure} & \text{Equation} & \text{Invariant to object geometry?} & \text{Invariant to highlights?} & \text{Opponent colors map on different clusters?} \\
\hline
\delta(\vec{x}_i, \vec{x}_j) & (5.12) & \text{No} & - & - \\
\Delta(\theta_{c,i}, \theta_{c,j}) & (5.21) & \text{Yes} & - & - \\
d(\vec{P}, \vec{L}) & (5.26) & \text{Yes} & - & - \\
d(\vec{P}, v) & (5.29) & \text{Yes} & \text{Yes} & \text{No} \\
\Delta(\theta_{h,i}, \theta_{h,j}) & (5.27) & \text{Yes} & \text{Yes} & \text{Yes} \\
\hline
\end{array}
\]

The uncertainty in \( q \) [19] is

\[
\sigma_q = \sqrt{\left( \frac{\partial q}{\partial u} \sigma_u \right)^2 + \cdots + \left( \frac{\partial q}{\partial w} \sigma_w \right)^2} \quad (5.44)
\]

The uncertainty in \( q \) is never larger than the ordinary sum

\[
\sigma_q \leq \left| \frac{\partial q}{\partial u} \right| \sigma_u + \cdots + \left| \frac{\partial q}{\partial w} \right| \sigma_w \quad (5.45)
\]

These two equations will be used in the next section to propagate uncertainties.

**5.4.1 Propagation of Uncertainties due to Photon Noise**

Modern CCD-cameras are sensitive enough to be able to count individual photons. Photon noise arises from the fundamentally stochastical nature of photon production. The probability distribution for counting \( \rho \) photons during \( t \) seconds is known to follow the Poisson distribution. The number of photons measured at pixel \( x \) is given by its average as

\[
\hat{h}(x) = \rho t \pm \sqrt{\rho t} \quad (5.46)
\]

Let \( \sigma_d \) denote the dark current uncertainty. Incorporating \( \sigma_d \) and the uncertainty of (5.46) in (5.2) gives

\[
c(x) \pm \sigma_c(x) = \gamma[\rho t \pm \sqrt{\rho t}] + [d(x) \pm \sigma_d] \quad (5.47)
\]
Our interest is in computing $\sigma_{c(x)}$. Let the dark current variance be denoted as $\text{var}(d) = \sigma^2_d$. Let the average image intensity measured over a homogeneously colored patch be $\bar{I} = \gamma pt$, then the associated variance $\text{var}(\bar{I}) = \gamma^2 pt$. We have the linear relation between $\bar{I}$ and $\text{var}(\bar{I})$ based on [15] as

$$\text{var}(\hat{I}) + \text{var}(\hat{d}) = \gamma \hat{I} + \text{var}(\hat{d}) \quad (5.48)$$

Linear regression among some intensity-variance pairs gives a robust estimation of the gain $\gamma$. It follows that the uncertainty in the number of photons measured at an arbitrary pixel $c(x)$ is given by

$$\sigma^2_c(x) = [\gamma \cdot c(x)]^2 + \sigma^2_d \quad (5.49)$$

### 5.4.2 Propagation of Uncertainty

The uncertainty in a pixel value is propagated to the uncertainty in polar angles as follows. First, it is assumed that the pixel values in the spectral image are independent. Therefore, using (5.45), the uncertainty due to the smoothing operation of (5.1) reduces to

$$\sigma^2_{\lambda_i}(x, \lambda) = \frac{1}{K} \sum_{i=y_A-[K/2]}^{y_A+[K/2]} \sigma^2_{\lambda_i}(x, \lambda_i) \quad (5.50)$$

From (5.10) it follows that the uncertainty in the white-balanced camera output is

$$\sigma^2_{c'}(x, \lambda_i) = \frac{c^2(x, \lambda_i)\sigma^2_{\lambda_i}(x, \lambda_i) + w^2(x, \lambda_i)\sigma^2_{\lambda_i}(x, \lambda_i)}{w^4(x, \lambda_i)} \quad (5.51)$$

where $c'$ denotes the white-balanced camera output and $c$ denotes the observed camera output.

For the general function $q(u, v) = \arctan(u/v)$ where the parameters $u, v$ are dependent and have associated uncertainties $\sigma_u, \sigma_v$, the uncertainty in output $\sigma_q$ is obtained using (5.45) as

$$\sigma_q \leq \left| \frac{\sigma_u}{u^2 + v^2} \right| + \left| \frac{\sigma_v}{u^2 + v^2} \right| \quad (5.52)$$

The function is shown in figure (5.3). Large uncertainties occur if $u$ and $v$ both approach the value zero. The polar angles of (5.24) are inter-dependent as each angle is obtained by division through the same value $\theta(\lambda_N)$. The uncertainty in chromaticity polar angle of (5.24) therefore follows straightforward from (5.52) by substituting $u = c(\lambda_i)$, $v = c(\lambda_N)$, and where both $\sigma_u$ and $\sigma_v$ are obtained from (5.51).

To obtain an estimate of the uncertainty of the hue polar angle, consider the term $c(\lambda_i) - [1 - \rho_x]$ of (5.36). The parameters $c(\lambda_i)$ and $\rho_x$ are assumed independent because the reflectance factor $c(\lambda_i)$ is assumed to be obtained independent from the
Figure 5.3: Uncertainty in the function \( \arctan(u/v) \) as function of \( u \) and \( v \). The uncertainties \( \sigma_u \) and \( \sigma_v \) are set equal to one. Large uncertainties occur if both \( u \) and \( v \) approach the value zero, indicating the instability of the function around the origin.

The reflectance factor \( \rho_s = 1 - \min\{c(\lambda_1), \ldots, c(\lambda_N)\} \). Thus, the resulting uncertainty is obtained using (5.44) as

\[
\sigma_{c-i}^2(\lambda_i) = \sigma_c^2(\lambda_i) + \sigma_{\rho}^2
\]

The uncertainty for the hue polar angle of (5.36) follows from (5.38). The exact number generated by (5.37) has no associated uncertainty, and therefore \( \sin(\phi(i, N)) \) has no associated uncertainty. However, the weights \( w_i = c(\lambda_i) \) do have uncertainty \( \sigma_c(\lambda_i) \), again specified by (5.51). The individual terms \( w_i \sin(\theta_i) \) are considered independent, because the reflectance factor \( w_i = c(\lambda_i) \) is assumed to be obtained independent from the reflectance factor at wavelength \( w_j = c(\lambda_j) \). Therefore, the uncertainty of the enumerator term \( u = \sum_i w_i \sin(\theta_i) \) is

\[
\sigma_u^2 = \sum_i (\sigma_c(\lambda_i) \cdot \sin(\phi(i, N)))^2
\]

A similar argument holds for the denominator term \( u = \sum_i w_i \cos(\theta_i) \) yielding \( \sigma_v \).
The uncertainty for (5.38) is then obtained by straightforward substitution of \( u, \sigma_u \) and \( v, \sigma_v \) in (5.52).

In summary, the uncertainty in the reflectance factors of a spectrum is determined in theory by converting a pixel color value into the number of photons counted at that pixel. Under the assumption that counting photons follows a Poisson distribution, the uncertainty associated with a pixel value is determined. The obtained uncertainty is propagated to the uncertainty in the two polar angle representations of the spectrum.

5.5 Photometric Invariant Region Detection by Clustering

In section 5.2 it was shown that uniformly colored objects of matte material draw half-rays in RGB and multispectral color space due to changes in the surface orientation, illumination intensity, and shading. In section 5.3 it was derived that the distance from a spectrum to such half-ray is a photometric invariant. Furthermore, in section (5.4), we derived for the \( i \)-th spectrum \( \bar{c}_i, i = 1, \cdots, n \) that the uncertainty \( \bar{\sigma}_i \) can be obtained using (5.51).

5.5.1 Robust K-means Clustering

Let a multispectral image consist of spectra \( \bar{c}_i, i = 1, \cdots, n \), with corresponding uncertainties \( \bar{\sigma}_i \). The well-known K-means clustering method [6] segments the image by minimizing the squared error criterion. A clustering is a partition \([ \bar{v}_1, \cdots, \bar{v}_K ]\) that assigns each spectrum to a single partition \( \bar{v}_j \), \( 1 \leq j \leq K \). The spectra assigned to \( \bar{v}_j \) form the \( j \)-th cluster.

We compute the cluster centre as the weighted average [19]. If \( M \) spectra \( \bar{c}_i \) with corresponding uncertainties \( \bar{\sigma}_i, i = 1, \cdots, M \), are assigned to a cluster, then the weighted average is computed as

\[
\bar{\mu} = \frac{\sum_{i=1}^{M} w_i \cdot \bar{c}_i}{\sum_{i=1}^{M} w_i}
\]

where the weights are the inverse squares of the uncertainties

\[
w_i = \frac{1}{\bar{\sigma}_i \cdot \bar{\sigma}_i}
\]

Since the weight attached to each measurement involves the square of the corresponding uncertainty \( \sigma_i \), any measurement which is much less precise than the others contributes very much less to the final answer (5.55). With \( \bar{\mu} \) the series of \( M \) spectra assigned to the \( j \)-th cluster, and with \( \bar{v}_j \) the weighted average of the spectra, the squared error for the \( j \)-th cluster is:

\[
e^2_j = \sum_{i=1}^{M} (\bar{c}_i - \bar{v}_j) \cdot (\bar{c}_i - \bar{v}_j)
\]
and the squared error for the clustering is:

$$E^2 = \sum_{i=1}^{K} e_i^2$$  \hspace{1cm} (5.58)

The objective of the $K$-means clustering method is to define, for given $K$, a clustering that minimizes $E^2$ by moving spectra from one cluster to another.

### 5.5.2 Photometric Invariant Segmentation

To obtain photometric invariant region detection, in this paper we propose to cluster on $K$ straight lines from the origin. Assume that the $N$-dimensional spectrum $\mathbf{c}$ is described by (5.11) with associated uncertainty $\sigma_c$ obtained from (5.51). The spectrum is transformed to chromaticity polar angles $\theta$ by (5.24) with associated uncertainty $\sigma_\theta$ obtained from (5.52).

To cluster in polar angle space, the angular distance of (5.26) replaces (5.57) and the weighted angular average of (5.38) replaces (5.55). Given $K$ clusters $v$, the spectrum is then assigned to the closest cluster. In the next step of the clustering algorithm, new partitions are obtained by moving spectra from one cluster to another. Kender [11] pointed out that color space transforms are unstable for sensor input values near the singularity. As is clear from (5.52), the instability of the polar angle transformation is the drawback of the polar angle representation of the spectrum. The instability is dealt with by updating the cluster by the weighted sum as defined by (5.38) where the weights $w_j$ are derived by (5.56). In other words, transformed polar angles with higher uncertainty contribute much less to the final estimate of the cluster than polar angles with small uncertainty. It was shown that the chromaticity polar angle representation is invariant to changes in the geometry of a uniformly colored object. Therefore, clustering in chromaticity polar angle space yields regions invariant to the geometry. In conclusion, using the uncertainties, we obtain segmentation results invariant to photometric effects and robust against noise.

Similarly, to find homogeneously colored surfaces from glossy materials, we cluster in the hue polar angle representation. For shiny surfaces, the spectrum is transformed to the hue polar angle $\theta_h$ by (5.36) with associated uncertainty $\sigma_{\theta_h}$ obtained from (5.52). Given $K$ clusters, the distance from the cluster $v_j$ to the spectrum is derived by (5.26). The spectrum is then assigned to the closest cluster $v_i$. The instability of the polar angle transformation is again dealt with by updating the cluster using the weighted sum (Equation (5.38)). In other words, transformed polar angles with higher uncertainty contribute much less to the final estimate of the cluster than polar angles with small uncertainty. In conclusion, it was shown that the hue polar angle representation is invariant to changes in the geometry and specularities. Therefore, clustering in hue polar angle space yields regions invariant to the geometry and specularities. Using the weighted sum for the updating of cluster centroids achieves robustness against noise.
5.6 Experiments

All multispectral images are grabbed using a Jain CV-M300 monochrome CCD-camera, Matrox Corona Frame-grabber, Navitar 7000 zoom lens and Inspectron V7 spectrograph under 500 Watt halogen illumination. The RGB images are grabbed using a Sony 3CCD color camera XC-003P and four Osram 18 Watt "Lumilux deLuxe daylight" fluorescent light sources.

To estimate the values of the electronic gain parameter $\gamma_e$ (5.3) and the value of the dark current variance of (5.48) for the monochrome camera, 19 images are taken of a white reference while varying the lens aperture such that each image has a different intensity as shown in figure (5.4). A line is fitted through the intensity-variance data yielding an electronic gain of $\gamma = 0.0069$, and dark current variance of $\sigma_d^2 = 0.87$.

The RGB-camera has a white-balancing option. The goal is therefore to establish the overall value of the camera gain $\gamma_i$ where $i \in \{R, G, B\}$. To that end, 26 images are taken of a white reference while repeating the procedure to obtain different intensity images. The data are shown in figure (5.5). Fitting three lines through a common
5.6.1 Propagation of Uncertainties in Transformed Spectra

Models were proposed in (5.49, 5.50, 5.51) to estimate uncertainties resulting from sensor noise in a spectrum for white-balanced camera systems. The goal of the experiment is to verify empirically the validity of the equations. Therefore, five multispectral images are taken from uniformly colored sheets of paper such that the entire spectral image exhibits one single color. The colors are red, yellow, green, cyan and blue.

Using the gain parameters, the uncertainty in the white-balanced camera output $\hat{s}_o(\lambda)$ can be estimated (5.49). The estimated uncertainties are averaged for each...
5.6. Experiments

<table>
<thead>
<tr>
<th>color</th>
<th>$\delta(\hat{\sigma}(\lambda), \sigma(\lambda))$ (5.61), Eq. (5.62)</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>$0.011 \pm 0.011$</td>
</tr>
<tr>
<td>yellow</td>
<td>$0.011 \pm 0.011$</td>
</tr>
<tr>
<td>green</td>
<td>$0.009 \pm 0.011$</td>
</tr>
<tr>
<td>cyan</td>
<td>$0.008 \pm 0.011$</td>
</tr>
<tr>
<td>blue</td>
<td>$0.006 \pm 0.011$</td>
</tr>
</tbody>
</table>

Table 5.2: Results differentiated for the estimated and real uncertainties in reflectance factors after the white-balancing operation for multispectral images of uniformly colored paper as indicated.

Table 5.2: Results differentiated for the estimated and real uncertainties in reflectance factors after the white-balancing operation for multispectral images of uniformly colored paper as indicated.

The estimation of the uncertainty in the chromaticity and hue polar angles by (5.52) is verified empirically in a similar way. The average of a series of $M$ angular

\[
\hat{\sigma}_c(\lambda) = \frac{1}{M} \sum_{i=1}^{M} \sigma_c(x_i, \lambda) \tag{5.59}
\]

for $M$ pixels along the one-dimensional spatial axis of the multispectral image. The real uncertainty is derived from the standard deviation of reflectance factors $c(\lambda)$ over the spatial range as

\[
\sigma^2(\lambda) = \frac{1}{M-1} \sum_{i=1}^{M} (c(x_i, \lambda) - \bar{c}(\lambda))^2 \tag{5.60}
\]

where $\bar{c}(\lambda)$ denotes the average reflectance factor. The absolute difference $\delta(\hat{\sigma}(\lambda), \sigma(\lambda))$ between the real and estimated error is obtained as

\[
\delta(\hat{\sigma}_c(\lambda), \sigma_c(\lambda)) = |\hat{\sigma}_c(\lambda) - \sigma_c(\lambda)| \tag{5.61}
\]

and then averaged over the wavelength range as

\[
\delta(\hat{\sigma}, \sigma) = \frac{1}{N} \sum_{i=1}^{N} \delta(\hat{\sigma}_c(\lambda_i), \sigma_c(\lambda_i)) \tag{5.62}
\]

where $N$ denotes the number of samples taken in the wavelength range. Due to the low sensitivity of the CCD-camera and low transmittance of the illuminant at lower wavelengths, the uncertainty is greater at the lower wavelengths than at higher wavelengths. The reflectance of a spectrum at a certain wavelength is expressed as the reflectance factor $c(\lambda)$ taking on values between 0 and 1. The difference between the estimated and real uncertainty in the reflectance factor is given in table (5.2) and is approximately 0.01, corresponding to one percent. Therefore, the table shows a very reasonable correspondence between the measured and real uncertainty. This conclusion can be confirmed visually by examination of figure (5.6).
Figure 5.6: Yellow paper. Experiment: Comparison of the estimated (dashed line) versus the real uncertainty (solid line) of the reflection factors for a yellow paper.

values $\theta_i$, $i = 1, \cdots, M$, with equal weights $w_i$ is computed using (5.38) and is denoted $\overline{\theta}$. The standard deviation is computed as

$$\sigma_\theta = \frac{1}{N-1} \sum_{i=1}^{N} [\Delta(\overline{\theta}, \theta_i)]^2, \quad 0 \leq \overline{\theta}, \theta_i < 2\pi (5.63)$$

where $\Delta$ is defined by (5.27). Similarly, the difference $\Delta(\bar{\sigma}_\theta(\lambda), \sigma_\theta(\lambda))$ between the real and estimated error between chromaticity angles at a certain wavelength is obtained using (5.27). The results are averaged over the wavelength range as

$$\delta(\bar{\sigma}_\theta, \sigma_\theta) = \frac{1}{N} \sum_{i=1}^{N} \Delta(\bar{\sigma}_\theta(\lambda_i), \sigma_\theta(\lambda_i)) (5.64)$$

The results of the experiment are given in table (5.3). The dimension of chromaticity polar angles is the number of spectral samples minus one. The second column of the table specifies the results for the spectrograph. The results are averaged over
5.6. Experiments

<table>
<thead>
<tr>
<th>color</th>
<th>multispectral $\delta(\hat{\alpha}, \sigma_\alpha)$</th>
<th>RGB $\delta(\hat{\alpha}, \sigma_\alpha)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>$0.6 \pm 0.8$</td>
<td>$1.26$</td>
</tr>
<tr>
<td>yellow</td>
<td>$0.5 \pm 0.7$</td>
<td>$0.01$</td>
</tr>
<tr>
<td>green</td>
<td>$1.6 \pm 2.4$</td>
<td>$0.36$</td>
</tr>
<tr>
<td>cyan</td>
<td>$0.7 \pm 0.7$</td>
<td>$0.11$</td>
</tr>
<tr>
<td>blue</td>
<td>$0.9 \pm 1.1$</td>
<td>$0.07$</td>
</tr>
</tbody>
</table>

Table 5.3: Results differentiated for the estimated and measured uncertainties in chromaticity polar angles using (5.64).

<table>
<thead>
<tr>
<th>color</th>
<th>multispectral $\delta(\hat{\theta}, \sigma_\theta)$</th>
<th>RGB $\delta(\hat{\theta}, \sigma_\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>red</td>
<td>$0.7$</td>
<td>$1.1$</td>
</tr>
<tr>
<td>yellow</td>
<td>$0.4$</td>
<td>$0.5$</td>
</tr>
<tr>
<td>green</td>
<td>$2.1$</td>
<td>$1.8$</td>
</tr>
<tr>
<td>cyan</td>
<td>$0.5$</td>
<td>$0.7$</td>
</tr>
<tr>
<td>blue</td>
<td>$1.7$</td>
<td>$0.5$</td>
</tr>
</tbody>
</table>

Table 5.4: Results differentiated for the estimated and measured uncertainties in hue polar angles using (5.27).

58 chromaticity angles, therefore the standard deviation is given as well. The third column specifies the results for the RGB-camera averaged over 2 chromaticity angles. The chromaticity angles are in the range of zero to 90 degrees, the difference between the estimated and real uncertainty is less than one percent. Consequently, there is a very reasonable correspondence between the measured and real uncertainty. A more detailed example is given in figure (5.7) for the results for the yellow color.

Similarly, for the hue polar angle, the results are given in table (5.4). The hue angles are in the range of zero to 360 degrees, the difference between the estimated and real uncertainty is less than one percent. Consequently, there is a very reasonable correspondence between the measured and real uncertainty.

5.6.2 Photometric Invariant Clustering

Multispectral Images Figure (5.8a) shows a multispectral image of a textile sample. The spectral information is on the vertical axis. The top of the picture corresponds to 410 nm, the bottom to 705 nm. The left-hand side of the image is from homogeneously red colored textile, the right-hand side is colored green. The structure of the textile is visible in intensity fluctuations occurring in otherwise homogeneous spectra. The result of clustering in the chromaticity polar angle space is shown in figure (5.9). The figure shows how the spectra form half-rays due to the geometry changes of the structure of the textile. Fitting of half-rays through the chromaticity
angle representation results in invariance for shadows and surface orientation changes. Figure (5.8b) shows the spectra of two plastic objects. The left-hand side object is colored orange, the right-hand side object is green. The objects are smooth and structure-less, but reflect specularities showing up as the vertical bright streaks in the spectral image. Furthermore, the intensity of the spectra gradually reduces towards the right hand side of the image due to a change in the surface orientation of the objects. The result of clustering in hue polar angle space is shown in figure (5.10). Clustering in hue polar angle representation results in independence to the highlights and surface orientation changes.

**RGB images** Figure 5.11 shows a RGB-image of several toys against a background consisting of four squares. The upper-left quadrant of the image consists of three uniformly painted matte cubes of wood. The upper-right quadrant contains two specular plastic donuts on top of each other. In the bottom-left quadrant a red highlighted ball and a matte cube are shown while the last quadrant contains two
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Figure 5.8: (a) shows a multispectral image of a textile sample. The spatial information is on the horizontal axis, the spectral information is on the vertical axis. The top corresponds to 410 nm wavelength, the bottom to 705 nm wavelength. The left hand side of the image is colored homogeneously red, the right hand side is colored green. The structure of the textile is visible through the intensity fluctuations occurring in the further homogeneous spectra. (b) shows the spectra of two plastic objects. The left hand side object is colored orange, the right hand side object is green. The objects are smooth and structure-less, but reflect specularities showing up as the vertical bright streaks in the spectral image. Furthermore, the intensity of the spectra gradually reduces towards the right hand side of the image due to a change in the surface orientation of the objects.

In figure (5.11b) the segmentation result is shown obtained by the \( K \)-means clustering method in \( RGB \)-data. False regions are detected due to abrupt surface orientations, shadows, inter-reflections and highlights. In contrast, the result of clustering in the chromaticity polar angle space is shown in figure (5.11c). Regions are detected insensitive for shadows and surface orientation changes but are affected by highlights. The result of clustering in hue polar angle space is shown in figure (5.11d). Here, computed region edges correspond to material boundaries discounting the disturbing influences of surface orientation, illumination, shadows and highlights. The difference between figures (5.11c) and (5.11d) is the invariance of the latter to the specularities reflected at the red ball.
Figure 5.9: Result of clustering in the chromaticity polar angle space for the multimage shown in figure (5.8a). The result is shown for the angle between the 510-nm and 710-nm wavelength. The spectra form half-rays due to the geometry changes of the structure of the textile. Fitting of half-rays through the chromaticity angle representation results in invariance for shadows and surface orientation changes.

5.7 Discussion

We combined a camera and image formation model to describe how light reflected by an object results in the observed sensor value. Based on the models, we then investigated how uniformly colored objects draw clusters in multispectral color space. We showed that uniformly colored objects of matte materials draw half-rays in mult-
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Figure 5.10: Result of clustering in hue polar angle space for the image shown in figure (5.8b). Clustering in hue polar angle representation results in independence to the highlights and surface orientation changes.

trispectral color space due to changes in the surface orientation, illumination intensity and shading. In contrast, due to specularities, shiny objects will draw triangularly-shaped planes in multispectral space. Therefore, we investigated computational models to detect clusters where the cluster shape is modeled as either a half-ray or a trianularly-shaped plane. To that end, different distance measures were examined and we concluded that the polar angular representations elegantly allow for photometric invariant detection of both matte and shiny surfaces. Alternatively, we showed that computing the distance of a point (here: color spectrum) to a plane in N-dimensional space has the undesired effect that opponent colors have the same distance to a cluster.
Figure 5.11: Segmentation results for the K-means clustering method. a. RGB-image b. Cluster model is a point, region detection is sensitive to intensity changes, shadows, geometry, highlights and color transitions. c. Cluster model is a half-ray, region detection is sensitive to highlights and color transitions. d. Cluster model is a trianually-shaped plane, region detection is sensitive only to color transitions. See also the color plates at pages 125 and 126.
However, we also showed that the polar angle representation may become unstable in the presence of sensor noise, a problem shared by other photometric invariant color spaces. To overcome this effect, we examined, in theory and in practice, the propagation of sensor noise to the computation of the polar angles. Taking the simplicity of our camera, noise and error propagation models into account, we showed that a striking correspondence exists between the estimated and real uncertainties. The estimated uncertainty was employed in the region detection method to be robust against sensor noise. Finally, we verified empirically that uniformly colored regions are detected by the proposed clustering method invariant to surface orientation changes, shadows and highlights.

Bibliography


