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Optimisation of coils for semi-continuous magnetic fields

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Abstract

The optimum size, characteristic time, necessary power and energy for magnets for a semi-continuous high field above 40 tesla are strongly dependent on the type of coil-system; several different types can be distinguished. The increase in temperature of the coil is, for a homogeneous coil, about proportional to the square of the maximum field, and nearly independent of *all* other parameters. The temperature rise and minimum power needed can be reduced considerably by using a *two-stage* coil-system or a coil system with *independently powered* sections. Some general properties of these latter types of coil systems will be discussed.

1. Introduction

This article is part of a continuing series about coils generating semi-continuous high magnetic fields in the range of 30 to 100 tesla, which can be regulated with high accuracy, during an interval of about 0.1 second; the first article appeared in 1965 [1]. This is a working out of some points, mentioned earlier in a more recent article presented at the 2nd International Symposium on High-Field Magnetism, Leuven, 1988 [2].

2. Types of coil systems

The first point in designing a magnet for pulsed fields is that one should distinguish very well between two types of power supply. The first is a *power-limited* supply, for instance a rectifier

coupled to the mains or a generator; the second is an *energy-limited* supply, for instance a charged capacitor battery. The latter is used for generating field pulses of short duration in many laboratories [3].

In this article, we now will consider a *power-limited* power supply only; there can be distinguished (at least) *four*, essentially different, types of power-limited high-field coil systems:

(A) *Homogeneous coils*, which are wound from the same wire and have the same current density everywhere; possible sub-coils, which are electrically connected in series, can be physically separated by internal steel shields for reinforcement. The 40 tesla coil in the High Field Facility of the University of Amsterdam is of this type (A).

(B) *Inhomogeneous coils*, where the outer parts are wound from a wire less strong (but with a better electrical conductivity) than the wire used in the inner parts. Generally, it is advantageous to choose the current density in the outer sub-coils slightly

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higher than in the inner ones; all sub-coils are, however, connected in series to the same power supply.

(C) *Independently powered coils.* This type becomes profitable when the coil becomes so large that its time constant (the L/R time) becomes considerably larger than the time during which the (constant) field is wanted. Now, firstly the outer sections (which have a relatively large self-inductance) are energised. Subsequently, when the current through the outer sections has (nearly) reached its maximum value, the inner sections are powered during a much shorter interval.

The duration of the current in these inner sub-coils is considerably less. The big advantage is that therefore a higher current density can be used, as compared to a coil system of type (B); even if this is not possible due to stress-limitation, anyhow much less energy is dissipated in the inner coils.

(D) *Subsequently powered coils.* This is type (C) brought to its limit. If the duration of the pulse, during which the inner coils are energised, is very small compared to the time constant of the outer coils, it does not matter very much whether these outer coils are energised or crow-barred during this short time. In that case, one can save in the investment for the power supply if one can crow-bar the outer coils, and use the *same* power supply subsequently for energising first the outer coils, and after that the inner coils. This design has been presented for the first time in Ref. [2]; this type of coil-system has, however, also considerable disadvantages.

It should be noted that, in coil systems of type (C) and (D), it cannot be prevented that due to the *mutual inductance* between the coils the current through an outer coil decreases somewhat when the current through an inner coil rises.

The following limitations are important for magnet coils of these types:

(1) *Heat* limitations, meaning that if the current or the duration of the current would be increased, parts of the coil would reach an unsafe temperature. For insulation between the windings consisting of an organic material, the temperature may not increase far above room temperature. Thermal stresses may also limit the increase in temperature.

(2) *Power* limitations, meaning that all of the available power is used, firstly during the rise of the field to store the magnetic energy, and secondly at the end of the constant current period to overcome the ohmic resistance of the coils.

(3) *Stress* limitations, meaning that some windings are stressed to their limit and would, at a slightly higher stress due to the Lorentz forces, deform plastically at each pulse.

(4) *Self-inductance* limitations, meaning that, due to the large time constant of the (sub)coil in question, the actual duration of the current pulse is always longer than the duration required by the experiment.

In most cases, we will have a combination of several of these limitations in one and the same coil system; for example the inner windings may be heat limited, and the outer windings self-inductance and stress limited.

There exist exact scaling laws for magnet coils of these types; it has been shown in Ref. [1] that, if all length dimensions are multiplied with a scaling factor α , a magnet is obtained with the same fields, temperatures and stresses on corresponding sites. The total power P is multiplied by α , the current density j by α^{-1} , the time intervals by α^2 , and the total stored energy by α^3 .

3. Calculation of the coil temperature

In some earlier calculations [1] the heat, dissipated during the *constant current period*, was taken as the principal contribution, and the effects of the rise (and decay) time of the field were considered as corrections. The higher the field, however, the larger the coils become and the larger the effect of the self-inductance will be. For coils for 60 tesla and higher, it is therefore a good approximation if one approaches the problem from the other side and neglects in first instance the duration of the constant-field period compared to the rise time of the field. (In the design of a 60 tesla magnet where the first one is about 0.1 second, the second one becomes longer than 1 second.) It appears that, within this approximation, several relations can be given without specifying the dimensions of the coil, the wire material, the number of windings etc.

If we assume that the voltage V which is applied to the coil at the start of the pulse, when the current increases and the *inductive* voltage $V_{\text{ind}} = L \delta I / \delta t$ has to be delivered, is about equal to the voltage at the end of the constant-current period, when the *ohmic* voltage $V_{\text{res}} = R I_{\text{max}}$ must be opposed, we find that the rise time of the field Δt_r is approximately equal to L/R .

(Δt_r would be the rise time, if the rate of increase of the current would stay at its initial value. In reality, the resistance of the coil extends the rise time; if we assume a function of the second degree for the current as a function of time, the time necessary for reaching the maximum value is equal to $2\Delta t_r$.)

For rectangular coils with homogeneous current density and a reasonable shape (the height has been taken equal to 1.19 times the outer radius), the self-inductance L and the resistance R are to a good approximation given by:

$$L \approx 0.31 \mu_0 N^2 a \quad \text{and} \quad R_f \approx \frac{3.2 N^2 \rho_f}{a \lambda}, \quad (1)$$

so that

$$\Delta t_r = L/R_f \approx 0.098 \frac{\mu_0 \lambda a^2}{\rho_f}. \quad (2)$$

Here a is the outer radius of the coil, and λ is the filling factor, i.e. the fraction of the volume of the coil that is filled by conducting wires; N is the number of windings, ρ_f and R_f are the resistivity and resistance at the end of the pulse. Note that the rise time is independent of N .

The first law of thermodynamics gives us for the energy dissipated as heat per volume of the windings:

$$\delta Q = c_v \delta T = j^2 \rho \delta t \quad (3)$$

with c_v the heat capacity of the wire material per volume, and j the current density (c_v equals C_v/V_M , where C_v is the molar heat capacity, and V_M the molar volume).

If we try to integrate the above equation in order to find the total heat, we estimate that $\int j^2 dt$, integrated over the rise time, is about equal to $j_{\text{max}}^2 \Delta t_r$, where j_{max} is the maximum current density. (Assuming that the current rises to its maximum value

following an inverted parabola, an exact calculation would give, for an integration from 0 to $2\Delta t_r$, a result of $16/15 j_{\text{max}}^2 \Delta t_r$.)

If we now take into account that, following Ampère's Law, the magnetic field in the centre of the coil is approximately equal to about $B \approx 0.72 \mu_0 \lambda j a$, we get the following general relation:

$$\begin{aligned} \Delta Q &= \int j^2 \rho dt \approx 0.098 \rho_{\text{av}} j_{\text{max}}^2 \frac{\mu_0 \lambda a^2}{\rho_f} \\ &\approx \frac{0.19}{\lambda} \frac{\rho_{\text{av}}}{\rho_f} \frac{B_{\text{max}}^2}{\mu_0}, \end{aligned} \quad (4)$$

with B_{max} the maximum value of the central field.

The rise in temperature is therefore:

$$\Delta T = \frac{\Delta Q}{c_{v,\text{av}}} \approx \frac{0.19}{c_{v,\text{av}} \lambda} \frac{\rho_{\text{av}}}{\rho_f} \frac{B_{\text{max}}^2}{\mu_0}. \quad (5)$$

Here we can insert some numerical values: for pure metals, where ρ depends heavily on temperature, ρ_{av}/ρ_f may be fairly small, but in most alloys this fraction will be close to 1. The value of c_v does not change very much for different metals and alloys; we will use the value for Cu at room temperature, $c_v = 3.5 \times 10^6 \text{ J/m}^3 \text{ K}$, for most other metals c_v will be less than that. For the filling factor λ we take 0.8; this gives the relation:

$$\Delta T \approx (0.06 \text{ K/T}^2) \frac{\rho_{\text{av}}}{\rho_f} B_{\text{max}}^2. \quad (6)$$

It is very surprising to see that under these assumptions the temperature rise of a magnet of type (A) depends *only* on the maximum field strength, and not on the dimensions of the coil or on the properties of the wire material!

If two nearly identical coils are compared, the only difference being that the wire of the second coil has a 10 times larger electrical resistivity ρ than the first, both coils can generate the same field, using the same current. From Eq. (6) it follows that both will suffer about the same temperature rise, and therefore dissipate about the same energy. The difference is, however, that for the second the pulse duration will be about 10 times shorter, and the needed power is therefore 10 times higher. If the same duration is wanted, the dimensions of the second coil should have to be enlarged by a factor of $\sqrt{10}$.

We must, however, make some additional remarks. Relation (6) gives a *minimum* value for ΔT , assuming that the duration of the constant-field period, Δt_c , is very small compared to the rise time Δt_r ; if this is not the case and we do not have a self-inductance-limited coil, the temperature rise will be larger by a factor $(\Delta t_c + \Delta t_r)/\Delta t_r$:

$$\text{if } \Delta t_c \gg \Delta t_r \text{ then } \Delta T = \frac{j_{\max}^2 \rho_{av} \Delta t_c}{c_{V,av}} \approx \frac{1.9 \rho_{av} B_{\max}^2 \Delta t_c}{c_{V,av} \mu_0^2 \lambda^2 a^2}. \quad (7)$$

Furthermore, in the case of a type (B) coil (with *inhomogeneous* current density), the above relation gives the *average* increase in temperature; in that case, the outer windings (from less strong and better conducting wire) will heat up less, but the inner ones considerably more than given by the relation above.

If someone would design a not too unrealistic magnet of type (A) or (B) for a semi-continuous field of 100 tesla, it seems unavoidable that at least the inner windings would become red-hot after the pulse, independent of the dimensions of the coil and the choice of the wire material (superconductors excluded).

Fortunately, there are two different ways to avoid these high temperatures. Firstly, a brute-force method: if the power available to charge the coil during the rise time is much higher than the power necessary during the constant-current phase (i.e. if the coil is not power limited), the rise time will be much shorter, and the rise in temperature too. (It seems a paradox that if one *increases* the power, the magnet becomes *less* hot!)

Secondly, one may consider coil systems of type (C) or (D) as discussed above; in these cases, the rise of temperature of a (independently powered) section of the coil system would depend only on the field strength *generated by that section*, where it should be noted that, at any rate for the inner sub-coil(s), the condition $\Delta t_c \gg \Delta t_r$ will hold, and the heating during the rise time can be neglected compared to the heating during the constant-current period.

4. Calculation of the required power

Now we will consider the *power* necessary for a type (A) magnet coil with outer radius a_2 and inner radius a_1 . The total height is assumed $1.19 a_2$. Since a_1 will, in this section, not be neglected as compared to a_2 , the volume becomes $1.19 \pi \lambda a_2 (a_2^2 - a_1^2)$. From Ampère's Law it follows that the central field can be approximated by $B \approx 0.72 \mu_0 \lambda j (a_2 - 1.4 a_1)$. The power can now be written as:

$$P \approx 3.7 \lambda j_{\max}^2 \rho_f a_2 (a_2^2 - a_1^2) \approx \frac{7.1 B_{\max}^2 \rho_f a_2 (a_2^2 - a_1^2)}{\mu_0^2 \lambda (a_2 - 1.4 a_1)^2}. \quad (8)$$

At first sight one observes that, if $a_1 \ll a_2$, P is about proportional to a_2 , so that a small coil seems advantageous. If a_1 is not much smaller than a_2 , the power P is minimal if:

$$a_2 \approx 3.845 \times a_1; \quad P \approx \frac{50 B_{\max}^2 \rho_f a_1}{\mu_0^2 \lambda}. \quad (9)$$

It is assumed here that ρ_f does not depend on the value of a_2 . ρ_f is a function only of the kind of wire material, its starting temperature, and the integral over time of the square of the current density. We will denote this latter quantity here by the symbol x , so $x = \int j^2 dt \approx j_{\max}^2 (\Delta t_r + \Delta t_c)$ where Δt_r is the rise time of the field, and Δt_c the constant current period. Since j scales with $1/a_2$ and Δt_r scales with a_2^2 , x and therefore ρ_f remain approximately independent of a_2 as long as $\Delta t_r \gg \Delta t_c$, x (and with it ρ_f) start to increase as soon as a_2 becomes so small that $\Delta t_r \approx \Delta t_c$.

Obviously, there exists an optimum value for a_2 , for which the power P is a minimum; this is the value at which the influence of Δt_c or a_1 just becomes significant. If the properties of the wire material are known, this value of a_2 , and from that those of j and ρ_f , can be calculated in the following way: The quantity ρ_f is a function of $x = \int j^2 dt$, which function depends only on the physical properties of the wire and the initial temperature; the temperature T of the wire can be integrated from Eq. (3): $\delta T = (\rho(T)/c_V(T)) \delta x$, and ρ is a function

of T only. This relation will be denoted here by $\rho = G(x)$:

$$\rho_f = G(x), \quad \text{with} \quad \frac{dG}{dx} = \frac{\rho}{c_v} \frac{d\rho}{dT}, \quad \int G(x) dx = Q. \quad (10)$$

The quantity x can be written as a function of a_2 :

$$x = \int j^2 dt \approx j_{\max}^2 (\Delta t_r + \Delta t_c) \approx \frac{1.9 B_{\max}^2}{\mu_0 \lambda^2 (a_2 - 1.4 a_1)^2} \left(\frac{0.098 \mu_0 \lambda a_2^2}{\rho_f} + \Delta t_c \right). \quad (11)$$

If the above expressions are substituted into Eq. (8), P can be expressed as a function of a_2 . Putting the derivative of P with respect to a_2 equal to zero gives the optimum solution which can be found numerically.

Now, in the case that $a_1 \ll a_2$, the optimum values can also be found with a simple graphical procedure, if the function $G(x)$ is known (see Fig. 1). In this case, Eq. (11) can be simplified to

$$x = x_\infty + x_1 \approx \frac{0.19 B_{\max}^2}{\mu_0 \lambda \rho_f} + \frac{1.9 B_{\max}^2 \Delta t_c}{\mu_0 \lambda^2 a_2^2}. \quad (12)$$

x_∞ is the value of x for an infinitely large coil. This value can be found by solving Eqs. (10) and (12) (without second term) for the variables x and ρ_f ; the value of x_∞ is given by the crossing of the hyperbola with the function $G(x)$ in Fig. 1.

Substituting Eqs. (10) and (12) into Eq. (8), and imposing the condition $\partial P / \partial a_2 = 0$, one gets after some calculation the very simple equation:

$$G(x) - 2 \frac{\partial G}{\partial x} x_1 = 0. \quad (13)$$

This means that we should find a point $x = x_\infty + x_1$ on the curve $\rho_f = G(x)$ where the tangent to this curve crosses the horizontal axis in a point at a distance $2x_1$ to the left, see the figure.

For many materials, the so-found optimum value for ρ_f appears to be approximately proportional to $B_{\max}$; since the optimum value of a_2 is about proportional to $B_{\max}^{0.5}$, the minimum necessary power P is therefore, for a given wire material, about proportional to $B_{\max}^{3.5}$.

Fig. 1. The G -function, as defined in Eq. (10), for pure copper and the strong copper-silver compound. These functions give the electrical resistivity as a function of $x = \int j^2 dt$, in this case for an initial temperature of 77 K. Only this function determines the power needed for a high-field coil made from the material in question. The drawn hyperbola corresponds to Eq. (12) for a very large value of a_2 and a field $B_{\max}$ of 60 T, and determines x_∞ . The drawn tangent line shows a graphical method to evaluate the optimum values of x and ρ for an adjustable value of the outer radius a_2 .

For maximum fields above about 50 tesla it appears, however, that for most materials the minimum-power solution corresponds to a final temperature well above room temperature. This means that the coil is, in fact, *heat limited*, and the maximum current density is given by Eq. (4).

5. Limitations caused by the stresses

Next to the power, the stresses in the wires of the magnet are very important. One should be sure that the huge stresses, caused by the Lorentz force acting on the windings, will never cause a winding to break, since in that case not only the magnet is destroyed, but also the stored magnetic energy, which may be in the order of magnitude of 10^7 – 10^8 joule, could give rise to an explosion. These stresses can be analyzed in an analytic [1] or semi-analytic [2,4] way. Moreover, finite element calculations have been done [5] which confirm our analytic results. The order of magnitude of the necessary yield strength of the windings can, however, also be found in the more simple way described below.

Firstly, we assume the magnet is built up from several thin coils, so that the change of the magnetic force over the thickness of the coil is small. These thin sub-coils can possibly be reinforced by surrounding steel shields. Furthermore, we assume that the windings are allowed to flow plastically everywhere the first time the magnet is energized, but that they do not flow back when the field is returned to zero.

The equilibrium condition gives for the derivative of the stress in the radial direction:

$$\frac{\partial \sigma_r}{\partial r} = \frac{\sigma_\theta - \sigma_r}{r} - \lambda F_r \approx \frac{\sigma_c}{r} - \lambda j B_z(r). \quad (14)$$

Here σ_r , σ_θ and σ_z are the principal stress components (in cylindrical coordinates), σ_c is the yield strength of the winding material, r is the distance from the axis, and $F_r = j B_z$ is the radial body force. The thickness and the elastic properties of the insulating material, which are different from those of the metal of the windings, are neglected here.

The pressure exerted on the steel shield around the coil equals approximately

$$p = -\Delta r \frac{\partial \sigma_r}{\partial r} = \Delta r \left(\lambda j B_z - \frac{\sigma_c}{r} \right), \quad (15)$$

where Δr is the total thickness of the sub-coil, i.e. $(a_2 - a_1)$.

In the most simple case (no external steel shielding), this pressure p should be zero, and the condition for the yield strength becomes:

$$\sigma_c \geq \lambda r j B_z(r). \quad (16)$$

We can decrease this value somewhat by using an external steel reinforcing shield. In that case, p will be positive and, since the shield should be relatively thin, it will exert this same pressure after the pulse on the not-energised coil, which could give rise to flow-back of the inner part of the coil.

Now, for a thin cylinder, pressurised from the outside, the condition for no repeated plastic flow-back is

$$\sigma_c \geq \frac{pr}{\lambda \Delta r}. \quad (17)$$

Eliminating p from both the above equations and solving σ_c gives the condition

$$\begin{aligned} \sigma_c &\geq \frac{\lambda}{1 + \lambda} r j B_z(r) \approx -1.16 \frac{r B_z(r) \partial B_z / \partial r}{\mu_0} \\ &= 1.16 \frac{B_z(r) (B_e - B_z(r))}{\mu_0}. \end{aligned} \quad (18)$$

In this expression the relation $\lambda j \approx -1.4(\partial B_z / \partial r) \mu_0$ has been used. From the simplified analysis above, it would follow that the numerical factor would be ≈ 0.8 ; if the effects of the space for the insulating material and the finite length of the coil both are incorporated in this factor, we find, for a realistic coil, a value of about 1.16.

B_e is the field, extrapolated linearly to the axis of the coil: $B_e = B_z(r) - r(\partial B_z / \partial r) = B_z(r) + 0.7 \mu_0 \lambda r j$.

Comparing Eqs. (16) and (18), the necessary yield strength can be decreased by about 30% by using a thin steel shield, independently of the strength of this shield; it must be realised, however, that this shield replaces some windings, so that the total efficiency of the magnet is also reduced by the shield. Generally an outer reinforcing shield is not advantageous, *unless the shield material is much stronger than the winding material.*

For a type (A) magnet, with homogeneous current density, the extrapolated field B_e is about equal to the actual field in the centre, $B_{\max}$. (Actually, $B_e > B_{\max}$, if the width of the bore is not to be neglected.) The maximum value of the yield strength occurs, in that case, when $B(r) = \frac{1}{2} B_e$, and has a value of $\sigma_c \geq 0.29 B_e^2 / \mu_0 \approx 0.29 B_{\max}^2 / \mu_0$.

If now, however, the current density j decreases with the distance r , the value of B_e becomes for the outer coils considerably smaller than $B_{\max}$, and a stable magnet with material with a much lower yield strength can be designed; the disadvantage is, of course, that the magnet becomes much larger in that case.

In fact, a stable type (B) magnet can be designed for any value of the yield strength; if σ_c is much smaller than $0.29 B_{\max}^2 / \mu_0$, it can be found by integrating Eq. (18) that in this case the current density should be adjusted so that the field satisfies the following relation:

$$B^2(r) = B_{\max}^2 - B_{\text{stress}}^2 \ln(r/a_1), \quad (19)$$

where $B_{\text{stress}} = 1.32 \sqrt{\mu_0 \sigma_c}$ and a_1 is the radius of the inner bore of the magnet. B_{stress} is a characteristic field, depending only on the yield strength of the wire material; if $\sigma_c = 1000$ MPa, $B_{\text{stress}} = 46.8$ T.

If the current density, which can be found from Eqs. (18) and (19) is used, in most cases the inner windings will become much too hot (viz. Eq. (7)), and j has to be reduced there for that reason. Even if this effect, which enlarges the coil considerably, is neglected, the outer radius a_2 of such a coil system would be about

$$a_2 \approx a_1 \exp\left(\frac{B_{\text{max}}^2}{B_{\text{stress}}^2}\right). \quad (20)$$

This means that, for example, a 100-tesla magnet system, wound from hard-drawn copper with a σ_c of 400 MPa, needs to have an outer diameter of at least $100\,000 \times$ the inner diameter. A similar relation has been derived earlier by Schneider-Muntau [6].

6. Design of a type (C) coil system

For maximum fields up to about 50 tesla type (A) or (B) coil systems can be considered; for higher fields one should use a type (C) system or, when getting enough power is a serious problem, a type (D) one. The coil system will consist of several nested coils, each with its own wire material and current density. It is advantageous to calculate the coil-system from the inside outwards, i.e. start with a desired field B_{max} at the centre, and subtract from that the contribution of each coil around the previous ones. This has to continue until a field zero is reached outside the outermost coil.

In the following discussion we first consider a type (C) coil system where all sub-coils are wound from the same material. In that case the innermost coil(s) will be often heat-limited; the permissible current density j_{heat} in them can be calculated from the resistivity and thermal properties of the wire and the wanted duration of the field pulse, as given by Eq. (7), where ΔT is the maximum permissible rise of the temperature, e.g. 250 K:

$$j_{\text{heat}} \leq \sqrt{\frac{c_{V,\text{av}} \Delta T}{\rho_{\text{av}} \Delta t_c}}. \quad (21)$$

6.1. Insufficient strength of the wire

If the strength of the wire is insufficient, i.e. $\sigma_c < \sim 0.29 B_{\text{max}}^2 / \mu_0$, one is obliged to design the next coils stress-limited; now generally the innermost (sub-)coils, for $a_1 < r < a_s$, will be heat-limited, the current density is constant and given by Eq. (21), and the field is $B_z = B_{\text{max}} - 0.72 \mu_0 \lambda j_{\text{heat}}(r - a_1)$. The outer coils are stress-limited, from Eq. (18) it follows that the maximum current density is in that case given by:

$$j_{\text{stress}} \leq \frac{(1 + \lambda) \sigma_c}{\lambda r B_z(r)}. \quad (22)$$

The boundary between the heat-limited and stress-limited case occurs at a radius a_s and a local field B_s , which are given by:

$$a_s \approx \frac{(1 + \lambda) \sigma_c}{\lambda B_{\text{max}}} \sqrt{\frac{\rho_{\text{av}} \Delta t_c}{c_{V,\text{av}} \Delta T}},$$

$$B_s^2 \approx B_{\text{max}}^2 - 2(1 + \lambda) \mu_0 \sigma_c. \quad (23)$$

From this and Eq. (19) it follows that for $r > a_s$ the field is given by:

$$B^2(r) = B_s^2 - B_{\text{stress}}^2 \ln(r/a_s) \\ \approx B_{\text{max}}^2 - B_{\text{stress}}^2 (\ln(r/a_s) + 1.0). \quad (24)$$

The outer radius, a_2 , of this coil system is therefore approximately :

$$a_2 \approx \frac{(1 + \lambda) \sigma_c}{\lambda B_{\text{max}}} \sqrt{\frac{\rho_{\text{av}} \Delta t_c}{c_{V,\text{av}} \Delta T}} \exp\left(\frac{B_{\text{max}}^2}{B_{\text{stress}}^2} - 1.0\right). \quad (25)$$

In the outermost sub-coil, where $B_z \Rightarrow 0$, the current density is limited by the total dissipated heat which depends on the time constant of the coil in question; from Eqs. (2) and (3) it follows:

$$j_{\text{selfind}} \leq \sqrt{\frac{c_{V,\text{av}} \Delta T}{0.098 \mu_0 \lambda r^2}} \quad (26)$$

(note that in this case the current density *only* depends on the heat capacity of the wire).

The total power needed is not calculated easily; if we assume that the power is dominated by the

power dissipated in the stress-limited part of the coil, we can estimate:

$$P \approx \frac{100 \sigma_c^2 \rho}{\mu_0 B_{\max}} \sqrt{\frac{\rho \Delta t_c}{c_{V,av} \Delta T}} \exp\left(\frac{B_{\max}^2}{B_{\text{stress}}^2}\right). \quad (27)$$

From the above equation it follows, that in a magnet system of this type, for very high fields and with wire with an insufficient strength, a high value of the yield strength σ_c is much more important in reducing the maximum needed power than a low resistivity.

It is also possible to fix the current density in the outermost coil by the demand of minimum power, if this results in a lower current density and a larger coil; in that case the optimum current density can be solved from Eqs. (8), (10) and (11). In all cases the value of j in most of the windings will decrease approximately with $1/r$ as a function of r .

6.2. Wire of sufficient strength

If, on the other hand, the strength of the wire is sufficient to withstand the stresses everywhere, we have only heat-limited windings in the innermost coil(s), where Eq. (21) holds, and self-inductance limited windings in the remainder of the coil system, where Eq. (26) holds. The boundary is at a radius a_s and a field B_s , which are given by:

$$a_s \approx \sqrt{\frac{\rho_{av} \Delta t_c}{0.098 \mu_0 \lambda}}, \quad B_s \approx B_{\max} - B_{\text{therm}}, \quad (28)$$

where $B_{\text{therm}} = 2.3 \sqrt{\mu_0 \lambda c_{V,av} \Delta T}$. B_{therm} is the thermal limit for the maximum field of an (A) type coil which is not stress-limited, see Eq. (6); for many materials B_{therm} is equal to about 65 T.

In the case of the outermost sub-coil only, some power can (possibly) be saved by making it somewhat larger than given by the heat-limitation. This does not hold for the inner coils: most power is consumed by the outermost coil, and the saving in power by optimising an inner coil is more than counterbalanced by the fact that, in that case, the outer coil needs a larger inner diameter to fit around the other coils, and therefore needs much more power.

For $r > a_s$, the field decreases logarithmically:

$$\begin{aligned} B(r) &\approx B_s - B_{\text{therm}} \ln(r/a_s) \\ &\approx B_{\max} - B_{\text{therm}} (\ln(r/a_s) + 1.0). \end{aligned} \quad (29)$$

The outer radius of this coil is then given by:

$$a_2 \approx \sqrt{\frac{\rho_{av} \Delta t_c}{0.098 \mu_0 \lambda}} \exp\left(\frac{B_{\max}}{B_{\text{therm}}} - 1.0\right). \quad (30)$$

For $B_{\max} \gg B_{\text{therm}}$, the necessary power can very roughly be estimated as:

$$P \approx \frac{20 c_{V,av} \Delta T \rho}{\mu_0 \lambda} \sqrt{\frac{\rho_{av} \Delta t_c}{0.098 \mu_0 \lambda}} \exp\left(\frac{B_{\max}}{B_{\text{therm}}}\right). \quad (31)$$

In both cases (insufficient and sufficient strength) the power increases exponentially with the maximum value of the desired field; in the first case, however, this increase is less extreme than in the second case, especially when $B_{\text{stress}} < B_{\text{therm}}$.

The necessary power and the dimension of the coils can be diminished if we allow *wires of different material* for the different sub-coils. In that case the composition of the wire material can be adjusted in such a way that all sub-coils (except possibly the outermost one) are simultaneously heat- and stress-limited. The analysis now strongly depends on the exchange rate of strength versus conductivity for a certain class of (composite) wire materials; this analysis has to be done numerically and will not be attempted here.

As mentioned at the derivation of Eq. (2), we always assumed that the maximum inductive voltage over a coil equals the maximum resistive voltage necessary. For a type (A) or (B) coil system, this is the most economic way. In a type (C) coil system, however, power may be saved if the inductive voltage is *higher* than the resistive voltage, at least for some of the heat- and self-inductance-limited, but not stress-limited, inner coils. In that case the current will build up more quickly, and therefore a higher current density may be used, resulting in smaller outer coils. The power saved for these outer coils can more than compensate the power wasted in the fast charging of the inner coils.

In any case it may be advantageous, both for a stress-limited and heat-limited magnet, to use

somewhat more power than the absolute minimum in order to charge the coils more quickly, without increasing the maximum current density. This saves much dissipated energy, and shortens the total duration of the pulse considerably.

7. Remarks about a type (D) coil system

As stated before, some power can be saved by crow-barring some outer coils during the energising of the inner coils. Minimising the total power used calls for a large self-inductance of these outer coils, and therefore large dimensions and large time constants. Some power (considerably less than 50%) can be saved in this way, but there are severe disadvantages for a coil of this type, as compared to a (smaller) type (C) magnet:

(a) Although the power supply of a type (D) magnet will be less expensive, the coils are larger and therefore more expensive. Moreover, these large coils operate at a considerable risk of malfunction, as compared to the risk to the power supply.

(b) As considerably much more magnetic energy will be stored in the large coils of a type (D) magnet, more measures should be taken to prevent

calamities in the case that a wire should break during the pulse.

(c) The costs of cooling the magnet between the pulses is considerably larger for a type (D) magnet.

(d) As the time interval from field zero to field zero is much longer, it becomes more difficult to do accurate measurements of, e.g., changes of a magnetic moment in a type (D) magnet.

We suggest therefore that, unless the limit of the available power makes a type (C) coil for the desired combination of field and duration impossible, one should not consider a type (D) coil system.

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