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**Rules and associations : hidden Markov models and neural networks in the psychology of learning**

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## Publications

The main chapters in this thesis have all been written as journal articles and have been published or are submitted. They have also all been co-authored by Maartje Raijmakers and Peter Molenaar.

Chapter 2: Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar, On the computational power and interpretation of subsymbolic processes, *Submitted for publication*.

Chapter 3 (slightly revised version): Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar (2001), in Robert M. French and Jacques P. Sourné, editors, *Connectionist models of Learning, Development and Evolution. The 6th Neural Computation and Psychology Workshop (NCPW6)*., p. 197–206, Springer-Verlag.

Chapter 4 (slightly revised and updated version): Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar (2000), *British Journal of Mathematical and Statistical Psychology*, 53, 317–327.

Chapter 5: Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar (2002), Fitting hidden Markov models to psychological data, *Scientific Programming, special issue on hidden Markov models*, accepted for publication, to appear summer 2002.

Chapter 6: Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar (2001), Reaction times and predictions in implicit sequence learning: Testing predictions by the simple recurrent network, *Under revision*.

Chapter 7: Ingmar Visser, Maartje E. J. Raijmakers and Peter C. M. Molenaar (2002), Associations and dissociations between direct and indirect measures of sequence learning, *Submitted for publication*.

Appendix A: Ingmar Visser (2001), Hidden Markov interpretations of neural networks, *Behavioral and Brain Sciences*, 23(4), 494–495 (Commentary on Connectionist Modelling in Psychology: A Localist Manifesto by Mike Page.)