An experimental approach to expectation formation in dynamic economic systems

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Chapter 3

Single-agent Treatment

3.1 Introduction

The objective of this chapter is to investigate how individual agents form expectations in an experimental cobweb economy, without knowledge of the underlying market equilibrium equations. In the experiment prices are generated by a nonlinear cobweb model, with expectations formed by individual participants. The realized market price in this treatment is to a large extent (up to a small noise term and some unanticipated demand shocks) determined by a single participants prediction, i.e. there is a one to one correspondence between the prediction of the participant and the realized market price. The cobweb economy in the experiment is unstable, i.e. under naive expectations prices diverge from the RE steady state and converge to a period 2-cycle. A motivation for conducting these single-agent experiments is that we use it as a benchmark for the multi-agent treatments of Chapter 4 where the realized market price is a function of the predictions of a group of individuals. This chapter is organized as follows. In Section 3.2 we describe the experimental design and we explain the main differences between the four treatments. The experimental results are presented in Section 3.3. Section 3.4 concludes and contains some remarks about the next chapters.

3.2 An experimental cobweb economy

The experiment was conducted in the computer laboratory of CREED and consists of four different treatments: noise-NoInfo, noise-Info, permanent shock-NoInfo and the permanent shock-Info. The subjects in the noise-NoInfo treatment and permanent shock-NoInfo treatment have to predict a 'value' between zero and ten while subjects in noise-Info treat-
ment and permanent shock-Info treatment have to predict a 'price' between zero and ten. The participants in both Info treatments are also given some general information about the market which will be explained in more detail below. Our objective for the distinction between a NoInfo and Info treatment is to investigate whether general market information improves prediction accuracy.

We start the experiment when everybody has finished reading their instructions (see Appendix 3.A). The experiment lasts 50 periods and every period lasts 30 seconds. Throughout this chapter we will call the subjects' predictions the predicted value/ predicted price and the realized value/price the real value. The subjects have no information about how the real value is obtained. Neither do they know that the market price (2.8) depends on their forecast of the price. They are informed that their prediction has to be between zero and ten. At the end of every period the subjects are informed about their last periods earnings, their total earnings and the real value. A time series of the real value and the predicted value is updated every period. Figure 3.1 shows the screen the subjects see during the experiment. The earnings are negatively related to the prediction error,

\[
\Pi = 1300 - 260(X - Y)^2, \tag{3.1}
\]

where \(Y\) is the predicted value and \(X\) is the real value. The expected value of this function is maximized by \(Y = EX\). Negative payoffs are not used; earnings are 0 if \(|X - Y| \geq \sqrt{5}\). At the end of the experiment the points are exchanged to Dutch guilders at a rate 1300 points = 1 guilder (a guilder corresponds to approximately 0.45 Euro). The maximum total earnings of a subject in the experiment is 50 guilders. At the end of every period
the subjects' screen is updated. Subjects are not informed about the market equilibrium equation that generates the real value as a function of the predicted value/price. The subjects participate only once. The real value is the market equilibrium price (2.8) in the cobweb model of Section 2.2, with expectations formed by a single subject, i.e.,

\[ p_t = \frac{a_t - \tanh(2(p_t^* - 6)) - 1}{0.25} + \epsilon_t, \]

where an (unknown) exogenous shock \( \epsilon_t \) is added. Notice that this is exactly the strongly unstable treatment with \( \lambda = 2 \) as discussed in Section 2.2.1. Table 3.1 reports the differences in parameters between the treatments.

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Table 3.1: Design parameters for the different treatments. The NoInfo treatments are treatments without market information while subjects in the Info treatments are given some general market information.

There are three main differences between the treatments:

- difference in market information,
- fixed versus a time varying RE steady state price,
- difference in noise level.

Firstly in the noise-NoInfo treatment and permanent shock-NoInfo treatment the subjects have no market information at all. Subjects are asked to predict a 'value' or a 'number' between 0 and 10 without knowledge where the 'value' comes from. In contrast, in the noise-Info and permanent shock-Info treatment the subjects have some general market information, i.e. they are informed that they have to predict a price. They also know that the price in the experiment is determined by equilibrium of demand and supply and that there is some uncertainty on the supply side. Secondly, as can be seen in Table 3.1, in both noise treatments the parameter \( a \) is fixed during the entire experiment. There are no large demand shocks but only a medium size noise term \( \epsilon_t \) in each time period. A motivation for this setup is that we wanted to see whether agents can learn the RE
steady state in a stationary environment. In contrast, in the other two treatments, the permanent shock-NoInfo and permanent shock-Info treatments, four different values of the parameter $a$ are used, implying that there are three (large) demand shocks (three shifts of the demand curve) within the 50 periods of the experiment. Each of these large demand shocks is permanent in the sense that it lasts for $10 - 15$ time periods until the next demand shock occurs. Since the parameter $a$ changes three times during these treatments the RE steady state price also changes three times, i.e. four different RE steady states have to be learned in 50 periods. The permanent shock treatment thus corresponds to a non-stationary world. Subjects have no information about any shocks occurring; in fact, as stated before, subjects do not even know that the time series are generated by an underlying cobweb model with feedback from their own expectations. The third difference between the treatments is the difference in exogenous shocks. In the permanent shock treatments there is small noise, $\epsilon_t$, drawn from a uniform distribution with small variance $\sigma_e^2 = 0.0033$. In the noise treatments there is medium size noise, $\epsilon_t$, drawn from a normal distribution with medium size variance, $\sigma_e^2 = 0.25$. The question now arises how these differences in the treatments affect the subjects predictions.

3.3 Experimental results

In this section we report the results of the single-agent treatments. We show the differences and similarities of the results between the different treatments. We first report the findings of the permanent shock-NoInfo and Info treatments. After that we give the results of both noise treatments.

3.3.1 Permanent shock treatments

The most striking result is that there are large differences in the earnings of the subjects. There are subjects earning about 52000 points (40 guilders) while there are other subjects earning about nothing. An obvious explanation for the large difference in earnings must be that subjects use quite different kind of expectation rules. By looking at the time series of the predictions and realized prices we categorize the subjects in three categories. The time series of the participants in the same category have qualitatively the same patterns. We obtain the following three categories:

1. subjects who seem to have some kind of adaptive expectations or AR(1) learning; an example of a participant in this category is given in Figure 3.2 (a) and 3.3 (b)
2. subjects who seem to have some special form of naive expectations, which we call Markov expectations; an example is given in Figure 3.3 (a)

3. subjects who do not seem to use a systematic forecasting rule

The subjects with adaptive expectations or AR(1) learning constantly update their predictions in the direction of the last realized value. In contrast, the subjects with naive expectations always predict the last realized value. Markov expectations means that a subject’s predicted value is either his last predicted value or the last real value, with some (fixed) probability of switching between these two options. A special form of markov expectations is naive expectations. The subjects we could not classify as having adaptive or Markov expectations are in category 3. Table 3.2 shows the average earnings of the subjects in the permanent shock treatment for the different categories. In total 39 subjects participated in these treatments, on average earning 15 guilders. From Table

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<th>category 3 (rest)</th>
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Table 3.2: Earnings of subjects in the permanent shock treatments. The top half of the table shows the results of the permanent shock-NoInfo treatment whereas the bottom half shows the results for the permanent shock-Info treatment. The second and fourth row show the number of subjects in the different categories and the corresponding average earnings.

3.2 we find that subjects in category 1 earned most money, while the difference in earnings between subjects in categories 2 and 3 is less obvious. Even though the subjects in the permanent shock-Info treatment had some general market information their average earnings are somewhat less than those of the subjects with the permanent shock-NoInfo treatment. Overall this suggests that general market information does not improve prediction performance. Note that the subjects with naive expectations or Markov expectations who according to the theory will always make systematic forecasting errors did earn some money. An explanation for this is that the theoretical cobweb model with naive expectations and parameters as in the experiment, leads to the stable steady state value in
Figure 3.2 (a) shows the time series of participant 31\(^1\) (permanent shock-NoInfo treatment), a typical example of a participant who we put in category 1. The dotted line represents the subjects' *predicted value* and the solid line represents the *real value*. From Figure 3.2 (a) we see that around period eight the participants’ prediction is close to the RE steady state value. At period sixteen an (unexpected) exogenous shock occurs (the parameter \(a\) changes from 2 to 3) causing the *real value* to become almost ten. At period 17 the participant adapts his *predicted value* too much in the direction of the *real value* causing the *real value* to decline sharply. But from that moment on he adapts with smaller steps and within four periods he has ‘learned’ the new steady state. After the second (period 29) and the third (period 41) exogenous shock this participants finds the RE steady state value even faster. Participant 31 is able to learn the unique RE steady state within 8 periods, without any knowledge of the market equilibrium equations. Even after a large unexpected exogenous shock, participant 31 is able to find the new RE steady state quickly. Figure 3.2 (b) shows that an adaptive forecasting rule with weight factor \(w = 0.2\) generates a time series similar to the time series of participant 31 in the experiment. The similarity between the two figures is striking and suggests that after 5 periods participant 31 indeed uses some kind of adaptive expectations forecasting rule with a

\(^{1}\)On entering the laboratory we assigned a number to the subjects. With this number we can identify the participants. The numbering of the participants is arbitrary.
Cobweb Model

small weight factor. From Table 3.2 we saw that only 12 of the 39 subjects use some kind of adaptive strategy while 16 subjects are classified as having markov expectations. Less than one third of all subjects is thus able to learn the RE steady state in the non stationary environment.

3.3.2 Noise treatments

The difference between the noise treatments and the permanent shock treatments is that there are no large shifts of the demand curve in the noise treatments. The parameter \(a\) is constant and fixed for 50 periods. Furthermore, in the noise treatments there are medium size normally distributed exogenous shocks, \(\epsilon_t \sim N(0, 0.25)\), opposed to the small size uniformly distributed shocks in the permanent shock treatments. Table 3.3 shows the results for both noise treatments. From the table we find that subjects from category

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<td>average earn.</td>
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<td>6.96</td>
<td>13.22</td>
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</table>

Table 3.3: Earnings of subjects in the noise treatments. The top half of the table shows the results of the noise-NoInfo treatment whereas the bottom half shows the results for the noise-Info treatment. The second and fourth row show the number of subjects in the different categories and the corresponding average earnings.

1 earn most money while the average earnings of the subjects in category 2 are almost zero. Notice that, in this case, general market information does lead to some prediction improvement: the average earnings in the noise-Info treatment are higher than in the noise-NoInfo treatment\(^2\). Furthermore, 13 of the 39 subjects are classified in category 1 and are able to learn the RE steady state. In contrast, two third of the participants is not able to learn the RE steady state in a strongly unstable stationary environment. Figure 3.3 (a) shows the time series of participant 102 (noise-Info treatment), a typical example of a participant we put in category 2, with a markov forecasting strategy.

\(^2\)In the next chapter we will no longer make a distinction between the single-agent NoInfo and Info treatments since we did not find a significant effect.
Participant 102 (almost) always expects either a high or a low price and consequently the

realized prices are always high/low. This subject's prediction is either his last prediction, \( p_t^* = p_{t-1}^* \), or the last realized price, \( p_t = p_{t-1} \). He never, not once in fifty periods, predicts a price between 4 and 6 and since the RE steady state price is 5.91 this participant does not earn any points. Apparently this participant does not learn from his systematic forecasting errors. In contrast, participant 110 which we classified in category 1 predicts the price very accurately as we can see from Figure 3.3 (b). We also did some simulations for the noise treatment, Figure 3.4 (a) shows the simulation results for markov expectations. In this simulation markov expectations means that a subject's predicted value is either his last predicted value or the last real value, with some probability of switching between these two options. This leads to the following time series, where the probability of predicting last price is equal to \( \frac{3}{4} \) and the probability of not changing the prediction is equal to \( \frac{1}{4} \). From a qualitative viewpoint, Figure 3.4 (a) is similar to the time series of participant 102 in Figure 3.3 (a). Markov expectations seems to be a good description of the forecasting rule participant 102 uses. Finally, Figure 3.4 (b) shows the time series of a simulation with SAC-learning as briefly discussed in Section 2.2.4. Despite the medium size permanent demand shocks, the learning algorithm converges quickly to the unique steady state equilibrium. This time series is similar to the time series of participant 110 (category 1). SAC-learning thus seems to be a reasonable description of the behavior of some of the participants in the experiment.

Figure 3.3: (a): Time series of participant 102. (b): Time series of participant 110. The realized price is the solid line whereas the dotted line represents the participants' prediction.
3.4 Concluding remarks

In this chapter we built an experimental environment to investigate expectation formation in a cobweb economy. In particular we investigate whether agents are able to learn the rational expectations steady state price in an unstable market when market equilibrium equations are not known and where the realization of the market price depends upon their own prediction alone. We find that in what may be seen as the simplest unstable dynamic economic expectations feedback system only about 35% of all individuals are able to learn the unique RE steady state. These subjects seem to use an adaptive learning rule. That is, they adapt their predictions with small steps into the direction of the realized price. Moreover, in some cases SAC-learning or OLS-learning describes expectation formation reasonably well. That a large fraction of the subjects is not able to learn the unique rational expectations equilibrium price/value, not even within 50 time periods is remarkable since we are dealing with what is perhaps the simplest dynamic economic model. Although there is a unique rational expectations steady state about 65% of all individuals are not able to learn this steady state when market equilibrium equations are unknown. Furthermore, information about the market situation does not improve the prediction results much.

In summary, we find that learning turned out to be quite difficult in an unstable cobweb economy, only one third of the participants is able to learn the RE steady state price. Furthermore, we observed a lot of convergence to the steady state price in the
permanent shock treatments between periods 29 and 40 suggesting that learning is easier in a stable cobweb economy.
3.A Instructions for single-agent treatments (Info)

This experiment lasts for 50 periods. Every period you have to give a prediction of a price of an unspecified good. Your task is to predict the price. We do not inform you about this price sequence. After you submitted your prediction, the computer reveals the real price to you. We will explain everything later in more detail. Every period you have 30 seconds to submit a prediction. Your prediction has to be between zero and ten. A maximum of two decimals is possible. After the twenty seconds have elapsed the real price is revealed to you. After ten more seconds the next period starts. Every period you can earn points. These points are convertible in to guilders at a rate of 1300 points is 1 guilder. The better your prediction the more you earn. On your desk is a pay-off table. For every possible prediction error the corresponding earnings are given. An example: your prediction is 3.42. The realized price turned out to be 2.13. Your prediction error then is: 3.42-2.13 = 1.29. From the table you can find that you have earned 867 points.

Screen: There are four blocks on your screen during the experiment. We will show them one after another. In the upper left corner a time series of your prediction and the corresponding realized price is shown. The realized price is in black while your prediction is in red. At the end of every period this graph is updated. Just below the graph of the time series is a block with information about last periods earnings, your total earnings and how much time you have this period to give a prediction. At the right side of your screen your last twenty prediction and the corresponding prices is shown. At the left bottom corner a block is shown in which you have to submit your prediction. After twenty seconds this block disappears.

If everybody has finished reading the instructions we will start the experiment. If you have any questions, now or during the experiment please raise your hand. Somebody will come and help you. Good luck.
## Payoff table

**Single-agent Treatment**

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