An experimental approach to expectation formation in dynamic economic systems

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Chapter 8

A Strategy Experiment in Asset Pricing

8.1 Introduction

In the previous chapter we have seen that in a simple experimental asset market populated by six participants, prices generally do not converge to the RE fundamental price, but (temporary) speculative bubbles emerge with asset prices growing at a constant rate. We find that simple prediction strategies describe participants’ behavior in the experiments well. In this chapter we employ the strategy method to investigate in more detail what kind of prediction strategies individuals use and whether, as participants gain more experience, prices settle down at the RE fundamental steady state.

In Chapter 5 we used the strategy method to investigate expectation formation in an experimental cobweb economy. In this chapter we will use the same procedures and methods to investigate how subjects form expectations in a simple asset pricing experiment. The strategy method provides a nice approach to investigate the way subjects form expectations. In most economic laboratory experiments subjects only make relatively few decisions and learning possibilities are limited.

The subjects of the strategy experiment first participate in an introductory laboratory experiment to gain experience in their forecasting task. In the instructions we inform the subjects that they are an advisor to a pension fund. The task of the advisors is to give an accurate prediction of the price of a risky asset. The pension fund bases its investment decision in the risky asset upon the price forecast of the advisor/participant. The market price of the risky asset is determined by equilibrium of demand and supply. At the end of the introductory experiment the subjects formulate and hand in their first strategy for the
strategy experiment. This strategy has to be a complete representation of the subject’s predictions in all possible states. Each period the price, $p_{t+1}$, has to be predicted. The strategy experiment consists of four rounds. In each round we simulate 1296 markets with six randomly drawn strategies for each market. At the end of each round the subjects receive feedback. The feedback for each participant consists of the price and the prediction of the strategy in 5 randomly selected simulations of fifty periods. The ranking of the strategies as well as a programmed version of their own strategy are also included in the feedback. The ranking of the strategies is based on the performance, measured by the quadratic forecasting rule.

After the subjects submit their fourth round strategies but before they receive the results of the fourth round we conduct a final laboratory experiment. The goal of this final experiment is to study the relationship between the actual behavior of subjects and the strategy they submit. We inform the subjects that they are in a market with five strategies from the third round. Since the subjects, in the final experiment, have information up till round 3 we can compare the prediction of the participant in the final experiment with the prediction of their fourth round strategy.

The main research questions of this chapter are: (1) What kind of strategies do subjects use? (2) Does the market price eventually converge to the fundamental price? (3) How does learning affect the price dynamics in the consecutive rounds? (4) How do individual strategies attribute to market stability or instability? (5) Is the submitted strategy an accurate description of the decisions subjects make in a regular experimental environment?

This chapter is organized as follows. Section 8.2 briefly recalls the asset pricing model of Chapter 6. In Section 8.3 the experimental design is explained. In Section 8.4 we study the similarities and differences between the submitted strategies, the short, medium and long run price dynamics and the results of the final experiment. We summarize and conclude in Section 8.5. Appendix 8.A contains all procedures and information given to the participants.

### 8.2 An asset pricing model

In this section we briefly recall the asset pricing model of Chapter 6 with $H$ different kinds of traders or strategies. Traders can choose between a risk free asset that pays a fixed return $r$ or a risky asset that pays an uncertain dividend $y_t$ in period $t$. Let $p_t$ denote the price of the risky asset. Market equilibrium in the asset pricing model with $H$ different
traders is given by:

\[
(1 + r)p_t = \frac{1}{H} \sum_{h=1}^{H} E_{h,t}(p_{t+1} + y_{t+1}),
\]

(8.1)

where \( r \) denotes the risk free rate of return, \( E_{h,t}(p_{t+1} + y_{t+1}) \) denotes the expectation or belief, formed at the beginning of date \( t \), of trader \( h \) about next periods price plus next periods dividend. In the strategy experiment \( H \) will be fixed at six. Notice that in this setup there is no robot trader and that there are no exogenous price shocks added to the equilibrium price.

We specialize on the simplest case of an independent and identically distributed process for dividends, that is,

\[
y_t = \bar{y} + \delta_t.
\]

(8.2)

We assume that expectations about future dividends are equal and correct for all traders, that is, we assume that \( E_{h,t}(y_{t+1}) = \bar{y} \) and \( \delta_t \sim N(0, \sigma^2_\delta) \).

Let us first recall the important benchmark when all traders have rational expectations. Notice that since all traders have correct expectations about future dividends the noise term \( \delta_t \) of the IID dividend process does not affect the realized market price which is given by

\[
p_t = \frac{1}{1 + r}(\bar{y} + \frac{1}{H} \sum_{h=1}^{H} E_{h,t}p_{t+1}).
\]

(8.3)

When expectations are homogeneous and all traders have rational expectations (with speculative bubble solutions excluded by the so called transversality condition), the rational expectations or fundamental price \( p^f_t \) is given by the discounted sum of expected future dividends, that is,

\[
p^f_t = \sum_{k=1}^{\infty} \frac{E_t(y_{t+k})}{(1 + r)^k}.
\]

(8.4)

Given that the dividend process is IID as in (8.2) the fundamental price \( p^f \) is constant and given by

\[
p^f = \frac{\bar{y}}{r}.
\]

(8.5)

### 8.3 Experimental design and procedures

The experiment lasted for eight weeks. The subjects are recruited from a course “Dynamical Systems”, a mathematical introduction to dynamical systems in the undergraduate econometrics program, and from the course “Micro Economics”, a course in the undergraduate economics program. Participation is not a course requirement. Students have
no prior knowledge about dynamic economic systems and the asset pricing model is not taught in the courses.

To gain some experience with their forecasting task the subjects first participate in an introductory laboratory experiment, similar to the NoRobot treatment asset pricing experiment of Chapter 7. Subjects participate sequentially in two different markets of 50 periods. In both markets they can earn 1300 points each period. The earnings are negatively related to the size of the quadratic forecasting error. The number of points earned in period $t$ by participant $h$ is given by the quadratic scoring rule

$$e_{ht} = \max\{1300 - \frac{1300}{49}(p_t - p_{ht})^2, 0\}, \quad (8.6)$$

where 1300 points is equivalent to 0.65 Dutch guilder. Notice that earnings in period $t$ are zero when $|p_t - p_{ht}| \geq 7$. The average earnings of the 21 participating subjects, in approximately 1.5 hours, were 31 Dutch guilders (14 Euro), where the maximum total earnings are 65 Dutch guilders (30 Euro).

**Introductory experiment.**

The goal of the introductory experiment was to give the subjects some experience in their forecasting task. The experiment was completely computerized and took place in the CREED experimental laboratory. The participants understanding of the instructions was checked by control questions. The forecasting task was presented as follows (this is a summary, for the complete instructions see Appendix 8.A):

- **You are a financial advisor** to a pension fund that wants to optimally invest a large amount of money. The pension fund has two investment options: a risk free investment and a risky investment. In each time period the pension fund has to decide which fraction of their money to put on the bank account and which fraction of the money to spend on buying stocks. In order to make an optimal investment decision the pension fund needs an accurate prediction of the price $p_t$ of stocks. As their financial advisor, you have to predict the stock market price $p_t$ (in guilder) during 51 subsequent time periods.

- **Information about the stock market**
  
The price $p_t$ of the stocks is determined by market equilibrium, that is, the stock market price $p_t$ in period $t$ will be the price for which aggregate demand equals supply. The supply of stocks is fixed during the experiment. The demand for stocks is mainly determined by the aggregate demand of a number of different pension funds active in the stock market.

- **Information about the investment strategies of the pension funds**
  
The precise investment strategy of the pension fund that you are advising and the investment strategies of the other pension funds are unknown. The return of the stock market
As the financial advisor of a pension fund you are not asked to forecast dividends, but you are only asked to forecast the price of the stock in each time period. Based upon your stock market price forecast, your pension fund will make an optimal investment decision. The higher your price forecast the larger will be the fraction of money invested by your pension fund in the stock market, so the larger will be their demand for stocks.

- **Forecasting task of the financial advisor**
  The only task of the financial advisors in this experiment is to forecast the stock market index \( p_t \) in each time period as accurate as possible. The price of the stock will in the first two periods always be between 0 and 100 guilders. The stock price has to be predicted two time periods ahead. After all participants have given their predictions for the first two periods, the stock market price \( p_1 \) in the first period will be revealed and based upon your forecasting error \( p_1 - p_1^f \) your earnings for period 1 will be given.

- **Summary of information**
  - past prices up to period \( t - 2 \): \( \{p_{t-2}, p_{t-2}, \ldots, p_1\} \)
  - past predictions up to period \( t - 1 \): \( \{p_{t-1}^f, p_{t-2}^f, \ldots, p_1^f\} \)
  - past earnings up to period \( t - 2 \)

- **Earnings**
  Earnings will depend upon forecasting accuracy only. The better you predict the stock market price in each period, the higher your aggregate earnings. Earnings will be according to the following earnings table. If you are the best advisor in the experiment, you can earn an additional bonus of DFL 50. This bonus will be given to the financial with the smallest average prediction error during the experiment.

Given the six individual predictions, the realized price is determined by the market equilibrium equation (8.3) with \( H = 6 \). In the introductory experiment subjects participate sequentially in two markets with different composition of subjects and different mean dividend \( \bar{y} \) and risk free interest rate \( r \). At the beginning of the markets the exact values of the mean dividend and interest rate are announced. In the first market the mean dividend and interest rate are \( \bar{y} = 3 \) and \( r = 5\% \) while in the second market they are \( \bar{y} = 2.4 \) and \( r = 6\% \). According to (8.5) the corresponding fundamental prices for the two markets are thus given by: \( p^f = \frac{3}{0.05} = 60 \) and \( p^f = \frac{2.4}{0.06} = 40 \).

**Strategy experiment**

The strategy experiment consists of four rounds. In each round the subjects have to submit a strategy. Participants can submit their strategy anytime before a biweekly deadline
but preferably after class. The number of participants decreased over the rounds, in round 1 all 21 participants submitted a strategy but in round 2 there were 19 participants who submitted a strategy. In rounds 3 and 4 we still had 17 and 16 submitted strategies, respectively. The first round started at the end of the introductory experiment when all participants had to submit their first strategy. The participants are given a folder with all necessary information about the procedures and the forecasting task. Appendix 8.A contains all information (translated from Dutch) included in the folder. The experimenters checked these strategies for clarity, completeness (whether the strategy provides a prediction in all possible situations), uniqueness (whether the strategy always provides exactly one prediction) and informational correctness (whether the strategy does not use information that is not available, such as the price $p_{t-1}$ when forecasting price $p_t$, future prices or previous predictions of other strategies). Every participant has his own personal code and we use this code to identify the subjects over the rounds. The participants do not know the personal codes of other participants.

In every round we simulate the experimental asset market 1296 times. Each market consists of six different, randomly drawn, strategies. For the simulations we use 9 different values of the mean dividend ($\bar{y} = 2, 2.25, 2.5, \ldots, 4$) and 9 different values of the interest rate ($r = 4\%, 4.25\%, 4.5\%, \ldots, 6\%$). The fundamental price is therefore always in the interval $[33\frac{1}{3}, 100]$. Every combination of mean dividend and interest rate we simulate 16 times, resulting in a total of 1296 simulations. The average number of simulations for a particular strategy is then 362, 409, 457 and 486 for rounds 1-4, respectively. The subjects are informed that the exact values of the mean dividend and the interest rate are not the same for every simulation but are available to the participating strategies at the beginning of each simulation. At the end of each round the subjects receive private information about how their strategy predicts in the simulations. The private information consists of a page with the programmed version of their strategy (programmed in Borland Pascal) and five pages containing for each subject five randomly chosen simulations in which his or her strategy has participated. They also receive public information, namely information given to all participants, about the ranking of the strategies by mean quadratic forecasting error. One week after they receive this information a new strategy has to be submitted.

We recognized that a similar incentive structure as in the introductory experiment and the asset pricing experiments of Chapter 7 in the strategy experiment might stimulate the subjects to cooperate and share their strategies and private information. For example, all subjects benefit from a fast convergence to the fundamental price. In the strategy experiment we therefore employ a tournament incentive structure: payment is based upon relative performance of the strategies. The performance is based upon the average
squared prediction error of the strategy over all simulations in that round. The participant with the strategy with the smallest average quadratic forecasting error receives 50 guilders (approximately 22.70 Euro) in rounds 1, 2 and 3. In the final round three prizes of 250, 150 and 50 guilders (113.60, 68.20 and 22.70 Euro respectively) are awarded. In addition to this students receive a flat fee of 5 guilders (2.25 Euro) for each strategy they submit.

A possible disadvantage of payments based upon relative performance is that subjects may try to destabilize markets in order to make it harder for the other market participants to forecast prices. However, it is easy to see that this cannot work. If all strategies predict the fundamental price except for one strategy that tries to destabilize the market by predicting a higher (lower) price, the realized price will be higher (lower) than the fundamental price but since the price is determined by the average of all predictions the price will always be closer to the fundamental price than to the prediction of the strategy that tries to destabilize the market. A single destabilizing strategy will thus end up with a larger quadratic prediction error than the other strategies in that market. Even more importantly, one can only affect realized prices in the market in which the strategy participates; an increasingly unstable market will cause a comparative advantage of the strategies active in the other markets. No subject ever mentioned (in the questionnaires\textsuperscript{1} or in class after the experiment ended) that he or she had tried to destabilize markets.

**Final experiment**

After subjects submit their final, fourth round strategy, but before they receive the final results we conduct another laboratory experiment. The main goal of this experiment is to investigate whether the strategies are a good representation of the actual behavior of the subjects in a laboratory experiment. In the final experiment the subjects are informed that they are in a market with five strategies from the third round. Since the participants have already submitted their fourth round strategy but did not receive the results of the fourth round simulations yet, we can compare their fourth round strategy with their behavior in the final experiment\textsuperscript{2}. For every participant we conduct two sequential markets of

\textsuperscript{1}Subjects fill in a small questionnaire every round. In this questionnaire they are asked about (among other things) their considerations when changing their strategy, the effect of the feedback upon their new strategy, whether they had talked with other subjects about the experiment and how well they thought their new strategy would perform.

\textsuperscript{2}The participants are informed about how their third round strategy predicts in round 3. They have no information about how their fourth round strategy predicts in the fourth round. Therefore their predictions in the final experiment are based upon the same information as their fourth round strategy. The only difference is that in the experiment the participants are more flexible since they can update and change their prediction strategies during the two markets of 50 periods each.
50 periods. A market consists of one participant and five third round strategies. The
five strategies that participate in each market are randomly drawn from the submitted
third round strategies. The fundamental price in the first market is different from the
fundamental price in the second market but in each of the two markets for all participants
the same. For the final experiment we also use the tournament structure for the earnings.
The winner, that is, the subject with the lowest average quadratic error over the two
markets, earns 100 guilders, the runner up earns 95 guilders etc..

8.4 Results

This section reports the results of the strategy experiment. We first look at the specific
characteristics of the strategies that are submitted. After that we will investigate the short
run dynamics in Section 8.4.2, the medium run price dynamics in Section 8.4.3 and the
long run price dynamics in Section 8.4.4. In Section 8.4.5 we investigate the behavior of
particular strategies in the simulations and in particular the effect of individual strategies
on the convergence of the market price. Finally in Section 8.4.6 we investigate whether
the submitted strategies are a good description of the behavior of the participants in an
experimental setting.

8.4.1 What kind of strategies do the participants use?

In most studies on economic dynamics researchers make assumptions about the expecta-
tions of the agents in the model. Usually the agents are assumed to be rational or
boundedly rational using a simple prediction rule. In this study we are interested in what
kind of strategies/prediction rules subjects actually use. In this section we will focus
on the general characteristics of the submitted strategies. We will not try to describe
every strategy separately in detail since the strategies differ a lot. Table 8.1 summarizes
the main characteristics of the strategies. In total 73 strategies are submitted. About
one quarter of all submitted strategies is continuous. The strategies are continuous if a
small change in the realized price will also result in a small change of the prediction. If a
strategy is not continuous it is in almost all cases conditional. The conditions on which
these strategies depend differ from past prices, past predictions, past prediction errors to
the mean dividend or the interest rate. An example of a conditional strategy is “if the

\[^3\text{In rounds 3 and 4 a couple of subjects only made small changes in their strategy.}\]
\[^4\text{The strategy of subject 6 in round 2 is the only strategy that is both conditional and continuous.}\]
\[^5\text{This strategy is conditional on the mean dividend. Given this initial condition on the mean dividend the}\]
\[^6\text{strategy is continuous.}\]
price in period \( t - 3 \) is smaller than the price in period \( t - 2 \) my prediction is ... if not my prediction is...". About 75% of all strategies is conditional. In 52 strategies the mean dividend \((\bar{y})\) is included; in most of these cases the interest rate is then also included. In only seven strategies the fundamental price \( p^* = \bar{y} \) is used for the prediction. In six of these strategies the fundamental price is only used in predicting the first and/or second price. Not a single participant had the fundamental price as its prediction for periods \( 3 - 50 \). All strategies include the price of two periods back (this is the last observed price). Almost 40% of all strategies is following a “trend”. For example if the last observed price, \( p_{t-2} \), was higher/lower than the price \( p_{t-3} \) the prediction of the price will be the last observed price plus/minus a term, which differs per individual\(^5\). There are even two strategies anticipating on cyclic behavior trying to detect a cycle. If the strategy observes a cycle, say of period 8, the strategy predicts the price eight periods back. The strategies get more complicated (measured by the total lines of program needed) until round 3; in round 4 there is a (slight) decrease of the complexity.

\(^5\)An example of a trend following strategy is strategy 14 in round 3: If \( p(t-2) < p(t-3) \) THEN \( V[T] = p(t-2) - \sqrt{\frac{1}{8} \sum_{k=3}^{6} (p(t-k) - p(t-k-1))^2} \) ELSE \( V[T] = p(t-2) + \sqrt{\frac{1}{8} \sum_{k=3}^{6} (p(t-k) - p(t-k-1))^2} \).
8.4.2 Short run dynamics

In this section we will study the winning strategies and the price behavior in the first 50 periods. We will refer to the first 50 periods as the short run dynamics. The incentives of the subjects and the ranking of the strategies are based upon the first 50 periods. Therefore the first 50 periods are of special interest. Figure 8.1 shows the average absolute distance, $|p_t - p^f|$, between the realized market price and the fundamental price for the first 50 periods for each of the four rounds averaged over 1296 simulations. We find that the average prices in the first round are close to the fundamental price (average absolute distance around 20).

![Figure 8.1: Average absolute distance from the fundamental averaged over 1296 simulations. All 81 pairs of mean dividend, $\bar{y}$, and interest rate, $r$, were simulated 16 times.](image)

In round 2 we see a slow increase in this distance in the first periods but then after period 10 we observe a fast increase until period 30 where the average price distance is about 200 from the fundamental. From period 30 on the average absolute distance remains more or less constant with prices far from the fundamental price. Another obvious result is that in rounds 3 and 4 the distance increases until period 13. After period 13 the distance to the fundamental price in both rounds 3 and 4 is more or less constant. In round 4, after period 13, the mean distance seems to gradually decrease over time. At period 50 the distance to the fundamental in round 4 is small and close to the distance in round 1. Furthermore, notice that the average absolute distance to the fundamental in the first
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period is almost the same in every round⁶.

Note that Figure 8.1 only gives us the average distance to the fundamental price and does not give much information about how well the strategies predict (since the strategies had to predict the realized price and not the fundamental price). Therefore we have to study the strategies more carefully. We will now investigate the prediction errors of the strategies, the price behavior and the winning strategies. Table 8.2 summarizes the mean quadratic forecasting error, the mean quadratic forecasting error of the winner, the mean variance of the prices and the winning strategies⁷.

<table>
<thead>
<tr>
<th>Round</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean quadratic forecasting error per period</td>
<td>87.54</td>
<td>7148.75</td>
<td>6486.76</td>
<td>1077.19</td>
</tr>
<tr>
<td>Mean quadratic error winner per period</td>
<td>20.64</td>
<td>659.58</td>
<td>965.76</td>
<td>600.79</td>
</tr>
<tr>
<td>Sample mean first 50 prices</td>
<td>56.99</td>
<td>192.24</td>
<td>101.34</td>
<td>78.65</td>
</tr>
<tr>
<td>Sample mean standard deviation first 50 prices</td>
<td>4.96</td>
<td>86.74</td>
<td>34.42</td>
<td>22.34</td>
</tr>
</tbody>
</table>

Winning strategy

<table>
<thead>
<tr>
<th>Subject</th>
<th>12</th>
<th>17</th>
<th>11</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_t^f = )</td>
<td>( \frac{p_{t-2} + p_{t-3} + p_{t-4}}{3} )</td>
<td>( p_t^f = \frac{p_{t-2} - \frac{10}{16}}{p_{t-2} + \frac{p_{t-3}}{2}} )</td>
<td>( p_t^f = \frac{p_{t-2}}{2} + 2(p_{t-2} - p_{t-3}) )</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.2: Results of the simulations (1296 simulations per round). In each simulation a market is formed with 6 randomly drawn strategies. The mean quadratic forecasting error is the average quadratic forecasting error over all strategies and all periods. The third and fourth row report the sample mean and the sample mean standard deviation of the realized prices over all simulations. The last row presents the prediction strategies of the winner. To limit the size of the table we only report the most relevant part of the strategy.

⁶An explanation for this may be that in each round the predictions in the first two periods have to be between 0 and 100

⁷All winning strategies are conditional and therefore too complicated and too long to report in the table. We report that part of the strategy for which the condition (for example an "if" condition) was satisfied most of the time.
The mean quadratic error in the first round is the smallest and the mean quadratic error in the second round is largest. After round 2 the mean quadratic error decreases considerably but at round 4 it is still much larger than in round 1 (around 12 times as large). Notice that, the mean quadratic error of the winner in round 2 is smaller than the mean quadratic error of the winner in round 3. The other strategies in round 2 perform much worse than in round 3.

From Figure 8.1 we saw that the prices in round 2 are on average far from the fundamental. Table 8.2 shows similar results. The sample mean price in round 2 is much higher than the sample mean prices in the other three rounds. Furthermore, from the fourth row of the table we find that the mean variance of the first 50 prices is lowest in round 1 and highest in round 2. There is little volatility in the market prices in round 1 compared to the volatility of the prices in round 2. In rounds 3 and 4 the prices are less volatile than in round 2 but still much more volatile than in round 1. The last row of Table 8.2 shows the winning strategies. The winning strategy in round 1 predicts the mean price over the last three periods. In round 2 an adaptive expectations rule with adaptation weight of 0.5 is the winning strategy. The winning strategy in round 3 is naive expectations minus a small constant depending on the interest rate. In round 4 the winner has a trend following strategy (conditional on $P[T-3] < P[T-2]$). Hence, in the relatively tranquil market the winning strategy predicts the mean price while in the very volatile market the winning strategy is an adaptive strategy. Furthermore, in the slightly volatile markets (rounds 3 and 4) the winning strategies are a trend following strategy and a naive strategy.

Table 8.3 shows the percentages of convergence over the rounds in the short run. We use the following measure of convergence: a price sequence converges if the maximum

<table>
<thead>
<tr>
<th>Short run convergence, sample period 25-50.</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>non-convergence</td>
</tr>
</tbody>
</table>

Table 8.3: Percentage of convergence to a steady state, period 2-cycle, cycles of periods 3-5 and non-convergence (i.e non-convergence and convergence to period 6 or higher-order cycles) of 1296 simulations, based upon periods 25-50.
difference between the highest and lowest price over the last 25 periods is smaller than 0.1. A similar criterion is used for cycles; for example in a 2-cycle the prices in all odd periods are within a 0.1-point range and the prices for all even periods are within a 0.1-point range (and we do not observe a steady state). The percentage of convergence to a steady state is very low, and decreases over the rounds from 6.9% in the first round to 1.8% in the fourth round. Furthermore, the percentage of non-convergence increases from 88.4% in round 1 to 98.1% in round 4. Although we would expect an increase in the percentage of convergence and a decrease of non-convergence over the rounds we observe the opposite.

In this section we found that the distance between the price and the fundamental price increased by a factor of almost 10 from round 1 to round 2. After round 2 the distance decreased but was in round 4 still larger than in round 1. A tentative explanation why round 2 is much worse than round 1 may be the following. The majority of the strategies in round 1 are cautious. However, in round 2, the exploration of the patterns in past prices leads to large deviations from the fundamental price due to overreaction. In rounds 3 and 4 the strategies get more cautious again due to the bad performance in round 2. Another striking result is the large difference between the mean standard deviation of prices in round 2 and round 1; the prices in round 2 are further away from the fundamental price, the sample mean of the prices is almost 200 in round 2 compared to 60 in round 1, and the prices in round 2 are also more volatile. In the last part of this section we found that there is almost no convergence to a steady state. Furthermore, the percentage of non-convergence increases over the rounds. An important question that remains is: how can we explain the low percentages of convergence in the short run?

8.4.3 Medium run dynamics

In the short run there is little convergence to a steady state. A question we therefore address in this section is: is the low percentage of convergence due to an inherently unstable steady state, or is it due to a slow speed of convergence to a stable steady state. In this section we will therefore study the medium run dynamics, i.e. the first 200 periods. Table 8.4 summarizes percentages of convergence to a steady state, convergence to a cycle and non-convergence in the medium run. From now on we use a slightly different measure of convergence than in the short run: A price sequence converges if the maximum difference between the highest and lowest price over the last 50 periods (instead of the 25
periods in the short run) is smaller than 0.1\(^8\). From Table 8.4 we find that in the medium run the convergence of the price to a steady state is highest in the third round and lowest in the first round while the non-convergence is highest in the first round and lowest in the second round. Table 8.4 shows that in rounds 1 and 2 the percentage of convergence to periodic cycles is still above 20\% (with about 14\% of 2-cycles). Moreover, in rounds 3 and 4 this fraction has decreased to about 5\% and 1.6\%, respectively. Furthermore, in round

<table>
<thead>
<tr>
<th>Medium run convergence, sample period 151-200</th>
<th>round 1</th>
<th>round 2</th>
<th>round 3</th>
<th>round 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>steady state</td>
<td>24.31%</td>
<td>43.13%</td>
<td>54.78%</td>
<td>49.31%</td>
</tr>
<tr>
<td>2 cycle</td>
<td>14.12%</td>
<td>13.73%</td>
<td>0.00%</td>
<td>0.15%</td>
</tr>
<tr>
<td>3 cycle</td>
<td>0.23%</td>
<td>0.15%</td>
<td>0.08%</td>
<td>0.08%</td>
</tr>
<tr>
<td>4 to 10 cycle</td>
<td>7.03%</td>
<td>9.12%</td>
<td>4.86%</td>
<td>1.39%</td>
</tr>
<tr>
<td>non-convergence</td>
<td>54.32%</td>
<td>33.87%</td>
<td>40.28%</td>
<td>49.07%</td>
</tr>
</tbody>
</table>

Table 8.4: Percentage of convergence to a steady state, a period 2-cycle, cycles of periods 3-10 and non-convergence (i.e non-convergence and convergence to periods 11 or higher-order cycles) of 1296 simulations, based upon the periods 150-200.

4 about 50\% of all simulations has settled down to a steady state in round 4, but also almost 50\% of all simulations does not converge within 200 periods. After 200 periods we observe much more convergence than after 50 periods. In the next section we will report the long run dynamics and study whether for some markets it takes even more than 200 periods for prices to converge to a steady state.

### 8.4.4 Long run dynamics

In the previous section we have shown that the percentage of convergence to a steady state is increasing over the rounds, but there are still about 50\% non-converging price sequences in round 4. One explanation for this phenomenon is that the steady state is stable but the learning process converges only slowly. Another explanation can be that the steady state is intrinsically unstable and that prices will never converge to the steady state. To investigate which of the scenarios explains the properties in the medium and

\(^8\)Our results do not change much if we take another measure of convergence. For example, if for convergence, the difference between the highest and lowest price must be smaller than 1 (instead of the 0.1) we find the same amount of convergence to a steady state and with a small period (smaller than 4) cycles. We do on the other hand observe more convergence to higher periods and therefore less non-convergence.
short run dynamics we study the long run dynamics. The long run dynamics is defined here as 1000 periods.

Table 8.5 shows the results for the long run dynamics. Convergence is defined in the same way as in the medium run analysis of Section 8.4.3, that is, a price sequence converges if the maximum difference between the highest and lowest price over the last 50 periods is smaller than 0.1. In Table 8.5 we also make a distinction between convergence to the fundamental steady state or to a steady state different from the fundamental price. We find that as the convergence to a steady state increases over the rounds the convergence to the fundamental price also increases. In the long run the majority of the simulations converges to a steady state. Where in round 1 we only observe approximately 40% convergence to a steady state, in round 4 the convergence has doubled to 80%. However, most of the simulations converging to some steady state, do not converge to the fundamental steady state. In round 4, 40% of all simulations converges to a steady state different form the fundamental steady state.

If we compare these results with the results from the medium run, we find that in the long run there is more convergence to a steady state and less non-convergence. Many of the non-converging sequences in the medium run have dissolved and converged in the long run. Hence, the speed of convergence is slow.
In this section, like in Section 5.3.4, we are also specially interested in the non-convergence and the possible occurrence of chaos and strange attractors. We apply the Wolf algorithm (Wolf et al. (1985)) to estimate the largest Lyapunov exponent over the period 1-1000. A positive Lyapunov exponent implies that the system exhibits sensitive dependence upon initial conditions and is chaotic (see Chapter 5 footnote 10). We find that almost 70% of the non-converging price sequences has a positive Lyapunov exponent, i.e. 16.95% of all simulations has a positive Lyapunov exponent. In contrast with Chapter 5, in this experiment we thus find a relatively low fraction of price sequences with a positive Lyapunov exponents. A possible explanation for this lower fraction of chaotic markets may be that the asset market is stable under naive expectations. In such a stable market chaos can only arise when there are strong trend extrapolating strategies.

We will now focus on the sequences that converge to a steady state price. We investigate whether these sequences converge to the fundamental price or to another steady state. Figure 8.2 shows the frequency distribution of the distance between the steady state price and the fundamental price after 1000 periods. Figure 8.2 shows that most of the price sequences converge to a price close to the fundamental price. Especially in round 2 there are a lot of sequences converging to a price far from the fundamental price (Figure 8.2 (b)). In round 1 the maximum distance of the steady state prices to the fundamental steady state price tends to be less than 10. In round 2 the maximum distance increases. A relatively large part of the steady state prices is far from the steady state. There are two peaks with prices far from the fundamental steady state, one at distance 400 and one at 700. The maximum distance in rounds 3 and 4 for most price sequences is again relatively small, 6 and 3 respectively. In 11% of all simulations in round 1 the sequence converges\(^9\) to the fundamental price. In round 2 12% of all simulations converges to the fundamental while in round 3 and 4 these percentages are 21% and 40%, respectively. In round 2 the standard deviation of the distance between the steady state price and the fundamental price is large, 300. The standard deviation in round 1, 3 and 4 is much smaller, 8.25, 2.02 and 1.40, respectively.

Summarizing, we find that a large fraction of the simulations converges to the fundamental price. For round 2 we find convergence to a steady state far from the fundamental.

---

\(^9\)We use the same definition of convergence as in Section 8.4.3, i.e. a price sequence converges to a steady state if the maximum difference between the highest and lowest price over the last fifty periods is smaller than 0.1. However, the percentage of convergence to the fundamental steady state is measured differently. A price sequence converged to the fundamental steady state if the price sequence converged and the distance to the fundamental price is smaller than 1 point. That is, for a fundamental steady state price of 50 all sequences that converged to a steady state price in the interval [49, 51] converged to the fundamental steady state.
Figure 8.2: Frequency distribution of the difference between the steady state price, at $t = 1000$, and the fundamental price. In round 1 500 simulations converged to a steady state, in round 2, 3 and 4 741, 781 and 1038 simulations converged to a steady state. The distance is zero if the price converges to the fundamental price $p^f = \frac{2}{r}$.

The fact that the underlying dynamics may have a steady state attractor far from the steady state may explain why in the short run the absolute distance to the fundamental steady state is large and why the mean squared forecasting error and the sample variance of prices are so high in round 2. Nevertheless a large part of the simulations converges to a price close to the fundamental. The fraction of convergence increases over the rounds and is even 80% in round 4. Furthermore, a small fraction of the price sequences has a positive Lyapunov exponent, implying sensitive dependence on initial conditions. In the next section we will investigate whether there are strategies that ‘destabilize’ the market. That is, if we remove a strategy that does not predict very well from the simulations will
we then observe more or less convergence or is there no difference?

8.4.5 Homogeneous strategy simulations.

In the previous section we found that about 20% of the price sequences does not converge within 1000 periods. On average only approximately 20% of all simulations converges to the fundamental steady state price. In this section, we study the cause of this non-convergence. We try to answer the following questions: "Is most of the non-convergence caused by the interaction of strategies or are there a couple of strategies that 'destabilize' the markets?" and "If a strategy does not converge in a homogeneous market, does such a strategy then contribute to the non-convergence in the heterogeneous simulations?". We first report the findings of the homogeneous simulations, i.e. simulations where the prediction of one strategy determines the price or, equivalently, all six prediction strategies are the same. Every round we simulate for all submitted strategies 81 markets (9 different values of the interest rate and 9 different values of the mean dividend). From Table 8.6 we find that the percentage of convergence to a steady state price, after 1000 periods, is almost constant over the rounds. On average in every round about 60% of the markets converge to some steady state price. The convergence to the fundamental steady state is approximately 40%. Notice that in round 2 the convergence to a non-fundamental steady state is 30%, much higher than in rounds 1, 3 and 4. The percentage of non-convergence is more or less constant over the rounds, ranging from 21% to 28%. There is hardly

<table>
<thead>
<tr>
<th>Convergence of homogeneous simulations.</th>
<th>round 1</th>
<th>round 2</th>
<th>round 3</th>
<th>round 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>fundamental</td>
<td>43.68%</td>
<td>42.69%</td>
<td>48.29%</td>
<td>39.89%</td>
</tr>
<tr>
<td>non-fundamental</td>
<td>19.17%</td>
<td>30.15%</td>
<td>16.99%</td>
<td>19.06%</td>
</tr>
<tr>
<td>steady state</td>
<td>62.85%</td>
<td>72.84%</td>
<td>65.29%</td>
<td>58.95%</td>
</tr>
<tr>
<td>2 cycle</td>
<td>9.64%</td>
<td>0.13%</td>
<td>0.44%</td>
<td>0.46%</td>
</tr>
<tr>
<td>3 cycle</td>
<td>0.18%</td>
<td>5.33%</td>
<td>5.95%</td>
<td>6.33%</td>
</tr>
<tr>
<td>4 to 10 cycle</td>
<td>1.17%</td>
<td>0.32%</td>
<td>0.43%</td>
<td>7.41%</td>
</tr>
<tr>
<td>non-convergence</td>
<td>26.16%</td>
<td>21.38%</td>
<td>27.89%</td>
<td>26.85%</td>
</tr>
</tbody>
</table>

Table 8.6: Percentage of convergence to a steady state, a period 2-cycle, cycles of periods 3-10 and non-convergence (i.e non-convergence and convergence to periods 11 or higher-order cycles) of the price within 1000 periods. Total number of simulations changes over the rounds since the number of submitted strategies changes. Each strategy is simulated 81 times for 9 different values of mean dividend and 9 different values of the interest rate.
any convergence to cycles of period 4-10 although there are some cycles of period 2 or 3. If we compare the long run convergence from Table 8.5 with the convergence of the homogeneous simulations from Table 8.6 we find that the percentage of convergence in rounds 1 and 2 is much higher in the homogeneous simulations. Furthermore, in rounds 1 and 2 the convergence to periodic cycles is much lower in the homogeneous simulations and the percentage of convergence to a steady state in round 4 is much lower.

Table 8.7 shows the non-converging strategies in the homogeneous simulations, i.e. the strategies for which there are simulations that do not converge in a specific round and the number of times that it does not converge. In the last column of Table 8.7

<table>
<thead>
<tr>
<th>Strategy</th>
<th>round</th>
<th>1</th>
<th>5</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>13</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>81</td>
<td>1</td>
<td>81</td>
<td>10</td>
<td>58</td>
<td>0</td>
<td>78</td>
<td>81</td>
<td>52</td>
<td>0</td>
<td>445</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>81</td>
<td>1</td>
<td>0</td>
<td>x</td>
<td>30</td>
<td>77</td>
<td>0</td>
<td>61</td>
<td>53</td>
<td>81</td>
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</tr>
<tr>
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<td>3</td>
<td>81</td>
<td>x</td>
<td>0</td>
<td>x</td>
<td>30</td>
<td>77</td>
<td>0</td>
<td>61</td>
<td>53</td>
<td>81</td>
<td>384</td>
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<tr>
<td></td>
<td>4</td>
<td>71</td>
<td>1</td>
<td>0</td>
<td>x</td>
<td>30</td>
<td>77</td>
<td>0</td>
<td>38</td>
<td>50</td>
<td>81</td>
<td>348</td>
</tr>
</tbody>
</table>

Table 8.7: For every strategy a homogeneous market is simulated 81 times. The first row shows the “non-converging strategies”. These are the strategies for which there are simulations that do not converge within 1000 periods. The second to the fifth row show for each round the number of times these strategies do not converge. The last column shows the percentages of non-convergence. The total number of simulations changes over the four rounds since the number of submitted strategies changes from 21 in the first to 16 in the fourth round. The last column shows the total number of markets that do not converge and the subsequent percentage of the total number of simulated markets.

the total number of non-converging simulations and the percentage of non-convergence is shown. From the table we find that there are a number of strategies that almost never converge in the homogeneous simulations. Note that convergence of a strategy in this homogeneous setting does not mean that the strategy is also converging in the heterogeneous setting. The strategies that are not listed in the table converge in every simulation. The strategies of subjects 1, 13 and 20 almost never converge. The strategies of Table 8.7 perform relatively poorly; most are in the bottom half of the ranking of the short run simulations. Only five of these 28 non-converging strategies end up in the top part of the ranking. Strategy 19 in round 1 is the winning strategy in that round and the third and fourth place are taken by strategies 11 and 18. In round 3 the winning strategy is strategy 11 which in 30 out of 81 simulations does not converge. From these results it is hard to conclude that the non-convergence in the heterogeneous markets is
caused by these non-converging strategies. For the next simulations we excluded some of these non-converging strategies to investigate whether the convergence increases without such a non-converging strategy. We excluded those strategies that participated in non-converging markets more than average (see Table 8.C.13 in Appendix 8.C). In round 1, for example, we find that strategies 9 and 17 are more than average in non-converging simulations. In round 2 strategy 13 converges only in 70% of the simulations while the average that round is 87%. Strategies 9 and 13 in round 3 also converge less than average to a steady state. In round 4 all strategies perform almost the same. In total we find nine ‘destabilizing’ strategies. These strategies belong to five participants\(^{10}\). Table 8.8 shows the percentages of convergence and non-convergence in the long run, where we excluded the destabilizing strategies (the bold faced strategies in Table 8.C.13). That is, we exclude the strategies that contributed most to the non-convergence in the long run simulations\(^{11}\). From Table 8.8 we find that the percentage of convergence to a steady state increases over the rounds. In round 1 the percentage of convergence to a steady state is 62.11% while in round 4 the percentage has increased to 90%. The percentages of

\(^{10}\)In round 1 we excluded three strategies, 9, 11 and 17. Strategies 13 and 19 were excluded in round 2. In round 3 and 4 strategies 9 and 13 were excluded.

\(^{11}\)We first excluded the strategy with the lowest percentage of convergence. After this we examined the results again and simulated with excluding one more strategy. If excluding one strategy resulted in more convergence to a steady state and less non-convergence we excluded that strategy. We followed this procedure until the moment where the results did not change anymore.

<table>
<thead>
<tr>
<th>Convergence of long run dynamics, sample period 951-1000.</th>
<th>round 1</th>
<th>round 2</th>
<th>round 3</th>
<th>round 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>fundamental</td>
<td>24.07%</td>
<td>10.57%</td>
<td>21.68%</td>
<td>42.67%</td>
</tr>
<tr>
<td>non-fundamental</td>
<td>38.04%</td>
<td>53.78%</td>
<td>52.93%</td>
<td>47.14%</td>
</tr>
<tr>
<td>steady state</td>
<td>62.11%</td>
<td>67.13%</td>
<td>74.61%</td>
<td>89.81%</td>
</tr>
<tr>
<td>2 cycle</td>
<td>22.76%</td>
<td>21.45%</td>
<td>0.08%</td>
<td>0.08%</td>
</tr>
<tr>
<td>3 cycle</td>
<td>0.00%</td>
<td>0.08%</td>
<td>0.08%</td>
<td>0.15%</td>
</tr>
<tr>
<td>4 to 10 cycle</td>
<td>7.03%</td>
<td>10.03%</td>
<td>11.57%</td>
<td>0.85%</td>
</tr>
<tr>
<td>non-convergence</td>
<td>8.10%</td>
<td>1.31%</td>
<td>13.66%</td>
<td>9.03%</td>
</tr>
</tbody>
</table>

Table 8.8: Percentage of convergence to a steady state, a period 2-cycle, cycles of periods 3-10 and non-convergence (i.e non-convergence and convergence to periods 11 or higher-order cycles) of the prices within 1000 periods of 1296 simulations without the ‘destabilizing’ strategies.
convergence to the fundamental price have increased a little in comparison with Table 8.5. However, the percentage of non-convergence has decreased enormously. In rounds 1, 3 and 4 the percentage of non-convergence is close to 10%. Note that in round 2 the percentage of non-convergence is almost zero. In round 4 there is almost no convergence to periodic cycles while in rounds 1-3 we still find 30%, 31% and 10% convergence to periodic cycles, respectively.

From the previous section we found that although some strategies do not converge in the homogeneous simulations this does not mean that these strategies are the main cause of the non-convergence. We even found some non-converging strategies that performed very well (based upon quadratic forecasting error) in the long run dynamics simulations. A better way to find the destabilizing strategies is by looking at the non-convergence of a strategy as a percentage of the total number of simulations a strategy participates in. By excluding the strategies that performed bad compared to the other strategies the percentage of convergence to a steady state increased. Furthermore the percentage of non-convergence decreased. In round 2 the percentage of non-convergence after excluding only two strategies is reduced to 1%. There are some strategies ('rotten apples') that contribute more than average to the non-convergence, by excluding these strategies from the simulations the percentage of convergence increases. However, when removing these unstable strategies the percentage of convergence to the fundamental steady state only increases a bit. The remaining non-convergence is probably caused by the interaction of the strategies.

8.4.6 Final Experiment

After the participants submit their fourth round strategy and before they receive the fourth round results we conduct a final laboratory experiment. The main goal of this final experiment is to study the relationship between actual behavior of subjects and the strategy they submit. Since the participants did not yet receive the fourth round results, the information in the final experiment is the same as for the third round strategies. We can, therefore, compare their predictions in the experiment with the third round predictions of their strategies. Just like in the introductory experiment in the final experiment we conduct two markets. The fundamental price in the first market is \( P^* = 60 \) while the fundamental price in the second market is \( P^f = 40 \). We employ a tournament incentive structure. The participant with the lowest average quadratic error over the two markets
markets, earns 100 guilders the second lowest earns 95 guilders etc.. Table 8.9 shows the average quadratic error of the participants (columns two to four) and the average quadratic error of the strategies (columns five to seven). A striking feature of Table 8.9 is that the strategies and the participants have a qualitatively similar average prediction error. More in particular large errors by the participants coincide with large errors by their strategies. This suggests that the strategy and the participants make on average the same mistakes. In 23 out of 30 cases the participants average quadratic prediction error is smaller than the error of the strategy. From the fourth column we find that participant 4 has the smallest average quadratic prediction error over the two parts of the experiment and therefore won the prize of 100 guilder (45 Euro); participant 1 was runner up and earned 95 guilders (43 Euro).

Table 8.10 shows the average quadratic distance per round between the strategy and the prediction of the participant. If this average quadratic distance is zero, the participant
in the experiment makes exactly the same prediction as his strategy. If the quadratic distance is larger than zero either the participant uses his strategy as a rule of thumb or he does something completely different. From the table we find that in 6 out of 30 cases

<table>
<thead>
<tr>
<th>Subject</th>
<th>quadratic distance</th>
<th>Total average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Part</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0.14</td>
<td>3.04</td>
</tr>
<tr>
<td>2</td>
<td>22.97</td>
<td>23090.18</td>
</tr>
<tr>
<td>4</td>
<td>1.19</td>
<td>6.18</td>
</tr>
<tr>
<td>7</td>
<td>2822.66</td>
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</tr>
<tr>
<td>9</td>
<td>28.01</td>
<td>7.86</td>
</tr>
<tr>
<td>11</td>
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<td>13</td>
<td>1.94</td>
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</tr>
<tr>
<td>15</td>
<td>0.7</td>
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</tr>
<tr>
<td>16</td>
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</tr>
<tr>
<td>17</td>
<td>0.37</td>
<td>38.13</td>
</tr>
<tr>
<td>18</td>
<td>5.4</td>
<td>7.9</td>
</tr>
<tr>
<td>19</td>
<td>7.23</td>
<td>12.25</td>
</tr>
<tr>
<td>20</td>
<td>9289.39</td>
<td>45.3</td>
</tr>
<tr>
<td>21</td>
<td>125.01</td>
<td>11.4</td>
</tr>
</tbody>
</table>

Table 8.10: The first column shows the participants number. The second to the fourth column show the average quadratic distance per period between the prediction of the strategy and the participants prediction in the two parts of the final experiment and the average total quadratic distance per period.

the average quadratic distance is larger than 965.76 per period\(^\text{13}\). The other strategies are on average close to the predictions of the participants. Table 8.11 shows the ranking of the participants and of the strategies based on the average quadratic prediction error. The difference between the ranking of the participant and the ranking of its strategy is never more than 2. That is, the strategy of the subjects who perform good/bad also perform good/bad. The Pearson correlation between the total average quadratic errors of the participant in the final experiment (column 4 in Table 8.9) and the total average quadratic errors of the strategy (column 7 in Table 8.9) in the experiment is 0.80. The prediction errors of the participants and their strategies are thus strongly correlated. The Pearson correlation of the rankings (the correlation between row 2 and row 3 in Table 8.11) is 0.96, i.e. the rankings after the final experiment of the strategies and of the subjects

\(^{13}\text{The mean quadratic error of the winner in round 3 equals 965.76.}\)
Table 8.11: The second row shows the ranking of the subjects based upon the average quadratic forecasting error in the final experiment, whereas the third row shows the ranking for the strategies. The difference between these rankings never exceeds 2.

is high and almost perfect. Another way to measure whether the strategy is a good representation of the participants behavior is to count the times that the strategy and the participant adapt their prediction in the same direction\textsuperscript{14}.

Table 8.12: The second row (sign perc.) shows the fraction that the participant adapts his prediction in the same direction as his strategy. The third row (pred. corr.) shows the average correlation (over the two parts) between the strategies predictions and the subjects predictions over 50 periods.

From Table 8.12 we find that 9 out of 15 participants adapt their prediction in the same direction as its strategies in more than 85\% of the times. From the third row of Table 8.12 we see that the correlation between the predictions of the strategy and the predictions of the participants is high. Table 8.12 summarizes the average correlation, over the two parts, between the prediction of the participant and its strategy. Participants 1 and 4 both have a correlation of 98\%, i.e. almost perfect correlation. This suggests that strategies are a fairly good representation of participants forecasts.

In summary, in this section we showed that the forecasting errors of the participants in the final experiment seem to be of the same magnitude as the errors of their strategies. The strategies seem to be a fairly accurate description of the actual forecasts of the participants in the experiment.

8.5 Concluding remarks

In this chapter we have reported the results of a strategy experiment. One of the goals of this study is to investigate what kind of prediction strategies participants use in an

\textsuperscript{14}We count the times that the prediction, $p_f$, is higher/lower than the previous prediction, $p_{f-1}$ for both the participant and the strategies.
experimental setting. We find that only few participants use simple linear strategies. Most of the participants submit complicated strategies and the strategies become more complicated over the rounds. Furthermore, the strategies in the first round predicted the market prices best.

We show that in the short run (within 50 periods) the prices have not yet converged to either the fundamental price or another steady state price or periodic cycle. In round 1 the distance to the fundamental is fairly small, the distance is much larger in round 2 and decreases again in rounds 3 and 4. However, in round 4 the distance to the fundamental is still larger than in round 1. A tentative explanation for this is that the strategies in round 1 are cautious but in round 2, while trying to improve predictions, the strategies become less cautious and there is more overreaction. As a result of the bad performance in round 2, the strategies in rounds 3 and 4 become more cautious again. Prediction errors decrease and prices are closer to the fundamental price.

We find that the number of converging price sequences increases over the rounds in both the medium and the long run. In the medium run the prices convergence to a steady state or periodic cycles but this percentage is not very high. However, in the long run in round 4 almost 40% of the price sequences converge to the fundamental value. Another 40% also converges to a steady state price but not to the fundamental value. These results suggest learning of the subjects over the rounds.

Whether individual strategies attribute to market stability or instability is difficult to say. A couple of strategies never converge in the homogeneous simulations. But whether these strategies cause the non-convergence in the normal simulations is hard to prove since some of the “non converging strategies” are doing very well in the normal heterogeneous agents simulations. Excluding destabilizing strategies from the simulations results in an increase of convergence to some steady state. However, the percentage of convergence to the fundamental value only slightly increases.

From the final experiment we find that the correlation between the strategies prediction and the participants forecast is high. Also the ranking of the strategies and the ranking of the participants based on the quadratic forecast errors are almost the same. This suggests that the strategies prediction is ‘close’ to the forecast of the participant. Furthermore, the average quadratic distance between the prediction of the strategy and participant is small. Together with the high percentages of adaptation in the right direction and the high correlation between the strategies and the predictions of the participants we conclude that the strategies are a good representation of what participants do in an experiment.

Summarizing, in round 1 only a small fraction (7%) of the prices converges to the fundamental. However, the realized market prices do not move too far from the funda-
mental. In contrast, in round 2 we find more convergence but the realized market prices are further away from the fundamental. This is probably due to an overreaction of the strategies to improve prediction accuracy. In rounds 3 and 4 the convergence results improve somewhat. The percentage of convergence to a steady state increases to 80% in round 4. More importantly, the convergence to the fundamental steady state is only 40%, in the long run, most markets converge to a steady state close to the fundamental, but the convergence is slow. Moreover, in a dynamic asset pricing model it turns out to be hard to learn the correct price level as determined by the underlying fundamentals.
8.A Instructions for formulating a strategy (translated from Dutch)

Contents of information-folder

- Time schedule
- Earnings
- Instructions for making strategies
  - The situation and available information
  - How to formulate a strategy
- How do the simulations work
- Examples of simulation results that you receive
- First round strategy form
- Questionnaire with first round strategy form
- Second round strategy form
- Questionnaire with second round strategy form
- Third round strategy form
- Questionnaire with third round strategy form
- Fourth round strategy form
- Questionnaire with fourth round strategy form
- Save a copy of your submitted strategy and the simulation results in this folder.
- Questions, problems or remarks address them to Henk van de Velden, tel: xxx, or email: veldenh@fee.uva.nl
Chapter 8: The Strategy approach

Time Schedule

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Wednesday January 10</td>
<td>Introductory Experiment. Earnings are payed right after the experiment</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Submit first round strategy</td>
</tr>
<tr>
<td>3</td>
<td>Wednesday January 17</td>
<td>Results first round</td>
</tr>
<tr>
<td></td>
<td>Wednesday January 24</td>
<td>Submit second round strategy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Before Friday January 26</td>
</tr>
<tr>
<td>5</td>
<td>Wednesday January 31</td>
<td>Results second round</td>
</tr>
<tr>
<td></td>
<td>Wednesday February 7</td>
<td>Submit third round strategy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Before Friday February 16</td>
</tr>
<tr>
<td>7</td>
<td>Wednesday February 21</td>
<td>Results third round</td>
</tr>
<tr>
<td></td>
<td>Wednesday February 28</td>
<td>Submit fourth round strategy</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Before Friday March 2</td>
</tr>
<tr>
<td>9</td>
<td>Wednesday March 7</td>
<td>Results fourth round</td>
</tr>
</tbody>
</table>

There are no classes in the week of 12 to 16 February

Confidential

Your submitted strategies will be confidential. During the experiment the other participants will get to know your strategy. We ask you not to talk about the experiment or strategies until after Wednesday March 7

Earnings

The goal of this project (the experiment and the formulations of the strategies) is scientific, we want to know how people form expectations in special cases. Because of the scientific purpose of this research there is money to motivate you. You can earn money:

- In the introductory experiment
- Every time you submit a strategy and the questionnaire you receive 5 guilders.
- The best strategy in every round earns 50 guilder
- The students who submitted every round a strategy make a change on winning a large price. The winning strategy of the last round wins 250 guilder! There is also a second prize of 150 guilder and a third prize of 50 guilder.
Your strategy has to predict prices in a situation that is much like the experiment in which you participated. Therefore we first summarize the essential features of that situation.

The situation

You are a financial advisor to a pension fund that wants to optimally invest a large amount of money. The pension fund has two investment options: a risk free investment and a risky investment. The risk free investment is putting all money on a bank account paying a fixed interest rate. The alternative risky investment is an investment in the stock market. In each time period the pension fund has to decide which fraction of its money to put on the bank account and which fraction of the money to spend on buying stocks. In order to make an optimal investment decision the pension fund needs an accurate prediction of the price $p_t$ of stocks. As their financial advisor, you have to predict the stock market price $p_t$ (in guilder) during 51 subsequent time periods. Your earnings during the experiment depend upon your forecasting accuracy. The smaller your forecasting errors in each period, the higher your total earnings.

Forecasting task of the financial advisor

The only task of the financial advisors in this experiment is to forecast the stock market index $p_t$ in each time period as accurate as possible. The price $p_t$ of the stock in the first period will always be between 0 and 100 guilder. The stock price has to be predicted two time periods ahead. At the beginning of the experiment, you have to predict the stock price in the first two periods, that is, you have to give predictions $p_1^f$ and $p_2^f$ for time periods 1 and 2. After all participants have given their predictions for the first two periods, the stock market price $p_1$ in the first period will be revealed and based upon your forecasting error $p_1 - p_1^f$ your earnings for period 1 will be given. After that you have to give your prediction $p_3^f$ for the stock market index in the third period. After all participants have given their predictions for period 3, the stock market index $p_2$ in the second period will be revealed and, based upon your forecasting error $p_2 - p_2^f$ your earnings for period 2 will be given. This process continues for 51 time periods. To forecast the stock price $p_t$ in period $t$, the available information thus consists of

- past prices up to period $t - 2 \{p_{t-2}, p_{t-2}, ..., p_1\},$
- past predictions up to period $t - 1 \{p_{t-1}^f, p_{t-2}^f, ..., p_t^f\},$
- past earnings up to period $t - 2$. 
Information about the stock market

The stock market price $p_t$ is determined by equilibrium between demand and supply of stocks. The supply of stocks is fixed during the experiment. The price $p_t$ of the stocks is determined by market equilibrium, that is, the stock market price $p_t$ in period $t$ will be the price for which aggregate demand equals supply.

Information about the investment strategies of the pension funds

The precise investment strategy of the pension fund that you are advising and the investment strategies of the other pension funds are unknown. The bank account of the risk free investment pays a fixed interest, the exact interest rate is announced at the beginning of the experiment. The holder of the stocks receives an uncertain dividend payment in each time period. These dividend payments are uncertain however and vary over time. Economic experts of the pension funds have computed the average dividend payment; the exact average dividend is announced at the beginning of the experiment. The return of the stock market per time period is uncertain and depends upon (unknown) dividend payments as well as upon price changes of the stock. As the financial advisor of a pension fund you are not asked to forecast dividends, but you are only asked to forecast the price of the stock in each time period. Based upon your stock market price forecast, your pension fund will make an optimal investment decision. The higher your price forecast the larger will be the fraction of money invested by your pension fund in the stock market, so the larger will be their demand for stocks.
**Strategy-experiment**

**Information**

Every simulation is a different market with different characteristics. Especially the mean dividend and the risk free rate of interest per period (constant during one simulation) can differ over simulations. This information is available at the beginning of a simulation. You have information about all past stock prices up till period $t - 2$ and all your past predictions up till period $t - 1$. You do not know: how many pension funds operate on the market in which your pension fund is active and you also do not know the predictions of other participants (strategies).

**How to formulate a strategy**

A strategy is a complete plan of action. If you would give your strategy to someone else, he or she should be able to make exactly the predictions that you yourself would have made.

Your strategy should comply with three requirements: your strategy should be complete, unambiguous and informational correct. The requirement of completeness means that your strategy should provide a prediction in all possible situations. The requirement of unambiguity means that your strategy should provide exactly one prediction that is a real number in all possible situations. The requirement of informational correctness means that your strategy only uses information that is available at that moment.

Example of an *incomplete* strategy

"In the first period my prediction is 40. In the next periods my prediction is 60 if the previous price was larger than 50 and 40 if the previous price was lower than 50". This strategy is not complete because it provides no prediction if the previous price was exactly 50.

Example of an *ambiguous* strategy

"In the first two periods my prediction is 70. In the next periods I will look at the period of the last realized price, $t - 2$. If in that period my prediction was too low, then my prediction for the price at period $t$ is my prediction for period $t - 2$ plus 10. If in that period $t - 2$ my prediction was too high then my prediction for period $t$ is my prediction of period $t - 2$ minus 10. If my prediction error in $t - 2$ was smaller than 5 my prediction for the price in period $t$ will be my prediction of period $t - 2$."

This strategy is ambiguous because it is unclear what the prediction should be if the
prediction at \( t - 2 \) was (for example) 3 above the realized price: should the prediction be maintained or decreased by 10? By indicating which rule has priority this strategy can be made unambiguous.

Example of an *informational incorrect strategy*

"In the first period my prediction is 45 and in the second period my prediction is 50. In the other periods my prediction depends on the price in period 5. If the price in period 5 was larger than 40 I predict 30 and otherwise I predict 70." This (rather strange) strategy is informational incorrect because in period 3 through 5 it is unknown what the price in period 5 will be.

In the examples above the strategies are described in words. However, it is easier to check whether a strategy complies to all requirements if everybody uses the same notation. The present period is indicated as \( t \). The information you can use in period \( t \) contains the realized prices of the previous periods \( p(i), 1 \leq i \leq t - 2 \) and the predictions of your strategy in the previous periods \( V(i), 1 \leq i \leq t - 1 \). You know the interest rate \( r \), which is constant during the entire simulation (51 periods), and also the mean dividend \( d \), which is also constant during the simulation (51 periods).

A strategy consists of two columns. In the first column you put the periods in which that part of your strategy is valid, in column 2 you put your strategy for these periods. You can use conditions when describing your strategy, like in the example of a (not necessarily successful) strategy below.

<table>
<thead>
<tr>
<th>Periods:</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t = 1 )</td>
<td>( V(t) = 50 + 2r - d )</td>
</tr>
<tr>
<td>( t = 2 )</td>
<td>( V(t) = 50 + 2 \times (r + d) )</td>
</tr>
<tr>
<td>( t = 3 )</td>
<td>( V(t) = p(t - 2) )</td>
</tr>
<tr>
<td>( t &gt; 3 )</td>
<td>if (</td>
</tr>
</tbody>
</table>

*Explanation*

The prediction of the first period can not depend on previous prices or predictions (because no information is yet available) in the second period the prediction can not depend on previous prices but can depend on the previous prediction. In this example a number is also given in period 2. In periods 4 to 51 this strategy first checks if the absolute value of the error in period \( t - 2 \) \( |V(t - 2) - p(t - 2)| \) was less than 10. If that is the case, the prediction is the price in period \( t - 2 \). If that is not the case the prediction is the mean
of the price of three periods ago and the prediction of three periods ago.

**Notation of more complicated strategies** If you want to construct more complicated conditional strategies, you have to use brackets. For example the strategy "If $|V(t-2) - p(t-2)| < 10$ then (If $p(t-2) > 60$ then $V(t) = p(t-2)$ else $V(t) = V(t-2)$) else $V(t) = (p(t-3) + V(t-3))/2". In this strategy the prediction depends on the last realized price, if the absolute error in the period $t-2$ was less than 10. If you have experience with programming in Pascal or Basic, you may also use the regular IF-THEN-ELSE statements of Basic or Pascal.

You may use all usual mathematical notations you need. If you are not sure whether your strategy will be clear for our programmer you should tell us when you hand in your strategy, and we will check the strategy immediately.

**How to check a strategy**

1 Check the left column. Does the strategy predict a price in all periods? If not, your strategy is not complete.

2 Check the right column. For each cell in this column (each sub-strategy) you should check the completeness and unambiguousness: is exactly one prediction generated in each possible situation? Check also the information that is used in each column: is this information indeed available? In the example above the sub-strategy in the bottom right cell uses the price of two periods ago, and such a strategy can only work from period 3 onwards.

**The computer simulations**

All submitted strategies will be programmed and several thousands of simulations will be run. Each simulation starts with the random draw of some strategies, these strategies will form a market for 51 periods. Subsequently some random parameters will be drawn which determine the demand and production curves. The market is run for 51 periods and for each participating strategy the quadratic prediction errors are calculated. After thousands of simulations for each strategy the mean quadratic prediction error is calculated and a ranking is made. At the top of this list is the strategy with the smallest mean quadratic error, and strategies below have an increasing mean quadratic error.

**Information about the simulations** As soon as the simulations are run, the ranking list will be made public on the website of CeNDEF. On this list all strategies are identified with a personal code.
A printed version of the ranking list will be distributed at the Monday classes. All participants will then also get a printout of the results of 5 simulations in which their strategy participated. These 5 simulations are randomly chosen.
8.B Example of a strategy form (translated from Dutch)

Strategy Form

Your personal code: ............

Your strategy:

<table>
<thead>
<tr>
<th>Periods:</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td></td>
</tr>
<tr>
<td>$t = 2$</td>
<td></td>
</tr>
</tbody>
</table>

Please use the following notation:
- $t$ period number
- $p(t)$ realized price in period $t$
- $V(t)$ your prediction in period $t$
- $r$ the risk free rate of interest
- $d$ mean dividend

The information you can use in period $t$ are the realized prices of the previous periods $p(i), 1 \leq i \leq t - 2$ and the predictions of your strategy in the previous periods $V(i), 1 \leq i \leq t - 1$.

We will do the best we can, but in case you later find out that the programmer did not program your strategy the way you meant it, there is nothing we can do about it.
The results of the simulations are final. Therefore, be sure to make exactly clear what you want your strategy to be and please write legible.

Don't forget to fill in the questionnaire!
<table>
<thead>
<tr>
<th></th>
<th>round 1</th>
<th>round 2</th>
<th>round 3</th>
<th>round 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>74%</td>
<td>90%</td>
<td>67%</td>
<td>80%</td>
</tr>
<tr>
<td>2</td>
<td>72%</td>
<td>89%</td>
<td>73%</td>
<td>83%</td>
</tr>
<tr>
<td>3</td>
<td>72%</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>4</td>
<td>78%</td>
<td>87%</td>
<td>71%</td>
<td>81%</td>
</tr>
<tr>
<td>5</td>
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<td>92%</td>
<td>x</td>
<td>86%</td>
</tr>
<tr>
<td>6</td>
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<td>91%</td>
<td>x</td>
<td>x</td>
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<tr>
<td>7</td>
<td>72%</td>
<td>88%</td>
<td>70%</td>
<td>82%</td>
</tr>
<tr>
<td>9</td>
<td>31%</td>
<td>82%</td>
<td>47%</td>
<td>73%</td>
</tr>
<tr>
<td>10</td>
<td>72%</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>11</td>
<td>64%</td>
<td>86%</td>
<td>74%</td>
<td>79%</td>
</tr>
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</tr>
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<td>72%</td>
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<td>55%</td>
<td>75%</td>
</tr>
<tr>
<td>14</td>
<td>70%</td>
<td>89%</td>
<td>70%</td>
<td>x</td>
</tr>
<tr>
<td>15</td>
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<td>87%</td>
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<td>85%</td>
</tr>
<tr>
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<td>69%</td>
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</tr>
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<td>87%</td>
<td>53%</td>
<td>x</td>
</tr>
<tr>
<td>average</td>
<td>68%</td>
<td>87%</td>
<td>68%</td>
<td>81%</td>
</tr>
</tbody>
</table>

Table 8.C.13: Percentages of convergence of the strategies, i.e. the number of converging simulations a strategy participates in as a percentage of the total number of simulations the strategy participates in.