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Managing unanchored, heterogeneous expectations and liquidity traps

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ABSTRACT

We study the possibility of (almost) self-fulfilling waves of optimism and pessimism and self-fulfilling liquidity traps in a New Keynesian model with a continuum of heterogeneous expectations. In particular, all agents choose, based on past forecasting performance, expectation values out of a distribution around the targets of the central bank. This framework allows us to explicitly model the “anchoring” of expectations as the variance of this distribution of possible expectation values. We find that when the zero lower bound on the nominal interest rate is not binding, adequate monetary policy can prevent waves of optimism and pessimism and exclude near unit root dynamics, even when expectations are unanchored. However, as shocks bring the economy to a situation with a binding zero lower bound, there is a danger of a long lasting self-fulfilling liquidity trap that can take the form of a deflationary spiral. This can be prevented if expectations are strongly enough anchored to the targets, or if the inflation target is high enough.

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1. Introduction

The aftermath of the recent financial crisis has highlighted the importance of the zero lower bound on the nominal interest rate and the possibility of self-reinforcing liquidity traps. Interest rates all over the world have been at low levels for a prolonged period of time, while recovery of output and inflation has been slow. While many factors may play a role in explaining this, low confidence is likely to be an important one. As consumers expect slow economic growth and a high risk of unemployment in the future, they are reluctant to increase their current consumption. This keeps aggregate demand low, and hence prevents a recovery in output. This problem is amplified by the fact that a combination of low inflation expectations and near zero nominal interest rates leads to a high real interest rate, which further reduces consumption. As a consequence, output stays even lower, further reducing confidence. The economy then is stuck in a liquidity trap with a binding zero lower bound and low output and inflation.

An important reason for increased transparency of many central banks and the adoption of inflation targeting, has been to *anchor* private sector expectations to a positive target value. While this policy has initially been very successful in stabilizing inflation, it seems that since the financial crisis private sector expectations have no longer been anchored to the target.

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Instead, many market participants consider the possibility that inflation will stay well below target for a prolonged period of time. Additionally, output expectations are no longer anchored to their long run growth path, and economists worry that economies might not return to this growth path any time soon. These unanchored expectations are crucial for explaining how a self-reinforcing liquidity trap as described above could arise.

While the widely used rational expectations paradigm may be useful for many applications, we believe that it is lacking in its ability to model dropping confidence and changes in the anchoring of expectations in a satisfactory way. Some successful attempts have been made at explaining expectations-driven liquidity traps by deviating, to some extent, from the traditional paradigm. Mertens and Ravn (2014) build a model where sunspot shocks can drive the economy temporarily towards a low inflation low output equilibrium with a binding zero lower bound. The existence of such a zero lower bound equilibrium was first highlighted by Benhabib et al. (2001a,b). Evans et al. (2008), Eusepi (2007, 2010), and Benhabib et al. (2014) take a different approach and study dynamics in a model where agents use adaptive learning to form expectations. In this framework, they find that deflationary spirals with ever decreasing inflation and output can arise. In all these models, expectations are however homogeneous and, therefore, either in line with the rational expectations equilibrium consistent with the central bank's targets, or deviating from this equilibrium.

We believe that more insight in the possibility of self-fulfilling liquidity traps can be obtained by allowing for *heterogeneity* in expectations in a framework where we can explicitly model the extent to which these heterogeneous expectations are *anchored* to the targets of the central bank and the corresponding desired rational expectations equilibrium. Besides giving a more intuitive notion to the anchoring of expectations, heterogeneous expectations are also arguably more realistic than the assumption of a representative agent. Both surveys of consumers and professional forecasters (e.g., Andrade et al., 2013; Carroll, 2003; Mankiw et al., 2003) and laboratory experiments with human subjects (e.g., Assenza et al., 2014; Pfajfar and Žakelj, 2018) show that in practice there exists considerable heterogeneity in the forecasts of macroeconomic variables.

In this paper, we apply the large type limit (LTL) concept of Brock et al. (2005) to the New Keynesian framework. Our agents consider a (continuous) distribution of expectation values around the targets of the central bank and the desired rational expectations equilibrium. When this distribution of expectation values has a small variance, we say that expectations are strongly anchored to the targets. When the variance is large and agents also consider expectation values that are far away from the targets, expectations are unanchored. In each period, each agent picks an expectation from the distribution of possible expectation values. They do this randomly, where the probability of choosing each expectation value depends on past realizations. That is, when agents that had pessimistic expectations turn out to be right (e.g., due to bad shocks to the economy), in subsequent periods, other agents are more likely to choose pessimistic expectations as well. However, there will always be heterogeneity present, with some agents expecting values close to the targets and other agents having more pessimistic or optimistic expectations.

We find that self-fulfilling waves of optimism and pessimism can arise, where agents coordinate on expectations above or below target. These low (high) expectations then result in low (high) inflation and output gap realizations, which in turn reinforce the pessimistic (optimistic) expectations. In normal times, such waves of pessimism can be limited, with aggressive enough monetary policy that provides strong mean reversion to the target steady state. Inflation and output gap will then remain relatively close to their targets, even when expectations are pessimistic. However, as the economy is driven to the zero lower bound on the nominal interest rate, monetary policy is no longer able to provide strong mean reversion, and self-fulfilling waves of pessimism can arise more easily. These self-fulfilling waves of pessimism can then even take the form of a deflationary spiral with ever decreasing inflation and output gap. This can only be prevented if expectations are relatively strongly anchored to the targets, and the inflation target is high enough. With our model, we are able to obtain analytical expressions for e.g. the region of values of inflation and output gap where the zero lower bound is binding, but recovery to the target steady state is still possible (the “corridor of stability” in the terminology of Benhabib et al., 2014; Eusepi, 2010; Evans et al., 2008).

Our model is related to the heterogeneous expectations models of Branch and McGough (2010), Massaro (2013), Gasteiger (2014) and Di Bartolomeo et al. (2016). These papers assume though, that agents can choose between two expectation formation rules, rather than a distribution of heterogeneous expectations. More closely related are the works of Anufriev et al. (2013), De Grauwe (2011), Agliari et al. (2017, 2015) and Pecora and Spelta (2017). However, where the focus in the above literature is on the desired, and in some cases optimal, specification of monetary policy when the zero lower bound is not binding, an important aspect of our paper is how self-reinforcing waves of pessimism can arise when conventional monetary policy can no longer be used due to the restrictions of the zero lower bound on the nominal interest rate. De Grauwe and Ji (2016) and Hommes and Lustenhouwer (2019) consider the zero lower bound in an heterogeneous expectations model. Our contribution differs from these papers in that De Grauwe and Ji (2016) rely on simulations only and do not derive analytical results, and Hommes and Lustenhouwer (2019) focus on the credibility of the central bank rather than the anchoring of expectations, in a model with only two expectation types.

The remainder of this paper is organized as follows. In Section 2 the economic model, and expectation formation mechanism are presented. Section 3 discusses the stability of the model and the occurrence of self-fulfilling waves of optimism and pessimism in normal times. In Section 4 we introduce the zero lower bound on the nominal interest rate and study self-fulfilling liquidity traps. Section 5 concludes

2. Model specification

2.1. Economic environment

We use a log-linearized New Keynesian model in line with [Woodford \(2003\)](#). Micro-foundations for this model when expectations are heterogeneous can be found in [Hommes and Lustenhouwer \(2019\)](#). The derivations in that paper largely follow [Kurz et al. \(2013\)](#).¹ Micro-foundations of the same New Keynesian model with heterogeneous expectations under alternative assumptions are derived by [Branch and McGough \(2009\)](#).

The model is given by a New Keynesian Phillips curve describing inflation, an IS curve describing output gap, and a rule for the nominal interest rate. Log-linearized output gap (x_t) and inflation (π_t) are given by

$$x_t = \bar{E}_t x_{t+1} - \frac{1}{\sigma} (i_t - \bar{r} - \bar{E}_t \pi_{t+1}) + u_t, \quad (1)$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t + e_t, \quad (2)$$

where κ , σ and β are model parameters, and $\bar{r} = \frac{1}{\beta} - 1$ is the steady state real interest rate. u_t and e_t are shocks to the economy, which we assume to be white noise. The expectation operator $\bar{E}$ indicates aggregate expectations that come from aggregating individual subjective expectations.

Finally, i_t is the nominal interest rate.² We assume the nominal interest rate is set according to a forward-looking Taylor type interest rate rule.³

$$i_t = \bar{r} + \pi^T + \phi_1 (\bar{E}_t \pi_{t+1} - \pi^T) + \phi_2 (\bar{E}_t x_{t+1} - x^T), \quad (3)$$

where π^T is the central banks inflation target, and $x^T = \frac{1-\beta}{\kappa} \pi^T$ is the output gap target consistent with π^T . Forward-looking rules generally perform well empirically (e.g., [Clarida et al., 1998](#)). Furthermore, [Evans and Honkapohja \(2003\)](#) show that an expectations based rule of the form (3), can be used to minimize an (add hoc) loss function containing discounted squared deviations of inflation and output gap from target under discretion, when the coefficients ϕ_1 and ϕ_2 are chosen in a certain way.⁴ [Branch and Evans \(2011\)](#) show that this “optimal” rule also works well in models with heterogeneous expectations, and [Hommes and Lustenhouwer \(2019\)](#) show that under heterogeneous expectations the general rule (3) can also work well with different coefficients.

Abstracting from shocks and plugging (3) into (1), gives the following model

$$x_t = \left(1 - \frac{\phi_2}{\sigma}\right) \bar{E}_t x_{t+1} - \frac{\phi_1 - 1}{\sigma} (\bar{E}_t \pi_{t+1} - \pi^T) + \frac{\phi_2}{\sigma} x^T, \quad (4)$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t. \quad (5)$$

2.2. Expectation formation

We deviate from the rational expectations hypothesis, and do not assume that all agents always exactly expect the same outcome of future variables. Instead, we assume that some *heterogeneity* is present, with some agents expecting values that are a bit higher and some agents expecting values that are a bit lower. This heterogeneity could be caused by agents making

¹ In the derivations in [Hommes and Lustenhouwer \(2019\)](#), the aggregation of individual decisions in terms of aggregates only (which is not the case in [Kurz et al., 2013](#)), hinges on a specific property of the way we model heterogeneous expectations. With the Heuristic switching model of [Brock and Hommes \(1997\)](#), in any period there is heterogeneity in expectations, with different fractions of agents using different heuristics. However, these fractions are updated in each period according to a probability distribution that depends on the relative past performance of each expectation formation heuristic. With this updating, each agent has in each period the same probability of following a particular heuristic as all other agents, independent of the heuristic the particular agent followed in the previous period. By assuming that agents are aware of this, their expectations about their own future consumption coincide with their expectations about the future consumption of any other agents, and therefore with their expectations about aggregate consumption, which is crucial for aggregation. See [Hommes and Lustenhouwer \(2019\)](#) for details. These micro-foundations are valid for any heterogeneous expectation formation process that relies on the heuristic switching model of [Brock and Hommes \(1997\)](#), which we also use in this paper. All that is stated is how agents make their decisions given their expectations and given their belief that all agents have an equal probability of forming expectations in a particular way in the future. How these expectations are formed and what e.g. the relation between inflation and output gap expectations are is taken as given, and can freely be altered without affecting the micro-foundations. Moreover, these expectations are not in any way required to be model-consistent, but can e.g., depend on past data.

² Note that different from [Hommes and Lustenhouwer \(2019\)](#), the nominal interest rate is here not defined in terms of deviation from steady state (which would give $\hat{i}_t = i_t - \bar{r}$). This makes the zero lower bound analysis in the current paper more intuitive.

³ All results in this paper carry over qualitatively to the case of a contemporaneous Taylor rule or a Taylor rule with lagged data. These results can be found in [Lustenhouwer \(2017\)](#). Further note that as in [Evans and Honkapohja \(2003\)](#) and other learning papers, we assume that the central bank can observe aggregate private sector expectations (e.g., through surveys), but that they take these aggregate expectations as given and do not know how these expectations are formed.

⁴ We assume here that the central bank can not observe and respond to contemporaneous shocks. Otherwise these show up as well in a rule derived from the loss function.

small mistakes. Alternatively, the heterogeneity can arise because some agents think they have reasons to be more optimistic or pessimistic in their predictions than is warranted by the publicly available information. More specifically, expectations are *distributed around the targets of the central bank*, π^T and x^T . This can be interpreted as some agents trusting the central bank and expecting inflation and output gap to be equal to their targets, while other agents expect that the central bank will not be able to exactly achieve its goals, but that inflation and output gap will be somewhat higher or lower. An alternative interpretation is that expectations are heterogeneously distributed around the minimum state variable (MSV) solution of the model. Since we assume no autocorrelation in shocks throughout most of the paper, the MSV solution coincides with the targets of the central bank in any period.

We further assume that when agents that were more optimistic or pessimistic in their predictions turn out to be right, then other agents will *learn* from this and adjust their expectations in the direction of the better performing agents. Agents may, for example, think that agents with smaller forecasting errors had additional information available to them, or just had more skills in analyzing the economic environment.

We implement this way of expectation formation with a heuristic switching model as in Brock and Hommes (1997), where agents switch between simple prediction rules, or heuristics. The heuristics in our model consist of deviations from the fundamental values of the economy. The fraction of agents using the heuristic with deviation, or bias, b_h in period t is updated according to the discrete choice model with multinomial logit probabilities (see Manski et al., 1981) given by

$$n_t^{z,h} = \frac{e^{\omega U_t^{z,h}}}{\sum_{h=1}^H e^{\omega U_t^{z,h}}}, \quad z = \pi, x. \quad (6)$$

Here H is the total number of prediction values, and $U_t^{z,h}$ is the fitness measure of heuristic h for variable z in period t . We will assume fitness measures to equal minus the last observed squared prediction error.⁵ ω is the intensity of choice. The higher the intensity of choice, the more sensitive agents become with respect to relative performance of prediction values, and the more forecasts of agents will be coordinated.⁶

Aggregate inflation expectations are now given by a weighted average of the predictions of all types

$$\bar{E}_t \pi_{t+1} = \pi^T + \sum_{h=1}^H b_h n_t^{\pi,h} = \pi^T + \sum_{h=1}^H b_h \frac{e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}{\sum_{h=1}^H e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}. \quad (7)$$

This equation can be written as

$$\bar{E}_t \pi_{t+1} = \pi^T + \frac{\frac{1}{H} \sum_{h=1}^H b_h e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}{\frac{1}{H} \sum_{h=1}^H e^{-\omega(\pi_{t-1} - b_h - \pi^T)^2}}. \quad (8)$$

Brock et al. (2005) show that if we let H go to infinity in this equation, the dynamics of our model can be closely approximated with the so called large type limit (LTL), where there is a *continuum* of prediction biases. Stated differently, the LTL is an accurate description of the evolutionary selection, (8), of H forecasting rules when H is large. The LTL can be obtained by replacing sample means by population means in (8). Assuming that the prediction biases b_h are normally distributed around zero with variance s^2 , this gives

$$\bar{E}_t \pi_{t+1} = \pi^T + \frac{\int_{-\infty}^{\infty} b e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-\frac{b^2}{2s^2}} db}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1} - b - \pi^T)^2} e^{-\frac{b^2}{2s^2}} db}. \quad (9)$$

Similarly, for output gap we obtain

$$\bar{E}_t x_{t+1} = x^T + \frac{\int_{-\infty}^{\infty} b e^{-\omega(x_{t-1} - b - x^T)^2} e^{-\frac{b^2}{2s^2}} db}{\int_{-\infty}^{\infty} e^{-\omega(x_{t-1} - b - x^T)^2} e^{-\frac{b^2}{2s^2}} db}. \quad (10)$$

The variance of the distribution of possible expectation values determines how likely it is that agents will choose expectation values that are far away from the target. When s^2 is very small, almost all agents will have expectations very close to the central bank's target, even if recent observations were very high or very low. If, however, s^2 is very high, then a considerable fraction of agents will increase its expectations to values far above the target after observing high realized inflation (or output gap), and decrease its expectations to low values after observing low inflation (or output gap). We can therefore interpret the parameter s^2 as a measure of the anchoring of expectations. When s^2 is low, expectations are strongly *anchored* to the targets and agents are not likely to let their expectations deviate by much from the target, even when shocks to the economy occur. When, on the other hand, s^2 is high, expectations are *unanchored* and can considerably deviate from the target when shocks to the economy have occurred. In this case, waves of optimism and pessimism could arise in the economy.

⁵ $U_t^{\pi,h} = -(\pi_{t-1} - E_{t-2}^h \pi_{t-1})^2$ and $U_t^{x,h} = -(x_{t-1} - E_{t-2}^h x_{t-1})^2$.

⁶ Note that with this way of forming expectations, agents do not rely on model equations (which they either do not know or do not believe to be fully accurate), but rather on what they are observing in the economy.

The large type limit can also be interpreted in terms of Bayesian updating. Agents then try to learn in each period about the correct value of b , with $N(0, s^2)$ as their prior. The likelihood function (the distribution of π_{t-1} given the true value of b) is normal, with a variance inversely related to the intensity of choice (ω). This means that the intensity of choice (ω) is inversely related to the perceived noise with which agents can observe the correct value of b . This interpretation is in line with the random utility model underlying the multinomial logit probabilities given in (6) (see Anderson et al., 1992). A more detailed comparison between the LTL and Bayesian updating is presented in Appendix A.

In Appendix B it is shown that (9) reduces to

$$\bar{E}_t \pi_{t+1} = \frac{\pi^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1}. \quad (11)$$

Analogously, we obtain the following equation for output gap expectations.

$$\bar{E}_t x_{t+1} = \frac{x^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} x_{t-1}. \quad (12)$$

In the LTL model, expectations thus are a linear combination of the central bank's target, and of past realizations. The weights on these two quantities depend on the intensity of choice (ω) and on the variance of the distribution of expectation values (s^2). As discussed above, the latter can be interpreted as the anchoring of expectations. We therefore have the intuitive result that when agent's expectations are more anchored to the targets, *aggregate expectations* will always stay closer to the targets, and when agent's expectations are less anchored, *aggregate expectations* will stay less close to the targets and rather follow the recent economic situation (the last observed inflation and output gap values). In the next section we show that the anchoring of expectations, s^2 , also plays a crucial role in determining the dynamics of the model and in determining what is required of monetary policy.

3. Monetary policy in normal times

Throughout the paper we will assume that the central bank picks an output gap target that is consistent with its inflation target and satisfies $x^T = \frac{1-\beta}{\kappa} \pi^T$. It then holds that aggregate expectations of $\bar{E}_t \pi_{t+1} = \pi^T$ and $\bar{E}_t x_{t+1} = x^T$ imply realized inflation and output gap that are equal to their fundamental values as well. Moreover, it then follows from (11) and (12) that expectations will again be equal to their targets in the next period. Hence, the fundamental values comprise a steady state in the two dimensional dynamical system defined by (4), (5), (11) and (12).

If this target steady is locally stable (all eigenvalues inside the unit circle) then convergence to the targets can occur. This is a first requirement for inflation and output gap to remain close to their targets. However, if one eigenvalue lies only just inside the unit circle (a *near unit root*), then convergence to the steady state will be very *slow*, and waves of optimism and pessimism can arise. Below, we first provide conditions for local stability in Section 3.1. Then we show in Section 3.2 that self-fulfilling waves of optimism and pessimism can arise even if monetary policy is such that local stability is assured. Finally, Section 3.3 discusses the occurrence of near unit roots and how these can be prevented. It turns out that the anchoring of expectations, s^2 , plays a crucial role in both the stability conditions and the occurrence of near unit roots.

3.1. Local stability

Proposition 1 states the conditions for local stability of the target steady state. There are two conditions, which respectively ensure that the first eigenvalue of the dynamical system, λ_1 , is smaller than +1, and that the other eigenvalue, λ_2 , is larger than -1.⁷ Proof of Proposition 1 is given in C.1.⁸

Proposition 1 (Stability LTL). (See Fig. 1) In the LTL model, the target steady state is locally stable if and only if

$$\phi_1 > 1 - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left(\frac{\sigma}{2\omega s^2 \kappa} + \frac{\phi_2}{\kappa} \right), \quad (\lambda_1 < +1), \quad (13)$$

and

$$\phi_1 < 1 + \frac{(1 + \beta)2\omega s^2 + 1}{2\omega s^2 + 1} \left(\frac{(4\omega s^2 + 1)\sigma}{2\omega s^2 \kappa} - \frac{\phi_2}{\kappa} \right), \quad (\lambda_2 > -1). \quad (14)$$

It follows from Proposition 1 that there is both a lower and an upper bound on how aggressive monetary policy responses can be.⁹ The conditions are presented in terms of the inflation policy coefficient, ϕ_1 , and are a function of the output gap policy coefficient, ϕ_2 , the anchoring of expectations, s^2 , and other model parameters.

⁷ Note that eigenvalues can also be complex, but that they always lie inside the unit circle when this is the case.

⁸ A similar proposition for a contemporaneous interest rate rule can be found in Pecora and Spelta (2017).

⁹ If monetary policy is so aggressive that Condition (14) is violated, the following occurs. If the CB responds too strongly, high inflation expectations lead too very low inflation realizations, which (depending on how strongly expectations are anchored), can lead to very low expectations in the subsequent period. These low expectations then again induce a strong policy response that leads to very high realizations of inflation, and high expectations in the subsequent period. The explosive overshooting then continues with both expectations and realizations reaching ever higher and lower values.

Fig. 1. Stability Conditions (13) and (14) of the Target steady state for different policy parameters (ϕ_1 , ϕ_2) and anchoring of expectations (s of the annualized model in percentage terms). The target steady state is locally stable above the plotted surface in panel (a) and below the surface in panel (b).

Fig. 1 plots the stability Conditions (13) and (14) as a function of s (the standard deviation of the distribution of expectation values) and ϕ_2 . In the figure, we use the calibration of [Woodford \(1999\)](#) with $\sigma = 0.157$, $\kappa = 0.024$ and $\beta = 0.99$. Moreover, we set the intensity of choice to $\omega = 60,000$. Because of different scaling of the fitness measure, this implies an intensity of choice in the same range as the estimated values of [Cornea-Madeira et al. \(2017\)](#) if we let the maximum forecast errors that should still be considered be 2%–4% (annualized).¹⁰

In panel (a) of Fig. 1, it can be seen that for low values of s (where expectations are strongly anchored to the targets), weak inflation policy ($0 < \phi_1 < 1$) does not lead to instability. For higher values of s (unanchored expectations), however, the central bank must respond strongly enough to inflation in order to satisfy (13). If we let s go to infinity, Condition (13) reduces to $\phi_1 > 1 - (1 - \beta) \frac{\phi_2}{\kappa}$, which is the well known *Taylor principle* that must be satisfied under rational expectations

¹⁰ What matters for the multinomial logit model, (6), is the difference in fitness between heuristics. When the fitness measures are multiplied by e.g., 100, then the intensity of choice should be divided by 100, to get the same fractions. [Cornea-Madeira et al. \(2017\)](#) use the relative forecasting error as fitness measure. For their fitness measure, using quarterly data, the absolute differences in fitness measures range up to 0.75. If we consider forecast errors up to annualized 2%–4%, then the maximum difference in fitness measures in our model at the quarterly frequency ranges up to 0.005^2 – 0.01^2 . Our intensity of choice should therefore be divided by something in the order of magnitude of $0.75/0.005^2$ – $0.75/0.01^2$. For an intensity of choice of 60,000, this gives an intensity of choice of 2–8, which is in the same range as the estimated values in [Cornea-Madeira et al. \(2017\)](#).

Fig. 2. Simulated time series of inflation and output gap for relatively unanchored (top panels) and relatively anchored (bottom panels) expectations in case of IID shocks. The corresponding rational expectations time series are plotted in dashed green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in order to obtain local determinacy, and that is necessary and sufficient for E-stability when agents use adaptive learning (Bullard and Mitra, 2002).¹¹

In panel (b) of Fig. 1 we can see that for low values of ϕ_2 , instability due to a too aggressive reaction of monetary policy is not an issue, even if ϕ_1 is as large as 20. However, if ϕ_2 is chosen too large, instability can arise. When expectations are relatively unanchored (high s), the target steady state is unstable for any positive ϕ_1 when ϕ_2 is larger than 0.4 (implying a reaction of the interest rate to output of 1.6 when annualized data are used). Such instability, however, does not occur when expectations are strongly anchored to the target (very low s). Finally, note that when expectations become fully unanchored (s going to infinity), (14) reduces to $\phi_1 < 1 + (1 + \beta) \left(\frac{2\sigma}{\kappa} - \frac{\phi_2}{\kappa} \right)$ and coincides with the upper bound for local determinacy under rational expectations (Bullard and Mitra, 2002).

3.2. Self-fulfilling waves of optimism and pessimism

Above, we found that when the central bank responds adequately to inflation and output gap, the target steady state is locally stable, no matter how anchored or unanchored expectations are. This does however not mean that the anchoring of expectations does not affect the model's dynamics in this case.

Fig. 2 plots in solid black the time series of inflation and output gap in case of IID shocks to both variables with a standard deviation of 1.5%. In the top panels we let expectations be relatively unanchored and set the annualized standard deviation of the distribution of expectation values to 5% (which implies $s^2 = (5/400)^2$). In the bottom panels we let expectations be relatively anchored and set the annualized standard deviation of the distribution of expectation values to 1% (i.e., $s^2 = (1/400)^2$). For comparison, we plot in all panels the case of rational expectations (which implies always expecting the target in case of IID shocks), in dashed green.

First focusing on the top panels of Fig. 2, it can be seen that in the case of relatively unanchored expectations, the time series of inflation and output gap are highly persistent (first order auto-correlation coefficient of 0.92 for inflation and 0.90 for output gap). Since there is no auto-correlation in shocks, this persistence is fully endogenous, and comes from the way agents form expectations. Positive or negative shocks can initiate *waves of optimism or pessimism* that are eventually reversed. If agents form expectations rationally, such drifts in inflation and output gap do not arise, as can be seen from the dashed green curves.

The intuition for the waves of optimism and pessimism, are (almost) self-fulfilling expectations. When shocks drive up inflation, this leads agents to increase their inflation expectations. Since the main determinant of inflation is expected inflation, inflation is likely to be high again in the subsequent period, leading again to high expectations. Meanwhile, the monetary authority responds to the high inflation expectations by increasing the interest rate, which depresses output gap. This leads to more pessimistic expectations about output gap, amplifying the recession.

¹¹ The result that stability is more easily achieved for low values of s , is similar to findings in models with constant gain learning. Evans et al. (2009) for example find that stability conditions become less stringent as the gain parameter goes to zero. Where, however, a gain parameter of zero implies full anchoring to initial beliefs, in our model, $s = 0$ implies full anchoring to the targets of the central bank.

Fig. 3. Simulated time series of inflation and output gap for relatively unanchored (top panels) and relatively anchored (bottom panels) expectations in case of autocorrelated shocks with autocorrelation coefficient 0.5 shocks. The corresponding rational expectations time series are plotted in dashed green. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

In our model, the waves of optimism and pessimism described above are somewhat limited by the anchoring of expectations. Even-though agents can always choose even higher (lower) expectations as realizations become higher (lower), the variance of the distribution of expectation types keeps most of the expectation mass relatively close to the target. That is, the anchoring of expectations prevents waves of optimism and pessimism from becoming too large.

This is clear in the bottom panels of Fig. 2, where expectations are considerably more anchored. The time series now stay much closer to the rational expectations benchmark, and endogenous waves of optimism and pessimism do not arise. Although some endogenous persistence is still present (with first order auto-correlation coefficients of 0.51 for inflation and 0.39 for output gap) expectations are not likely to deviate too much from the targets and endogenous drifts in inflation and output gap are always swiftly reversed.

Next, we illustrate what happens if the endogenous persistence from unanchored expectations interacts with exogenous persistence from auto-correlated shocks. Fig. 3 plots time series of output and inflation for the case where shocks follow an AR(1) process with auto-correlation coefficient 0.5. For comparison, the case of fully rational expectations is again plotted in dashed green.

Comparing this figure with Fig. 2, it can be seen that the waves of optimism and pessimism in case of unanchored expectations (top panels) have a considerably larger amplitude. Moreover, inflation and output gap now also deviate considerably more from their targets in case of relatively anchored expectations (bottom panels), and in case of rational expectations (dashed green). We find that expectations amplify the auto-correlation that comes from the shocks. In case of relatively unanchored expectations the sample auto-correlation coefficients are now increased to 0.97 for inflation and 0.96 for output. For relatively anchored expectations they become 0.84 for inflation and 0.78 for output gap.

The larger volatility and persistence that we find in our model with bounded rationality is in line with findings of other learning models such as in Orphanides and Williams (2004, 2007) and Hommes and Zhu (2014). The former authors find that under adaptive learning more hawkish monetary policy can mitigate this. A similar conclusion under heterogeneous expectations is obtained by Gasteiger (2014, 2017). Below we investigate what the central bank can do in our model to reduce the persistent waves of optimism and pessimism.

3.3. Preventing (near) unit-roots

When expectations are perfectly self-fulfilling, the system has a unit root (eigenvalue equal to one). In this case, the model has a continuum of steady states, as is discussed in Proposition 2. The proof of Proposition 2 is given in C.2.

Proposition 2. Ignoring the zero lower bound on the nominal interest rate, the fundamental steady state is unique, unless ϕ_1 is exactly equal to the value of (13). At this knife edge case where one eigenvalue equals +1, there is a continuum of steady states. The steady state output gap level corresponding to inflation level $\tilde{\pi}$ is then given by

$$\tilde{x} = \frac{1}{\kappa} \left(\left(1 - \frac{2\omega s^2 \beta}{2\omega s^2 + 1} \right) \tilde{\pi} - \frac{\beta \pi^T}{2\omega s^2 + 1} \right). \quad (15)$$

Fig. 4. Absolute value of eigenvalue λ_1 as function of ϕ_2 for different values of ϕ_1 . Panel (a) depicts the case of relatively unanchored expectations and panel (b) corresponds to relatively anchored expectations.

From [Proposition 2](#) it follows that if one eigenvalue equals $+1$, expectations are perfectly self-fulfilling for a continuum of inflation and output gap values (all comprising a steady state). This implies that in the absence of shocks, the model could stay forever at any level of inflation as long as agents expect this to happen and expect the corresponding output gap level to realize.

When monetary policy is slightly more aggressive, the largest eigenvalue is slightly smaller than $+1$ and the continuum of steady states disappears. Convergence to the fundamental steady state then occurs from all initial inflation and output gap levels. However, as long as the eigenvalue is close to $+1$ (near unit root), expectations are still *almost* self-fulfilling for a continuum of non-fundamental values, and convergence to the fundamental steady state will be very slow. That is, in the absence of further shocks, realizations of inflation would be closer to the target than in the previous period, but not by much. Therefore, expectations would also only slowly adjust, and mean reversion back to the steady state would go quite slowly. When new shock innovations are added to this near unit root process, the near random walks of the top panels of [Fig. 2](#) arise, with waves of optimism and pessimism.

[Fig. 4](#) plots the absolute value of the eigenvalue λ_1 , defined in [\(13\)](#), as a function of ϕ_2 , for different values of ϕ_1 .¹² Panel (a) of [Fig. 4](#) shows the relatively unanchored case of the top panels of [Fig. 2](#) with $s^2 = (5/400)^2$, while panel (b) shows the relatively anchored case of the bottom panels of [Fig. 2](#) with $s^2 = (1/400)^2$. Panel (a) illustrates that when expectations are not so much anchored to the targets, then a low value of ϕ_2 implies a near unit root (absolute value of eigenvalue close to 1), no matter how aggressive the monetary authority responds to inflation (high ϕ_1). However, when ϕ_2 is chosen larger,

¹² We plot only λ_1 , and not λ_2 because we are interested here in investigating the highly persistent near unit root dynamics observed in the previous section, that can be interpreted as self-fulfilling waves of optimism and pessimism. When λ_2 comes close to the unit circle it gets close to -1 , so that such dynamics do not arise. Instead we would then observe highly cyclical dynamics which are less empirically relevant.

a near unit root can still occur if ϕ_1 is not large enough, or if ϕ_2 is chosen too large. We can conclude that a *well tuned combination* of ϕ_1 and ϕ_2 is required to prevent nearly self-fulfilling waves of optimism and pessimism in this case.

If, however, expectations are strongly anchored to the target, as illustrated in panel (b) of Fig. 4, then near unit roots do not occur. This is the case even if both the responses to inflation and to output gap in the Taylor rule are low. Hence we can conclude that the anchoring of expectations plays an important role in determining what is required of monetary policy in normal times.

4. Zero lower bound on the interest rate

In Section. 3 we saw that self-fulfilling waves of optimism and pessimism can arise in our model, even in the absence of the zero lower bound on the nominal interest rate, and that the anchoring of expectations plays a crucial role in determining whether this will occur or not. In this section, the effect of a zero lower bound (ZLB) on the nominal interest rate is investigated. This lower bound turns out to have important consequences for the dynamics of our model, and make the role of anchoring of expectations even more important.

When the zero lower bound is binding, the policy coefficients ϕ_1 and ϕ_2 can no longer provide any feedback mechanism that prevents optimistic or pessimistic expectations from becoming self-fulfilling. Low inflation and output gap can then induce a self-fulfilling deflationary spiral with ever decreasing inflation and output gap, unless expectations are strongly enough anchored.

In normal times the interest rate is still given by (3). However, when this rule implies that $i_t < 0$, then i_t is instead set equal to 0. When the zero lower bound on the nominal interest rate is a binding constraint we say that the system is in the “ZLB region”. Otherwise the system is in the “positive interest rate region”.

In the ZLB region the model is described by

$$x_t = \bar{E}_t x_{t+1} + \frac{1}{\sigma} \bar{E}_t \pi_{t+1} + \frac{\bar{r}}{\sigma}, \quad (16)$$

$$\pi_t = \beta \bar{E}_t \pi_{t+1} + \kappa x_t. \quad (17)$$

The dynamics in the zero lower bound region turn out to crucially depend on the existence and position of a non-fundamental steady state in the ZLB region. This is discussed in Proposition 3. Proof of the proposition is given in C.3.

Proposition 3. *The only steady state under the zero lower bound is given by*

$$\pi^{zlb} = \frac{(\sigma(1 + (1 - \beta)2\omega s^2) + \kappa(2\omega s^2 + 1))\pi^T + \kappa(2\omega s^2 + 1)^2 \bar{r}}{\sigma(1 + (1 - \beta)2\omega s^2) - \kappa(2\omega s^2 + 1)2\omega s^2}, \quad (18)$$

$$x^{zlb} = \frac{((1 - \beta)\frac{\sigma}{\kappa}(1 + (1 - \beta)2\omega s^2) + (2\omega s^2 + 1))\pi^T + (1 + (1 - \beta)2\omega s^2)(2\omega s^2 + 1)\bar{r}}{\sigma(1 + (1 - \beta)2\omega s^2) - \kappa(2\omega s^2 + 1)2\omega s^2}. \quad (19)$$

This steady state is an unstable saddle-point, and exists if and only if

$$\omega s^2 > \frac{1}{4} \left(\sqrt{\left(1 - (1 - \beta)\frac{\sigma}{\kappa}\right)^2 + 4\frac{\sigma}{\kappa}} - \left(1 - (1 - \beta)\frac{\sigma}{\kappa}\right) \right). \quad (20)$$

When the steady state exists, all initial conditions above the stable eigenvector through (π^{zlb}, x^{zlb}) imply recovery to the positive interest rate region, while all initial conditions below it result in a deflationary spiral. This eigenvector has slope

$$-\frac{1 - (1 - \beta)\frac{\sigma}{\kappa} + \sqrt{(1 - (1 - \beta)\frac{\sigma}{\kappa})^2 + 4\frac{\sigma}{\kappa}}}{2\sigma}. \quad (21)$$

Fig. 5 illustrates the “recovery region” and the “deflationary spiral region” in the (π, x) -plane. Here we set $\pi^T = 2\%$, and $s^2 = (5/400)^2\%$ (relatively unanchored expectations). The red line in Fig. 5 depicts the zero lower bound. For values of inflation and output gap above this line, the nominal interest rate is positive (positive interest rate region) and convergence to the target steady state can occur. For combinations of inflation and output gap below this line, the zero lower bound is a binding constraint (ZLB region). The black line indicates the stable eigenvector through the unstable ZLB saddle steady state of Proposition 3. Combinations of inflation and output gap above (i.e., to the right of) this line are not too low, so that recovery to the positive interest region occurs. Inflation and output gap combinations below the black line lead to falling inflation and output gap and hence a deflationary spiral.¹³

¹³ Our “recovery region” is strongly related to the “corridor of stability” discussed in Benhabib et al. (2014) and Eusepi (2010). These papers consider global dynamics of a New Keynesian model under adaptive learning, and find numerically that there is a region around the ZLB steady state from which the economy goes to the target steady state and a region from which inflation and output go to minus infinity. In our log-linearized model we are able to find an analytical expression for the curve that separates these two regions. The drawback of using a linearized model is of course that it only gives reasonable approximations for inflation and output gap not too far from the steady state.

Fig. 5. Regions of recovery and a deflationary spiral in the annualized (π, x) -plane. The thick red line indicates the ZLB. The black dot at 2% inflation indicates the target steady state, while the black dot at the bottom of the figure depicts the unstable ZLB saddle steady state from Proposition 3. The unstable eigenvector through this saddle is depicted by the dashed line, while the stable eigenvector is depicted by the solid black line. For initial conditions to the left of this black line a deflationary spiral occurs, and for initial conditions to the right of this line inflation and output gap will recover and increase to the positive interest rate region. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Fig. 6. Stable target steady state (solid blue) and ZLB saddle steady state (dashed green) in LTL model for different levels of anchoring of expectations. Points above the ZLB saddle steady state imply convergence to the target steady state, while points below it lead to a deflationary spiral. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The position of the black line, and hence the size of the recovery region, depends on the position of the ZLB steady state, which in turn depends on the anchoring of expectations and the inflation target. Fig. 6 shows how exactly the position of the ZLB steady state depends on the anchoring of expectations, with s on the horizontal axis and inflation on the vertical axis. The dashed green curve represents the unstable ZLB steady state. Initial conditions above the dashed green curve imply recovery to the target steady state, while initial conditions below it imply a deflationary spiral. The (stable) target steady state is indicated by the solid blue line. In order to first isolate the effect of the anchoring of expectations, the inflation target is set to zero. Below we will show what happens as the inflation target is increased.

Taking the limit $s^2 \rightarrow \infty$ in (18) gives (for any value of the inflation target) $-r$. Therefore, the green line in Fig. 6 has a horizontal asymptote at $-r$.¹⁴ The intuition for this result is that the (log-linearized) period t real interest rate (given by $i_t - \bar{r} - \bar{E}_t \pi_{t+1}$) is positive under a binding zero lower bound if and only if $\bar{E}_t \pi_{t+1} < -\bar{r}$. It follows from Eq. (1), that when this is the case, output gap realizations will be lower than output gap expectations. It then follows from (2) that for reasonable calibrations of κ and β the same holds for inflation. This implies that in case of unanchored expectations,

¹⁴ This is also the position of the steady state under homogeneous rational expectations.

Fig. 7. Bifurcation diagram of LTL model in π^T . The blue line represents the target steady state, and the green dashed line depicts the unstable ZLB saddle steady state. Points above the ZLB steady state imply convergence to the target steady state, while points below it lead to a deflationary spiral. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

inflation realizations below the negative of the steady state real interest rate lead to expectations that are pessimistic enough to lead to even lower realizations, leading again to lower expectations and a subsequent deflationary spiral.

In Fig. 6 we see that as expectations become more anchored (s is decreased) the ZLB steady state that separates the deflationary spiral region from the recovery region lies at a lower and lower inflation value. This reflects that even though in this model it is always possible that agents choose lower and lower expectation values as they observe lower realizations, the probability for this to happen becomes smaller as the variance (and standard deviation) of the distribution of expectation values decreases. Realizations of inflation and output must then be very low to induce agents to choose lower expectations, and, at some point, even *infinitely low* inflation and output gap will no longer decrease expectations enough to induce a deflationary spiral. Looking at Fig. 5, this implies that as s becomes smaller, the black line is considerably moved to the left, so that the deflationary spiral region decreases in size. When s no longer satisfies (20) the deflationary spiral region disappears altogether and the target steady state is *globally stable*.

Since agent's expectations are distributed around the targets, another way the central bank could reduce the likelihood of a self-reinforcing liquidity trap, is by choosing a higher inflation target. As can be seen in Eq. (18), the inflation level of the ZLB steady state depends linearly on the inflation target (π^T). This is illustrated in Fig. 7. Here we plot the inflation values of the target steady state (solid blue curve) and the ZLB steady state that separates the deflationary spiral region from the recovery region (dashed green curve) as a function of the inflation target (π^T), for the case of $s^2 = (5/400)^2$ (relatively unanchored expectations). Note that, as the inflation target is chosen higher, the target steady state has an equivalently higher inflation level as well, making the solid blue curve upward sloping. The ZLB steady state, on the other hand, has a (slightly) decreased inflation value as the inflation target is increased. An increased inflation target thus increases the recovery region and makes it less likely that negative shocks bring the model from the target steady state to the deflationary spiral region.

The above result is similar to findings under rational expectations, where liquidity traps can be prevented by choosing the inflation target high enough, so that large shocks in the model do not lead to a severely binding zero lower bound and a high real interest rate.¹⁵ Our model however emphasizes the role of expectations rather than that of exogenous shocks. Low expectations might come about because of large shocks, but could also arise slowly because of declining confidence after a period where small negative shocks were relatively more prevalent.

5. Conclusion

We built a model with heterogeneous expectations to gain more insight in the possibility of self-fulfilling liquidity traps, and its relation with the anchoring of expectations. In our model, there is a continuum of normally distribution expectation values that agents can choose from, centered around the central bank's targets. The variance of this distribution is our measure of the anchoring of expectations. All agents then choose an expectation value in each period, with the probability of choosing an expectation value being higher when the last observed realization was close to this value.

We found that when inflation and output gap expectations are relatively unanchored from their targets (large variance in the distribution of possible expectation values), small non persistent shocks can initiate waves of optimism and pessimism. This can then only be prevented if the response to inflation expectations in the Taylor rule is large enough and the response to output gap expectations is neither too large, nor too small. The central bank then provides strong enough mean reversion

¹⁵ See e.g., Ball (2013).

to the target steady state to exclude a near unit root, so that almost self-fulfilling waves of optimism and pessimism cannot arise.

Moreover, we find that bad shocks that drive the economy to the zero lower bound on the nominal interest can initiate a self-fulfilling wave of pessimism that brings the economy into a deflationary spiral. This can be prevented if expectations are strongly enough anchored to the targets, or if the inflation target is high enough. Note however, that we have abstracted in our analysis from potential costs that an increased inflation target can bring in terms of increasing volatility in the economy and decreasing the anchoring of expectations.

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Appendix A. LTL and Bayesian updating

Eq. (9) can be written as

$$\bar{E}_t \pi_{t+1} - \pi^T = \frac{\int_{-\infty}^{\infty} b e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}} db}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}} db} = \int_{-\infty}^{\infty} b \psi(b) db. \quad (\text{A.1})$$

Here, $\psi(b) = \frac{e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}}}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}} db}$ can be interpreted as agents' (posterior) probability density function of choosing bias b . This facilitates a comparison between the large type limit and Bayesian updating.

To make the relation between LTL and Bayesian updating more apparent, we multiply the numerator and denominator of $\psi(b)$ by $\frac{\sqrt{2\omega}}{\sqrt{2\pi}}$ and by $\frac{1}{s\sqrt{2\pi}}$ to get

$$\psi(b) = \frac{\frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1}-b-\pi^T)^2} \frac{1}{s\sqrt{2\pi}} e^{-\frac{b^2}{2s^2}}}{\int_{-\infty}^{\infty} \frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1}-b-\pi^T)^2} \frac{1}{s\sqrt{2\pi}} e^{-\frac{b^2}{2s^2}} db}. \quad (\text{A.2})$$

We now obtain a perfect mapping to the posterior of a Bayesian updating process, given by

$$P(b|\pi_{t-1}) = \frac{P(\pi_{t-1}|b)P(b)}{\int_{-\infty}^{\infty} P(\pi_{t-1}|b)P(b)db} \quad (\text{A.3})$$

First of all, the posterior $P(b|\pi_{t-1}) = \psi(b)$ gives the probability density of choosing bias b , given the observed value of π_{t-1} . The prior distribution of b is given by $P(b) = \frac{1}{s\sqrt{2\pi}} e^{-\frac{b^2}{2s^2}}$, which is the assumed distribution of the possible biases over the real line. The likelihood of observing realization π_{t-1} , given a value of the bias b , is now given by $P(\pi_{t-1}|b) = \frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1}-b-\pi^T)^2}$. That is, agents believe that the data generating process (DGP) of π_{t-1} is equal to a constant, $\pi^T + b$, plus a normally distributed error term with variance $\frac{1}{2\omega}$. They then try to learn the true constant of the DGP by observing the noisy time series and applying Bayesian updating. Note that the true constant value in principle is allowed to change over time. What the agents try to infer in the above two equations is what the true value was likely to be at the moment that π_{t-1} was generated.

When ω goes to zero, the variance of the noise in the perceived DGP goes to infinity, so that the “likelihood” $\frac{\sqrt{2\omega}}{\sqrt{2\pi}} e^{-\omega(\pi_{t-1}-b-\pi^T)^2}$ goes to zero for all values of π_{t-1} . This means that agents believe that no useful information about b can be inferred from observing π_{t-1} . As a consequence the posterior should just equal the prior, which is indeed the case according to Eq. (A.2) (all types get equal weight which means that the distribution of mass over values on the real line is fully determined by the distribution of types over the real line).

When $\omega = +\infty$ the variance of the noise in the perceived DGP goes to zero and the likelihood $e^{-\omega(\pi_{t-1}-b-\pi^T)^2}$ becomes degenerate: it equals 1 when the observation (π_{t-1}) equals the true value $\pi^T + b$, and 0 for all other possible values of π_{t-1} . This implies that observing π_{t-1} is perceived as being fully informative about the true value of the constant. After observing π_{t-1} agents immediately know what the true constant value was, and the posterior distribution should be degenerate as well, with all its mass on the true value $\pi^T + b$. This is indeed what Eq. (A.2) tells us, i.e., for infinite intensity of choice it holds that $\psi(b) = 1$ for $b = \pi_{t-1} - \pi^T$ and $\psi(b) = 0$ everywhere else (all agents switch with probability 1 to expecting the last observed value π_{t-1}).

We can conclude that if we interpret the LTL as Bayesian updating, then the intensity of choice parameter determines how informative observations of π_t are about the expectations that agents should choose. The larger the intensity of choice, the less “noise” agents think they encounter, and the more clear it is to them, what prediction value they should choose in the next period.

Appendix B. Derivation of large type limit

Eq. (19) can be written as

$$\bar{E}_t \pi_{t+1} - \pi^T = \frac{\int_{-\infty}^{\infty} b e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}} db}{\int_{-\infty}^{\infty} e^{-\omega(\pi_{t-1}-b-\pi^T)^2} e^{-\frac{b^2}{2s^2}} db} = \frac{\int_{-\infty}^{\infty} b e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db}{\int_{-\infty}^{\infty} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db}, \quad (\text{B.1})$$

where we have divided both numerator and denominator by $e^{-\omega(\pi_{t-1}-\pi^T)^2}$ to obtain the final expression.

In order to simplify this expression we first write

$$\frac{d}{db} \left[e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} \right] = \left[-2b\left(\omega+\frac{1}{2s^2}\right) + 2((\pi_{t-1}-\pi^T)\omega) \right] e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)}, \quad (\text{B.2})$$

which implies that

$$b e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} = \frac{\frac{d}{db} \left[e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} \right]}{-2\left(\omega+\frac{1}{2s^2}\right)} + \frac{(2\omega s^2(\pi_{t-1}-\pi^T))}{(2\omega s^2+1)} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} \quad (\text{B.3})$$

Plugging this in in the numerator of (B.1) gives

$$\begin{aligned} \bar{E}_t \pi_{t+1} - \pi^T &= \frac{\int_{-\infty}^{\infty} \frac{d}{db} \left[e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} \right] db}{-2\left(\omega+\frac{1}{2s^2}\right) \int_{-\infty}^{\infty} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db} \\ &\quad + \frac{(2\omega s^2(\pi_{t-1}-\pi^T))}{(2\omega s^2+1)} \frac{\int_{-\infty}^{\infty} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db}{\int_{-\infty}^{\infty} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db} \end{aligned} \quad (\text{B.4})$$

Or,

$$\bar{E}_t \pi_{t+1} - \pi^T = \frac{\left[e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} \right]_{-\infty}^{\infty}}{-2\left(\omega+\frac{1}{2s^2}\right) \int_{-\infty}^{\infty} e^{-b^2\left(\omega+\frac{1}{2s^2}\right)+2b((\pi_{t-1}-\pi^T)\omega)} db} + \frac{(2\omega s^2(\pi_{t-1}-\pi^T))}{(2\omega s^2+1)} \quad (\text{B.5})$$

$$= \frac{(2\omega s^2(\pi_{t-1}-\pi^T))}{(2\omega s^2+1)}, \quad (\text{B.6})$$

where the last equality follows from evaluating the numerator of the first term which gives $e^{-\infty} - e^{-\infty} = 0$

It now follows that

$$\bar{E}_t \pi_{t+1} = \pi^T - \frac{2\omega s^2 \pi^T}{(2\omega s^2+1)} + \frac{(2\omega s^2 \pi_{t-1})}{(2\omega s^2+1)} = \frac{\pi^T}{(2\omega s^2+1)} + \frac{(2\omega s^2 \pi_{t-1})}{(2\omega s^2+1)}. \quad (\text{B.7})$$

Appendix C. Proofs propositions

C1. Proof Proposition 1

The Jacobian in the fundamental steady state equals

$$B \begin{pmatrix} 1 - \frac{\phi_2}{\sigma} & -\frac{\phi_1-1}{\sigma} \\ \kappa(1 - \frac{\phi_2}{\sigma}) & \beta - \kappa \frac{\phi_1-1}{\sigma} \end{pmatrix},$$

with

$$B = \frac{2\omega s^2}{2\omega s^2+1}. \quad (\text{C.1})$$

The eigenvalues therefore are given by

$$\lambda_{1,2} = \frac{B}{2} \left(\left(1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1-1}{\sigma} \right) \pm \sqrt{\left(1 + \beta - \frac{\phi_2}{\sigma} - \kappa \frac{\phi_1-1}{\sigma} \right)^2 - 4\beta \left(1 - \frac{\phi_2}{\sigma} \right)} \right).$$

Local stability requires $\lambda_1 < 1$ and $\lambda_2 > -1$. By keeping only the square root on one side of the equation and taking squares, $\lambda_1 < 1$ can be written as

$$\phi_1 > 1 + \left(-\beta B - \frac{1}{B} + 1 + \beta\right) \frac{\sigma}{\kappa} + (\beta B - 1) \frac{\phi_2}{\kappa}, \quad (\text{C.2})$$

Filling in B from (C.1) results in Condition (13).

Similarly, $\lambda_2 > -1$ can be written as

$$\phi_1 < 1 + (1 + \beta + \beta B + \frac{1}{B}) \frac{\sigma}{\kappa} - (1 + \beta B) \frac{\phi_2}{\kappa}, \quad (\text{C.3})$$

from which Condition (14) can be obtained.

C2. Proof Proposition 2

It follows from Eq. (4) that in a steady state the model under the forward-looking Taylor rule satisfies

$$x = \frac{\frac{x^T}{2\omega s^2 + 1} (1 - \frac{\phi_2}{\sigma}) + \frac{\phi_2}{\sigma} x^T - \frac{\phi_1 - 1}{\sigma} \left(\frac{2\omega s^2}{2\omega s^2 + 1} \pi + \frac{\pi^T}{2\omega s^2 + 1} - \pi^T \right)}{\left(1 - (1 - \frac{\phi_2}{\sigma}) \frac{2\omega s^2}{2\omega s^2 + 1} \right)}, \quad (\text{C.4})$$

Plugging this in (5), using $x^T = \frac{1-\beta}{\kappa} \pi^T$ and rearranging results in

$$\pi \left(1 - \beta \frac{2\omega s^2}{2\omega s^2 + 1} + \kappa \frac{(\phi_1 - 1)2\omega s^2}{\sigma + 2\omega s^2 \phi_2} \right) = \pi^T \left(1 - \beta \frac{2\omega s^2}{2\omega s^2 + 1} + \kappa \frac{(\phi_1 - 1)2\omega s^2}{\sigma + 2\omega s^2 \phi_2} \right). \quad (\text{C.5})$$

This has as a solution $\pi = \pi^T$, so that the fundamental steady state where average expectations equal the rational expectations equilibrium values always exists.

Alternatively, if the part in brackets in (C.5) is 0, any inflation level is a steady state. This is the case if and only if

$$\phi_1 = 1 - \frac{1 + (1 - \beta)2\omega s^2}{2\omega s^2 + 1} \left(\frac{\sigma}{2\omega s^2 \kappa} + \frac{\phi_2}{\kappa} \right), \quad (\text{C.6})$$

which is exactly the value where the fundamental steady state loses stability and one eigenvalue equals +1 (see Proposition 1).

For any inflation level $\tilde{\pi}$, corresponding steady state output gap $\tilde{x}$ then follows from (5) and is given by (15).

C3. Proof Proposition 3

When the zero lower bound is binding the LTL model becomes

$$x_t = \frac{x^T}{2\omega s^2 + 1} + \frac{2\omega s^2}{2\omega s^2 + 1} x_{t-1} + \frac{1}{\sigma} \frac{\pi^T}{2\omega s^2 + 1} + \frac{1}{\sigma} \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1} + \frac{\tilde{r}}{\sigma}, \quad (\text{C.7})$$

$$\pi_t = \beta \frac{\pi^T}{2\omega s^2 + 1} + \beta \frac{2\omega s^2}{2\omega s^2 + 1} \pi_{t-1} + \kappa x_t. \quad (\text{C.8})$$

Solving for the steady state of this model results in (18) and (19). Steady state output gap and inflation both are negative if and only if

$$\sigma(1 + (1 - \beta)2\omega s^2) - \kappa(2\omega s^2 + 1)2\omega s^2 < 0, \quad (\text{C.9})$$

which can be rewritten as (20).

The eigenvalues of the system defined by (C.7) and (C.8) are given by

$$\lambda_{1,2} = \frac{\omega s^2}{2\omega s^2 + 1} \left(\left(1 + \beta + \frac{\kappa}{\sigma} \right) \pm \sqrt{\left(1 + \beta + \frac{\kappa}{\sigma} \right)^2 - 4\beta} \right). \quad (\text{C.10})$$

This implies that the steady state is an unstable saddle if and only if

$$\omega s^2 > \frac{1}{\beta - 1 + \frac{\kappa}{\sigma} + \sqrt{\left(1 + \beta + \frac{\kappa}{\sigma} \right)^2 - 4\beta}}, \quad (\text{C.11})$$

which, after some algebraic manipulation, reduces to (20). Therefore, when (20) does not hold the system has a unique attractor that lies outside the ZLB region. This implies that from all initial conditions inflation and output gap will go towards this attractor and cross the zero lower bound. Recovery then always occurs.

When (20) holds, initial conditions below the stable eigenvector through the steady state given by (18) and (19) lead to ever decreasing inflation and output gap, while initial conditions above it lead to increasing inflation and output gap, and thereby to recovery. The slope of this eigenvector is given by (21).

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