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THE ECONOMIC VALUE OF AUTONOMY

by

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THE ECONOMIC VALUE OF AUTONOMY

Abstract

We develop an economic theory of “autonomy”, which we interpret as the discretion or ability to make a decision that others disagree with. We show that autonomy is essentially an option for the decisionmaker, and can be valued as such. The value of the autonomy option is decreasing in the extent to which the decisionmaker’s future decision-relevant opinion is correlated with the opinions of others who may be able to impede the decision.

We argue that autonomy drives economic decisions in a significant way. Even though autonomy is similar to control, it is quite different from the “private benefits of control” that the literature has analyzed extensively; we explicitly limit our examination of autonomy to situations in which the decisionmaker and those who can object to the decision are maximizing the same objective function. Explicit consideration of autonomy generates results that are strikingly different from those one would obtain in an agency or private-control-benefits setting. We illustrate the differences through an application involving an entrepreneur’s choice of autonomy in raising financing for a project. We also discuss a variety of other applications of autonomy.
THE ECONOMIC VALUE OF AUTONOMY

“There is only one success – to be able to spend your life in your own way”

Christopher Morley

1. INTRODUCTION

Casual observation tells us that people value autonomy. Managers sometimes refer to it as “elbow room”, the independence to be able to alter operating decisions and change strategic direction when circumstances change. Microsoft, which has significant liquidity and no long-term debt, argues that this gives senior management the flexibility to exploit future opportunities that may arise. This could be interpreted as Microsoft’s management “purchasing” some degree of autonomy from investors in the sense that the company can finance new investments without having to raise capital from the market. Banks routinely build up capital levels well in excess of regulatory capital requirements. For example, since 1995, essentially none of the 100 largest U.S. banking firms have been constrained by regulatory capital standards (see Flannery and Rangan (2002)). This provides bank managers with some autonomy in dealing with regulators. Within organizations, more senior executives are given greater discretion over decisions, which mean they have more autonomy to do what they want, relative to those at lower levels. This is sometimes referred to as “real authority” (e.g. Aghion and Tirole (1997)).

But what exactly is “autonomy”? We address this question by developing an economic theory of autonomy that pivots on the intuition that autonomy is the ability to make decisions one thinks are best when others disagree. This is not a question of who is right and who is wrong. It is a question of divergent beliefs about the optimal course of action, with insufficient time or objective data for beliefs to converge. It is a matter of intuition. The decisionmaker may be convinced that a particular action is optimal, and yet other reasonable people may feel equally strongly that it is not. The ability to take the desired action in the face of such disagreement is what autonomy is all about.

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There is a close relationship between autonomy and the notion of financial flexibility described by this example. The notion of flexibility has been used in a variety of contexts in economics. For example, Tadelis (2002) uses flexibility in a very different sense from ours by focusing on the ability to make ex post changes with incomplete contracting in the context of the make-or-buy decision.
We find that autonomy is isomorphic to an option. The option is worthless to the decisionmaker when others agree with the decisionmaker; it is “in the money” precisely when there is disagreement. And the value of this option is increasing in the likelihood that others will disagree with the decisionmaker.

One could argue that people value autonomy for its own sake. That is, they derive positive utility from getting their way when others disagree with them because it signifies a form of power. While we interpret autonomy as organizational power in one of our applications, we show that it is not necessary to attach an exogenous utility benefit to autonomy for it to be valued by the decisionmaker.

It is important to recognize that autonomy has nothing to do with asymmetric information (Akerlof (1970)) problems *per se*. The decisionmaker desires autonomy not because he has better information than those who delegate decisions to him. Rather, the decisionmaker recognizes the possibility of future disagreement even when all concerned are conditioning their opinions on the same incremental information set. Thus, an essential ingredient of our theory is the possibility that the same piece of information may be processed very differently by different people. The simplest way to think about this is that agents have different priors (e.g., Allen and Gale (1995) and Morris (1995)), but a variety of other factors that we discuss later may also account for this.

Autonomy also differs from agency problems, which are created by a divergence of interests between the agent and the principal (e.g. Holmstrom (1979)). In our analysis, we explicitly assume that both the decisionmaker and those delegating to him are maximizing the *same* terminal payoff. That is, their interests are perfectly aligned. But their beliefs are not.

Having said this, we do believe that interesting insights may emerge from juxtaposing autonomy with problems of asymmetric information and agency. We do this in some of our applications.

Autonomy is closely linked to flexibility in decisionmaking. When a decisionmaker has more autonomy, he also has greater flexibility to make the decisions he thinks are best. In many financial applications, flexibility often seems a more apt description than autonomy. For example, keeping more liquidity within the firm provides management with greater operating flexibility – albeit at a cost – and also some autonomy from investors in deciding where to invest the liquidity.
A literature somewhat related to our concept of autonomy is the debate about rules versus discretion, largely in the context of macroeconomic policy (see Modigliani (1964), Kydland and Prescott (1977), Goldfeld (1982), Barro (1986)). In this literature, a discretionary policy seeks to select the best decision, given the information available at the time. Rules, by contrast, are seen as a form of commitment to a binding contract that prescribes the actions that someone will take, possibly contingent on some publicly-observable exogenous variables. Thus, discretion is a special case of a rule or contract in which none of today’s provisions restrict a person’s future actions.

The standard argument in favor of discretion is that it may be difficult to anticipate all possible state contingencies in advance and hence a rigid rule may preclude taking advantage of information that arrives unexpectedly after the rule was formulated. The argument in favor of rules is that they represent precommitments to avoid problems caused by the dynamic inconsistency of optimal plans.

The view of discretion in this literature is very different from our notion of autonomy. In particular, in the rules-versus-discretion literature, discretion is seen as an optimal control problem where, at each point in time, the decision selected is best, given the current state and given that decisions will be optimally selected in the future. While discretion in this literature means autonomy for the decisionmaker to do what is best given the information available, it is not the disagreement-based autonomy that we study. We view discretion as the ability to take the desired action in the face of disagreement.\(^2\)

We show in our analysis that autonomy is valuable to the decisionmaker, and our premise is that autonomy considerations affect a variety of decisions. The more uncertain the decisionmaker is about the future value of a decision-germane payoff and the higher the probability of disagreement, the more autonomy is valued. Since the potential applications of autonomy are far too numerous for us to exhaustively discuss in this paper, we examine a few applications to illustrate how we can use autonomy

\(^2\) In a contracting context, Boot, Greenbaum and Thakor (1993) examine the choice between “illusory promises” – contracts that are honored only at the discretion of one of the two contracting parties – and legally-binding financial contracts. In contrast to their notion of discretion as having to do with contract-honoring incentives, we view autonomy as the ability one contracting party has to do what it believes is optimal for both parties, in the face of divergent beliefs about the optimal course of action.
to gain insights into various economic phenomena.

The first application, and the one we develop in detail formally, is the entrepreneur’s choice of initial financing. The entrepreneur’s autonomy is his ability to make decisions that financiers may object to. The degree of autonomy from financiers is an _ex ante_ choice variable for the entrepreneur. The entrepreneur’s tradeoff in this choice is as follows. On the one hand, greater autonomy benefits the entrepreneur because it increases the value of the autonomy option to him, and on the other hand, it leads to a higher cost of external capital.

We consider this application in the context of both debt and equity financing. In each case, we compare the results we get when the driving force of the dissonance between the entrepreneur and external financiers is potential disagreement and when it is an agency problem involving a private benefit of control for the entrepreneur. We obtain strikingly different results across the autonomy and private benefit-of-control cases for both debt and equity financing. This analysis shows that while an examination of autonomy can be viewed as a way to endogenize the benefits of control, there are important differences between autonomy and exogenous _private_ control benefits.

In addition to exploring this application in depth, we discuss numerous other applications of autonomy. The first of these is the design and issuance of financial securities. We argue that equity provides the firm’s manager greater autonomy from direct investor intervention than debt, but carries a higher cost of capital. Thus, which security the manager decides to issue will be dictated by the value of autonomy to the manager at that point in time.

Our next application is secured lending. The potential loss of autonomy due to lender intervention creates a role for collateral as a device for the borrower to recapture autonomy since the lender’s incentive to intervene in the borrower’s decision is weakened by the security collateral provides. We show that this insight is important for explaining the borrower’s choice between secured and unsecured loans.

We then apply the concept of autonomy to “power” or “real authority” in organizations. We argue that one interpretation of power is to have greater autonomy to make decisions that one thinks are
right even when these decisions conflict with organizational rules or the views of others. We examine the implications of our theory for the determinants of the value of powers, and we present a framework in which employees who are higher up in the organizational hierarchy are given greater autonomy.

In our next application, we explain how autonomy can be used to understand why people conform excessively in organizations, i.e., why we have so many yes-men.

Finally, we discuss how autonomy considerations may affect a firm’s choice between private and public ownership. We argue that a firm with a sufficiently low stock price, reflecting high disagreement between management and investors, may prefer private ownership over public ownership.

The upshot of our discussion is that an explicit analysis of the tradeoffs involved in autonomy yields implications that are at odds with conventional wisdom and also illuminates some economic phenomena that have not yet been explained. The rest of the paper is organized as follows. Section 2 develops the framework to assess the economic value of autonomy. Section 3 discusses the first application: the entrepreneur’s choice of autonomy. Section 4 takes up other applications. Section 5 concludes. All proofs are in the Appendix.

2. AN ECONOMIC MODEL OF AUTONOMY

In this section we present and analyze a general model of autonomy.

2.1 The Players and Their Preferences

There are two key players in the model: the decisionmaker and those whose endorsement of the decision must be obtained by the decisionmaker. We will refer to the decisionmaker as the “insider” and those who must endorse the decision as “outsiders.” The reasons for why the insider must seek the outsiders’ endorsement are quite transparent in the applications we discuss. Everybody is risk neutral; this assumption is innocuous and serves merely to simplify the analysis.

2.2 Model Description

The key assumptions are that the insiders and outsiders may disagree, and that outsiders may veto the insider’s choice. The “autonomy option” is the value that the insider is willing to pay to reverse or
prevent a veto by outsiders. We call this an option because autonomy here pertains only to the situation in which the insider wishes to undertake an investment that outsiders do not. The asymmetry that creates the option is that if the insider thinks the investment is a bad idea, it will never be pursued, regardless of what outsiders think, and if the insider and outsiders both agree it is a good idea, the insider will make the investment. That is, autonomy is the option the insider has to disregard the opinion of outsiders. We model this as follows. At $t = 0$, a potential decision is anticipated at $t = 1$. This can be a course of action, say an investment. Both the insider and the outsiders start out with the same priors about project value (the terminal payoff from the investment). At $t = 1$, the insider observes a signal $z$ about the terminal payoff from the investment and has to decide on the investment opportunity. The insider randomly draws a prior belief about the precision of $z$, and updates to arrive at a posterior belief about the terminal payoff. This posterior belief leads the insider to interpret the expected terminal payoff as $x$. The insider wants to undertake the investment if $x \geq S$, where $S$ is a particular threshold. We could interpret $S$ as the investment, where $x - S$ is the NPV of the investment. In an options context, $S$ is the strike price of the option. The terminal payoff on the investment is observed at $t = 2$.

If the insider decides to invest in the project, he must seek the endorsement of outsiders. If this happens, outsiders see the same signal $z$. However, outsiders as a group also randomly draw a prior belief about the precision of $z$, which leads them to arrive at their own posterior belief about the expected terminal payoff, which we refer to as the outsiders’ interpretation of project value, $y$. If $y \geq S$, they permit the investment. If $y < S$, they seek to veto the investment. The interpretations $x$ and $y$ are possibly correlated, with a joint probability density function $q(x, y)$. However, both the insider and outsiders base their decisions solely on their own interpretations. The difference between $x$ and $y$ is therefore a measure of the extent of disagreement between the insider and outsiders at $t = 1$. Both insiders and outsiders seek to maximize the expected terminal payoff at $t = 2$, so their disagreement over the investment decision is not due to a divergence of interests or objective functions, i.e., it is not an agency problem.
The difference of opinions here is also not due to asymmetric information; if it were, the less-informed party would immediately update its information set to that of the better-informed party, with the latter being coaxed to reveal this information (Revelation Principle). Nor is the disagreement due to a lack of information aggregation, since the insider and outsiders are all observing the same signal $z$. Rather, it is due to different interpretations generated by different prior beliefs about the precision of $z$, and there is no reason why anybody should revise his prior beliefs solely on the basis of someone else’s prior beliefs. It is useful then to pause and ask why these priors about the precision of $z$ may differ. One reason may be that $z$ consists of hard facts that the insider and the outsider agree on and “soft” or subjective information whose interpretation may differ across individuals because it is affected by intuition (e.g., Clark and Mackaness (2001)). The psychology literature has provided numerous other reasons. One is a “propensity for uniqueness” that causes the insider or the outsider to ignore historical statistics in judging $z$ (Kahneman and Lovallo (1993)). Another is the tendency of people to ignore information that contradicted earlier beliefs. Yet a third reason, identified by Stumpf and Dunbar (1991), is differences in “personality types” that lead to differences in cognitive biases, such as selective

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3 Kandel and Pearson (1995) show that a model of trade around public announcements of information in which agents interpret the same public information differently explains the relationship between the volume of trade and stock returns around public announcements. Bamber, Barron and Stober (1997) also investigate the relationship between trading volume and earnings announcements and find that trading volume arises because of investor disagreement caused by differences in prior beliefs.

4 Thus, the setting here is different from that in Aumann’s (1976) famous proof that two “like-minded” individuals cannot disagree forever (see also related literature surveyed by Bonanno and Nehring (1997)). However, even Aumann’s thesis that rational learning must lead to convergence has been challenged by Miller and Sanchirico (1997), who point out that the foundations of the “rational learning” literature go back to the Blackwell and Dubins (1962) theorem that if $Q$ and $P$ are measures on an infinite product space and $Q$ is absolutely continuous with respect to $P$, then $Q$ and $P$ will almost surely “merge” in the sense that the “distance” between these revised probabilities, conditional on the full continuation of the sequence, will limit to zero as the sequence unfolds. Kalai and Lehrer (1993) rely on this result to claim that rational learning leads to Nash equilibrium. In a subsequent paper, Kalai and Lehrer (1994) introduce “weak merging”, which is convergence of conditional probabilities based on only the next period’s draw as opposed to the infinite future. What Miller and Sanchirico (1997) show is that the assumptions that lead to the convergence result in the rational-learning literature are perverse in the sense that assuming absolute continuity is the same as assuming merging, so that the inevitability of agreement with rational learning is more an assumption than a result; obtaining agreement as a consequence of Bayesian updating requires “extra rational” restrictions on human behavior.

5 Studies of the decision-making process that contributed to the escalation of the Viet Nam war have shown that leaders paid more attention to information that was more compatible with their earlier decisions (White (1997)).
perception, overconfidence, social desirability, reasoning-by-analogy, or even ideology (as in Mullainathan and Shleifer (2002)).

The potential disagreement is anticipated at \( t = 0 \). The key question now is what value the insider attaches at \( t = 0 \) to autonomy, i.e., to being able to make the decision he wants over the objection of outsiders. In what follows, it will be convenient to work directly with the joint distribution of the posterior means, \( x \) and \( y \). Given below is the sequence of events in the model.

**Figure 1: Sequence of events**

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Autonomy option obtained and assessed</td>
<td>- Insider observes signal ( z ) and interprets it as ( x ).</td>
<td>- Outputs, if any, are realized</td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Outsiders observe ( z ) and interpret it as ( y ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Insider seeks to invest if ( x = S )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>- Outsiders veto if ( y &lt; S ) (assuming they have veto power)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Outputs, if any, are realized</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.3 **Analysis**

We proceed as follows. With a zero discount rate between \( t = 0 \) and \( t = 1 \), the total expected net present value of the investment opportunity *without* an outsider veto as assessed by the insider is

\[
E(V) = \int_{S}^{\infty} [x - S] g(x) \, dx
\]

where

\[
g(x) = \int_{-\infty}^{\infty} q(x, y) \, dy
\]

and \( q(x, y) \) is the joint density function of \( x \) and \( y \), with cumulative distribution function \( Q(\cdot, \cdot) \). Similar to \( g(x) \), the marginal density of \( y \) is

\[
h(y) = \int_{-\infty}^{\infty} q(x, y) \, dx.
\]

The value \( E(V) \) can be decomposed as follows (we let the inner integral be over \( x \) and the outer integral over \( y \)).

---

\[6\] This may be one way to interpret the evidence provided by Fuchs, Krueger and Poterba (1997) who surveyed economists at leading U.S. research universities and found that their positions on policy proposals disagreed due to their personal values rather than their estimates of relevant economic parameters.
\[ E(V) = \int_{-S}^{S} \int_{-S}^{S} [x-S]q(x,y)dx
dy + \int_{-S}^{S} [x-S]q(x,y)dx
\]  \hspace{1cm} (2)

\[ \equiv A + D \]

where

\[ A \equiv \int_{-S}^{S} \int_{-S}^{S} [x-S]q(x,y)dx
dy, \]

(3)

represents the expected value of the investment opportunity over states in which there is agreement, and the expression \( D \),

\[ D \equiv \int_{-S}^{S} \int_{-S}^{S} [x-S]q(x,y)dx
dy \]

(4)
is the expected value of the investment opportunity over states in which there is disagreement; it represents the value of the autonomy option.

\( D \) measures the expected loss in NPV of the investment when disagreement leads to a veto by outsiders. From (4) we see that the autonomy option derives its value from states in which the insider has a favorable assessment, \( x \in (S, \infty) \) and wishes to invest, while outsiders have an unfavorable assessment, \( y \in (-\infty, S) \), and wish not to invest.

To analyze the properties of the value of the autonomy option, we assume that \( q(x,y) \) is bivariate normal, with correlation \( \rho \in [-1,1] \) between \( x \) and \( y \). One benefit of this is that the correlation between two normal random variables uniquely defines their joint distribution. In the more general case, one would have to work with the conditional probabilities of events rather than correlation\(^7\), which would make the comparative statics analysis of the properties of \( D \) rather difficult. Thus, we assume

\[ q(x,y | \rho) = \left\{ \left[ \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \right] \times \exp \left\{ -\frac{1}{2[1-\rho^2]} \left[ \left( \frac{x-\mu_x}{\sigma_x} \right)^2 - 2\rho \left( \frac{x-\mu_x}{\sigma_x} \right) \left( \frac{y-\mu_y}{\sigma_y} \right) + \left( \frac{y-\mu_y}{\sigma_y} \right)^2 \right] \right\} \right\} \]

(5)

\(^7\) See Schweizer and Wolff (1981), for example. Since we are working directly with the joint distribution of the posterior means \( x \) and \( y \), it should be understood that \( \rho \) will be related to the correlation in the prior beliefs about the precision of \( z \) that are drawn randomly for the insider and outsiders.
where $\mu_x$ and $\sigma_x^2$ are the mean and variance of $x$, related to its marginal density $g(x)$, and $\mu_y$ and $\sigma_y^2$ are the mean and variance of $y$, related to its marginal density $h(y)$. Because we do not want to assume that the insider is any more optimistic or pessimistic than outsiders or any different in (over)confidence, we will assume that $\mu_x = \mu_y$, and $\sigma_x = \sigma_y$. Further, we will assume that, in an expected value sense, the option to invest is “in the money” at $t = 0$, i.e.,

$$\mu_x = \mu_y > S.$$  \hspace{1cm} (6)

We can now state our first result.

**Theorem 1:** The value of the autonomy option, $D$, is:

(i) strictly decreasing in $\rho$, the correlation between $x$ and $y$ for all $\rho \in [-1,1]$;

(ii) strictly increasing in $\sigma_x^2$, the variance of $x$, and $\sigma_y^2$, the variance of $y$; and

(iii) increasing in the $\mu_x$, the mean of $x$, and decreasing in $\mu_y$, the mean of $y$.

The first part of this theorem is intuitive. The autonomy option is most valuable to the insider when the insider and outsiders are most likely to disagree. This is the case when the correlation between $x$ and $y$ is the lowest.

The intuition behind the second part is as follows. $D$ is increasing in the variance of $x$ for the usual reason that the value of a call option increases with the variance in the value of the underlying asset. As for the variance of $y$, note that $y$ does not directly enter the integrand in (4) except through the density function $q(x, y | \rho)$. Intuitively, therefore, we can focus on what a higher $\sigma_y^2$ does to the probability that $y < S$. Because $S < \mu_y$ and the marginal density of $y$ is normal, an increase in the variance of $y$ increases the probability that $y < S$. Consequently, $D$ increases.

Because our analysis has both a real option to invest (i.e., the entrepreneur wishes to invest when $x > S$) as well as an autonomy option (being able to invest when $x > S$ even though $y < S$), it is useful at

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8 To be technically precise, we assume these equalities hold at the points at which we examine the relationship of $D$ to changes in any variables.
this stage to separate the two. This separation is readily made when $x$ and $y$ are uncorrelated, i.e., $\rho = 0$. Then,

$$D = \Pr(y < S)C(x, S)$$

where $C(x, S)$ is the value of a call option with a strike price of $S$ and written on a security with value given by $x$. Here $C(x, S)$ is the real option embedded in $D$, and $\Pr(y < S)$ represents the disagreement component of the autonomy option $D$. Other than in the $\rho = 0$ case, the value of the autonomy option does not separate like this, but this case provides useful intuition to think about the more general case.

The intuition behind the third part, dealing with the means of $x$ and $y$, is as follows. As the mean of $x$ increases, the insider’s assessment of the future value of the investment opportunity also increases, so the value of being able to invest even in the face of disagreement is augmented as well. As for the mean of $y$, an increase in this variable means that outsiders are more likely to interpret the public signal as indicating that the investment opportunity is good, which diminishes the likelihood of disagreement for any fixed correlation coefficient; here the asymmetric nature of autonomy – the outsiders’ interpretation $y$ is irrelevant when $xS$ – plays a role.\(^9\) Hence, the value of the autonomy option declines.

Thus far we have dealt with autonomy as a binary variable, in that either the insider has it (thereby availing of its full value $D$, with the value of investment being $A + D$), or he does not (in which case the value of the investment is merely $A$). In reality, the insider’s autonomy is much more likely to be between these two extremes. There may be instances in which the insider will be able to overcome disagreement by outsiders and instances in which outsiders will successfully veto the decision. Thus, it may be more reasonable to depict autonomy as a continuous variable, say a probability $\eta \in [0, 1]$, which represents the likelihood that the insider will be able to implement his decision even when there is disagreement. We will henceforth refer to $\eta$ as the “autonomy probability”. In our analysis, we initially take $\eta$ as exogenously given. In practice, it is likely to be affected by a host of factors, such as the type\(^9\)

\(^9\) Without this asymmetry, an increase in the mean of $y$ would decrease the likelihood of disagreement conditional on $x \geq S$ and increase this likelihood conditional on $x < S$. 
of contract between the insider and outsiders, the reputation of the insider based on past decisions and so on. In later analysis, we will treat \( \eta \) as an endogenous variable.

We can then write (2) as:

\[
E(V|\eta) = A + \eta D
\]

(7)

where \( A \) and \( D \) are defined in (3) and (4) respectively. The outsiders’ assessment of the value of the investment depends on the decision of the insider, and equals:

\[
E(V_o|\eta) = A_o - \eta D_o
\]

(8)

where

\[
A_o = \int\int [y-S]q(x,y|\rho)\,dxdy
\]

(9)

\[
D_o = \int\int [S-y]q(x,y|\rho)\,dxdy.
\]

(10)

where \( A_o \) is the outsiders’ assessment of the value of the investment in the case of agreement and \( D_o \) is the cost of the autonomy option to outsiders. The following result is now immediate.

**Lemma 1:** An increase in the autonomy probability \( \eta \) leads to a strict increase in the value of the investment opportunity to the insider (i.e., \( \frac{\partial E(V|\eta)}{\partial \eta} > 0 \)) and a strict decrease in the value of the investment opportunity to outsiders (i.e., \( \frac{\partial E(V_o|\eta)}{\partial \eta} < 0 \)).

The intuition is that an increase in \( \eta \) means a higher probability that the insider will prevail when there is disagreement. This increases the expected value of the investment opportunity, as assessed by the insider. The opposite is true for outsiders. This lemma highlights the tension between the value of autonomy to the insider and its cost to outsiders who may disagree with the insider. In our applications, we will be maximizing the insider’s objective function, \( E(V|\eta) \), subject to an ex ante participation constraint for outsiders. Consequently, the cost of autonomy to outsiders is something that will, in equilibrium, be passed on to the insider ex ante. This means that the insider will have to trade off the ex post benefit of autonomy against the ex ante cost of compensating outsiders for this autonomy. This tradeoff will be an important aspect of the applications discussed next.
Our previous analysis has focused on the comparative statics of $D$. This is useful in applications in which the decisionmaker has an innate preference for autonomy. However, in general the insider’s decisions will be driven by their effect on $A + \eta D$. Thus, it is also useful to know the properties of $A$.

**Theorem 2:** The value of the investment opportunity to the insider when there is agreement, $A$, is:

(i) increasing in the agreement parameter, $\rho$;

(ii) increasing in $\sigma_x^2$, the variance of $x$, and decreasing in $\sigma_y^2$, the variance of $y$;

(iii) increasing in $\mu_x$, the mean of $x$, and increasing in $\mu_y$, the mean of $y$.

The first part is very intuitive. As the agreement parameter increases, there is a larger number of states in which there is agreement, and hence the expected value of the investment opportunity with agreement is higher. As for the second part, the higher value of $A$ due to an increase in $\sigma_x^2$ is due to the real-option effect discussed in connection with the impact of $\sigma_x^2$ on $D$ in Theorem 1. An increase in $\sigma_y^2$ decreases the probability that $y > S$ and hence the measure of the agreement region. This causes $A$ to fall. As for the third part, it is clear that an increase in $\mu_x$, increases the probability that $x > S$ and an increase in $\mu_y$, increases the probability that $y > S$. Hence, the measure of the agreement region increases in either case and the value of $A$ goes up.

**Theorem 3:** The cost of the autonomy option to outsiders, $D_o$, is:

(i) decreasing in the agreement parameter, $\rho$;

(ii) decreasing in $\sigma_x^2$, the variance of $x$, and increasing in $\sigma_y^2$, the variance of $y$; and

(iii) increasing in $\mu_x$, the mean of $x$, and decreasing in $\mu_y$, the mean of $y$.

The intuition behind part (i) is the same as that for the negative relationship between $D$ and $\sigma$. As for part (ii), the intuition underlying why $D_o$ is decreasing in $\sigma_x^2$ is as follows. Because the density function of $x$ is symmetric around its mean and $S < \mu_x$, an increase in the variance of $x$ decreases the probability that $x > S$. Thus, outsiders perceive a lower probability that the insider will exercise the
autonomy option, which reduces $D_o$. It is for a similar reason that $D_o$ is increasing in $\sigma_y^2$. An increase in $\sigma_y^2$ increases the probability that $y < S$, thus increasing the expected cost of autonomy as perceived by outsiders. Turning to the third part of the theorem, the intuition behind $\partial D_o/\partial \mu_x > 0$ is that an increase in $\mu_x$ makes it more likely that the insider will exploit the investment opportunity without making it more likely that outsiders will want it exploited. This increases the expected cost outsiders perceive related to the insider investing in a state in which outsiders view the investment as a bad idea. As for the intuition behind $\partial D_o/\partial \mu_y < 0$, an increase in $\mu_y$ makes the investment opportunity more valuable to outsiders and hence lowers the cost of the autonomy.

Next we examine the properties of $A_o$.

**Theorem 4:** The value of the investment opportunity to outsiders when there is agreement, $A_o$, is:

(i) increasing in the agreement parameters $\rho$;

(ii) decreasing in $\sigma_x^2$, the variance of $x$, and increasing in $\sigma_y^2$, the variance of $y$; and

(iii) increasing in $\mu_x$, the mean of $x$, and increasing in $\mu_y$, the mean of $y$.

The intuition behind part (i) is the same as that underlying the positive relationship between $A$ and $\rho$. As for part (ii), an increase in $\sigma_x^2$ decreases the probability that $x > S$ and hence makes it less likely that the insider will invest in the project. This decreases the value of $A_o$. An increase of $\sigma_y^2$ leads to an increase in $A_o$ because it is an increase in the volatility of the underlying asset on which the option to invest is valued by the outsider. Finally, $A_o$ is increasing in $\mu_x$ and $\mu_y$ because an increase in either variable represents an increase in the measure of the region over which the insider and the outsider agree that the projects should be taken.

Having examined the properties of $D$, $A$, $D_o$ and $A_o$, we will now explore applications of autonomy.
3. APPLICATION OF AUTONOMY TO THE EXTERNAL FINANCING PROBLEM: AUTONOMY VERSUS PRIVATE CONTROL BENEFITS

In this section we develop in detail the application of autonomy to the entrepreneur’s problem of what kind of contractual arrangement to use in raising external capital. In particular, we focus on the tradeoff between autonomy and the cost of capital. Our analysis examines first the entrepreneur’s problem when external financing is raised with equity and then when it is raised with debt. In each case, we consider the problem from two perspectives: when the friction is potential disagreement between the entrepreneur and the financiers and when it is an exogenous private benefit of control for the entrepreneur. The purpose of analyzing the problem this way is to clarify how disagreement and the associated autonomy concerns differ from the usual agency setting with private benefits of control. The importance of this is underscored by the fact that it is natural to think of control when thinking of autonomy.

For this application, it is useful to think in terms of the total value of the project (or the firm) rather than just the NPV. That is, think of $S$ as the value of assets (perhaps cash) that the firm has currently, and $x$ (or $y$) as the signal about the value of the new opportunity that is mutually exclusive with the assets in place. That is, if one takes advantage of the new opportunity, then one invests $S$ and the value of the firm becomes $x$ as viewed by the insider and $y$ as viewed by outsiders. If one passes up the new opportunity, the value of the firm is $S$. Let us designate the total firm value as TV, so that the value to the insider equals:

$$ E(TV \mid \eta) = A + \eta D + S $$

and the value to the outsider is:

$$ E(TV_{\alpha} \mid \eta) = A_{\alpha} - \eta D_{\alpha} + S $$

3.1 The Entrepreneur’s Financing Problem with Equity

Consider the following situation. At date $t = 0$, an entrepreneur (insider) raises funds in the amount of $SI$ from venture capitalists (VCs) whom we shall view as outsiders. At this time, a contract is negotiated between the entrepreneur and the VCs which specifies: (a) how much equity ownership, $\alpha \in (0,1)$, the VCs receive in exchange for providing $SI$, and (b) the autonomy probability $\eta$. While we
initially assume that capital is raised with equity, we will later also examine debt.

The entrepreneur operates his business by investing $I$ to hire employees and acquire office space, furniture and computers. There is no uncertainty about this activity. However, at some future date, say $t = 1$, a new investment opportunity will arise whose value will be $x$ as assessed by the entrepreneur and $y$ as assessed by the VCs. At this point, $I$ is a sunk cost and the firm’s assets in place are worth $S$. Investing in this opportunity will tie up all of the assets in place. The autonomy probability $\eta$ will determine whether the entrepreneur is able to take advantage of this opportunity if the VCs object. Let $R$ represent the VCs’ reservation rate of return for providing funds to the entrepreneur.

At $t = 0$, the entrepreneur’s problem with equity can be written as

$$\max_{\eta \in [0,1]} \{A + \eta D + S\} \{1 - \alpha\}$$

subject to

$$\alpha \{A_o - \eta D_o + S\} \geq I[1 + R]$$

The following result is immediate.

**Lemma 2:** The VCs’ ownership fraction $\alpha$ is increasing in the autonomy probability $\eta$.

This simple result highlights an interesting point. The entrepreneur faces a tradeoff between autonomy and the cost of capital. As he seeks higher autonomy, the ownership he has to surrender to the VCs goes up, i.e., the cost of capital increases. Thus, the “cost” of the entrepreneur’s autonomy that the VCs absorb ex post is borne *ex ante* by the entrepreneur. Note, however, that both $\alpha$ and $\eta$ are endogenous in this model. So we now present a result about how these are endogenously determined.

**Lemma 3:** The optimal ownership share surrendered by the entrepreneur to the VC is given by

$$\alpha^* = \frac{I[1 + R]}{A_o - \eta^* D_o + S}$$

where $\eta^*$, the unique solution to the maximization program in (13) – (14), exists and is given by

$$\eta^* = \frac{A_o + S}{D_o} - \sqrt{\frac{I[1 + R][D[A_o + S] + D_o[A + S]]}{DD_o^2}}$$

This lemma shows that the entrepreneur’s tradeoff between the benefit of autonomy and the
higher cost of capital that greater autonomy brings with it leads to an interior optimum with respect to the amount of autonomy the entrepreneur seeks. We will now examine some of the comparative statics properties of $\alpha^*$ and $\eta^*$.

Lemma 4: The optimal degree of autonomy sought by the entrepreneur is decreasing in the VC’s reservation rate $R$.

The intuition is that an increase in $R$ makes each increment of autonomy more expensive for the entrepreneur, so he seeks less autonomy. As is well known, VCs often finance entrepreneurs using contracts that restrict the entrepreneur’s autonomy significantly by giving the VCs the ability to intervene and exercise control that is disproportionate relative to the ownership share held by the VCs (see Admati and Pfleiderer (1994), Chan, Siegel and Thakor (1990), and Gompers and Lerner (1998,1999)). That is, these contracts appear to explicitly separate financial ownership from autonomy.\(^\text{10}\)

It is natural to expect that the correlation between the entrepreneur’s and VCs’ opinions will affect both the value and the cost of financing of the project to the entrepreneur, as shown below.

Theorem 5: For any given level of autonomy, $\eta$, the equilibrium cost of capital for the entrepreneur, as measured by $\alpha^*$, is decreasing in the agreement parameter $\rho$ and increasing in $\sigma_x^2$, the variance of $x$.

The first part of the theorem is intuitive. An increase in $\rho$ increases the value of the project to the VC and hence lowers $\alpha^*$ for any given autonomy parameter $\eta$. An important implication of this result is that entrepreneurs will pay a higher cost of capital with equity when they are funding relatively new projects about which there is less familiarity and hence greater potential for disagreement.

The second part of the theorem about the impact of $\sigma_x^2$ on $\alpha^*$ is counterintuitive when considered within the usual real-options context without disagreement. As equity is a call option on the project, an increase in the variance of the project value should lead to an increase in the value of equity, and hence an increase in the value of the VC’s share of the project value. Consequently, for any given $\eta$,

\(^{10}\) For a paper that empirically examines the consequences of post-funding disagreement between the VC and the entrepreneur, see Higashide and Birley (2002).
the entrepreneur should be able to raise capital with a lower $\alpha$, i.e. his cost of equity capital should be lower. With disagreement, however, this result is turned on its head. The reason why this happens is that the probability that the entrepreneur will invest in the project decreases as $\sigma_y^2$ increases, so the probability mass attached to the state in which the entrepreneur invests and the VC agrees it is a good investment declines. This diminishes the value of the project to the VC since the VC assesses the value of the project using his interpretation $y$ and attaches no real-option benefit related to the higher probability of higher $x$ values that go along with an increase in $\sigma_y^2$.

We now address the relationship between the entrepreneur’s net expected payoff and some of the parameters of the model.

**Theorem 6:** The equilibrium net expected payoff of the entrepreneur is increasing in the agreement parameter $\rho$ and in the means of $x$ and $y$, $\mu_x$ and $\mu_y$.

The economic intuition is that disagreement in our model is purely dissipative, so an increase in the likelihood of agreement improves welfare. Moreover, an increase in the interpretation of project value by either the entrepreneur or the VC leads to a larger probability mass over which agreement will occur, hence increasing the entrepreneur’s welfare.

### 3.2 Debt Financing With Disagreement

Instead of equity, suppose the entrepreneur goes to a bank and borrows $I$ using a debt contract. Let $r_d$ be the repayment obligation to the bank. Let $R$ represent the (competitive) bank’s cost of funds. Again we assume that the entrepreneur interprets the project value as $x$ and the bank interprets it as $y$.

The entrepreneur’s maximization program now becomes:

$$\max_{\eta_d \in [0,1]} \{ A + \eta_d D + S - r_d \}$$

subject to

\[11\] This explanation focuses on the effect of $\sigma_y^2$ on $A_y$. While an increase in $\sigma_y^2$ also decreases $D_y$, the effect on $A_y$ swamps the effect on $D_y$. 

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where we will assume \( r_d < S \). We can now prove the following:

**Lemma 5:** The optimal degree of autonomy retained by the entrepreneur with a debt contract is \( \eta^*_d \), which is the unique solution to the maximization program in (17) – (18) and satisfies:

\[
\left[ 1 - \eta^*_d \right] \int \int \limits_{-s}^{s} q(x, y) \, dxdy + \int \int \limits_{-s}^{s} \{y, \min \{y, r_d\} + [1 - \eta_d] r_d \} \, dxdy + \int \int \limits_{-s}^{s} r_d q(x, y) \, dxdy \geq I \left[ 1 + R \right]
\]  

(18)

The equilibrium repayment obligation of the entrepreneur is given by

\[
r_d^* = \frac{I \left[ 1 + R \right] - \eta^*_d \int \int \limits_{-s}^{s} yq(x, y) \, dxdy}{1 - \eta^*_d \int \int \limits_{-s}^{s} q(x, y) \, dxdy}
\]  

(19)

We now have the analog of Lemma 4.

**Lemma 6:** The optimal degree of autonomy sought by the entrepreneur with a debt contract, \( \eta^*_d \), is decreasing in the bank’s reservation return \( R \).

As another comparative static property of this equilibrium, we have the analog of Theorem 5.

**Theorem 7:** An increase in \( \sigma^2_x \), the variance of \( x \), leads to a lower equilibrium borrowing cost, \( r_d^* \), for the entrepreneur for any \( \eta_d \).

This theorem stands in contrast to Theorem 5. As the variance of the entrepreneur’s assessment of the value of the project increases, the borrowing cost for the entrepreneur declines. The reason for the contrast with equity and the intuition for this result is as follows. With equity, the entrepreneur’s cost of capital increases with \( \sigma^2_x \) because the VC’s assessment of total project value diminishes as \( \sigma^2_x \) increases, and the VC has a linear share of this project value. With debt, the bank cares only about the probability.
that the entrepreneur will exercise the autonomy option when the bank thinks it is a bad idea. Since this probability decreases when \( \sigma_i^2 \) increases, the entrepreneur’s borrowing cost also declines with \( \sigma_i^2 \).

### 3.3 External Financing With Private Benefits of Control

To distinguish our theory with disagreement from one with a private benefit of control, we now outline a model in which the entrepreneur and external financiers agree on their interpretations of the value of the project, which we will refer to as \( x \). However, we will now assume that the entrepreneur can invest in a lemon project with an observable payoff of 0 and a private control benefit of \( B \) for the entrepreneur. We will assume \( B > S \). This control benefit is non-contractible and accrues entirely to the entrepreneur. Thus, there will be a cutoff value of \( x \), say \( \hat{x} \), such that the entrepreneur will prefer the innovative project if \( x \geq \hat{x} \) and the lemon if \( x < \hat{x} \). It is clear that with equity financing \( \hat{x} = B [1 - \alpha]^{-1} \).

Since there is no disagreement over the value of the innovative project, we now interpret the autonomy probability \( \eta \) as the probability with which the entrepreneur can invest in the lemon when \( x < \hat{x} \); clearly, when \( x \geq \hat{x} \), the entrepreneur and the external financier both wish to invest in the innovative project. When \( x \in (S, \hat{x}) \), external financiers prefer the innovative project, and when \( x < S \), they prefer the mundane project.

With equity financing, the entrepreneur’s maximization program becomes (using the subscript \( B \) to denote the “private benefit of control” case)

\[
\begin{align*}
\text{Max} \left[ 1 - \alpha \eta \right] & \left[ - \int_{\hat{x}}^{\hat{x}} \left[ x - S \right] g(x) \, dx \right] \\
& + \eta \int_{\hat{x}}^{\hat{x}} \left[ B \{1 - \alpha\}^{-1} - S \right] g(x) \, dx \\
& + \left[ 1 - \eta \right] \int_{S}^{\hat{x}} \left[ x - S \right] g(x) \, dx + S
\end{align*}
\]

subject to

\[
\alpha \left[ 1 - \eta \right] \int_{S}^{\hat{x}} \left[ x - S \right] g(x) \, dx + \int_{\hat{x}}^{\hat{x}} \left[ x - S \right] g(x) \, dx + S \geq I \left[ 1 + R \right]
\]
As in the case with disagreement, we can solve for the optimal $\eta$ and $\alpha$ here since the objective function is globally concave in $\eta$ once $\alpha$ is substituted into it from the binding pricing constraint (22). We skip those details here and get straight to a comparative static result on the cost of capital. The results with private benefits of control that correspond to those with autonomy will be given the same numbers as in the autonomy case but with asterisks.

**Theorem 5*:** An increase in $\sigma_x^2$, the variance of $x$, leads to a lower equilibrium cost of equity capital for the entrepreneur for any $\eta$.

This result is a consequence of the real-option effect associated with the choice of the innovative project. As the variance of $x$ increases, even though it becomes more likely that the entrepreneur will invest in the lemon, the call-option effect of increasing the variance makes the project more valuable to the VC. This lowers the cost of equity capital for the entrepreneur.

This result is in sharp contrast to Theorem 5 which says that the cost of equity capital with disagreement is *increasing* in the variance of $x$. The reason for this reversal is that with disagreement, the VC does not see the real-option benefit associated with a higher variance of $x$, but considers only the smaller probability of investment in the innovative project occurring when $\sigma_x^2$ goes up. Hence, the VC associates a lower value with the project when $\sigma_x^2$ increases. By contrast, with a private benefit of control, the VC has the *same* interpretation of project value as the entrepreneur and hence internalizes the higher call-option value associated with a higher $\sigma_x^2$. A comparison of Theorems 5 and 5* thus provides a clear way to distinguish between disagreement-based autonomy and private benefits of control via the distinction between the real option and the disagreement-based autonomy option.

We now turn to debt financing. Let $r_b$ be the repayment obligation on debt with control benefits. The cutoff value of $x$ such that the entrepreneur will switch from the innovative to the lemon project if $x < \hat{x}$ is given by $\hat{x} = B + r_b$. Note that if $x \geq \hat{x}$, the entrepreneur invests in the innovative project. If $x \in (S, \hat{x})$, the entrepreneur will invest in the lemon if the bank is unsuccessful in preventing it from
doing so; this happens with probability $\eta_{h_d}$ (we use the subscript “Bd” to denote the autonomy probability with debt with a private benefit of control), which is the autonomy parameter. If $x \in \{\hat{x}, \hat{x}\}$ and the bank is successful in preventing the entrepreneur from investing in the lemon (probability $1 - \eta_{h_d}$), then the innovative project is taken. If $x \leq S$, the entrepreneur will invest in the lemon with probability $\eta_{h_d}$ and in the mundane project (payoff $S$) with probability $1 - \eta_{h_d}$. Thus, the entrepreneur’s maximization program can be written as:

$$\max_{\eta_{h_d} \in [0,1]} \left\{ \int_{\frac{1}{2}}^{\frac{3}{2}} \left[ x - S - r_g \right] g(x) dx + \eta_{h_d} \int_{\frac{1}{2}}^{\frac{3}{2}} \left[ B - S \right] g(x) dx \right\}$$

subject to

$$\int_{\frac{1}{2}}^{\frac{3}{2}} r_g g(x) dx + \left[ 1 - \eta_{h_d} \right] \int_{-\infty}^{\frac{1}{2}} \left[ x - S - r \right] g(x) dx \geq I \left[ 1 + R \right]$$

Again we can solve for the optimal $\eta_{h_d}$ and $r_g$ in this case, but do not present those details in the interest of space. The following result can be proved.

**Theorem 7**: The equilibrium cost of debt financing for the entrepreneur is increasing in the variance of the value of the innovative project, $\sigma^2_x$, for any $\eta$.

This result is in contrast to Theorem 7, which says that with disagreement the cost of debt financing decreases with $\sigma^2_x$. With a private benefit of control, the entrepreneur pays a higher cost of debt if the variance $\sigma^2_x$ increases. The intuition here is that the entrepreneur and the bank interpret the value of the innovative project the same way, so that an increase in $\sigma^2_x$ increases the probability that both attach to the event that $x < \hat{x}$. Since the entrepreneur prefers the lemon project only when $x < \hat{x}$, his borrowing cost increases with $\sigma^2_x$.

To summarize, with private benefits of control, the costs of financing with debt and equity behave in the usual manner with respect to risk. As uncertainty about project value increases, the cost of equity
financing decreases and the cost of debt financing increases. This happens because the shareholders benefit from the enhanced real-option value of the project. With disagreement and autonomy, the opposite holds: the cost of equity financing increases and the cost of debt financing decreases when the entrepreneur’s assessment of project value becomes more uncertain. This happens because an increase in this form of uncertainty does not cause the financiers to change their assessments of the project but they do revise downward their assessment that the entrepreneur will undertake the innovative project. The cost of debt declines because the bondholders now perceive less risk, but shareholders demand higher ownership because they see less upside potential. Disagreement is therefore at the heart of what drives the difference between autonomy and an exogenous private benefit of control.

Given the stark contrasts between the costs of equity and debt financing with disagreement on the one hand and exogenous private control benefits on the other, it is useful to ask when one should expect disagreement to be more likely and when private control benefits are likely to predominate. We believe that disagreement is more important when one is dealing with entrepreneurs who are bringing forth relatively new technologies/projects for funding. Financiers are likely to be relatively unfamiliar with them and the absence of objective historical data will suggest high potential disagreement. A pure private-benefits-of-control problem without disagreement seems applicable to entrenched managers used to enjoying the “perks” of office and who are seeking funding for well-known technologies/projects with relatively low potential disagreement over project value.

4. OTHER APPLICATIONS OF AUTONOMY

In this section we discuss how autonomy could be used to gain insights into other economic phenomena such as the design and issuance of financial securities, the role of collateral, the notion of power in organizations, conformity and the choice of private versus public ownership.

4.1 The Design and Issuance of Financial Securities

The problem analyzed in the previous section can be generalized to that of designing financial securities and deciding which to issue. We believe that, in addition to specifying a particular sharing rule
for financial payoffs, every financial contract stipulates a degree of autonomy. Equity provides management greater autonomy and is a more flexible contract than debt partly because it gives management the flexibility to pay whatever dividends it chooses, whereas debt contractually obligates the firm to make periodic interest payments. One could even distinguish between bank debt and public debt along these lines (Berlin and Mester (1992)). Since different financial securities endow the issuer with varying degrees of autonomy from investor intervention and hence varying degrees of flexibility to the issuer, the choice of which security to raise financing with will also be driven by how much flexibility the issuer needs at a given point in time and the tradeoff between the value of autonomy and the impact of autonomy on the cost of capital for that security. Our theory predicts that securities with greater autonomy for issuers will carry a higher cost of capital, controlling for priority/seniority, even if there are no tax-treatment differences, and that the impact of autonomy on the cost of capital and hence the issuer’s net expected payoff will be affected by the agreement parameter and the expected future value of the project being financed (Theorem 6). Thus, when a firm with a relatively high equity agreement parameter and fairly restrictive debt covenants in place anticipates having access to future investment opportunities, it will assess a relatively high net payoff with equity (Theorem 6) and may perceive a “pecking order” of securities, with equity being the most attractive, followed by preferred stock, bank debt and public debt. Contrast this with the asymmetric–information–based pecking order derived by Myers and Majluf (1984) in which equity is at the bottom of the pecking order. According to our analysis, the firm may choose the Myers and Majluf (1984) pecking order when it anticipates future investment opportunities with a relatively low equity agreement parameter.12

4.2 The Role of Collateral in Lending

Bank loans are either secured or unsecured. While secured loans are more common than unsecured loans, banks routinely make unsecured loans. Collateral lowers the cost of debt financing (see

12 It appears then that autonomy offers a new way to approach security design that is different from the risk-sharing and information-based approaches currently in vogue. See for example Allen and Gale (1994), Boot and Thakor (1993), Chowdhry, Grinblatt and Levine (2002), and Fulghieri and Lukin (2001).
Rajan and Winton (1995)), so it is somewhat puzzling why all loans are not secured. That is, what exactly is the cost the borrower perceives in pledging an asset as collateral?

Our theory suggests an answer. A borrower who takes an unsecured loan is subject to being blocked by the bank if there is disagreement about the worth of a new project. Collateral can reduce the measure of the states in which the bank will block the borrower’s project choice even if there is disagreement, because collateral makes the bank’s payoff less sensitive to the project payoff. Thus, collateral can be viewed as a way for the borrower to overcome disagreement.

Given that the borrower has access to only a limited amount of collateral, it must choose how to allocate this scarce resource across its many loans. We know from Theorems 3 and 4 that the cost of external financing with disagreement is decreasing in the agreement parameter \( \rho \). Thus, the marginal value of collateral will be relatively low for loans involving future investment decisions that are less likely to involve disagreement, and relatively high for loans with greater future potential for disagreement. This implies that a borrower with limited collateral will take unsecured loans to finance “familiar” projects on which future disagreement with the bank has a relatively low likelihood, and reserve collateral for securing loans to finance relatively “unfamiliar” projects. In other words, the borrower may choose an unsecured loan even when it has collateral available.

4.3 Power in Organizations

A simple interpretation of autonomy is that it represents power in an organizational setting. That is, autonomy is “real authority” in the sense of Aghion and Tirole (1997). If we interpret \( y \) as the recommendation based on the rules and procedures of the organization, then power is the ability to make decisions outside the boundaries defined by organizational rules. Alternatively, if we interpret \( y \) as the consensus opinion of those other than the decisionmaker, then power is the ability to do something that others in the organization disagree with.

Interpreted this way, power is typically an inverted pyramid in an organization – it increases as

\[ \rho \]

While those results are proved with equity financing, they are true for debt financing as well.
you move up the hierarchy. Why? Our theory offers a possible explanation. Suppose that there are two types of insiders: competent and incompetent. Competent insiders have good intuition in that their interpretation $x$ is a more precise indicator of the eventual outcome than $y$. Incompetent insiders have bad intuition in that $x$ is a less precise indicator of the eventual outcome than $y$. Over time, insiders will develop reputations for competence and a sorting will occur based on these reputations. Those judged to be more competent will get promoted over those judged to be less competent. An insider’s human capital will increase every time he makes a decision based on $x$ (his intuitive judgment) that conflicts with $y$ (the rules) and the outcome reveals ex post he was right. Those at the top of the organizational hierarchy will have the best reputations for their intuitive judgment or competence. Consequently, they will also be given the most power or flexibility to “break the rules.”

This perspective reveals that the sorting of individuals based on perceived competence will be the slowest in organizations that are the most “rule-bound”, i.e., where individuals have the least autonomy. Thus, relatively adaptable organizations like Microsoft or Cisco should sort out individuals based on intuitive judgment (or leadership capability) faster than a government bureaucracy.

Our analysis also allows us to address the question about the determinants of the value of power to an individual. Theorem 1 tells us that the value of the autonomy option is decreasing in $\rho$ and increasing in $\sigma_x^2$ and $\sigma_y^2$. This means that a decisionmaker will value power more when others are more prone to disagree with him and when interpretations of decision-relevant signals are the most noisy. Moreover, Theorem 1 also tells us that the value of the autonomy option is increasing in $\mu_x$ and decreasing in $\mu_y$. This implies that power will be valued more when the decisionmaker has a higher expectation of the value of the investment opportunity and others have a lower expectation of its value.

4.4 Conformity in Organizations

There is now a substantial literature that seeks to explain why people tend to agree too much, especially with their superiors (see, for example, Prendergast (1993) and the references therein). Considerations of autonomy provide a simple explanation.
There are two ways to understand conformity. If, as in the previous subsection, there is an organizational sorting so that those with better intuition are more likely to be promoted, then a subordinate will wish to agree with his boss, even if his boss is breaking the rules the subordinate believes in, simply because the boss is likely to be right. Alternatively, the boss may have an innate preference for autonomy which may be manifested in a negative utility associated with not being able to make the decision she wants because others disagree. The subordinate may therefore agree with the boss when others disagree, in order to increase the probability the boss will prevail (i.e., contribute to an increase in $\eta$). Thus, when the boss has an innate preference for power, “yes-men” will conform more when the value of the autonomy option to the boss is higher. According to Theorem 1, this is when interpretations are the most subject to error and the potential for disagreement is the highest, i.e., the value of information aggregation will be lost precisely when its potential contribution is the greatest.

4.5 Private Versus Public Ownership

Autonomy is also likely to be a consideration in the firm’s decision of whether to be publicly or privately owned. When a firm is publicly owned, it cannot choose the degree of autonomy ($\eta$) associated with equity. This is likely to be determined by the rules of corporate governance for publicly-traded firms as well as the firm’s past financial performance and the reputation of its senior management. The better the firm’s past performance and the more reputable its management, the higher is $\eta$ likely to be because the Board of Directors and institutional investors will tend to provide management more “elbow room” in this case.

By contrast, with private equity, management can choose financiers who are willing to provide it with the desired autonomy. But this greater autonomy often looks like a Faustian bargain since private equity financiers typically have a higher ability to replace management. That is, management can choose its desired tradeoff between autonomy on the one hand and the cost of equity and personal risk on the other, opting for greater autonomy possibly by accepting a higher equity cost of capital and a higher probability of being fired. For the financiers, the lower diversification and liquidity that accompany
private ownership are traded off against the higher expected payoff they perceive from investing in a firm whose management they have confidence in and can more easily replace if that confidence evaporates. Private financiers usually play a different role compared to the corporate governance imposed by public shareholders who are more distant and have less control in firing management; they can typically only hamper management in the case of disagreement. Thus, a key distinction between public and private ownership may be that the degree of autonomy in the former is “market-determined”, whereas it may be a choice variable in the latter. From Lemma 3 we know that there is an interior optimal degree of autonomy for management, so that there will be instances in which the exogenously-set degree of autonomy with public ownership is so far away from what is optimal that private ownership is preferred. We would expect intuitively that publicly-traded firms with better past performance and more reputable CEOs would enjoy higher degrees of autonomy, lower costs of equity financing and higher stock prices, so that the degree of autonomy managers in these firms enjoy is closer to what they consider optimal. Conversely, low stock prices may indicate a lack of investor confidence in the CEO and a lower degree of autonomy for the CEO. This will diminish the benefits the CEO perceives with public ownership, and will increase the relative attractiveness of private ownership at low stock prices.

An example of this is provided by the recent experience of small publicly-owned companies in the European Union that have been viewed as facing a somewhat hostile environment in the financial markets that has adversely affected their strategic and operational flexibility. For example, an insider in one of these companies stated, “With equity markets consistently ignoring the merits of small and mid-cap corporates, the option to exit the public market and go private is becoming more and more alluring.” This is consistent with the implications of our theory, which suggests that when the autonomy associated with public equity is sufficiently low, it might become more attractive for management or (possibly hostile) bidders to take the firm private and “manage” its degree of autonomy.

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5. CONCLUSION

The goal of this paper has been to provide economic content to “autonomy.” The starting point of our analysis has been the premise that different people may perceive the same reality in different ways, so that two agents observing the same information may disagree on the optimal course of action suggested by that information. Faced with such disagreement, autonomy for the decisionmaker is defined as the ability to make the decision he thinks is best even though others view it as a mistake.

We have shown that autonomy is analogous to an option whose value increases with the potential for disagreement. This perspective on autonomy paves the way for examining a variety of different applications of the concept. We analyze in detail how autonomy considerations can affect the terms and mix of external financing chosen by an entrepreneur. We then discuss how autonomy can be used: to analyze a firm’s security issuance decision, determine the choice between secured and unsecured loans, improve our understanding of power and conformity in organizations, and analyze the firm’s choice between private and public ownership.

We believe that an examination of the economic content of autonomy is more than just another way of looking at the private benefits of control. We believe that considerations of disagreement and autonomy play a fundamental role in implicit and explicit contracting of various sorts, and generate predictions that are quite different from those available with other approaches. Examples of such predictions that our analysis has highlighted are:

1. With disagreement, the cost of external equity is increasing in the entrepreneur’s assessment of the variance of the total underlying project payoff. With a standard private-benefits-of-control framework, the cost of equity capital is decreasing in this variance.

2. With disagreement, the cost of debt financing is decreasing in the entrepreneur’s assessment of the variance of the total underlying project payoff. With a standard private-benefits-of-control framework, the cost of debt financing is increasing in this variance.

3. Unsecured loans will be more common for financing projects with payoff structures that are relatively easy to estimate and interpret using historical data (such as working capital loans) and secured loans...
will be more common for financing projects with less-familiar payoff structures (such as new plant investments).

(4) With disagreement, the pecking order for security issuance may have equity before debt.

(5) Power will be most highly valued by the decisionmaker when the potential for disagreement is the highest, interpretations are the most subject to error and the decisionmaker’s prior assessment of the project value is the highest.

Our initial exploration of autonomy suggests that there is a rich agenda for future research. One possibility is to dive more deeply into the topics we have discussed as applications, and think of other applications of autonomy. For example, autonomy may provide an alternative way to think about the design of political institutions and the related literature on “choosing how to choose”. In that literature, the issue is the determination of the socially-optimal checks and balances for elected leaders, which then affects how much unchecked power leaders have.\textsuperscript{15} Other examples include a firm’s decisions regarding its payout policy and the organization of its capital budgeting decisions.

\textsuperscript{15}For example, Aghion and Bolton (1997), Maskin and Tirole (2001), and particularly Aghion, Alesina and Trebbi (2002), where the tradeoff is between the increased likelihood of good reforms being blocked if leaders are given too little power and the increased likelihood of expropriation if they are given too much power.
Appendix

Proof of Theorem 1: Let us first prove part (i). The value of $D$ is given by (4).

Now define $u \equiv \frac{x - \mu_x}{\sigma_x}, \quad w \equiv \frac{y - \mu_y}{\sigma_y}$ (A-1)

Substituting (A-1) in (4) and writing $q(x, y|\rho)$ as a bivariate normal density using (5), we can express (4) as

$$D = \int_{-\infty}^{\infty} \int_{[s-\mu]^{\rho,\mu}} \left[ u\sigma_x + \mu_x - S \right] \tilde{q}(u, w) du dw$$

where

$$\tilde{q}(u, w) = \exp\left\{ -\frac{1}{2} \left[ 1 - \rho^2 \right] \left[ u^2 - 2\rho uw + w^2 \right] \right\}$$

which upon simplification yields:

$$\tilde{q}(u, w) = \frac{\exp\left\{ -\frac{1}{2} \left[ 1 - \rho^2 \right] \left[ (w - \rho u)^2 + (1 - \rho^2) u^2 \right] \right\}}{2\pi \sqrt{1 - \rho^2}}$$

(A-3)

Now substitute $\alpha = \frac{w - \rho u}{\sqrt{1 - \rho^2}}$ and $d\alpha = \frac{dw}{\sqrt{1 - \rho^2}}$ in (A-2) to write:

$$D = \int_{s-\mu}^{\rho,\mu} \left\{ \int_{-\infty}^{\infty} \frac{e^{-\alpha^2/2}}{\sqrt{2\pi}} \left[ u\sigma_x + \mu_x - S \right] e^{-\alpha^2/2} du \right\} d\alpha$$

where

$$j(\rho, u) = \frac{\left( (S - \mu_x)/\sigma_x \right) - \rho u}{\sqrt{1 - \rho^2}}$$

(A-5)

Also define

$$p(J(\rho, u)) = \int_{-\infty}^{\infty} \frac{e^{-\alpha^2/2}}{\sqrt{2\pi}} d\alpha$$

(A-6)
Note that
\[ d\hat{P}/d\rho = \left[ \partial\hat{P}/\partial\hat{J} \right] \left[ \partial\hat{J}/\partial\rho \right] \]
and \( \partial\hat{P}\hat{J} > 0 \).

Now,
\[ \partial\hat{P}/\partial\rho = \int_{|S - \mu|/\sigma} [d\hat{P}/d\rho] [u\sigma_x + \mu_x - S] e^{-u^2/2} \frac{du}{\sqrt{2\pi}} \]
(A-7)

Since \( u\sigma_x + \mu_x - S > 0 \forall u \) in the range of integration, we know that the sign of the integrand in (A-7) and hence the integral itself will be the same as the sign of \( d\hat{P}/d\rho \). Further, since \( \partial\hat{P}\hat{J} > 0 \), the sign of \( \partial\hat{P}/\partial\rho \) will be the same as the sign of \( \partial\hat{J}/\partial\rho \).

Now,
\[ \partial\hat{J}/\partial\rho = \frac{\left( \hat{P}\left( S - \mu, \sigma_x \right) - \mu \right)}{\left[ 1 - \rho^2 \right]^{1/2}} \]
(A-8)

Since \( u > [S - \mu] / \sigma_x = [S - \mu] / \sigma_y \), we see that \( \partial\hat{J}/\partial\rho < 0 \), which means \( \partial\hat{P}\hat{J} > 0 \).

Part (ii) can be proved by differentiating (A-4) with respect to \( \sigma_x \). Note that
\[ \partial\hat{P}/\partial\sigma_x = \int_{|S - \mu|/\rho} \hat{P}(\rho, u) e^{-u^2/2} du > 0. \]

Similarly,
\[ \partial\hat{P}/\partial\sigma_y = \int_{|S - \mu|/\rho} \frac{e^{-u^2/2}}{\sqrt{2\pi}} \left[ S - \mu \right] \left[ u\sigma_x + \mu_x - S \right] e^{-u^2/2} du \]
\[ > 0 \text{ since } \mu_x > S. \]

To prove that (iii), Differentiate (A-4) with respect to \( \mu_x \) to obtain:
\[ \partial\hat{P}/\partial\mu_x = \int_{|S - \mu|/\rho} \hat{P}(\rho, u) e^{-u^2/2} du > 0. \]
Differentiating (A-4) with respect to $\mu_y$ yields:

$$\frac{\partial D}{\partial \mu_y} = \int_{|\mu|} \left[ \frac{d\hat{P}}{d\mu_y} \right] \left[ u\sigma_y + \mu_y - S \right] e^{-\sigma^2/\hat{J}} du$$  \hspace{1cm} (A-9)$$

where $d\hat{P}/d\mu_y = \left[ \frac{\partial \hat{P}}{\partial \hat{J}} \right] \left[ \frac{\partial \hat{J}}{\partial \mu_y} \right]$, with $\partial \hat{P}/\partial \hat{J} > 0$. Now,

$$d\hat{P}/d\mu_y = \frac{e^{-\sigma^2/\hat{J}}}{\sqrt{2\pi}} \left[ \frac{-1}{\sigma_y \sqrt{1 - \rho^2}} \right] < 0,$$

which means $\partial D/\partial \mu_y < 0$.  

\textbf{Proof of Lemma 1:} Immediate upon differentiating $E(V_j|\eta)$ and $E(V_o|\eta)$ in (7) and (8) respectively.  

\textbf{Proof of Theorem 2:} As in the proof of Theorem 1, we can make the appropriate substitutions and express (3) as:

$$A = \int_{|\mu|} \tilde{P} \left( \tilde{J}(\rho,u) \right) \left[ u\sigma_y + \mu_y - S \right] e^{-\sigma^2/\tilde{J}} du$$  \hspace{1cm} (A-10)$$

where

$$\tilde{P} \left( \tilde{J}(\rho,u) \right) = \int_{j(\rho,u)} e^{-\sigma^2/\tilde{J}} d\sigma$$  \hspace{1cm} (A-11)$$

and all the other variables are defined as in the proof of Theorem 1.

We now tackle the first part of the theorem. It is apparent from (A-11) that $\partial \tilde{P}/\partial \tilde{J} < 0$.

Moreover, we showed in the proof of Theorem 1 that $\partial \tilde{J}/\partial \rho < 0$. Thus it follows that $\partial A/\partial \rho > 0$.

Now consider the second part. Differentiating (A-10) with respect to $\sigma_y$ gives:

$$\frac{\partial A}{\partial \sigma_y} = \int_{|\mu|} \tilde{P} \left( \tilde{J}(\rho,u) \right) u e^{-\sigma^2/\tilde{J}} du > 0$$

Differentiating (A-10) with respect to $\sigma_y$ yields:
\[
\frac{\partial A}{\partial \sigma_y} = \int \left[ \frac{\partial \bar{P}}{\partial \bar{J}} \right] \left[ \frac{\partial \bar{J}}{\partial \sigma_y} \right] \left[ u \sigma_y + \mu_y - S \right] \frac{e^{-\bar{J}^2}}{\sqrt{2\pi}} \, du
\]

where \( \frac{\partial \bar{J}}{\partial \sigma_y} = \frac{\left[ \mu_y - S \right] \sigma_y^2}{\sqrt{1 - p^2}} > 0 \). Since \( \frac{\partial \bar{P}}{\partial \bar{J}} < 0 \), we have shown that \( \frac{\partial A}{\partial \sigma_y} < 0 \).

We now turn to the third part of the theorem. Differentiating (A-10) with respect to \( \mu_x \) yields:

\[
\frac{\partial A}{\partial \mu_x} = \int \left[ \frac{\partial \bar{P}}{\partial \bar{J}} \right] \left[ \frac{\partial \bar{J}}{\partial \mu_x} \right] \left[ u \sigma_x + \mu_x - S \right] \frac{e^{-\bar{J}^2}}{\sqrt{2\pi}} \, du > 0.
\]

Differentiating (A-10) with respect to \( \mu_y \) yields:

\[
\frac{\partial A}{\partial \mu_y} = \int \left[ \frac{\partial \bar{P}}{\partial \bar{J}} \right] \left[ \frac{\partial \bar{J}}{\partial \mu_y} \right] \left[ u \sigma_y + \mu_y - S \right] \frac{e^{-\bar{J}^2}}{\sqrt{2\pi}} \, du
\]

where \( \frac{\partial \bar{J}}{\partial \mu_y} = \frac{-1}{\sigma_y \sqrt{1 - p^2}} < 0 \)

Since \( \frac{\partial \bar{P}}{\partial \bar{J}} < 0 \), this means that \( \frac{\partial A}{\partial \mu_y} > 0 \).

\begin{flushright}
\textbf{Proof of Theorem 3:} We know that \( D_o \) is given by (10). Substituting \( u = \frac{x - \mu_x}{\sigma_x} \) and \( w = \frac{y - \mu_y}{\sigma_y} \) as in the proof of Theorem 1, we can write:
\end{flushright}

\[
D_o = \int_{[s-u],[e_u]} \int_{[s-u],[e_u]} \left[ S - u \sigma_x - \mu_x \right] \bar{q}(u,w) \, du \, dw
\]

Now substitute

\[
\xi = \frac{u - \rho w}{\sqrt{1 - \rho^2}} \quad \text{and} \quad d\xi = \frac{du}{\sqrt{1 - \rho^2}}
\]

to write

\[
D_o = \int_{[s-u],[e_u]} P \left( J \left( \rho, w \right) \right) \left[ S - u \sigma_y - \mu_y \right] \frac{e^{-\bar{J}^2}}{\sqrt{2\pi}} \, dw \quad \text{(A-12)}
\]

where
\[ P(J(\rho, w)) = \int_{\mathcal{S}(\rho, w)} e^{-x^T J_2} d\xi \]  
(A-13)

\[ J(\rho, w) \equiv \frac{[S - \mu_x]/\sigma_x - \rho w}{\sqrt{1 - \rho^2}} \]  
(A-14)

We will now show that \( \partial D_x / \partial \rho < 0 \). Differentiating (A-12) yields:

\[
\partial D_x / \partial \rho = -\int - \frac{[S - \mu_x]/\sigma_x - \rho w][S - w\sigma - \mu] e^{-x^T J_2}}{\sqrt{2\pi [1 - \rho^2]}} dw
\]

(where we have substituted \( \mu_x = \mu_y = \mu \) and \( \sigma_x = \sigma_y = \sigma \))

Since

\[
\int - \frac{[S - \mu_x]/\sigma_x - \rho w][S - w\sigma - \mu] e^{-x^T J_2}}{\sqrt{2\pi [1 - \rho^2]}} dw
\]

\[
> 0,
\]

we have \( \partial D_x / \partial \rho < 0 \)

Let us now turn to part (ii) of the theorem. Using (A-14), we see that

\[
\partial J(\rho, w) / \partial \sigma_x = -\frac{[S - \mu_x]}{\sigma^2} > 0 \text{ since } S < \mu_x.
\]

Thus, we see from (A-13) that

\[
\partial P(J(\rho, w)) / \partial \sigma_x < 0,
\]

which implies from (A-12) that \( \partial D_x / \partial \sigma_x < 0 \). Differentiating \( D_x \) with respect to \( \sigma_y \) yields:

\[
\partial D_x / \partial \sigma_y = \int P(J(\rho, w))[-w] e^{-x^T J_2}}{\sqrt{2\pi}} dw > 0
\]

since \( w < 0 \) over the range of integration.
We now prove part (iii) of the theorem.

Now,

\[ \partial D_{\xi} \theta \mu_{\gamma} = \int_{-\infty}^{\infty} \left[ \partial P / \partial \mu_{\gamma} \right] \left[ S - w \sigma_{\gamma} - \mu_{\gamma} \right] e^{-w^2/\sqrt{2\pi}} dw \]

where

\[ \partial P / \partial \mu_{\gamma} = \left[ \partial P / \partial J \right] \left[ \partial J / \partial \mu_{\gamma} \right] = \frac{-1}{\sigma_{\gamma} \sqrt{1 - \rho^2}} \left[ -\frac{e^{-J^2/\sqrt{2\pi}}}{\sqrt{2\pi}} \right] = \frac{ce^{-J^2/\sqrt{2\pi}}}{\sqrt{2\pi}} \]

where \( c = \frac{1}{\sigma_{\gamma} \sqrt{1 - \rho^2}} > 0 \)

Thus,

\[ \partial D_{\xi} \theta \mu_{\gamma} = \int_{-\infty}^{\infty} ce^{-[J^2+w^2]/\sqrt{2\pi}} \left[ S - w \sigma_{\gamma} - \mu_{\gamma} \right] dw \]

\[ = c \int_{-\infty}^{\infty} e^{-[J^2+w^2]/\sqrt{2\pi}} \left[ S - w \sigma_{\gamma} \right] dw - c \int_{-\infty}^{\infty} e^{-[J^2+w^2]/\sqrt{2\pi}} w \sigma_{\gamma} dw \]

Define \( \lambda \equiv -w \) and write the above as:

\[ \partial D_{\xi} \theta \mu_{\gamma} = -c \int_{-\infty}^{\infty} \left[ S - \mu_{\gamma} \right] e^{-[J^2+\lambda^2]/\sqrt{2\pi}} d\lambda - c \int_{-\infty}^{\infty} \lambda \sigma_{\gamma} e^{-[J^2+\lambda^2]/\sqrt{2\pi}} d\lambda \]

\[ = c \int_{-\infty}^{\infty} \left[ \lambda \sigma_{\gamma} + S - \mu_{\gamma} \right] e^{-[J^2+\lambda^2]/\sqrt{2\pi}} d\lambda \]

\[ > 0 \text{ since } \lambda \sigma_{\gamma} + S - \mu_{\gamma} > 0 \text{ almost everywhere over the range of integration.} \]

Finally,

\[ \partial D_{\xi} \theta \mu_{\gamma} = \int_{-\infty}^{\infty} P \left( J \left( p, w \right) \right) \left[ -1 \right] e^{-w^2/\sqrt{2\pi}} dw < 0 . \]

\textbf{Proof of Theorem 4:} We will first prove that \( \partial A_{\gamma} / \partial \rho > 0 \). Note that after substituting \( u \) and \( w \) and then \( \xi \) as in the other proofs, we can write:
\[ A_0 = \int_{[s,w]} p(J(p,w)) \left[ \omega \sigma_y + \mu_y - S \right] \frac{e^{-w/\rho}}{2\pi} dw \]  \hspace{1cm} (A-15)

where \( P \) and \( J \) are defined as earlier in the proof. Thus,

\[ \frac{\partial A_0}{\partial \sigma} = -\int_{[s,w]} e^{-w/\rho} \left[ \omega \sigma_y - \left( s - \mu \right) \right] \frac{e^{-w/\rho}}{\sqrt{2\pi(1-\rho^2)}} dw \]

> 0

We now prove part (ii). Differentiating (A-15) with respect to \( \sigma \), we get:

\[ \frac{\partial A_0}{\partial \sigma} = -\int_{[s,w]} e^{-w/\rho} \left[ \omega \sigma_y - \left( s - \mu \right) \right] \frac{e^{-w/\rho}}{\sqrt{2\pi(1-\rho^2)}} dw < 0. \]

Moreover,

\[ \frac{\partial A_0}{\partial \mu} = \int_{[s,w]} \frac{wP(J(p,w))e^{-w/\rho}}{\sqrt{2\pi}} dw > 0. \]

We now turn to the third part of the theorem. From (A-13) and (A-14) we see that \( \partial P/\partial J < 0 \) and \( \partial J/\partial \mu < 0 \). Thus, \( \partial P/\partial \mu > 0 \). Using (A-15), we now see that \( \partial A_0/\partial \mu > 0 \). Further,

\[ \frac{\partial A_0}{\partial \mu} = \int_{[s,w]} \frac{P(J(p,w))e^{-w/\rho}}{\sqrt{2\pi}} dw > 0. \]

**Proof of Lemma 2:** Note that in equilibrium, (14) should be binding, i.e.,

\[ \alpha \{ A_o - \eta D_o + S \} = I[1 + R] \]

Thus, the equilibrium value of \( \alpha \) is given by

\[ \alpha^* = \frac{I[1 + R]}{\{ A_o - \eta D_o + S \}} \]

Differentiating \( \alpha^* \) with respect to \( \eta \) gives:

\[ d\alpha^*/d\eta = \frac{I[1 + R]D_o}{\{ A_o - \eta D_o + S \}^2} > 0 \]

\[ \checkmark \]
Proof of Lemma 3: Note first from Lemma 2 that $\alpha^*$ is given by (15). Substituting for $\alpha^*$ from (15) into the objective function (13) gives us:

$$\{A + \eta D + S\} \left[1 - \frac{1[1+R]}{[A_y - \eta D_o + S]}\right]$$

Differentiating with respect to $\eta$ yields:

$$D[A_y - \eta D_o + S]^2 = 1[1+R]\left[D[A_y + S] + D_o[A + S]\right]$$

This equation has two solutions:

$$\eta^* = \frac{A_y + S}{D_o} - \sqrt{\frac{1[1+R]\left[D[A_y + S] + D_o[A + S]\right]}{DD_o}}$$

and

$$\eta^{**} = \frac{A_y + S}{D_o} + \sqrt{\frac{1[1+R]\left[D[A_y + S] + D_o[A + S]\right]}{DD_o}}$$

Note, however, that only $\eta^*$ satisfies the second-order condition for a unique maximum:

$$2 D[A_y - \eta D_o + S][-D_o] < 0,$$

hence it is the maximizing solution we need.

Proof of Lemma 4: Obvious upon differentiating $\eta^*$ with respect to $R$, with $\eta^*$ given by (16).

Proof of Theorem 5: We know from Lemma 3 that $\alpha^*$ is given by (15). It is apparent from (15) that

$$\frac{\partial \alpha^*}{\partial \left[A_y - \eta^* D_o\right]} < 0.$$ 

Now, from Theorem 3 we know that $\partial D_o/\partial \rho < 0$ and from Theorem 4 we know that $\partial A_o/\partial \rho > 0$. Thus, $\partial \left[A_y - \eta^* D_o\right]/\partial \rho > 0$ for any $\eta^*$, which implies that $\partial \alpha^*/\partial \rho < 0$ for any $\eta^*$.

As for the effect of $\sigma_{\alpha^*}$, we know from Theorem 3 that $\partial D_o/\partial \sigma_{\alpha^*} < 0$, and from Theorem 4 that $\partial A_o/\partial \sigma_{\alpha^*} < 0$. From the expressions in Theorem 1 and 2 we can show that $\partial \left[A_y - \eta D_o\right]/\partial \sigma_{\alpha^*} < 0$,

which implies from (15) that $\partial \alpha^*/\partial \sigma_{\alpha^*} > 0$.◆
Proof of Theorem 6: Using the various expressions in the proofs of Theorems 1 and 2 and the fact that $S < \mu_s = \mu_s$, we can show that $\partial[A + \eta D]/\partial \rho > 0$, $\partial[A + \eta D]/\partial \mu_s > 0$ and $\partial[A + \eta D]/\partial \mu_s > 0$.

Similarly, using the various expressions in Theorems 3 and 4 we can show that $A_o - \eta D_o$ is increasing in $\rho$, $\mu_s$ and $\mu_s$. Moreover, from Lemma 3 we know that $\partial \alpha^*/\partial [A_o - \eta D_o] < 0$. Thus, it follows that the entrepreneur’s net expected payoff is increasing in $\rho$, $\mu_s$ and $\mu_s$.

Proof of Lemma 5: Note that (18) can be written as

$$r_d \left\{ \Pr(x > S, y > S) + \Pr(x > S, y \in (r_o, S)) \right\} + \Pr(x < S) + \int_{-\infty}^{\mu} \left( \eta x + [1 - \eta x] r_o \right) g(x, y) dxdy$$

$$= I[1 + R]$$

which is equivalent to:

$$r_d - \eta_d \int_{-\infty}^{\mu} [r_o - y] g(x, y) dxdy = I[1 + R]$$

Solving this equation yields

$$r^*_d = \frac{I[1 + R] - \eta_d \int_{-\infty}^{\mu} y q(x, y) dxdy}{1 - \eta_d \int_{-\infty}^{\mu} q(x, y) dxdy}$$

Substituting $r^*_d$ back in the objective function (17) and differentiating with respect to $\eta_d$ yields the first-order condition that can be rearranged to obtain (19). Now to verify that the $\eta_d^*$ given by (19) is indeed a global maximum, we differentiate with respect to $\eta_d$ to obtain:

$$2 \left[ 1 - \eta_d \int_{-\infty}^{\mu} g(x, y) dxdy \right] \left[ - \int_{-\infty}^{\mu} q(x, y) dxdy \right]$$
<0.

**Proof of Lemma 6:** Note that \( \eta_d^* \) is the unique solution to (19). It is apparent from this condition that 
\[
\frac{\partial \eta_d^*}{\partial R} < 0.
\]

**Proof of Theorem 7:** The equilibrium borrowing cost is \( r_s^* \) and is given by (20). Upon rearrangement, (20) can be written as:
\[
r_s^* = \eta_s^* \int_{-S}^{1} \left[ r_s^* - y \right] q(x, y) dx dy + I[1 + R].
\]
Differentiating with respect to \( \sigma_s^2 \) yields
\[
\frac{\partial r_s^*}{\partial \sigma_s^2} = \eta_s^* \int_{-S}^{1} \left[ \frac{\partial r_s^*}{\partial \sigma_s^2} \right] q(x, y) dx dy + \int_{-S}^{1} \left[ r_s^* - y \right] \left[ \frac{\partial q(x, y)}{\partial r_s^*} \right] dx dy,
\]
which implies
\[
\frac{\partial r_s^*}{\partial \sigma_s^2} = \frac{\eta_s^* \int_{-S}^{1} \left[ r_s^* - y \right] \left[ \frac{\partial q(x, y)}{\partial r_s^*} \right] dx dy}{1 - \eta_s^* \int_{-S}^{1} q(x, y) dx dy} \quad (A-16)
\]
Since the numerator in (A-16) is negative, we see that \( \frac{\partial r_s^*}{\partial \sigma_s^2} < 0 \).

**Proof of Theorem 5*: Since (22) holds tightly in equilibrium, we can write the equilibrium \( \alpha_s \) as the solution to the following equation:
\[
\alpha_s^* = \frac{I + R}{\int_{i(\alpha_s)}^{S} \left[ x - S \right] g(x) dx + \int_{i(\alpha_s)}^{1 - \eta_s^*} \left[ x - S \right] g(x) dx + S}
\]
where \( \eta_s^* \) is the equilibrium value of \( \eta_s \) and \( \hat{x} = B \left[ 1 - \alpha_s^* \right]^{-1} \).

Now, the denominator of the above expression can be written as:
Now we know that \( \int_{S}^{x} g(x) \, dx \) is the value of a call option and is hence increasing in \( \sigma^2 \). If \( \int_{S}^{x} g(x) \, dx \) is decreasing in \( \sigma^2 \), the denominator will be increasing in \( \sigma^2 \) for any given \( \eta_a \) and hence we will be done, since at the optimal \( \eta^* \) corresponding to the higher \( \sigma^2 \), \( \alpha^* \) cannot be any higher.

If \( \int_{S}^{x} g(x) \, dx \) is increasing in \( \sigma^2 \), then the maximum negative impact on the rate at which the denominator increases with \( \sigma^2 \) that is created by \( \int_{S}^{x} g(x) \, dx \) will be with \( \eta^* = 1 \).

With this value, the denominator becomes
\[
\int_{S}^{x} g(x) \, dx + S,
\]
which is clearly increasing in \( \sigma^2 \) for any given \( \alpha^* \). Hence, \( \alpha^* \) is decreasing in \( \sigma^2 \).

**Proof of Theorem 7**: We know that the pricing constraint (24) holds tightly in equilibrium. Using the equilibrium value of \( \eta_{bd} \), we can express (24) as:
\[
r^* - \eta_{bd} \int_{\frac{S}{r^n}} \eta^n g(x) \, dx = I[1 + R].
\]
Differentiating with respect to \( \sigma^2 \) yields:
\[
\frac{\partial r^*}{\partial \sigma^2} = \frac{\eta_{bd} \int_{\frac{S}{r^n}} \left[ \frac{\partial g(x)}{\partial \sigma^2} \right] \, dx}{1 - \eta_{bd} \int_{\frac{S}{r^n}} g(x) \, dx} > 0
\]
since the numerator is positive.
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