Numerical time integration on sparse grids

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Since many real-life processes from engineering, physics, economics and a range of other disciplines can be described with differential equations, there is a need for practical methods for solving differential equations. Only in rare cases can differential equations be solved analytically. For the majority of differential equations, at best approximate solutions can be computed with the help of computers. With the power of modern computers and the sophistication of current algorithms this can often be done in a straightforward manner. However, there still exist numerous problems for which the numerical solution of the underlying differential equations is not straightforward.

A well-known example of a numerically difficult problem is the solution of the full Navier-Stokes equations, especially under turbulent conditions. Another example is that of global atmospheric transport models used for modeling pollution or forecasting the weather. In these models the number of unknowns required to accurately capture the spatial variations of the solutions can be excessively large.

The main focus of this thesis lies on a method that holds the promise of alleviating the restriction of excessively large numbers of unknowns. This is the sparse grid combination technique, which aims to solve a set of differential equations using significantly fewer unknowns. In fact, in the limit of high accuracy, the number of unknowns required by the sparse grid combination technique is independent of the spatial dimensionality of the problem. E.g., asymptotically a spatially 3D problem requires the same order of unknowns as a spatially 1D problem.

The sparse grid combination technique can be understood as a multivariate extrapolation technique. Instead of solving a set of differential equations on a single grid, solutions are obtained on a number of semi-coarsened grids. After solving these semi-coarsened problems, the solutions are combined to obtain a single, more accurate solution. In Chapters 2, 3 and 4 of this thesis error expressions are derived that measure the approximation error due to the sparse grid combination technique. Furthermore, test cases are considered numerically to validate these expressions and to test the applicability of the technique for a number of problems.

It becomes apparent that the sparse grid combination technique can be highly efficient for some problems. Especially problems that contain locally lower dimensionality are well suited for the sparse grid combination technique. E.g., in Chap-
The sparse grid combination technique is shown to be effective for a 2D-flow problem containing locally 1D solution layers. However, it also becomes apparent that the sparse grid combination technique is less well suited for other problems. E.g., in Chapter 3 it is shown that for a model problem without locally lower dimensionality the sparse grid combination technique is less efficient than Richardson extrapolation.

In Chapter 5 of this thesis a mixed gradient-diffusion problem is considered. The motivating application for this problem is that of axon growth studied in neurobiology. In biological experiments it is observed that axons bundle and debundle during their growth. An initial model is considered which assumes that the axons secrete chemical substances through which they communicate with each other. It is shown that this initial model can already predict the bundling and debundling behavior, albeit for a small range of parameters. Furthermore, an important complication inherent in the model is pointed out. I.e., in a numerical implementation there exists a danger that the axons blind themselves with their own chemical emissions. A condition is presented that the numerical scheme must satisfy in order to avoid this self-blinding of the axons.