Viscoelastic behavior of dental restorative composites during setting
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Citation for published version (APA):
MODELING OF AXIAL STRESS-STRAIN DATA

Based on the article:

Abstract

This paper describes an engineering approach to the analysis of axial stress-strain data by means of mechanical models. Two 2-parametric models (Kelvin and Maxwell) and one 3-parametric model (Standard Linear Solid) which are possible candidates for describing the viscoelastic behavior of dental restorative material during setting are presented. An identification procedure was developed by which the material parameters can be calculated by matching the model’s stress response to experimental stress data. In addition, an evaluation procedure was developed to calculate the model’s response to axial shrinkage strain only. The identification procedure was validated and the effect of white noise on the parameter identification was investigated. The results showed that the identification procedure is free from error, and that it is capable of identifying two parameters from sinusoidal stress-strain data with a good degree of accuracy. The identification of three parameters from axial stress-strain data is only possible when the contribution of shrinkage strain to the applied strain is substantial.
**Introduction**

The literature overview in chapter 2 emphasis on the importance to gain knowledge of the mechanical behavior development of dental resin composites during setting. In this research project, the changes in mechanical behavior during setting were monitored by dynamic stress-strain tests, as described in chapter 3 in this thesis. The mechanical behavior can be quantified by (i) stress-strain analysis, revealing the storage modulus ($E'$) and loss modulus ($E''$) of the material, or by (ii) mathematical modeling [1]. The advantages of using a mechanical model are several. The most important advantage is that for a model the cause (strain) and effect (stress) is well known. With the aid of stress-strain data, recognizable material parameters such as Young’s modulus (E) and viscosity ($\eta$), can be identified. When factors such as resin formulation, initiation system, C-factor, etc on the shrinkage stress development are studied, then the development of these material parameters can be compared to each other, whereby the most suitable approach can be singled out.

Another advantage of using models is that the stress within the restorative material can be predicted on basis of the strain and material parameters as input for the model. Finite Element Analysis (FEA) is a popular simulative modeling technique by which the stress development with respect to a restoration design can be studied relatively straightforward.

The process for representing the mechanical behavior of shrinking resin composites in a model is called mathematical modeling (Fig. 4.1). As stated earlier, the mechanical behavior of resin composites during setting is hidden in stress-strain data. Therefore, a properly defined stress-strain law, the so-called constitutive equation, must be used as mechanical model. This

![Diagram of mathematical modeling](image_url)

**Figure 4.1** Scheme of mathematical modeling.
type of model can be visualized by elementary elements, because the stress-strain law is based on an observation of nature. For example, the linear elastic solid equation (Hooke’s law), which is visualized by a spring, is based on the observation that the relative deformation is proportional to the stress. This lead to a material parameter (Young’s modulus $E$), which can be identified by matching the model’s stress to experimental stress data.

Three mechanical models (Maxwell, Kelvin, and Standard Linear Solid model) are candidates for representing the mechanical behavior of shrinking resin composites. The equations representing these models are differential equations, which contain derivatives of both the stress and strain with time. In the course of this research project, several modeling procedures were developed to match the model’s equation to experimental data. The first procedure was able to identify the material parameters by performing a least square method to a system of normal equations derived from the differential equation [2]. A drawback of this procedure was that the differential equation was used, not solved. In this way, it was not possible to evaluate the model response; i.e., to calculate the model’s stress response on basis of the strain and calculated parameters.

The second modeling procedure was based on the identification procedure of Hübsch [3]. In this so-called parameter identification procedure, the differential equation was solved and the parameters were identified by an optimization procedure. In this study, certain aspects of this procedure have been improved. First, the differential equation can be solved analytically, which greatly reduced computation time and enhanced the accuracy of the results. Second, the procedure provides beside the parameter value also its error estimate.

The third modeling procedure was able to evaluate the model response under shrinkage conditions. With the aid of this so-called evaluation procedure, we were able to accept or reject the model for describing the mechanical behavior of shrinking resin composites.

This chapter describes the modeling approach on a step-by-step basis, as depicted in Figure 4.1. The parameter identification (step 4) and evaluation procedure (step 5) is described in more detail. In addition, the results of the validation i.e., checking the implementation of the algorithm used in the procedure, and the influence of noise on the parameter identification are presented. A study on the effect of noise on the parameter identification was necessarily, since the experimental data recorded in the dynamical test method are not exact, but contains white noise (Fig. 3.15).
Step 1 – Mechanical behavior of shrinking composites

In this research project, the mechanical behavior of shrinking dental restoratives was monitored by experiments with the universal testing machine [4]. For mathematical modeling, therefore, a properly defined stress-strain law, the so-called constitutive equation, must be used as model. Tests on shrinking resin composites showed that these materials do not only store but also dissipate mechanical energy, because applied deformation cycles result in hysteresis in stress-strain data (Fig. 3.11), a typical phenomena for viscoelastic materials.

In a co-operative study with the Department of Engineering of the University of Wales, many viscoelastic models were investigated on the basis of axial stress-strain data provided by our laboratory [3]. Adequate modeling results were achieved with the Maxwell model if only the build-up of stress through polymerization shrinkage was taken into account. This encouraging result pave the way to study the mechanical behavior of shrinking resin composites with simple mechanical models. In this research project, not only the Maxwell model, but also the Kelvin model and Standard Linear Solid model were investigated.

Step 2 – Modeling considerations

Some assumptions have to be made to predict the complexity of the real mechanical behavior by a limited, but useful, model. First, the mechanical properties of dental resin composites are considered isotropic; i.e., the material property does not depend on the location within the resin composite, but is the same in all directions. Further, any possible plasticity behavior of the restorative material is disregarded. Another important assumption is that the deformations applied to resin composites do not negatively influence the structural integrity and the polymerization reaction during setting. Results in Table 3.2 justify that the structural integrity is not being altered by strains of amplitude $\leq 0.04 \%$ and frequencies $\leq 0.02$ Hz. Finally, the mechanical behavior of the setting material is considered constant when identifying the parameters. The benefit of this assumption is described in the next section.

Step 3 – Formulate a mathematical model

The theory of linear viscoelasticity is applicable to dental composites, because the experimental data were generated at small strains ($<0.5 \%$) [5].
Linear viscoelasticity can be described using simple mechanical models consisting of springs and dashpots, which can model the process of energy storage and dissipation respectively. All models were investigated in one dimension only, because the stress-strain data were monitored in one direction. The models must be kept simple, because by modeling uniaxial data, only a restricted number of material parameters can be fitted in a unique way. As a consequence, the validity of the qualitative and quantitative viscoelastic behavior studied is confined to the stress, strain, and strain rate range covered by the experiment. To meet the requirement of isotropic shrinkage behavior of dental composites, the specimens were made as long as possible; in case of light-activated composites without exceeding the depth of cure (2 mm) [6].

![Diagram of viscoelastic models](image)

**Figure 4.2** Linear viscoelastic models between two extremes: pure fluid behavior (left) and pure elastic behavior (right).

The mechanical behavior of resin composites during setting changes from fluid behavior before, to solid behavior after setting. When this mechanical behavior is modeled by small time intervals in the stress-strain data, then the material parameters can be assumed to be constant. As a result, the fundamental relationship between stress, strain, and time for a linear viscoelastic material can be represented by the general equation:
\[
\sum_{n=1}^{\infty} a_n \frac{d^n \sigma}{dt^n} = \sum_{m=1}^{\infty} b_m \frac{d^m \varepsilon}{dt^m}
\]  
(4.1)

in which \(a_n\) and \(b_m\) are constants. Some mechanical models can be obtained from Equation (4.1):

**Hooke's law:**

\[a_0 \sigma = b_0 \varepsilon\]  
(4.2)

**Newton's law:**

\[a_0 \sigma = b_1 \frac{d \varepsilon}{dt}\]  
(4.3)

**Maxwell model:**

\[a_0 \sigma + a_1 \frac{d \sigma}{dt} = b_0 \frac{d \varepsilon}{dt}\]  
(4.4)

**Kelvin model:**

\[a_0 \sigma = b_0 \varepsilon + b_1 \frac{d \varepsilon}{dt}\]  
(4.5)

**Standard Linear Solid model:**

\[a_0 \sigma + a_1 \frac{d \sigma}{dt} = b_0 \varepsilon + b_1 \frac{d \varepsilon}{dt}\]  
(4.6)

The derivation of the differential equations is fairly straightforward, and is well documented in the literature [5, 7, 8]. The material parameters of interest are the Young's modulus (E), which represents the stiffness in axial tension and compression, and the extensional viscosity (\(\eta\)), which is inversely related to viscous flow [9].

The above expressions in the form of differential equations are equivalent to describing the linear viscoelastic behavior by mechanical models constructed with springs, which obey Hooke's law, and dashpots, which obey Newton's law of viscosity. The Maxwell model consists of a spring and a dashpot in series (Fig. 4.2). When this model is strained, the spring will react instantaneously and proportional to the stress. The model represents a viscoelastic liquid because the stress energy stored in the spring will be slowly dissipated by the dashpot. After removal of the stress, the elastic component will recover immediately and completely, while the strain of the dashpot remains intact.

The Kelvin model consists of a spring and a dashpot in parallel. The response on a strain is retarded by the resistance of the dashpot, and after removal of the stress the dashpot recovers slowly, thus with a time delay, and completely. Finally, the Standard Linear Solid model
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consists of a spring and a Maxwell model in parallel. When this model is strained, both springs will be deformed equally. With time, the stress energy in spring $E_1$ will be dissipated through its connecting dashpot, so that the stress contributed by this arm of the model will decay. Ultimately, the resulting stress will be due to spring $E_2$ and this is termed the “equilibrium stress”. After removal of the stress, the dashpot recovers slowly, thus with a time delay, and completely. The model can exhibit a viscoelastic liquid property ($E_2=0$), as well as a viscoelastic solid property.

As stated earlier, the polymerization reaction results in a drastic change of the mechanical properties of the resin composites. As viscoelastic behavior takes place between the two extremes (Fig. 4.2), it is imaginable that more than one viscoelastic model is necessary to model the mechanical behavior of resin composites during setting.

**Step 4 – Parameter identification procedure**

In this section, the parameter identification procedure is described. The identification procedure does not only provide the value of the material parameters but also its error estimate. To reach this goal, the differential equations (4.4-4.6) of the models were solved allowing the stress to be expressed as a function of the strain and unknown parameters. The material parameter values were then adjusted until the model response did fit the experimental observation as closely as possible. A scheme of the parameter identification procedure is shown in Figure 4.3.

**Mathematical solution of the model**

The differential equations (4.4-4.6) of the viscoelastic models contain time derivatives of both the stress and the strain. Allowing these equations to be solved, one of the variables, either the stress or the strain, has to be declared the independent variable. The choice of making the strain the independent variable seems natural, because the mechanics of the experiment were such that the strain was externally controlled. As a consequence, using the strain as input for the model means that, if the model under consideration is representative for the shrinking resin composite, the model’s output should match the stress observed in the experiment.

As the functional form of the applied strain (sine and linear) with time is known, the differential equations (4.4-4.6) can be solved analytically,
yielding the stress as a function of strain and the unknown material parameters (appendix A). To achieve independence from the points at which the strain data were collected, a sine and linear fit was applied to respectively the dynamical strain and shrinkage strain of the isolated time interval. Finally, the strain values in the interval were chosen equidistantly in order not to bias the parameter identification procedure towards closer approximation in certain regions of the interval.

![Diagram](image)

**Figure 4.3** Parameter identification procedure applied on a small time interval in stress-strain data.

For the final stress equation of the Maxwell or Standard Linear solid model, the stress at the beginning of the interval, the so-called initial stress $\sigma(t_0)$, has to be taken into account. This initial stress can be obtained from experimental stress data or calculated with the aid of the initial strain $\varepsilon(t_0)$. In this study, the initial stress was obtained from experimental stress data, because evaluation of the initial stress by integrating the initial strain and the variable material parameters over the time period $[0,t_0]$ prior to the isolated interval requires extensive computation.

**Extracting the material parameters**

To be able to assess how well a model with a certain set of material parameters approximates the real mechanical behavior of shrinking resin composites, a quantitative measure of the difference between the
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stress computed from the model ($\sigma_{\text{model}}$) and the stress measured in the experiment ($\sigma_{\text{exp}}$) over a small time interval has to be defined. The nonlinear least squares algorithm is useful for this purpose, whereby the quantitative measure can be defined as [10]:

$$\delta = \sum_{i=1}^{k} \alpha_i (\sigma_{\text{model}}(t_i) - \sigma_{\text{exp}}(t_i))^2$$  \hspace{1cm} (4.7)

That is, the model’s stress approximates the stress measured in the experiment at selected k points in the time domain of the isolated interval. No weighting of the data was applied ($\alpha_i = 1$). A cubic spline fit was applied to the experimental stress data. With spline interpolation, the stress values were chosen at the same points in time ($t_i$) as in the case for the strain values.

The final material parameters associated with the viscoelastic model were calculated by an optimization procedure, in which the residual (\(\delta\)), as defined in Equation (4.7), was minimized. For this purpose, the Levenberg-Marquardt method was selected. In this method, the line-search direction strategy was performed with cubic polynomial algorithm [11]. The method has been tested on some nonlinear problems and it has been proven to be more robust than the Gauss-Newton method.

The main difficulty in the implementation of the Levenberg-Marquardt method is an effective strategy for controlling the step-size so, that it is efficient in finding the minimum value of the residual. The initial step-size in the search direction was set to one, and, at each iteration, the step-size was determined and updated by a standard procedure implemented in the optimization procedure. Iterating to convergence (to machine accuracy or to the roundoff limit) is generally wasteful, time consuming, and unnecessary, because the minimum for \(\delta\) is at best only a statistical estimate of the parameters. In practice, the condition for stopping the iteration process was the first or second occasion that the residual (\(\delta\)) decreased by a negligible amount, say either less than 0.1 absolutely or some fractional amount like $10^{-3}$.

The final value of the ‘square difference’ between the experiment and the model stress may be viewed as an indicator as to how well the chosen model can be made to fit the real material behavior. It cannot be the indicator, because it may be that the model describes the mechanical behavior of the material very well, but that the parameters associated with the model cannot be determined accurately from the experimental stress, resulting in a high value of the residual. Therefore, the residual is rather an indicator of the ‘usefulness’ of a model than of the modeling capability.
The parameter identification provides (i) the parameters, (ii) the error estimate on the parameters, and (iii) the residual ($\delta$) that is a quantitative measure of the difference between experimental and model stress. For the calculation of the error estimate on the parameters, the error was assumed to be normally distributed [11].

Validation of parameter identification procedure

To verify the validity of the algorithm of the parameter identification procedure presented above, a set of artificial data was generated for the three viscoelastic models by solving the differential equation analytically, using prescribed material parameters and a known input strain. The material parameters were chosen to produce stresses similar to the ones measured in the experiment. For the Kelvin and Maxwell model a sinusoidal strain was used, while for the Standard Linear Solid model two different strain functions were used: (i) sinusoidal and (ii) linear and sinusoidal (Fig. 4.4).

![Diagram](image)

**Figure 4.4** Response of Standard Linear Solid model on (a) sinusoidal strain (amplitude=0.04 % & period=30 s) and linear strain (slope= 0.002 %/s) with exact parameters $E_1=2.00$ GPa, $E_2=0.10$ GPa, and $\eta=25.00$ GPa.s. (b) White noise was added to the model stress response (SNR=0.85).

The linear strain represents artificial shrinkage strain for a small interval in time. In reality, the slope of the linear strain; i.e., the shrinkage strain rate of setting composites, changes continuously. Therefore, the effect of the slope in the linear strain on the identification of the three parameters...
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of the Standard Linear Solid model was studied at five realistic shrinkage strain rate values (Fig. 4.5).

Finally, white noise was added to the artificial stress response of the three models to investigate the influence of the noise level, defined as the Signal-to-Noise Ratio (Fig. 4.6), on the parameter identification procedure. The effect of noise was studied solely on the stress signal (Fig. 4.4), because the load signal of the load cell (1 kN) was far more affected to white noise than the deformation signal of the sensitive LVDT transducers.

![Graph](Image)

**Figure 4.5** Shrinkage strain rate curve of a two-paste resin composite (see Fig. 3.9). Identification of the prescribed parameters of the Standard Linear Solid model was performed at five selected slope values for the linear strain: 0.003, 0.002, 0.001, 0.0005, and 0.0001 %/s.

![Graph](Image)

**Figure 4.6** Definition of Signal-to-Noise Ratio (SNR) [12].

**Evaluation procedure**

In this section, a procedure to calculate the model response on the input of axial shrinkage strain and calculated material parameters is described. With this procedure, the appropriateness of the various mechanical models can be evaluated by comparing the model response with the axial shrinkage stress of resin composites, as measured with the test system (Fig. 3.7).

Basically, the evaluation procedure was derived from the parameter identification procedure. No optimization routine was carried out, because the material parameters were already known. In the procedure, the model was loaded, on a sequential interval-step (Δt) basis, with a set
of material parameters and shrinkage strain data (Fig. 4.7). The shrinkage strain in the small interval was assumed to increase linear with time. Small time intervals in the setting time domain were applied to the model, because the differential equations (4.4-4.6) used in this procedure are only valid for constant material parameters. The material parameter values were obtained by cubic splines interpolation on the mean parameter values. The material parameter values were selected in the middle of the time interval. The stress solution at the end of each interval was taken as the initial stress condition for the next interval.

![Diagram](Image)

**Figure 4.7** Scheme of evaluation procedure. The model stress response was calculated for small intervals ($\Delta t$) sequentially in time. The stress solution at the end of each interval was taken as the initial stress condition for the next interval.

The cubic splines interpolation, parameter identification procedure, and evaluation procedure were performed on a Pentium (200 MHz) desktop computer with MATLAB software (version 5.3, Mathworks) under Windows® 98 platform.

**Results and discussion**

**Validation**

The parameter identification procedure applied to the artificial stress data of each model converged quickly to the true value of the material parameter (Table 4.1); indicating that the implementation of the algorithm for the parameter procedure is valid for all models. The small
standard deviation in the parameter is due to the round-off error of the computer. In spite of these good results, a remark concerning the use of least squares is in order here. A striking limitation of a least squares optimization is the need for a good initial estimate of the material parameters. It was found that when the initial material parameters were poorly chosen; i.e., out of a certain range, a local shallow minimum was found and the calculated parameters were incorrect. In this situation, the optimization algorithm sticks in a very flat region of the δ landscape. Therefore, the optimization was routinely started from different initial material parameter values and the optimization results were compared. The global minimum always corresponded to the lowest δ value and the best graphic fit.

The parameters of the Standard Linear Solid model could only be identified from sinusoidal stress data when the initial estimate of the parameters was close to the exact value. Under normal identification conditions - using arbitrary chosen initial estimates - the procedure was not able to determine the prescribed parameters accurately (Table 4.2). The explanation for this observation is that in the case of stress data generated by a sinusoidal strain of one frequency only, no more than two independent parameters can be determined, because only two

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Kelvin model</th>
<th>Maxwell model</th>
<th>SLS model</th>
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</thead>
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<td>Exact</td>
<td>Calculated</td>
<td>Exact</td>
</tr>
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<td>$2.00 \times 10^9$</td>
<td>$2.00 \times 10^9$</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>$2.50 \times 10^{10}$</td>
<td>$2.50 \times 10^{10}$</td>
<td>$2.50 \times 10^{10}$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>—</td>
<td>—</td>
<td></td>
</tr>
<tr>
<td>$\delta^8$</td>
<td>$1.79 \times 10^{-17}$</td>
<td>$2.21 \times 10^{-19}$</td>
<td></td>
</tr>
</tbody>
</table>

$^a\delta$=quantitative measure of the difference between experimental and model stress.

Table 4.1 Validation results of the parameter identification procedure for the three mechanical models with standard error in parenthesis. The artificial stress data were generated with sinusoidal strain (amplitude=0.04 % & period=30 s). The procedure was started with $1.00 \times 10^9$ as initial value for the two-parametric models. No white noise was added to the stress.
Table 4.2 Validation results of the parameter identification procedure for the Standard Linear Solid model with different initial values for the parameters. The artificial stress data were generated with two different input strain: (i) sinusoidal (amplitude=0.04 % & period=30 s) and (ii) sinusoidal and linear (slope=0.002 %/s) with the prescribed parameters $E_1=2.00$ GPa, $E_2=0.10$ GPa, and $\eta=25.00$ GPa.s. No white noise was added to the stress.

<table>
<thead>
<tr>
<th>Initial value</th>
<th>Sinusoidal stress</th>
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<th>Sinusoidal and linear stress</th>
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<tr>
<td>$E_1$ $E_2$ $\eta$</td>
<td>$E_1$ $E_2$ $\eta$ $\delta^a$</td>
<td>$E_1$ $E_2$ $\eta$ $\delta^a$</td>
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</tr>
<tr>
<td>1.00 0.10 20.0</td>
<td>1.61-10^9 (2.92-10^{-6}) 0.59-10^9 (4.64-10^{-6}) 14.50-10^9 (7.74-10^{-5}) 2.32-10^{-15}</td>
<td>2.00-10^9 (2.26-10^{-7}) 0.10-10^9 (1.88-10^{-7}) 25.00-10^9 (7.14-10^{-6}) 8.42-10^{-12}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00 1.00 10.0</td>
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<td>2.00-10^9 (3.86-10^{-6}) 0.10-10^9 (4.21-10^{-6}) 25.00-10^9 (2.14-10^{-6}) 2.75-10^{-13}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00 1.00 1.00</td>
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<td>2.00-10^9 (2.04-10^{-7}) 0.10-10^9 (1.69-10^{-7}) 25.00-10^9 (6.32-10^{-6}) 7.11-10^{-12}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.10 10.0 1.00</td>
<td>2.20-10^9 (0.76-10^{-4}) 1.62-10^9 (1.28-10^{-5}) 3.54-10^9 (3.95-10^{-5}) 2.27-10^{-15}</td>
<td>2.00-10^9 (1.86-10^{-6}) 0.10-10^9 (2.42-10^{-6}) 25.00-10^9 (3.74-10^{-5}) 2.98-10^{-10}</td>
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<td></td>
</tr>
</tbody>
</table>

$\delta^a$=quantitative measure of the difference between experimental and model stress
Table 4.3 Validation results of the parameter identification procedure for the Standard Linear Solid model with $1.00 \cdot 10^9$ as initial value for the three parameters. The artificial stress data were generated with sinusoidal (amplitude=0.04 % & period=30 s) and linear strain (five different slope values) with the prescribed parameters $E_1=2.00$ GPa, $E_2=0.10$ GPa, and $\eta=25.00$ GPa.s. No white noise was added to the stress.

| Slope linear strain (%/s) | Number of iterations | Material parameters (without noise) $E_1$ $E_2$ $\eta$ $\delta^a$ |
|---------------------------|----------------------|--------------------------------------------------|-----------------|-----------------|-----------------|
| 0.003                     | 220                  | 2.00$\cdot 10^9$ (2.01$\cdot 10^{-7}$) 0.10$\cdot 10^9$ (2.14$\cdot 10^{-7}$) 25.00$\cdot 10^9$ (6.24$\cdot 10^{-6}$) 7.11$\cdot 10^{-12}$ |
| 0.002                     | 348                  | 2.00$\cdot 10^9$ (2.25$\cdot 10^{-7}$) 0.10$\cdot 10^9$ (2.47$\cdot 10^{-7}$) 25.00$\cdot 10^9$ (7.44$\cdot 10^{-6}$) 8.76$\cdot 10^{-12}$ |
| 0.001                     | 554                  | 2.00$\cdot 10^9$ (5.75$\cdot 10^{-7}$) 0.10$\cdot 10^9$ (6.97$\cdot 10^{-5}$) 25.00$\cdot 10^9$ (2.12$\cdot 10^{-5}$) 2.03$\cdot 10^{-11}$ |
| 0.0005                    | 994                  | 2.01$\cdot 10^9$ (3.28$\cdot 10^{-5}$) 0.10$\cdot 10^9$ (3.07$\cdot 10^{-5}$) 25.02$\cdot 10^9$ (7.42$\cdot 10^{-4}$) 8.05$\cdot 10^{-9}$ |
| 0.0001                    | 1249                 | 2.05$\cdot 10^9$ (2.88$\cdot 10^{-3}$) 0.11$\cdot 10^9$ (2.87$\cdot 10^{-3}$) 25.08$\cdot 10^9$ (6.91$\cdot 10^{-3}$) 6.03$\cdot 10^{-4}$ |

$^a\delta$=quantitative measure of the difference between experimental and model stress

Independent variables are generated, namely a load signal and a phase angle. To obtain an accurate prediction of the prescribed parameters, the stress to be modeled must be generated by a multi-wave strain.

Addition of linear strain to the sinusoidal strain proved adequate for this propose, because the procedure was capable in finding the exact value of the material parameters using different initial estimates (Table 4.2). The lower the slope in the linear strain, the longer time it took to calculate the material parameters (Table 4.3). At a shrinkage strain rate value of 0.0001 %/s the identification procedure was able to calculate the parameter values within acceptable computation time only, when less stringent termination criteria were applied. These criteria could be stated less stringent, because the artificial stress data was free of noise. At shrinkage strain rates smaller than 0.0001 %/s the procedure failed to calculate the parameters exactly.
Figure 4.8 Effect of white noise on the calculated Maxwell parameters (■ E, △ η) compared to the prescribed values (line). The residual (δ) of each pair of Maxwell parameters is scaled on the secondary x-axis (top). The Signal-to-Noise Ratio (SNR) is defined as the amplitude of the load signal divided by the amplitude of the noise signal (Fig. 4.6). The gray area represents the SNR region in which the load signal (0-100 N) of the universal testing machine experiments was recorded. The error bars indicate the relative standard error in the parameters.

White noise in stress data

White noise added to the artificial stress data influenced the calculation time of the parameter identification procedure; the lower the Signal-to-Noise Ratio (SNR) value, the longer it took to find the material parameters. The results of the calculated Maxwell parameters at different white noise levels are visualized in Figure 4.8. For each SNR value (primarily x-axis), the calculated Maxwell parameters were plotted with the residual (secondary x-axis) to match. The gray area represents the SNR region for the load signal range (0-100 N), in which the load signal of resin composites was recorded in the universal testing machine experiments. The results for the Kelvin model resembles the Maxwell model and are therefore not reported.

The parameter identification results of artificial stress data with different levels of noise reveal several interesting features. First, down to a Signal-Noise-Ratio (SNR) value of two, the calculated material para-
meter values of the 2-parametric models were very good approximations of the exact values. As the load measurements in the experiments of resin composites take place within 10% of the load cell capacity, the lower and upper boundaries of the practical SNR values, as measured using the steel specimen, were 2.20 and 2.76 respectively. The position of the practical SNR values, shown as a gray area in Figure 4.8, revealed that when the viscoelastic behavior of resin composites is described by the Maxwell or Kelvin model, then it was possible to obtain precise material parameter values for these models.

![Graph](image)

**Figure 4.9** Effect of slope in linear strain on the calculated Standard Linear Solid parameters (■ $E$, △ $\eta$) compared to the prescribed values (line). The artificial stress data were generated with sinusoidal (amplitude=0.04% & period=30 s) and linear strain with five different strain rate values. In addition, noise (SNR=2.2) was added to the stress data. The identification procedure was started with $1.00E10^9$ as initial value for the three parameters. The error bars indicate the relative standard error in the parameters.

A second feature is that the value of $\delta$, which measures the difference between the experimental and the model stress, increases as the noise level of the stress data increases. Since the calculated material parameter values are good approximations of the exact values, the noise carried by the stress data in the SNR region down to two has no decisive influence on finding the global minimum of the $\delta$ parameter. As soon as the SNR value dropped under the value of two, then a remarkable phenomenon was visible for the Maxwell model. Although the standard error in both
parameters increased rapidly, the calculated Young's modulus parameter still approximated the exact value, while the viscosity parameter deviated markedly from the exact value. Since the high level of noise carried by the experimental stress data has a decisive influence on the viscosity parameter of the Maxwell model, caution is advisable with respect to the calculated value of this parameter at low SNR values. In absence of white noise, the identification procedure was able to calculate the three parameters of the standard Linear Solid model exactly down to a linear strain slope value of 0.0001 %. For practical application, it is of interest to known if this slope limit holds when the stress data contains white noise with SNR=2.2; i.e., at the lower boundary of the practical SNR region of the dynamic test system. Figure 4.9 shows that this is not the case.

Down to a slope value of 0.0003 %, the identification procedure was capable in finding the material parameters within an acceptable deviation (<5 %) from the exact value with a nearly constant value of δ (7.5-10⁻³). At 0.0003 % and lower, the procedure was not capable to calculate the three parameters on this acceptable basis.

**Conclusions**

In this chapter, an engineering approach to the modeling of axial stress-strain data is described step-by-step. Three mechanical models representing linear viscoelasticity are introduced as possible candidates for the modeling of the mechanical behavior of resin composites during setting. In addition, a parameter identification procedure and an evaluation procedure are presented in detail. Validation results show that the software implemented in MATLAB is free of error. On the basis of sinusoidal stress-strain data, the parameter identification procedure is capable of finding the two parameters associated with the Maxwell and Kelvin model with a good degree of accuracy. When performing the procedure on sinusoidal stress data with a high level of noise, the calculated viscosity value of the Maxwell model must be regarded as questionable. To obtain an accurate prediction of the three material parameters of the Standard Linear Solid model, the stress to be modeled must be generated by a multi-wave strain. For this moment, the addition of linear strain with slopes larger than 0.0003 %/s to the sinusoidal strain proved adequate for this purpose.
Chapter 4 | Modeling stress-strain data

References

4. See chapter 3 of this thesis.
9. See chapter 2 of this thesis.