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DOI

[10.1287/orsc.8.3.310](https://doi.org/10.1287/orsc.8.3.310)

Publication date

1997

Published in

Organization Science

[Link to publication](#)

Citation for published version (APA):

Peli, G., & Masuch, J. M. F. (1997). The logic of propagation strategies: Axiomatization a fragment of organizational exology in firstorder logic. *Organization Science*, 8(3), 310-331. <https://doi.org/10.1287/orsc.8.3.310>

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Organization Science, Vol. 8, No. 3. (May - Jun., 1997), pp. 310-331.

Stable URL:

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The Logic of Propagation Strategies: Axiomatizing a Fragment of Organizational Ecology in First-Order Logic

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First-order logic provides a rigorous, formal method to assess the conceptual validity of a theory. This paper introduces the method and applies it to propagation theory within organizational ecology. It identifies important limitations in this apparently straightforward and widely accepted theory about first movers and efficient producers. If this is true for such a simple theory, imagine how logically fragile more complicated organization science theories must be!

Andrew H. Van de Ven

Abstract

As a part of a larger effort to apply formal logic to organization science, we axiomatize the theory of propagation strategies (life history strategies) of Organization Ecology. We provide an axiomatic system in *first-order* logic that derives the theory's predictions as theorems from a set of underlying assumptions. First-order logic is widely seen as a standard language for the formalization of scientific theories; it couples considerable expressive power with acceptable computational properties. Our axiomatization increases the understanding on the scope and the limits of Organization Ecology, and yields a consistent theory of propagation strategies with a sound explanatory structure.

(*Logic; Formal Theory; Organizational Ecology; Computer Science*)

1. Introduction

In the past, theories were perceived as propositional systems. They were held to be axiomatizable by self-evident "first principles" and derivable from those principles through "logical reasoning" (Nagel 1961, Stegmüller 1976). A basic tool for making a theory contradiction-free, axiomatization would also bring clarity, simplicity, and coherence; in addition, it would

further the development of more general theories. In reality, however, few theories were ever thoroughly axiomatized; increasing specialization led to increasingly specialized, mutually incompatible kinds of knowledge; and logic as a systematic tool in theory building was largely abandoned.

Today's researchers are in a bind, especially in the social sciences. The standard epistemology still assumes that theories are representable as propositional systems and require logical properties, such as consistency and contingency, as preconditions of meaningful empirical testing (Popper 1959). But today's researchers rarely use formal tools for establishing these logical properties. Few researchers are happy with inconsistent, incoherent, or unclear theories, and logic is one of the most frequently used words in science. But most researchers, having taken good note of the demise of logical positivism, are convinced that metaphor, analogy, and other mechanisms of creative thinking hold sway. Considerable effort still goes into unification and systematization. But most theories in the social sciences are stated in natural language (e.g., English), which provides remarkable expressive power, but also many ways for getting lost in ambiguity and confusion, and hence no proper basis for the systematization, or unification, of theories.

We want to get out of this bind. We think that logic is still the way to go, because it provides a set of appropriate tools for an axiomatization, but believe one should gain some experience in using formal logic before passing judgment on its applicability.

Logic, and in particular the *first-order logic* (FOL) employed in this axiomatization, occupies a specific position among mathematical techniques. FOL comes with a formal notion of logical consequence, which is crucial if the derivability of theorems is at stake. Next, FOL involves weaker semantic requirements than the standard mathematical machinery of calculus or (linear) algebra; as a consequence, logic may work as a formalization tool where those techniques fail (because they require, among other things, specific quantitative information that is not always available)—still, logic allows for the same rigor as mathematics. As a consequence, logic is a powerful tool for theorizing: verbal theoretical arguments can be translated into a set of logical formulas, and this set can be examined for important properties, such as consistency (does the argument involve a contradiction) or logicity (are specific propositions really derivable or not). Also, this examination can be automated through the use of *theorem provers*, computer applications that relieve the nonlogician of the burden of finding the proof himself. We use such a theorem prover, Metafor, extensively in the course of formalization (Ó Nualláin 1993).¹

Logical formalization, once achieved, provides a transparent inference structure that makes the assumptions of a given theory explicit, including the background assumptions that are often neglected in informal reasoning. The explicit inference structure may suggest generalizations of the theory (because some assumptions said to be required are, in fact unnecessary) or to a restriction of the theory's scope (because more assumptions are needed than originally thought). At the end of the day, a clearer picture of the theory evolves—hence this study.

Our study is part of a larger effort to formalize a relevant part of organization theory (Péli et al. 1994, Péli 1997, Masuch and Huang 1996, Pólos 1995, Bruggeman 1997). We provide a logical axiomatization of a part of Organizational Ecology (OE), one of the best-known modern theories of organization (Hannan and Freeman 1977, 1989; Carroll and Hannan 1994), and a serious contender against standard contingency theory (Thompson 1967, Mintzberg 1979, Grandori 1987). The part we investigate is usually labelled “life history strategies” (Brittain and Freeman 1980, Hannan and Freeman 1989), which we call propagation strategies. The theory of propagation strategies describes the

dynamics of environmental selection for two types of organizations, *first movers* and *efficient producers*.

This paper is organized as follows. First, we give a brief outline of the formal machinery. Second, we sketch the propagation part of organizational ecology and list the relevant questions that the theory should answer. Third, we develop an axiomatization of propagation strategies by answering each relevant question with one or more theorems. In each case, we try to derive the theorem from simpler assumptions, so that the final axiomatization yields an explanatory theory. The discussion section evaluates the logical properties of this axiomatization.

2. The Formal Apparatus

We use first-order predicate logic (FOL) as the language of our formalization. Simpler propositional logic aside, FOL is the best known and also best understood formal logic. Partly because of that, FOL is the only logic for which mature theorem provers are available, i.e., computer applications that help to automate the logical derivation of theorems from a given set of premises. A large variety of new logics (higher order, modal, default, etc.) has been developed in the last few decades, that typically exceed FOL's expressive power. But these logics are difficult to handle, and their computational properties are less well-understood, hence our choice for FOL.

First-Order Logic is based on the following classes of symbols:

- *constant names*: names for objects in the domain (e.g., FM, EP);
- *predicate constants*: names for properties of, or relations between, objects. We use capitalized strings of symbols (e.g., *Selection_favors*)
- *quantifiers*: “for all” ($\forall$) and “there exists” ($\exists$)
- *variables*: e.g., x, y
- *logical connectives*: symbols that allow one to build complex expressions from simple expressions. The five standard connectives are “ $\neg$ ” (negation, not), “ $\vee$ ” (disjunction, or), “ $\wedge$ ” (conjunction, and), “ $\rightarrow$ ” (conditional, if-then), and “ $\leftrightarrow$ ” (biconditional, if and only if).

A first-order language may also include function symbols. For example, the $g(\text{FM}, t)$ function evaluates to the growth rate of first movers at time point t . FOL can do without function symbols, but they render the formulas much shorter and more readable. For a more detailed account, the reader is referred to standard textbooks, such as Gamut (1991), or Barwise and Etchemendy (1990).

Types of Formulas

The formalization generates a set of expressions in FOL (“formulas”). This set can be ordered according to the role of each formula in the theory. Some formulas represent theorems, others represent premises required to explain (“derive”) theorems. Formulas may also play both roles simultaneously (e.g., lemmas, theorems used to explain other theorems). Among the premises one can distinguish (1) definitions, (2) meaning postulates, and (3) material assumptions. A definition fixes the content of a concept in terms of other concepts, a meaning postulate does so partially, i.e., it puts restrictions on the meaning of a concept (e.g., saying that a bachelor must be male amounts to a meaning postulate). Material assumptions delineate the properties of the domain of the (formal) theory; they establish the domain-specific relations that supposedly hold between the concepts. Meaning postulates are helpful if one cannot characterize a concept completely.

Axiomatization

We propose a set of theorems that should answer the basic questions of the propagation part of OE. The axiomatization is developed along the explanatory structure of the original theory fragment. We start with a candidate theorem and then look for appropriate premises of each theorem. Such premises may come from three sources:

- (1) statements of organizational ecology, (e.g., “the disbanding rate of first movers exceeds the disbanding rate of efficient producers”);
- (2) general ecological considerations, (e.g., “more resilient agents are less likely to die when resources are scarce”);
- (3) common-sense knowledge (e.g., “the number of organizations cannot be negative”).

To keep the construction transparent, we present the derivation of the first theorem as a proof tree. Such a tree consists of layers of premises, each of which gives rise to a conclusion (which appears at the next layer), the final conclusion being the theorem (compare Figure 3 for the proof tree of Theorem 1).

To repeat, we begin with a formal representation of a candidate theorem and then look for suitable premises. This procedure is applied recursively: once a set of premises is found, we move to the next layer by looking for new premises that explain the assumptions of the previous layer. The recursion must halt somewhere, of course. We try to persist until we find assumptions that correspond to general ecological or

common background knowledge (as it turns out, we won’t be always successful). The idea is that each explanation should be ultimately based on safe grounds, and that, with respect to organizational ecology, both common-sense knowledge and general ecological assumptions can be assumed as given.

Common sense assumptions are usually trivial for the reader (e.g., “if something increases, then it does not decrease at the same time”), but not for a theorem prover that never had a chance to watch *Sesame Street*. To avoid information overload, we do not spell out the background-assumptions in the main text. (The appendix provides the complete premise set of Theorem 1 as an illustration. The premise list of the other theorems is available upon request at g.peli@bdk.rug.nl.)

It should be clear that our axiomatization is not the only possible one. But by trying to make the argumentation structure transparent, we help the reader to decide whether the axiomatization is reasonable, what can be learned from it, and how it might be improved.

3. Organizational Ecology

Most organizational theories regard organizations as agents that rationally adapt structure to changing environments (Scott 1982, Robbins 1990). Organizational ecology, in contrast, de-emphasizes the role of rational adaptation. Organizational structures are seen to evolve as a function of environmental selection. When environmental conditions change, new organizations or new forms of organizations emerge, while maladapted organizations die.

OE employs analogies from biology, such as the notion of a population. Organizations with similar form may constitute a population. Members of a population might include organizations that differ with respect to properties that do not constitute differences in form, and one such property may concern the way in which they are founded.

First Movers and Efficient Producers

The categories *first mover* (FM) and *efficient producer* (EP) are introduced by an analogy. In bioecology, a species is said to follow an *r*-strategy if its growth rate is very high under conditions of resource abundance (MacArthur and Levins 1964), (the letter *r* usually denotes the *intrinsic* growth rate). The investment in each reproductive event is low, but the offspring’s survival chance is low under normal conditions of re-

source scarcity. Conversely, a species is said to follow a *K*-strategy if there are fewer offsprings but individuals are more resilient when resources are scarce. *K*-strategists typically do better than *r*-strategists when the *carrying capacity* of the environment is approximated (the letter *K* denotes carrying capacity in ecological models). Flies are *r*-strategists, elephants are *K*-strategists.

According to organizational ecology, first movers appear quickly when new opportunities (resource configurations) arise. They have simple, easy-to-assemble structures. Conversely, efficient producers take more time to assemble, but their structures are more complex, and, because of that, more resilient. As a consequence, the death rate of EP is typically lower than the death rate of FM. The long gestation period is the price EP have to pay for their resilience: the elaboration of a sophisticated structure takes time and resources.

We use the terms “*r*-strategist” and “first mover” synonymously in this article, that is, we assume that first movers enter new environments through *founding* events. Note, however, that on a broader view, first movers may also grasp new opportunities by migration or an extension of activities. Such organizations may have a sophisticated structure that would allow for market entry under quite different environmental conditions. Haveman (1993), for example, found that first-entrant organizations are often characterized by large *size*, a property that hardly matches the image of ordinary *r*-strategists.

Questions

As indicated above, our strategy is to answer the relevant questions concerning the propagation fragment by means of theorems. The two crucial questions to be answered by the theory are obviously: (1) which environmental conditions favor FM, and (2) which conditions favor EP? These questions branch out into several directions. One direction is organizational birth, but it may transpire that the corresponding questions are already answered implicitly, since they are implied by the meaning of the concepts “first mover” and “efficient producer.” A second direction is organizational death; we would like to know if some conditions will lead to the disappearance of FM (3), and that of EP (4). A third direction is competition: (5) under which conditions will FM force out EP, and, conversely, (6) EP force out FM. Finally, we would like to know (7) whether the classification in terms of FM and EP is exhaustive.

4. The Axiomatization of Propagation Strategies

We need to establish a set of material assumptions about the conditions of a new resource configuration (e.g., a new market). The overall scenario is approximately the following:

First movers are the first to exploit a new resource configuration, but the new opportunities also give rise to the initiation of efficient producers that appear later on the scene. In the process, the number of new organizations, and hence the size of the new population, will grow until the environment’s carrying capacity is approached. EP will eventually come to dominate the scene, as they should be more efficient than FM and hence able to outcompete them. And this story is bound to repeat itself whenever a new environment replaces the old one.

Whether FM or EP are going to dominate in the long run will primarily depend on the speed of environmental change (i.e., the frequency of succession of resource-environments), and on the relative growth rates of both subpopulations. If change is fast, FM will dominate, because they are always quick to appear. If change is very slow, EP get time enough to develop, and eventually replace, FM. The growth of each subpopulation depends on the interaction of two factors: entries and exits. A high exit rate (“mortality rate”) may be counterbalanced by a high entry rate (“birth rate”); conversely, low exit rates will only provide a selection advantage if entry rates are sufficiently high.

Ecological Considerations

As pointed out above, we need some assumptions that rest on general ecological considerations. We have to deal with organizations, which face more or less changing environmental conditions. Each specific environment provides a particular configuration of renewable resources for (a group of) specific agents, so a change of environmental conditions means a change in the resource configuration. Once in a while, a *new* resource configuration appears (imagine a new invention providing new business opportunities); simplifying the language, we may speak of a “new” environment. One can assume that entrepreneurs grasp the opportunity and found new organizations. This assumption constitutes the first axiom. Let $\text{Env}(e)$ denote the fact that e represents a resource configuration; let $\text{During}(e, t)$ denote the fact that t occurs while the resource configuration exists; let the function $\text{appear}(O, e)$ evaluate to the time point at which the first organizations appear in e ; and let the function $\text{num}(e, t)$ evaluate to the

number of organizations present in e at t ; then we can write:

A1. *Some organizations persist in populated environments.*

$$\forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \wedge t \geq \text{appear}(O, e) \rightarrow \text{num}(e, t) > 0].$$

(For all e, t , if e is an environment, and t occurs during e , and t does not precede the appearance of organizations, O , in e , then the number of organizations in e at time t is bigger than zero.)

Entrepreneurs may try to grasp the new opportunity fast, doing a quick, but perhaps somewhat dirty job, and initiate *first movers*. Conversely, entrepreneurs may take more time and effort, and initiate efficient producers. K -strategists won't become serious contenders against r -strategists unless they carry the advantage of using resources more efficiently, or, given the same level of available resources, are more likely to survive than r -strategists. The second axiom expresses this idea. Let the $\text{resil}(x)$ function evaluate to the resilience of a group of organizations x , then we get:

A2. *Efficient producers are more resilient than first movers.*

$$\text{resil}(\text{EP}) > \text{resil}(\text{FM}).$$

(The resilience of the first movers, FM, is higher than the resilience of the efficient producers, EP.)

Since FM are easy-to-assemble, they grasp new environmental opportunities quickly; they are the first to make use of a new resource configuration. As a consequence, their appearance in the new environment precedes the appearance of efficient producers:

A3. *First movers appear sooner in the environment than efficient producers.*

$$\forall e [\text{Env}(e) \rightarrow \text{appear}(\text{EP}, e) > \text{appear}(\text{FM}, e)].$$

(For all e , if e is an environment, then the appearance of FM in e precedes the appearance of EP.)

We may assume that objects under selective pressure depend on their environments for resources. While resources are abundant, populations can be expected to grow. As they grow, however, they use more and more resources. Resources are conceptualized as renewable, yet finite at any particular point in time. With

no or few organizations depending on a particular resource configuration, unbound resources are relatively abundant, so their availability is large. As a population grows, however, more and more resources are used up and the *resource availability* decreases; growth will be brought to a halt when the environment's carrying capacity is reached. Once this happens, the system approaches an equilibrium, i.e., a state in which the stream of offered resources is matched by the size of the resource-consuming population. And unless the system is disturbed, equilibrium will be maintained, so the availability of free resources remains constant (zero). Let the function $\text{res}(e, t)$ evaluate to the resource availability of e at t , let the function $\text{carr}(e)$ evaluate to the time point at which the carrying capacity is reached, and let the predicates $\text{Decreasing}(\text{res}(e, t))$ and $\text{Const}(\text{res}(e, t))$ denote the facts that the resource availability of e at t is decreasing, or constant. Furthermore, let $\text{Stable}(e)$ denote the fact that e persists for a relevant duration of time. Then we get the following two axioms:

A4. *Resource availability decreases until the carrying capacity is reached.*

$$\begin{aligned} \forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \wedge \text{num}(e, t) > 0 \rightarrow \\ (t < \text{carr}(e) \rightarrow \text{Decreasing}(\text{res}(e, t))) \\ \wedge (t \geq \text{carr}(e) \rightarrow \text{Const}(\text{res}(e, t)))]. \end{aligned}$$

(For all e, t , if e is an environment and time point t occurs during e , and the number of organizations in e at t is bigger than zero, then if t precedes $\text{carr}(e)$, the time when the carrying capacity of e is reached, then the resource availability in e is decreasing at t , and it remains constant past $\text{carr}(e)$.)

A5. *The carrying capacity is reached in stable environments.*

$$\begin{aligned} \forall e [\text{Env}(e) \wedge \text{Stable}(e) \\ \rightarrow \exists t [\text{During}(e, t) \wedge t \geq \text{carr}(e)]]. \end{aligned}$$

(For all e , if e is an environment, and e is stable, then there exists a t , such that t occurs during e , and t does not precede $\text{carr}(e)$, the time when the carrying capacity of e is reached.)

Along the lines of the second and fourth axiom, we may also assume that less resilient subpopulations are more affected by decreasing resource availability. That

is, the gap between their survival perspectives increases as free resources get scarcer. Let $\text{Subpops}(s1, s2, e, t)$ denote the fact that $s1$ and $s2$ are nonempty subpopulations of the same population in e at t , and let the function value $d(x, t)$ denote the disbanding rate of x at t ; then we get:

A6. *Less resilient subpopulations are more affected by decreasing resource availability.*

$$\begin{aligned} & \forall e, s1, s2, t [\text{Env}(e) \wedge \text{Subpops}(s1, s2, e, t) \\ & \quad \wedge \text{resil}(s2) > \text{resil}(s1) \rightarrow \\ & \quad (\text{Decreasing}(\text{res}(e, t)) \\ & \quad \rightarrow \text{Increasing}(d(s1, t) - d(s2, t))) \\ & \quad \wedge (\text{Const}(\text{res}(e, t)) \rightarrow \text{Const}(d(s1, t) - d(s2, t)))]. \end{aligned}$$

(For all $e, s1, s2, t$, if e is an environment, and $s1$ and $s2$ are nonempty subpopulations in e at time t , and $s2$ is more resilient than $s1$, then a decreasing resource availability in e at t implies an increasing difference between the disbanding rate of $s1$ and $s2$ at t , and constant resource availability implies a constant difference.)

We assume that the population grows in number until it reaches a ceiling, the carrying capacity:

A7. *The number of organizations increases until the carrying capacity is reached.*

$$\begin{aligned} & \forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \rightarrow \\ & \quad (t < \text{carr}(e) \rightarrow \text{Increasing}(\text{num}(e, t))) \\ & \quad \wedge (t \geq \text{carr}(e) \rightarrow \text{Const}(\text{num}(e, t)))]. \end{aligned}$$

(For all e, t , if e is an environment and time point t occurs during e , then if t precedes the time when the carrying capacity of e is reached, $\text{carr}(e)$, then the number of organizations in e is increasing at t , and it remains constant past $\text{carr}(e)$.)

It is important to note that, in reality, resource consumption depends on the “size” of the organizations involved. Note that A7 excludes significantly inhomogeneous size distribution. We’ll reiterate this point later.

This concludes our preliminary list of axioms based on general ecological considerations (Figure 1). The basic idea is to assume a new resource configuration (called environment for short), that lasts for a relevant

Figure 1 Ecological Assumptions

period of time. Initially, the environment is empty, that is, no organization depends on its resources. Unless the resource configuration changes, we assume that an established population will grow until the carrying capacity is reached (which is assumed to be positive in order to avoid vacuous cases). In the process, more resources are consumed, so that the resource availability decreases as the population grows. As resource availability diminishes, the gap between the survival perspectives of the more and the less resilient subpopulations increases.

In the next step of the formalization process, we try to derive the answers to the initial questions about propagation from the list of axioms established above, adding additional axioms if necessary. Figure 2 provides a preview of the results. The first two questions (about selection preferences) will be answered by the Theorems 1–3 for the case of stable environments and by Theorems 8 and 9 for dynamic environments. Questions 3 and 4 (about organizational death) will be answered by Theorems 6 and 7. Questions 5 and 6 (about competition) are addressed by Theorems 4 and 5. Question 7, finally, is addressed by Theorem 10.

Selection's Favor: Theorems 1 to 3

The first two questions are about the effect of selection on FM and EP. We consider the case of stable environmental conditions first. As it stands, we expect the theory to postulate that stable environmental conditions provide enough time for EP to thrive, so there should be a theorem to the effect that stable environments favor EP past a certain point in

Figure 2 The Inference Structure of Theorems 1 Through 9

time—not immediately, however, since new opportunities (new environments) are first seized by FM. $\text{Selection_favors}(\text{EP}, \text{FM}, t)$ stands for the fact that EP are favored by selection above FM.

THEOREM 1. *In stable environments, selection favors efficient producers above first movers past a certain point in time.*

$$\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow$$

$$\exists t_0 [\text{During}(e, t_0) \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge$$

$$t \geq t_0 \rightarrow \text{Selection_favors}(\text{EP}, \text{FM}, t)]]].$$

(For all e , if e is an environment and e is stable, then there exists a time point t_0 , such that t_0 occurs during e , and for all time points t , if FM and EP are nonempty subpopulations in e at t , and t is greater or equal to t_0 , then selection favors EP above FM at t .)

As outlined above, our strategy is to build the axiomatization by finding explanations for each theorem. In case of Theorem 1 (and in the case of other theorems as well) this explanation depends on the meaning of the predicate *selection favors*. In the propagation fragment (as well as in other parts of organizational ecology), selection preferences are stated in terms of growth rates: selection favors (sub)population A above B if they share the same environment and A 's growth rate exceeds B 's growth rate. The notion of selection may have other connotations, so we refrain from a complete definition (that would exclude other possibili-

ties) and give a partial characterization, a meaning postulate, instead:

MEANING POSTULATE 1. *Selection favors subpopulations with higher growth rates.*

$$\forall e, s1, s2, t [\text{Env}(e) \wedge \text{Subpops}(s1, s2, e, t) \wedge$$

$$g(s2, t) > g(s1, t) \rightarrow \text{Selection_favors}(s2, s1, t)].$$

(For all $e, s1, s2, t$, if e is an environment, and $s1$ and $s2$ are nonempty subpopulations in e at t , and $s2$ assumes higher growth rate than $s1$ at t , then selection favors $s2$ above $s1$ at time t .)

Using Meaning Postulate 1, Theorem 1 can now be stated in terms of growth rates, or, more precisely, Theorem 1 is deducible from Lemma 1 plus Meaning Postulate 1:

LEMMA 1. *The growth rate of efficient producers exceeds the growth rate of first movers past a certain time in stable environments.*

$$\forall e [\text{Env}(e) \wedge \text{Stable}(e)$$

$$\rightarrow \exists t_0 [\text{During}(e, t_0) \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge$$

$$t \geq t_0 \rightarrow g(\text{EP}, t) > g(\text{FM}, t)]]].$$

(For all e , if e is an environment, and e is stable, then there exists a time point t_0 , such that t_0 occurs during e , and for all time points t , if FM and EP are nonempty subpopulations in e at t , and t is greater or equal to t_0 , then EP assume a higher growth rate than FM at time t .)

Growth rates are composites of founding rates and disbanding rates, and the theory tells us that the disbanding rates of EP should undercut the disbanding rates of FM. So we assert a new, simpler statement (also presented as a lemma because we expect it to be explained in terms of the differential resilience of contending subpopulations):

LEMMA 2. *The disbanding rate of first movers exceeds the disbanding rate of efficient producers.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ \rightarrow d(\text{FM}, t) > d(\text{EP}, t)].$$

(For all e, t , if e is an environment, and FM and EP are nonempty subpopulations in e at t , then FM's disbanding rate is higher than EP's disbanding rate at t .)

In fact, Lemma 2 is explicitly stated as an assumption in the theory (Hannan and Freeman 1989, p. 120), but we can derive it from more basic ecological assumptions and from a (weaker) extra assumption, i.e., Assumption 9.

This covers the disbanding rates for the time being. How about founding rates? To arrive at Lemma 1, we assume that the founding rate of FM does not exceed the founding rate of EP past a certain time, since a higher growth rate is implied by a lower disbanding rate only if the disbanding rate's effect is not counterbalanced by a higher founding rate of first movers. The function value $f(x, t)$ is the founding rate of x at t .

A8. *The founding rate of first movers does not exceed the founding rate of efficient producers past a certain time in stable environments.*

$$\forall e [\text{Env}(e) \wedge \text{Stable}(e) \\ \rightarrow \exists t_0 [\text{During}(e, t_0) \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge \\ t \geq t_0 \rightarrow f(\text{EP}, t) \geq f(\text{FM}, t)]]].$$

(For all e , if e is an environment and e is stable, then there exists a time point t_0 , such that t_0 occurs during e , and for all time points t , if FM and EP are nonempty subpopulations in e at t , and t is bigger than t_0 , then EP's founding rate is greater or equal to FM's founding rate at t .)

Obviously, we would like to have A8 as a lemma, not as an assumption, since it seems to require an explana-

tion. In fact, the ecological intuition might suggest that FM are more easily and hence more frequently founded, regardless of the circumstances. But there is no explanation for A8 in organizational ecology. The only alternative is to do without A8, yet without some kind of restriction on FM's founding rate, Theorem 1 cannot be proven. What to do? Well, we carry on. We have a gap in the theory, but we live with it for the time being, and accept A8 as an axiom. In fact, we can justify A8 by stepping somewhat outside organizational ecology and using an argument depending on rational adaptation. Assume that entrepreneurs adapt rationally to the environment. Then they are less likely to found unsuccessful organizations, because that would be irrational. So, if an organizational type is not successful and entrepreneurs adapt rationally to changing conditions, then this fact should have a negative effect on its founding rate, and A8 could be explained. Hannan and Freeman stipulate a similar feed-back mechanism between organizational survival perspectives and the frequency of foundings when dealing with the *density dependence* of demographic rates (Hannan and Freeman 1989, pp. 132, 141–142). Delacroix and Carroll (1983) allow for a similar mechanism when studying the rate dependence of organizational founding.

With Assumption 8 and Lemma 2, we can now derive Lemma 1, the intuition being that the growth rate of EP must exceed the growth rate of FM if the lower disbanding rate of EP is not counterbalanced by a higher founding rate of FM.

The next task is to derive Lemma 2. To do so, we must consider how the carrying capacity is reached. As the new population grows, more and more resources are used (A4). Selective pressure increases, hence resilience against selective pressure is becoming more important. And this means that the gap in disbanding rates between first movers and efficient producers should at least not decrease as time passes (A6). This consideration yields the next lemma:

LEMMA 3. *The difference between the disbanding rates of first movers and efficient producers does not decrease with time.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t))].$$

(For all e, t , if e is an environment, and FM and EP are nonempty subpopulations in e at t , then the difference between EP's and FM's disbanding rate is not decreasing at t .)

One might expect to be able to derive Lemma 2 from Lemma 3 now, but the proof requires some additional information that should be in the ecological background assumptions yet isn't: namely that FM's disbanding rate is also higher in the initial stages of a new environment. Let the function $\text{init}(e)$ evaluate to the earliest time point when FM and EP are present in the new environment, then we can write:

A9. *The disbanding rate of first movers initially exceeds the disbanding rate of efficient producers.*

$$\forall e [\text{Env}(e) \rightarrow d(\text{FM}, \text{init}(e)) > d(\text{EP}, \text{init}(e))].$$

(For all e , if e is an environment, then FM's disbanding rate exceeds EP's disbanding rate at the initial time of e , $\text{init}(e)$.)

At first sight, Assumption 9 and Lemma 3 seem to yield Lemma 2, but so far we cannot be assured that FM and EP are present. So we fix this by stating:

A10. *If first movers and efficient producers are present in the environment at time points $t1$ and $t2$, then they are also present during the period between $t1$ and $t2$.*

$$\forall e, t, t1, t2 [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t1)$$

$$\wedge \text{Subpops}(\text{FM}, \text{EP}, e, t2) \wedge t1 \leq t \leq t2$$

$$\rightarrow \text{Subpops}(\text{FM}, \text{EP}, e, t)].$$

(For all $e, t, t1, t2$, if e is an environment, and FM and EP are nonempty subpopulations in e at time points $t1$ and $t2$, and time point t occurs between $t1$ and $t2$, then FM and EP are nonempty subpopulations in e at t .)

Assumption 10 may seem fairly simple, but it spells out a nontrivial constraint: extinct subpopulations cannot revive unless the resource configuration changes, that is, a new environment opens. Lemma 3 and Assumptions 9 and 10 are sufficient to derive Lemma 2.

The next task is to justify Lemma 3. To do so, we have to ponder the growing gap in life perspectives. This gap, as we know from ecological considerations, is due to growing competition for resources: as subpopulations grow, resources are less easily available.

LEMMA 4. *Decreasing resource availability affects the disbanding rate of first movers more than the disbanding rate of efficient producers.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow$$

$$(\text{Decreasing}(\text{res}(e, t))$$

$$\rightarrow \text{Increasing}(d(\text{FM}, t) - d(\text{EP}, t)))$$

$$\wedge (\text{Const}(\text{res}(e, t))$$

$$\rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t)))].$$

(For all e, t , if e is an environment, and FM and EP are nonempty subpopulations in e at t , then a decreasing resource availability in e at t implies an increasing difference between the disbanding rates of EP and FM at t and a constant resource availability implies a nondecreasing difference.)

Note that other factors, for instance *legitimation*, might also affect growth rates (in the specific case of legitimation, the stated tendency would be reinforced). Be this as it may, Lemma 4, in conjunction with Assumption 4 yields Lemma 3.

We have finally reached the bottom of the recursive spiral (Figure 3). Lemma 4 can be deduced by using A2 and A6: the less resilient first movers are more affected by deteriorating resource conditions than efficient producers. So Theorem 1 is now derivable from Assumptions 2, 4, 6, 8, 9, and 10, Meaning Postulate 1, plus some additional common sense assumptions (compare the appendix).

Theorem 2

Theorem 1 gives a partial answer to the first two questions regarding the propagation fragment: stable conditions favor efficient producers over first movers in the long run. But what happens in the short run? Are first movers favored? We might expect so, but the explanation is less obvious than one may think.

To hone in on the conditions that separate states favoring first movers from states favoring efficient producers, we introduce a hypothetical point in time (called *critical point*), at which selection's favor switches from first movers to efficient producers. We define the critical point as the *earliest* time point from which efficient producers assume permanently higher growth rate than first movers (Definition 1).

Figure 3 The Proof Tree of Theorem 1

DEFINITION 1. A point is the critical point of an environment, if and only if it is the earliest time point past which efficient producers assume permanently higher growth rate than first movers.

$$\begin{aligned} \forall e, t_0 [\text{Env}(e) \rightarrow \\ (t_0 = \text{crit}(e) \leftrightarrow \neg (g(\text{EP}, t_0) > g(\text{FM}, t_0)) \\ \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge \\ t > t_0 \rightarrow g(\text{EP}, t) > g(\text{FM}, t)])] \end{aligned}$$

(For all e , if e is an environment, then a time point t_0 is the critical point of e , if and only if, EP's growth rate does not exceed FM's growth rate at t_0 , and for all t , if FM and EP are nonempty subpopulations in e at t and t exceeds t_0 , then EP's growth rate exceeds FM's growth rate at t .)

Unfortunately, we cannot be sure that both FM and EP are always around. Since FM appear prior to EP (A3), there is an EP-less period early on. What would the meaning of "selection favors" be if one of the compared groups is not present at all? One solution could be to characterize empty groups as having a zero growth rate. This would imply, however, that selection favors nonexistent groups to existing groups with negative growth rates. Another option could be to postulate that the growth rate of an empty group is always lower than the growth rate of any existing group. Though this would work in a technical sense, it sounds strange. So, instead of insisting on the comparison of growth rates, we resort to a new meaning postulate. Let the predicate $\text{O_group}(x, e, t)$ denote the fact that x is a potentially empty subpopulation in environment e at time t :

MEANING POSTULATE 2. *Selection favors organizational groups with members above groups without members.*

$$\begin{aligned} \forall e, s1, s2, t [\text{Env}(e) \wedge \text{O_group}(s1, e, t) \\ \wedge \text{O_group}(s2, e, t) \wedge \text{num}(s1, t) > 0 \\ \wedge \text{num}(s2, t) = 0 \rightarrow \text{Selection_favors}(s1, s2, t)] \end{aligned}$$

(For all $e, s1, s2, t$, if e is an environment, and $s1$ and $s2$ are organizational groups in e at time t , and the number of organizations in $s1$ at t is bigger than zero, and the number of organizations in $s2$ at t is zero, then selection favors $s1$ above $s2$ at t .)

The technicality of empty groups out of the way, we can now move to Theorem 2. We want to prove that:

THEOREM 2. *Selection favors first movers above efficient producers until the appearance of efficient producers.*

$$\begin{aligned} \forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \wedge \\ \text{appear}(\text{FM}, e) \leq t < \text{appear}(\text{EP}, e) \\ \rightarrow \text{Selection_favors}(\text{FM}, \text{EP}, t)] \end{aligned}$$

(For all e, t , if e is an environment, and time point t occurs during e , and t does not precede FM's appearance in e , but t precedes EP's appearance in e , then selection favors FM above EP at t .)

Theorem 2 seems trivial, yet we won't be able to prove it from the knowledge base already established unless we also assert that no propagation strategists besides FM and EP exist in the environment. But how do we know? In fact, it might very well be that there

are other propagation strategies besides FM and EP. Yet, if we want to go on, we have to exclude this possibility for the time being, so we bring in another auxiliary axiom:

A11. *The population contains only first movers and efficient producers.*

$$\forall e, s, t [\text{Env}(e) \wedge \text{O_group}(s, e, t) \\ \wedge \text{num}(s, t) > 0 \rightarrow s = \text{EP} \vee s = \text{FM}].$$

(For all e, s, t , if e is an environment, and s is an organizational group in e at time t and the number of organizations in s is bigger than zero, then s is either an FM or an EP subpopulation.)

Theorem 2 now follows from Assumptions 1, 3, and 11 and from Meaning Postulate 2.

Theorem 2 does not tell us what is going to happen once EP have appeared, and Theorem 1 only refers to cases past the critical point. So we need an additional theorem covering the time period between the appearance of EP and the critical point. We expect that selection still favors FM during that period, so we want to prove:

THEOREM 3. *Selection favors first movers above efficient producers past the appearance of efficient producers until the critical point is reached.*

$$\forall e, t [\text{Env}(e) \wedge \text{During}(e, \text{crit}(e)) \wedge \\ \text{appear}(\text{EP}, e) \leq t < \text{crit}(e) \\ \rightarrow \text{Selection_favors}(\text{FM}, \text{EP}, t)].$$

(For all e, t , if e is an environment, and the critical point of e occurs during e , and t does not precede EP's appearance in e , but t precedes the critical point of e , then selection favors FM above EP at t .)

To prove Theorem 3, we intend to demonstrate that the growth rate of FM permanently exceeds the growth rate of EP during the period under investigation. We have a problem similar to the one that led to A8. We apply the same solution, and add another restricting assumption:

A12. *The difference between the founding rates of first movers and efficient producers decreases with time.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ \rightarrow \text{Decreasing}(f(\text{FM}, t) - f(\text{EP}, t))].$$

(For all e, t , if e is an environment, and FM and EP are nonempty subpopulations in e at t , then the difference between EP's and FM's founding rate is decreasing at t .)

Assumption 12 is just as problematic as Assumption 8, but it could be justified in a similar way: as entrepreneurs perceive declining perspectives for first movers, they are less likely to found such organizations, bringing their founding rate down. Theorem 3 now follows from L3, MP1, A10, and A12, plus the definition of the critical point, D1.

Competition

Theorem 1 argues that selection favors efficient producers past a certain time in stable environments. However, the theory claims more: efficient producers also *outcompete* first movers, provided they have enough time to do so (Hannan and Freeman 1989, p. 119):

THEOREM 4. *Efficient producers outcompete first movers past a certain time in stable environments.*

$$\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow$$

$$\exists t_0 [\text{During}(e, t_0) \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t)$$

$$\wedge t \geq t_0 \rightarrow \text{Outcompete}(\text{EP}, \text{FM}, t)]]].$$

(For all e , if e is an environment, and e is stable, then there exists a time point t_0 , such that t_0 occurs during e , and for all time points t , if FM and EP are nonempty subpopulations in e at t , and t is greater or equal to t_0 , then EP outcompete FM at t .)

As it turns out, we can derive Theorem 4 from the already established knowledge base if we add a definition of the verb "to outcompete." It is clear that if an organizational group A outcompetes another group B , then selection also favors A to B . However, selection's preference says nothing about the signs of the growth rates; both groups may grow or decline (of course, A must grow faster or contract slower). Yet, *A outcompeting B* means something more; it means that A affects B 's fate directly (Barnett and Carroll 1987), namely that A forces B out of its environment. As a result, the number of organizations in group B must decline. So, let the predicate $\text{Outcompete}(x, y, t)$ stand for the fact that x outcompetes y at t , then we can write the following definition:

DEFINITION 2. A subpopulation outcompetes another subpopulation in a given environment, if and

only if, the first one has a nonnegative growth rate and the second one has a negative growth rate.

$$\begin{aligned} \forall e, s1, s2, t [& \text{Env}(e) \wedge \text{Subpops}(s1, s2, e, t) \rightarrow \\ & (g(s1, t) \geq 0 \wedge g(s2, t) < 0) \\ & \leftrightarrow \text{Outcompete}(s1, s2, t)] . \end{aligned}$$

(For all $e, s1, s2, t$, if e is an environment, and $s1$ and $s2$ are nonempty subpopulations in e at t , then $s1$ outcompetes $s2$ at t , if and only if the growth rate of $s1$ is greater or equal to zero at t , and the growth rate of $s2$ is smaller than zero at t .)

Theorem 4 is based on the intuition that competition becomes a *zero sum game* once the carrying capacity is reached: if one of the two subpopulations grows, the other must decline. Theorem 4 follows from Assumptions 4, 5, 7, 11, Lemma 1, and Definition 2.

Addressing the competition between FM and EP, the last theorem claims that EP are the winners in the long run: they outcompete first movers past a certain point in time in stable environments. But how about the short run? Are there situations when first movers, at least temporarily, can outcompete efficient producers? The knowledge base established so far allows for both outcomes, and organizational ecology provides no additional information to decide the question immediately. So we proceed tentatively and consider both alternatives. The first alternative would be:

THEOREM 5. *First movers never outcompete efficient producers.*

$$\begin{aligned} \forall e, t [& \text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ & \rightarrow \neg \text{Outcompete}(\text{FM}, \text{EP}, t)] . \end{aligned}$$

(For all e, t , if e is an environment, and FM and EP are nonempty subpopulations in e at a time t , then FM do not outcompete EP at t .)

It would be enough for a proof of Theorem 5 to assume that more resilient organizational groups do not decrease under circumstances when less resilient ones can expand under similar resource conditions:

A13. *If a less resilient organizational group does not decrease in a given environment, then a more resilient group does not decrease either in the same environment.*

$$\begin{aligned} \forall e, s1, s2, t [& \text{Env}(e) \wedge \text{During}(e, t) \wedge \neg(g(s1, t) < 0) \\ & \wedge \text{resil}(s2) > \text{resil}(s1) \rightarrow \neg(g(s2, t) < 0)] . \end{aligned}$$

(For all $e, s1, s2, t$, if e is an environment and time point t occurs in e , and the growth rate of $s1$ is nonnegative at t and $s2$ is more resilient than $s1$, then the growth rate of $s2$ is nonnegative at t .)

Efficient producers are more resilient, so Assumption 13 implies that their growth rate is not negative when the growth rate of first movers is not negative. Theorem 5 follows from Assumptions 2 and 13 and Definition 2.

The second alternative would be the negation of Theorem 5. Call it Theorem 5*:

THEOREM 5*. *First movers may outcompete efficient producers in some environments.*

$$\begin{aligned} \exists e, t [& \text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ & \wedge \text{Outcompete}(\text{FM}, \text{EP}, t)] . \end{aligned}$$

(There exist e, t , such that e is an environment, and FM and EP are nonempty subpopulations in e at time t , and FM outcompete EP at t .)

Not surprisingly, Theorem 5* depends on the negation of Assumption 13:

A13*. *Efficient producers may decrease in members even when first movers grow or stagnate.*

$$\begin{aligned} \exists e, t [& \text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ & \wedge g(\text{FM}, t) \geq 0 \wedge g(\text{EP}, t) < 0] . \end{aligned}$$

(There exist e, t , such that e is an environment, and FM and EP are nonempty subpopulations in e at time t , and the growth rate of FM is nonnegative at t , and the growth rate of EP is negative at t .)

Having A13*, rather than A13, we would have to conclude that there are periods when first movers force out efficient producers. If such a period is long enough, then efficient producers might become extinct temporarily, before achieving dominance over first movers (T1, T4). This does not conform with A10, so we decide to add A13 to the knowledge base, rather than A13*.

With Theorems 4 and 5 we have answered the questions about the competition between the two subpopulations for the case of stable environments.

Population Death

The next questions concern the extinction of populations. We would expect EP, once established, to survive for as long as the environment remains unchanged, and

we can prove the corresponding theorem without an extension of the knowledge base:

THEOREM 6. *Once they have appeared in an environment, efficient producers do not disappear.*

$$\forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \wedge t > \text{appear}(\text{EP}, e) \rightarrow \text{num}(\text{EP}, t) > 0].$$

(For all e, t , if e is an environment and time point t occurs during e and the appearance of EP in e precedes t , then the number of EP is positive at t .)

Theorem 6 follows from Assumptions 1, 2, 4, 7, 11, and 13. Conversely, we expect FM to die out in the long run as a consequence of the competition with a stronger subpopulation:

THEOREM 7. *First movers disappear past a certain point in time in stable environments.*

$$\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow \exists t_0 [(\text{During}(e, t_0) \wedge t_0 > \text{appear}(\text{FM}, e) \wedge \text{num}(\text{FM}, t_0) = 0)].$$

(For all e , if e is an environment, and e is stable, then there exists a time point t_0 , such that t_0 occurs in e and t_0 occurs after the appearance of FM in e and the number of FM is zero at t_0 .)

The problem is that monotonically decreasing functions may approximate a positive number asymptotically (allowing for cases when the number of FM stays positive on the long run). In order to exclude this possibility, we have to make sure that our organizational groups are numbered in nonnegative integers, so that any decrease of the subpopulation between two observations must be greater or equal to one. Assumptions 3 and 5, Lemma 1 and Theorem 6 now prove Theorem 7.

Theorem 7 may come as a surprise to some readers. They may argue that FM might stabilize their position before efficient producers appear in the environment. With time, these organizations may improve their reproducibility, and hence their survival chances (Hannan and Freeman 1989, pp. 74–75). So, some first movers may be around even in the long run (think of Apple Computers Inc. as a possible example). Alternatively, first movers may stabilize their market position by acquiring special legitimacy, i.e., legitimacy related to their status as a subpopulation (Carroll and Harrison 1994), so the fact that Theorem 7 excludes these possibilities indicates some limits of the given conceptualization.

With Theorem 7 in place, we have now answered the first six questions of the propagation fragment, but

Figure 4 Selection Events in Stable Environments

only for the case of unchanging resource configurations (Figure 4).

Changing Environments

How about changing resource configurations? Change in this context would mean that we observe a sequence of different resource configurations. Assuming a fixed observation period (during which the sequence occurs), we can express the rapidity of change by the frequency of shifts from one configuration to the next (Figure 5).

Since individual adaptation is assumed to have only a marginal positive effect on survival, we must expect that the new resource configuration will give rise to new forms while old forms, now unfit, must disappear. If environmental change is slow (environments persist for long periods of time), then efficient producers ultimately dominate each environment (T8). Conversely, if environmental change is fast (environments follow each other in rapid succession) then first movers dominate each environment (T9). But how do we formalize this intuition?

As a first step, we have to extend the meaning of the verb phrase “selection favors” to the case of changing environmental circumstances. The idea is that the effect of selection in a given environment is captured by the result at the end of that environment, and that the effect of selection in a sequence of environments is given by the sequence of results. We shall say that a propagation strategy is favored during a given observation period only if this strategy prevails finally in all the environments occurring in this period.

MEANING POSTULATE 3. *If selection favors a certain propagation strategy s_1 above another propagation strategy s_2 at the endpoint of all environments of the observation period, then it favors s_1 above s_2 during the observation period.*

$$\begin{aligned} \forall s_1, s_2 [\forall e [\text{Env}(e) \wedge \text{During}(\text{obs}, e) \\ \rightarrow \text{Selection_favors}(s_1, s_2, \text{end}(e))] \\ \rightarrow \text{Selection_favors}(s_1, s_2, \text{obs})]. \end{aligned}$$

(For all s_1, s_2 , if for all environment e that occurs in observation period obs selection favors s_1 above s_2 at the end of e , then selection favors s_1 above s_2 in obs.)

Figure 5 Slow vs. Rapid Environmental Change in the Observation Period

"env#" denotes a resource configuration (environment) of the observation period.

As a second step, we have to characterize *slow* and *rapid* environmental change. Unfortunately, organizational ecology does not tell us where to draw the line between fast and slow environmental change, and without this information T8 and T9 are not derivable from the premise set. What we can do, however, is to ponder what kind of characterizations of these notions would allow us to derive T8 and T9 from the knowledge base we already have. This way of conceptualization is well-established in maths and in logic. A good example is the proof analysis of Euler's polyhedron theorem (Lakatos 1976), where the desired theorem helps to clarify the meaning of the word "polyhedron."

If selection favors efficient producers under slowly changing environmental conditions, then slow environmental change should mean that selection favors efficient producers above first movers in each environment of the observation period (cf. MP3). But this is only going to happen if each environment extends long enough to contain a critical point (D1). So, it seems reasonable to characterize slow change with respect to an observation period as a sequence of environments such that each contains a critical point. It should be obvious that we should not attempt to give a full definition of *slow* or *fast* but stick to meaning postulates:

MEANING POSTULATE 4. *If environmental change is slow during an observation period, then all the environments in the observation period contain a critical point.*

$$\text{Slow_change}(\text{obs}) \rightarrow \forall e [\text{Env}(e) \wedge \text{During}(\text{obs}, e) \rightarrow \exists t [\text{During}(e, t) \wedge t > \text{crit}(e)]].$$

(If environmental change is slow during observation period obs, then for all e , if e is an environment and e

occurs in obs, then there exists a time t , such that t occurs during e and t occurs after the critical point of e .)

Rapid environmental change can be characterized along the same lines (MP5):

MEANING POSTULATE 5. *If environmental change is rapid during an observation period, then no environment in the observation period contains a critical point.*

$$\begin{aligned} \text{Rapid_change}(\text{obs}) \rightarrow \forall e [\text{Env}(e) \wedge \text{During}(\text{obs}, e) \\ \rightarrow \neg \text{During}(e, \text{crit}(e))]. \end{aligned}$$

(If environmental change is rapid during observation period obs, then for all e , if e is an environment and e occurs during obs, then the critical point of e does not occur during e .)

Note that Meaning Postulates 3–5 allow for circumstances where selection has no significant preference. If environments change moderately fast, selection may favor FM in some environments and favor EP in others.

We expect organizational ecology to postulate that selection favors efficient producers not only in stable, but also in relatively stable (i.e., slowly changing) environmental conditions. Conversely, rapidly changing conditions should favor FM. So we want to prove:

THEOREM 8. *Selection favors efficient producers above first movers if environmental change is slow.*

$$\text{Slow_change}(\text{obs}) \rightarrow \text{Selection_favors}(\text{EP}, \text{FM}, \text{obs}).$$

(If environmental change is slow in observation period obs, then selection favors EP above FM in obs.)

THEOREM 9. *Selection favors first movers above efficient producers if environmental change is rapid.*

$$\text{Rapid_change}(\text{obs}) \rightarrow \text{Selection_favors}(\text{FM}, \text{EP}, \text{obs}).$$

(If environmental change is rapid in observation period obs, then selection favors FM above EP in obs.)

As it turns out, we can prove Theorem 8 from Meaning Postulates 1–4, Definition 1, and Theorem 6. However, we cannot yet derive Theorem 9. Why? The answer is that it takes some time until new organizations appear in a new environment. If environmental change is extremely fast, then some environments will be finished before any organizations have a chance to make its appearance. To get around this problem, we restrict Theorem 9 to cases in which environmental change is not extremely fast. We call such change *not*

extreme. If the change during an observation period is not extreme, then all resource configurations during the observation period persist long enough to give rise to new populations (Meaning Postulate 6).

MEANING POSTULATE 6. *If environmental change is not extreme during an observation period, then no environment stays empty during this period.*

$\neg \text{Extreme}(\text{obs})$

$\rightarrow \forall e[\text{Env}(e) \wedge \text{During}(\text{obs}, e) \rightarrow \neg \text{Empty}(e)].$

(If environmental change is not extreme in observation period obs, then for all e , if e is an environment and e occurs in obs, then e does not remain empty.)

Meaning Postulates 3, 5, 6 and Theorems 2, 3 imply the “not extreme” version of Theorem 9. Call it T9*.

THEOREM 9*. *Selection favors first movers above efficient producers if environmental change is rapid but not extreme during the observation period.*

$\text{Rapid_change}(\text{obs}) \wedge \neg \text{Extreme}(\text{obs})$

$\rightarrow \text{Selection_favors}(\text{FM}, \text{EP}, \text{obs}).$

(If environmental change is rapid but not extreme in observation period obs, then selection favors FM above EP in obs.)

Two Propagation Strategies?

Organization ecology takes only two propagation strategies into account, but it does not claim to have a complete typology. Can we infer the existence of any other propagation strategies (Theorem 10)? ($\text{O}(x)$ denotes that object x is an organization, $\text{FM}(x)$ and $\text{EP}(x)$ predicate that organization x belongs to the corresponding type. Note the difference with the terms FM and EP, which serve as names for first movers and efficient producers.)

THEOREM 10. *There are nonfirst mover and nonefficient producers organizations.*

$\exists x[\text{O}(x) \wedge \neg \text{FM}(x) \wedge \neg \text{EP}(x)].$

(There exists x , such that x is an organization, and x is not a first mover, and x is not an efficient producer.)

As it turns out, we cannot infer Theorem 10 on the basis of the given knowledge base. In fact, to gain clarity as to the existence of other propagation strategies, we have to make an excursion into other parts of organization ecology. We know that the superior performance of efficient producers is based on the fact that their *routines* are highly elaborated (Hannan and Freeman 1989). This gives us a link to the part of organizational ecology known as “generalists vs. specialists,” which provides more information on routines. In the light of this information, we introduce the distinction between “complementary” and “alternative” routine patterns, the idea being that specialists using primarily complementary routines, and generalists primarily alternative routines. As the word indicates, complementary routines complement each other (e.g., marketing routines complement production routines), whereas alternative routines can be substituted for one another (e.g., one production routine may alternate with another production routine). Alternative routines are required to adapt to fluctuating environments.

Now, organizational routines are repetitive patterns of activity. They are the source of *reproducibility* of performance. But their development and maintenance require investments. This is the point at which we have a link with propagation strategies. Since first mover organizations are assembled fast, they have to put together routines quickly; efficient producers, in contrast, can concentrate on the fine-tuning of routines, since they have more time. As a result, efficient producers can be supposed to have elaborated routines at their founding (A14). $\text{Elab}(x)$ means that x has elaborated routines and *founding* denotes the founding time of x .

A14. *Efficient producer organizations have elaborated routines at their founding.*

$\forall x[\text{O}(x) \wedge \text{EP}(x) \rightarrow \text{Elab}(x, \text{founding})].$

(For all x , if x is an organization and x is an efficient producer, then x has elaborated routines at its founding.)

Since first movers are assembled fast, they should not feature many alternative routines initially, that is, when they are founded (the $\text{num_routine}(x, t)$ function value tells whether x features a high or a low number of alternative routines at time t):

A15. *First mover organizations have a low number of routines at their founding.*

$\forall x[\text{O}(x) \wedge \text{FM}(x) \rightarrow \text{num_routine}(x, \text{founding}) = \text{low}].$

(For all x , if x is an organization and x is a first mover, then the number of routines of x is low at its founding time.)

However, some organizations are generalists at birth, such as small-town general stores. Since the parallel development of many routines is time consuming, some of these organizations come to birth with only modestly elaborated routines (A16):

A16. *Some organizations come to birth with many but poorly elaborated routines.*

$$\exists x[\text{O}(x) \wedge \text{num_routine}(x, \text{founding}) = \text{high} \\ \wedge \neg \text{Elab}(x, \text{founding})].$$

(There exists x , such that x is an organization and the number of routines of x is high at its founding, and the routines of x are not elaborated at its founding.)

Assumptions 14–16 prove Theorem 10. In short, we can conclude that the typology of propagation strategies requires an extension, provided that the constraints embodied in A14–16 are warranted. (Note, however, that on a broader interpretation to the notion “first mover,” A15 need not be warranted any more. For example, organizations may enter new environments by activating one of their existing alternative routines or by building a new routine. Such “first mover” generalists may have several routines even at birth (see A16), falsifying A15.)

5. Discussion

Our aim was to base our axiomatization on general ecological considerations and common-sense knowledge. We started with seven ecological axioms, but discovered that they were insufficient. Consider the theorems in sequence:

Theorem 1. Already the first theorem (Selection ultimately favors EP above FM in stable environments) required three additional assumptions: A8, A9, and A10. Axiom 8 is really problematic. In stipulating that the founding rate of FM does not exceed the founding rate of EP past a certain point in time, A8 goes against the grain of organizational ecology. If agents are not guided by some rational considerations, then they are likely to continue founding FM at a higher rate than EP even when FM do badly, since founding FM is easier than founding EP. We saw, however, that A8 could be justified by rational adaptation: founders are more likely to found successful organizations, and EP are more successful in the long run. The second new

axiom (A9) only claims that first movers are more likely to die than efficient producers even in the initial stages of an environment. Axiom 10 posits that the presence of FM and EP at two points of time, t_1 and t_2 , implies that they are also present during the period between t_1 and t_2 . A10 does not go against the grain of organizational ecology either, but it makes a simplifying assumption: once extinct, FM and EP cannot reappear unless the resource configuration changes.

Theorem 2. The second theorem (Selection favors FM until EP appear) required one additional, simplifying assumption (A11) that excludes propagation strategies other than FM and EP. This simplification is essential since the theory considers only two propagation strategies, despite the fact that one may conceive of alternative strategies (Theorem 10).

Theorem 3. The third theorem also required one new assumption, A12. Axiom 12 is just as problematic as A8, but it too can be justified by rational adaptation.

Theorems 4–7. The fourth theorem (EP ultimately outcompete FM in stable environments) relies only on already established axioms, and the additional assumption spelled out for the fifth theorem (A13) assumes a link between resilience and growth. The sixth and the seventh theorems (once appeared, EP won't disappear for as long as the resource configuration persists; FM become extinct past a certain point in time in stable environments) require no extra assumptions.

Theorems 8–10. The last three theorems play a different role; Theorems 8 and 9 are about changing resource configurations, and Theorem 10 is a metatheorem about the completeness of the typology of propagation strategies. T8 and T9 do not require new assumptions, but they do rely on new meaning postulates which extend the interpretation of selection preference to the case of changing environmental conditions and fix a meaning for the concepts *slow*, *fast*, and *extreme* change. In addition, Theorem 9 required an additional meaning postulate, since “extremely fast” environmental change may be too rapid to allow any group to appear. Being a metatheorem, Theorem 10 must rely on new assumptions outside the scope of the propagation fragment, and it remains an open question whether these assumptions are strictly in line with organizational ecology. Be this as it may, Theorem 10 draws attention to a broader interpretation of “first movers,” since generalists that enter swiftly new environments by activating alternative routines might also be consid-

ered first movers, provided one is willing to drop the (simplifying) assumption that new resource configurations are exclusively conquered by new organizations. This simplifying assumption out of the way, A15 (FM have a low number of routines at founding) is no longer warranted, and Theorem 10 no longer follows. In short, under the broader interpretation of the concept “first mover,” it remains an open question whether or not there are more than two propagation strategies.

A Second Conceptualization

The present formalization assesses selection's favor in terms of growth rates. This seems to be the natural way of thinking about natural selection, yet it is not the only one. One could also conceptualize natural selection in terms of “population mass” (the accumulated size of all agents in the population) as Glenn Carroll suggested in a personal communication. We briefly sketch the basic ideas of this conceptualization.

Think of each subpopulation as a basin (a stock in economic terms), with the level of each basin representing the total size of a subpopulation (Figure 6). The role of demographic rates is taken over by the inflows and outflows respectively, and the total size (i.e., the level in the basin) becomes the criterion for selection preferences. Consequently, the carrying capacity is no longer interpreted as the maximal number of organizations, but as the maximal organizational mass sustainable in a given environment. Note that on this interpretation, carrying capacity appears as a property of the environment (just as in bioecology), and not as property of the size distribution of the organizations in the environment.

As far as the technical implementation is concerned, the old characterization of selection preferences (MP1)

has to be replaced by a new one that claims that selection prefers the subpopulation with the bigger accumulated mass. Analogous modifications are required in Definitions 1 and 2 (on the critical point and on “outcompeting”). The formulas on founding and disbanding rates also have to be replaced by new ones on mass inflows and outflows.

The mass-comparison model leaves room for size concentration processes: there is no need to assume that the size distribution of organizations is homogeneous (required for Assumption 7 and Definition 2). If the size concentration can grow in a subpopulation, then the number of organizations may decrease, even if the subpopulation is favored by selection (think of the automotive industry as a possible example)—the aggregate mass is not fully determined by the actual number of organizations. Furthermore, in the borderline case of a homogeneous size distribution, the inflow of mass is proportional to the number of foundings, and the outflow of mass to the number of disbandings, respectively.

An additional subtle advantage of this conceptualization is that growth rate comparison involving empty subpopulations cannot exclude the possibility that selection would favor an empty group to a non-empty one with negative growth rate. This inconvenience disappears in case of mass comparison, since mass cannot be negative. As a consequence, there is no need for MP2 in the second conceptualization, and the difference between the $\text{Subpops}(s_1, s_2, e, t)$ and the $\text{O_group}(s, e, t)$ predicates also goes away.

There is another reason for having two different conceptualizations. With both of them, we get basically the same results. The macrotheory of first movers and efficient producers does not depend on whether a subpopulation's dominance is measured by demographic rates or by the aggregate mass of the composite organizations in this subpopulation, and this is an indication for the *robustness* of the theory of propagation strategies (Stinchcombe 1991, Hannan 1992).

However, there is a counterargument against the mass-based approach as well. The change of the size distribution in a population is not necessarily a zero sum game: the carrying capacity for the whole population may increase because some organizations are exceptionally large (Hannan and Freeman 1989, p. 127). For example, investigating telephone companies, Barnett and Amburgey (1990) found that the increasing size of some companies had beneficial effects for the rest of the industry, since the big guys “pioneered” the way for the others.

Figure 6 The Mass-flow Model of Selection

On balance, it is hard to tell whether the rate-dependence or the mass-dependence approach should be “favored” in organizational ecology. Perhaps that some combination of both approaches is possible, or that yet another conceptualization of selection preferences comes up.

6. Results

The quest for axiomatic theories raises many expectations. By axiomatizing a theory, one would gain clarity, create order, assure logicity, coherence, and consistency; one would even establish scientific truth—provided one would find self-evident axioms without violating logic in the process. These expectations have been largely frustrated in the past, either because theories lived and died without axiomatization, or because the axiomatizations did not live up to the expectations. It is possibly one of the more sizable paradoxes of western thought that the axiomatic program was rarely questioned until the 20th century, yet rarely carried through (Stegmüller 1976). Would our axiomatization make a difference?

It depends, since the potential effects of an axiomatization depend on the structure and presentation of the target theory. For example, if the target theory is already presented in an assumption-theorem structure (as is the case for the inertia-part and the density-dependence part of Organizational Ecology), then matters of logicity and parsimony loom large. If, on the other hand, a theory is presented informally (as is the case for the propagation part of OE), other aspects come to the fore. One such aspect is a better understanding of the target theory.

Scopes and Limits of the Propagation Fragment

Our axiomatization contributes to a better understanding of the scope, and the limits, of the theory of propagation strategies in the context of OE. Perhaps the most important result is that the standard interpretation of the concept of selective advantage requires the import of rational adaptation into the theory.

An important limit of the theory is given by the dichotomization of propagation strategies into two classes: first movers and efficient producers. The propagation fragment does not consider other propagation strategies besides FM and EP. As it turned out, however, we could not take this suggestion for granted. Bringing in considerations from other parts of organizational ecology, we showed that the typology is incomplete, so other propagation strategies could exist.

A more subtle point, related both to rational adaptation and the underlying typology of propagation strategies, concerns the possibility of a gradual metamorphosis of individual first movers. Other parts of organizational ecology, notably the part about organizational inertia, suggest the possibility that some first movers, by gradually enhancing their reproducibility, are accumulating more and more elaborate routines, which, in turn, increase their efficiency. This possibility may represent, in a sense, a third way of revealed organizational behavior—guided perhaps by a combination of rational adaptation and “blind” selection. One way to introduce this “third way” organically into organizational ecology would be by assuming selective pressure at the level of microbehavior (Nelson and Winter 1982); in case, such selective pressure would favor more reliable routines and hence constitute a general trend leading to the gradual metamorphosis of the surviving first movers into efficient organizations.

We planned to follow a specific strategy in axiomatizing the propagation fragment by moving from theorems to assumptions that do not depend on the specific content of organizational ecology. As the need for the simplifying assumptions shows, we have not been able to carry through this strategy without compromising, still we argue that our simplifications are defensible: every theory makes simplifying assumptions to get things done, and making required simplifications explicit does not weaken the theory but improves our understanding of it.

There is at least one additional problem, related to the pace of environmental change. The notions *slow* and *rapid* environmental change appear in core statements of the theory fragment (Hannan and Freeman 1989, p. 118) but they are not sufficiently described. As a consequence, we had to use an auxiliary construction in the run-up to the Theorems 8 and 9, in that we relied on the distinction between FM and EP in order to distinguish between slow and fast change. We went ahead because we saw no other way to proceed for lack of independent information regarding the metric of environmental change. Finally, there is the possibility of very fast change, that had to be ruled out in order to derive Theorem 9, and this implies yet another simplifying assumption.

Other Results

Clarity of the Language. Using a well-defined formal language, our axiomatization has created clarity—but

only in a sense. Many questions about the meaning of the basic concepts in OE (e.g., environment, population) remain, and such questions become more apparent in the context of a formalization. We have sidestepped these questions by using plausibility arguments in the presentation of the ecological background assumptions, because any attempt at discussing reasonable alternatives would have overloaded the paper. Aficionados of Organizational Ecology may sense, however, that the theory is sensitive to the way these basic concepts are understood, and this is borne out by related work (Péli et al. 1994, Bruggeman 1997, Péli 1997).

Logicity: Is the Explanation Logically Sound or Not?

It has often been claimed that axiomatic systems provide scientific explanations as a side effect of their deductive structure. Modern epistemology has shown this claim to be exaggerated, but not meaningless, in that scientific explanation includes both deductive and non-deductive reasoning (Nagel 1961). The intended explanations of theories may prove unsound for many reasons. Establishing the soundness of a theory in terms of natural language is quite problematic, because natural language has no formal criteria for deductive reasoning. Since the explanatory structure of the propagation fragment has only been hinted at in the original literature, we cannot claim definitive improvement; the explanatory structure of our axiomatization is simply more explicit. From a different angle, however, we can be quite confident; we have a formal machinery in place that provides sound inference rules for deductions. The reasoning that leads from the premises to the theorems is purely computational; it is just as sound as, say, a computation in elementary arithmetic.

Consistency. Consistency concerns whether a given theory is contradiction-free (technically speaking, whether it is possible to derive a contradiction from the theory). In the past, consistency held sway as an informal, but undebatable criterion of scientific effort; along with the use of formal languages, consistency has now acquired more formal standing. However, establishing the consistency of the theory by informal means is just as impossible as establishing the soundness of its explanatory structure. A formal language with certain computational properties is required. FOL has these properties—up to a point. We can formally prove the consistency of the established premise set. However, contradictions may still hide in the nonformalized parts of OE; for example, one cannot exclude that adding

some “trivial” background knowledge formulas to the premise set would not generate a contradiction.

Self-evidence. Using empirical confirmation as a relative truth criterion for theories is relatively modern; for most of occidental scientific history, truth was sought via other routes, notably via the self-evidence of a theory’s basic assumptions. In developing our logical rendering, we followed the strategy of searching for assumptions which are either generally accepted bio-ecological background knowledge, or common sense knowledge. Having been forced to make simplifying assumptions, we have not been able to carry this strategy through without compromise, but in the process we have learned more about the scope and limits of the theory. To avoid any misunderstanding: we are not trying to reestablish antique truth criteria. A formal theory facilitates the execution of empirical tests, and hence the application of modern truth criteria.

7. Future Directions

Progress along the lines of this research will depend on more than methodological promises; it will depend on results. Further research should ensure that such results are to be had. This can be done in different ways.

Axiomatization in FOL

The effort to axiomatize organizational ecology continues. We have more or less definitive axiomatizations of the inertia part, the density-dependence part, and the generalist-specialist part of the OE (Péli 1993, 1997; Péli et al. 1994, Bruggeman 1997) and are working on the (partial) unification of these parts. Also, there is work in progress on the axiomatization of Thompson’s *Organizations in Action* (1967), a representative text of contingency theory (Masuch and Huang 1996, Pólos 1995). So far, we have not observed incommensurability of the two approaches. Indeed, organizational ecology does, in places, invoke rational adaptation; conversely, contingency theory can be interpreted in terms of social selection, and this interpretation may yield a more coherent theory by providing an answer to the question how environmental conditions are translated into optimal organizational structures.

Theorem-building

So far, we have built axiomatizations by establishing an assumption-theorem structure in natural language (if such a structure is not already given explicitly), and subsequently formalized this structure in FOL. So, we did a rational reconstruction of a given fragment and

knew the theorems we wanted to prove in advance. However, one may want to go a step further; there may be more theorems implicit in a given set of assumptions than are “visible” at first sight (Kamps and Masuch 1997.) What is needed are “strong” heuristics that help us to identify promising candidate theorems.

Acknowledgments

Thanks are due to Jeroen Bruggeman, Glenn Carroll, Scip Garling, Babette Greiner, Michael Hannan, Melina Huyg, Jaap Kamps, Breannán Ó Nualláin, László Pólos, Andrew H. van de Ven, and the anonymous *OS* referees for comments on previous versions of this paper. This research was supported by a “PIONIER” grant of the Dutch National Science Foundation (# PGS 50-334) and by a TEMPUS grant from the European Union (# HUT 263-92). This research was presented at the Academy of Management Dallas 1994 meeting and a brief version of this paper was published in the *Best Papers Proceedings of the Academy of Management*, Dallas 1994, Dorothy P. Moore, Ed.

Appendix

The Premises of Theorem 1

Formulas denoted by “B” express background knowledge.

Theorem 1

MEANING POSTULATE 1. *Selection favors subpopulations with higher growth rates.*

$$\begin{aligned} &\forall e, s1, s2, t [\text{Env}(e) \wedge \text{Subpops}(s1, s2, e, t) \wedge \\ &\quad g(s2, t) > g(s1, t) \\ &\quad \rightarrow \text{Selection-favors}(s2, s1, t)]. \end{aligned}$$

LEMMA 1. *The growth rate of efficient producers exceeds the growth rate of first movers past a certain time in stable environments.*

$$\begin{aligned} &\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow \exists t_0 [\text{During}(e, t_0) \\ &\quad \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge t \geq t_0 \rightarrow g(\text{EP}, t) > g(\text{FM}, t)]]]. \end{aligned}$$

THEOREM 1. *In stable environments, selection favors efficient producers above first movers past a certain point in time.*

$$\begin{aligned} &\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow \exists t_0 [\text{During}(e, t_0) \\ &\quad \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \\ &\quad \wedge t \geq t_0 \rightarrow \text{Selection_favors}(\text{EP}, \text{FM}, t)]]]. \end{aligned}$$

Lemma 1

LEMMA 2. *The disbanding rate of first movers exceeds the disbanding rate of efficient producers.*

$$\begin{aligned} &\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ &\quad \rightarrow d(\text{FM}, t) > d(\text{EP}, t)]. \end{aligned}$$

A8. *The founding rate of first movers does not exceed the founding rate of efficient producers past a certain point in time in stable environments.*

$$\begin{aligned} &\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow \exists t_0 [\text{During}(e, t_0) \\ &\quad \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge t \geq t_0 \rightarrow f(\text{EP}, t) \geq f(\text{FM}, t)]]]. \end{aligned}$$

B. *If EP have lower disbanding rate and not lower founding rate than FM, then EP have higher growth rate than FM.*

$$\begin{aligned} &\forall t [d(\text{FM}, t) > d(\text{EP}, t) \wedge f(\text{EP}, t) \geq f(\text{FM}, t) \\ &\quad \rightarrow g(\text{EP}, t) > g(\text{FM}, t)]. \end{aligned}$$

B. on “greater or equal to.”

$$\forall x, y [x \geq y \leftrightarrow x > y \vee x = y].$$

LEMMA 1. *The growth rate of efficient producers exceeds the growth rate of first movers past a certain time in stable environments.*

$$\begin{aligned} &\forall e [\text{Env}(e) \wedge \text{Stable}(e) \rightarrow \exists t_0 [\text{During}(e, t_0) \\ &\quad \wedge \forall t [\text{Subpops}(\text{FM}, \text{EP}, e, t) \wedge t \geq t_0 \rightarrow g(\text{EP}, t) > g(\text{FM}, t)]]]. \end{aligned}$$

Lemma 2

A9. *The disbanding rate of first movers initially exceeds the disbanding rate of efficient producers.*

$$\forall e [\text{Env}(e) \rightarrow d(\text{FM}, \text{init}(e)) > d(\text{EP}, \text{init}(e))].$$

A10. *If FM and EP are present in the environment at time points $t1$ and $t2$, then they are also present during the period between $t1$ and $t2$.*

$$\begin{aligned} &\forall e, t, t1, t2 [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t1) \\ &\quad \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t2) \wedge t \geq t1 \wedge t2 \geq t \\ &\quad \rightarrow \text{Subpops}(\text{FM}, \text{EP}, e, t)]. \end{aligned}$$

LEMMA 3. *The difference between the disbanding rates of first movers and efficient producers does not decrease with time.*

$$\begin{aligned} &\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ &\quad \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t))]. \end{aligned}$$

B. *The initial time of an environment is the earliest time point when both FM and EP are present in the environment.*

$$\begin{aligned} \forall e, t [\text{Env}(e) \rightarrow \\ (\text{During}(e, \text{init}(e)) \rightarrow \text{Subpops}(\text{FM}, \text{EP}, e, \text{init}(e))) \\ \wedge (\text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow t \geq \text{init}(e))]. \end{aligned}$$

B. *If the positive gap between the disbanding rates of FM and EP at t_1 does not decrease on interval $[t_1, t_2]$, then FM's disbanding rate stays higher at t_2 .*

$$\begin{aligned} \forall t_1, t_2 [\forall t [t_2 \geq t \geq t_1 \wedge d(\text{FM}, t_1) > d(\text{EP}, t_1) \\ \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t))] \\ \rightarrow d(\text{FM}, t_2) > d(\text{EP}, t_2)]. \end{aligned}$$

B. *If first movers and efficient producers are present in an environment at a certain point of time, then this time point occurs during the environment persists.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow \text{During}(e, t)].$$

B. *The initial time of an environment does not precede the opening of this environment.*

$$\forall e [\text{Env}(e) \rightarrow \text{init}(e) \geq \text{beg}(e)].$$

B. *If time point t_1 occurs after the opening of the environment, and a later time-point, t_2 , occurs during the environment, then t_1 also occurs during the environment.*

$$\begin{aligned} \forall e, t_1, t_2 [\text{Env}(e) \wedge t_1 \geq \text{beg}(e) \wedge t_2 > t_1 \\ \wedge \text{During}(e, t_2) \rightarrow \text{During}(e, t_1)]. \end{aligned}$$

B. *Transitivity.*

$$\forall x, y, z [x > y \wedge y > z \rightarrow x > z].$$

B. *On "greater or equal to."*

$$\forall x, y [x \geq y \leftrightarrow x > y \vee x = y].$$

LEMMA 2. *The disbanding rate of first movers exceeds the disbanding rate of efficient producers.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow d(\text{FM}, t) > d(\text{EP}, t)].$$

Lemma 3

A4. *Resource availability decreases until the carrying capacity is reached.*

$$\begin{aligned} \forall e, t [\text{Env}(e) \wedge \text{During}(e, t) \wedge \text{num}(e, t) > 0 \rightarrow \\ (\text{carr}(e) > t \rightarrow \text{Decreasing}(\text{res}(e, t))) \\ \wedge \neg (\text{carr}(e) > t) \rightarrow \text{Const}(\text{res}(e, t))]. \end{aligned}$$

LEMMA 4. *Decreasing resource availability affects the disbanding rate of first movers more, than the disbanding rate of efficient producers.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow$$

$$\begin{aligned} (\text{Decreasing}(\text{res}(e, t)) \rightarrow \text{Increasing}(d(\text{FM}, t) - d(\text{EP}, t))) \\ \wedge (\text{Const}(\text{res}(e, t)) \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t)))]. \end{aligned}$$

B. *If first movers and efficient producers are present in an environment at a certain point of time, then this time point occurs during the environment persists.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow \text{During}(e, t)].$$

B. *If first movers and efficient producers are present in an environment at a certain point of time, then the environment is not empty at this time.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow \text{num}(e, t) > 0].$$

B. *If something increases, then it does not decrease.*

$$\forall x [\text{Increasing}(x) \rightarrow \neg \text{Decreasing}(x)].$$

B. *"Greater or equal to."*

$$\forall x, y [x \geq y \leftrightarrow x > y \vee x = y].$$

LEMMA 3. *The difference between the disbanding rates of first movers and efficient producers does not decrease with time.*

$$\begin{aligned} \forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \\ \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t))]. \end{aligned}$$

Lemma 4

A6. *Less resilient subpopulations are more affected by decreasing resource availability.*

$$\begin{aligned} \forall e, s_1, s_2, t [\text{Env}(e) \wedge \text{Subpops}(s_1, s_2, e, t) \wedge \text{resil}(s_2) > \text{resil}(s_1) \\ \rightarrow (\text{Decreasing}(\text{res}(e, t)) \rightarrow \text{Increasing}(d(s_1, t) - d(s_2, t))) \\ \wedge (\text{Const}(\text{res}(e, t)) \rightarrow \text{Const}(d(s_1, t) - d(s_2, t)))]. \end{aligned}$$

A2. *Efficient producers are more resilient than first movers.*

$$\text{resil}(\text{EP}) > \text{resil}(\text{FM}).$$

B. *If something is constant, then it is not decreasing.*

$$\forall x [\text{Const}(x) \rightarrow \neg \text{Decreasing}(x)].$$

LEMMA 4. *Decreasing resource availability affects the disbanding rate of first movers more, than the disbanding rate of efficient producers.*

$$\forall e, t [\text{Env}(e) \wedge \text{Subpops}(\text{FM}, \text{EP}, e, t) \rightarrow$$

$$\begin{aligned} (\text{Decreasing}(\text{res}(e, t)) \rightarrow \text{Increasing}(d(\text{FM}, t) - d(\text{EP}, t))) \\ \wedge (\text{Const}(\text{res}(e, t)) \rightarrow \neg \text{Decreasing}(d(\text{FM}, t) - d(\text{EP}, t)))]. \end{aligned}$$

Endnote

¹A public domain theorem prover, Otter, is available at the Internet site: <ftp://info.mcs.anl.gov/pub/Otter>.

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Accepted by Andrew H. Van de Ven, former Departmental Editor; received December 19, 1994. This paper has been with the authors for two revisions.