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How well determined is the $N - \Delta$ electromagnetic transition form factor at high Q^2 ?

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Abstract

We explore the sensitivity of the $N - \Delta(1232)$ electromagnetic transition form factor, extracted from the inclusive electroproduction data, to the choice of background. We find that the extraction of the transition form factor depends critically on it and the existing inclusive data are consistent with a transition form factor much larger than previous values, showing no serious discrepancy with the Bloom-Gilman duality.

The electromagnetic transition amplitudes to the excited states of the nucleon provide important tests for QCD. At low four-momentum transfer squared, negative Q^2 , these amplitudes are useful in illuminating corrections to the SU(6) symmetry of the quark model, for example, due to the color hyperfine interaction. The interest at high Q^2 is in the transition to the perturbative QCD (pQCD) regime. Based on a recent analysis of the world supply of inclusive electron scattering data at high Q^2 in the resonance region, Stoler [1] has concluded that the nucleon- $\Delta(1232)$ electromagnetic transition form factor exhibits a fall off much faster than the dipole behavior expected from pQCD [2], while transition form factors for other dominant resonances appear to behave quite *normally*. Although the behavior of the $N - \Delta$ transition form factor is not necessarily in disagreement with the pQCD counting

rules [3], the $\Delta(1232)$ is appearing to fall faster than its underlying background in the Q^2 region from 3 to 10 GeV², thus, violating the Bloom-Gilman duality [4,5]. On the other hand, an analysis [5] of the structure function in the $\Delta(1232)$ resonance region, without a separation into background and resonance pieces, is in agreement with the Bloom-Gilman duality. These two seemingly contradictory behaviors can be reconciled only if the background would rise relative to the resonance as the resonance falls, a rather bizarre occurrence. In many pQCD motivated hadron models [6], such an odd behavior of the nucleon-Delta electromagnetic transition form factor is expected. Thus, a critical examination of this transition form factor is a fundamental issue in baryon physics. This is the central focus of this Letter.

In contrast to Stoler's work [1], a different conclusion regarding the importance of the Δ at high Q^2 is reached in the work of Olsson, Osypowski and Monsay (OOM) [7]. A typical result of their work is shown in Fig. 1 at $Q^2 = 5.86$ GeV². The dashed-dotted line is

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Fig. 1. The total inclusive cross section at 5.86 GeV^2 compared with the model of OOM [7] (dashed-dotted) line. The dashed and dotted lines are the contributions from the Δ and PV Born terms, respectively. The dashed-dot-dot line arises from an additional background of the form $(s-u)^3$. The total response, including the additional background, is given by the solid line. The data are from Ref. [8].

the total inclusive cross section, while the dashed and dotted lines are the Δ and background contributions, respectively. The data are from Ref. [8]. In Fig. 1, two main features are evident; first, the Δ is playing a significant role up to the resonance energy; second, there is some missing strength at higher W , which OOM attribute to tails of higher resonances. In other words, the OOM conclusion is that additional production mechanisms are becoming important *above* the Δ resonance energy, which should be contrasted with Stoler's conclusion that additional production mechanisms are important even near threshold.

The difference between OOM and Stoler can be traced to the different background contributions. Stoler includes a phenomenological background, with some assumed energy dependence, added incoherently to the Breit-Wigner resonance piece. The OOM model is based on the effective Lagrangian approach with the background given by the pseudovector (PV) Born terms including the fairly well-known nucleon and pion electromagnetic form factors. It is evident from Fig. 1 that this background is too small to explain the data, and hence the need in this approach for a rela-

tively large Δ contribution. As we discuss in more detail below, it is possible to add an additional, smooth, phenomenological background contribution (dashed-dot-dot line in Fig. 1) to the OOM model which gives an excellent fit to the data (solid line in Fig. 1) in this energy region.

The purpose of the Letter is to examine in more detail the OOM approach and the dependence in extraction of the transition form factor from the inclusive data on a particular choice of the background. We will restrict ourselves to $W \leq 1.31 \text{ GeV}$ (W the total cm energy), since the inclusive data should be dominated by the one pion channels. This also has a theoretical advantage in that a unitary treatment of the amplitude may be used. While an extended coverage in W is desirable, unitarization of the amplitude in the higher W range is poorly understood.

The model we use is that of OOM [7], supplemented by an additional phenomenological background needed to describe the data at $W \geq 1.23 \text{ GeV}$. The latter may be interpreted as arising from tails of higher resonances or from t -channel meson exchanges. The $\gamma N \Delta$ vertex is described in terms of one gauge coupling, the g_1 term of Ref. [9]. Of course, two additional couplings are needed to describe the most general $\gamma N \Delta$ vertex and these couplings should be important at low Q^2 . However, from the inclusive data it is not possible to make a separation of these couplings, or equivalently, the M1, E2 and C2 contributions. Furthermore, the two additional gauge couplings can be shown to vanish faster than g_1 at large Q^2 assuming the validity of the pQCD counting rules.

Following Ref. [10], the *resonant* contribution to the total inclusive cross section at the resonance energy is

$$\begin{aligned} \sigma^R &\equiv \sigma_T^R + \epsilon \sigma_L^R \\ &= 2\pi\alpha \frac{q_L^2}{MK} \frac{1}{(1 + Q^2/M_+^2)} \frac{1}{M_\Delta \Gamma_\Delta} \\ &\quad \times [|G_M|^2 + 3|G_E|^2 + \epsilon |G_C|^2 Q^2 / 4M_\Delta^2], \end{aligned} \quad (1)$$

where q_L is the photon three-momentum in the lab frame, M is the nucleon mass, M_Δ is the Δ mass, $K = (M_\Delta^2 - M^2)/(2M)$, $M_+ = (M_\Delta + M)/2$, α is the fine structure constant, and ϵ is the virtual photon polarization. In terms of the gauge coupling g_1 , we have

Table 1

The amplitude A_H ($\text{GeV}^{-1/2}$) at various Q^2 extracted from the inclusive data with different assumptions for the background

Q^2	Model 1		Model 2		Stoler A_H
	A_H	χ^2_{DF}	A_H	χ^2_{DF}	
2.91	0.0546 ± 0.0010	1.10	0.0516 ± 0.0010	1.10	0.0482 ± 0.0006
3.91	0.0361 ± 0.0008	1.41	0.0340 ± 0.0007	1.14	0.0288 ± 0.0012
5.86	0.0176 ± 0.0003	1.19	0.0152 ± 0.0004	1.44	0.0127 ± 0.0008
7.93	0.0104 ± 0.0004	1.76	0.0080 ± 0.0005	0.75	0.0055 ± 0.0011
9.84	0.0062 ± 0.0004	1.16	0.0038 ± 0.0006	0.53	0.0026 ± 0.0014
10.8	0.0059 ± 0.0005	0.22	0.0044 ± 0.0015	0.24	

Model 1 has a background of the form $(s - u)^3$ while model 2 has a background of the form $(s - u)$. Also given are the chi-squared per degree of freedom, χ^2_{DF} . The final column contains the results of Stoler [1].

$$\begin{aligned}
 G_M &= \frac{g_1}{6M_\Delta(M_\Delta + M)} \\
 &\times [(3M_\Delta + M)(M_\Delta + M) + Q^2], \\
 G_E &= \frac{g_1}{6M_\Delta(M_\Delta + M)} [M_\Delta^2 - M^2 - Q^2], \\
 G_C &= \frac{2M_\Delta g_1}{3(M_\Delta + M)}. \quad (2)
 \end{aligned}$$

We have not assumed $\sigma_L^R = 0$, in contrast to Stoler [1]. From (1) and (2), we find a quite reasonable value of $\sigma_L^R/\sigma_T^R = 0.11$ at $Q^2 = 5 \text{ GeV}^2$. To make connection with A_H defined in Ref. [1], we have

$$\sigma_T^R + \epsilon\sigma_L^R = \frac{2M}{M_\Delta\Gamma_\Delta} |A_H|^2. \quad (3)$$

In the K-matrix and Noelle methods of unitarization [9], use of the total inclusive cross section on the left-hand side of Eq. (3) gives an upper bound on $|A_H|$.

Our model for the background is not purely phenomenological as in [1]. We take the effective Lagrangian approach of OOM to define our basic background, supplemented by an additional phenomenological piece, discussed earlier, needed to describe the inclusive electroproduction data above $W = 1.23 \text{ GeV}$. We introduce the latter in a unitary and crossing symmetric manner by adding to the A_4^- invariant amplitude [11] a real contribution of the form $(s - u)$ or $(s - u)^3$, s and u the usual Mandelstam variables, with the strength determined by a fit to the inclusive data. The amplitude is projected into multipoles and unitarized via the K-matrix method as described in Ref. [9]. Our various background contributions evidently have different energy dependences and, as will be seen, lead to quite different results for A_H . They also serve

to demonstrate the arbitrariness in these choices. However, they are no better or worse than any other choice in this energy region, in our opinion. We thus have two parameters to fit to the data: the strength of the additional background term and the g_1 gauge coupling. These two parameters are fitted to the W data in the range from threshold to 1.3 GeV at each Q^2 . The form factors relevant to the PV Born terms are as in Ref. [7].

The results for A_H at various Q^2 are given in Table 1, along with the results of Stoler's [1] analysis of the SLAC data [8]. Our model 1 has an additional background of the form $(s - u)^3$; in model 2, the background is of the form $(s - u)$. A typical result for model 1 is shown in Fig. 1. Statistically the fits are very good and must be deemed acceptable parameterizations of the data. The errors are defined in terms of $\chi^2_{\text{MIN}} + 1$. At the lowest Q^2 values, our fitted results are 15% to 25% larger than Stoler's results, but the discrepancy increases at higher Q^2 . At $Q^2 = 7.93$ and 9.84 GeV^2 , the results of model 2 are about 50% larger than Stoler's, while the results of model 1 are a factor of 2–3 times larger than Stoler's results. As Stoler's fit also looks reasonable (no chi-squared are given in Ref. [1]), one may conclude that a very large range of A_H is compatible with the data. This conclusion should come as no surprise. Even at the real photon point, where multipole analyses exist, the separation into background and resonance is difficult and model dependent. In the case of inclusive data, the separation is very difficult, if not impossible.

In Table 2 we show the results for the ratio σ^R/σ , where σ^R is the resonant contribution to the inclusive cross section, and σ is the total inclusive cross section

Table 2

The ratio of resonant cross section (σ^R) to the total cross section (σ) at various Q^2 for the models discussed in the text. Also shown are Stoler's results [1]

Q^2	σ^R/σ		
	Model 1	Model 2	Stoler
2.91	0.86 ± 0.06	0.77 ± 0.07	0.67 ± 0.05
3.91	0.80 ± 0.08	0.71 ± 0.07	0.51 ± 0.06
5.86	0.76 ± 0.05	0.56 ± 0.04	0.40 ± 0.05
7.93	0.66 ± 0.07	0.39 ± 0.06	0.18 ± 0.07
9.84	0.56 ± 0.08	0.21 ± 0.07	0.10 ± 0.08
10.8	0.56 ± 0.15	0.31 ± 0.23	

at the resonance energy. Evidently, this ratio is also a measure of the resonance to background ratio. It is seen that the Δ plays a significant role in model 1, even at 10 GeV^2 ; in model 2 and in Stoler's work, the Δ is becoming much less important as Q^2 increases. Thus, in model 1, the resonance is falling roughly as fast as its underlying background, in agreement with the Bloom-Gilman duality, while in model 2 and in Stoler's work the Δ is falling faster than the background. To make this more quantitative, we find in all three cases the ratio is well described by

$$\sigma^R/\sigma = A + B/Q^2. \quad (4)$$

If the ratio scales or is approaching the scaling limit, then the $1/Q^2$ term should be small compared to the constant term. In fact, it is quite reasonable to assume that if the scaling limit is being approached, then B should be on the order of the typical hadron mass scale squared $\sim 1 \text{ GeV}^2$. In model 1, this is what happens, we find $A = 0.53$, $B = 1.02 \text{ GeV}^2$ with a χ^2 of 2.4 for 6 points. In model 2, we find $A = 0.14$, $B = 2.07$ with a χ^2 of 8.5, the major contribution to the χ^2 coming from the point at 9.84 GeV^2 . If this point is excluded, we find $A = 0.24$, $B = 1.67$ with a χ^2 of 2.7. In either case, the $1/Q^2$ term is very important and contributes about 50% to the ratio at 10 GeV^2 . Thus, duality for the resonance is clearly violated in this model. For Stoler's results, we have $A = -0.02$, $B = 2.06$ and $\chi^2 = 3.5$, i.e., σ^R/σ found by Stoler are falling like Q^{-2} , a clear violation of the Bloom-Gilman duality.

We now turn to the Bloom-Gilman duality ratio [4,5] using either the Bjorken scaling variable x or the Nachtmann variable ξ . We integrate from the pion threshold up to x (or ξ) corresponding to $W = 1.3 \text{ GeV}$.

Table 3

The results for the Bloom-Gilman duality ratio at various Q^2 obtained from the two models described in the text

Q^2	Model 1		Model 2	
	x	ξ	x	ξ
2.91	4.86 ± 0.13	3.07 ± 0.08	4.31 ± 0.17	2.72 ± 0.11
3.91	4.98 ± 0.23	3.37 ± 0.15	4.43 ± 0.19	2.99 ± 0.13
5.86	3.99 ± 0.14	2.94 ± 0.10	2.96 ± 0.17	2.19 ± 0.12
7.93	3.49 ± 0.27	2.73 ± 0.21	2.07 ± 0.25	1.62 ± 0.19
9.84	2.39 ± 0.25	1.94 ± 0.20	0.92 ± 0.28	0.74 ± 0.23
10.8	2.92 ± 0.51	2.40 ± 0.42	1.59 ± 1.12	1.31 ± 0.92

The results for model 1 and model 2 are shown in Table 3, where it is evident that model 2 shows much more Q^2 variation in the ratio than does model 1. Above 4 GeV^2 in model 1, there is some Q^2 variation, but it does not seem significant for the following reasons. First, the four points above 4 GeV^2 all agree within 20% of their average, comparable to, but slightly larger than, results found in other works [4,5]. Second, the integration region considered here is smaller than in previous works, and thus one would expect larger Q^2 variations. Finally, as in previous works, we have ignored the Q^2 variation of the structure function in the "scaling" region and have calculated the ratio using structure functions of Ref. [12], valid at $Q^2 \approx 5 \text{ GeV}^2$. In fact, one should use the structure function in the scaling region at a Q^2 higher than the Q^2 in the Δ region. The Q^2 variation of the structure function in the scaling region can be quite important for the Δ , since the integration is concentrated around $x \approx 1$. Although it is not clear how one should choose the relevant Q^2 in the scaling region, if we adopt the simple ansatz that Q^2 in the scaling region increases as Q^2 increases in the Δ region, then the ratio at higher Q^2 will be enhanced compared to the results in Table 3. This is because the valence quarks dominate near $x = 1$; as Q^2 increases, the valence quark distribution shifts to smaller x . The integral over the structure function in the scaling region thus decreases and enhances the ratio. Therefore, the results of model 1 show no serious discrepancy with the Bloom-Gilman duality.

For all three cases considered here, the Bloom-Gilman ratio for the *background* is nearly constant at large Q^2 (the constants obtained for different models are of course different), and thus this ratio cannot be used to rule out any of the choices of background de-

scribed above. If the Δ is falling rapidly relative to the background as a function of Q^2 , for a fixed W around 1.23 GeV, then the imaginary part of the electroproduction amplitude should be also falling rapidly. In particular, at large Q^2 , the imaginary parts of the resonant multipoles at resonance should be nearly zero. This should be contrasted with the situation at the real photon point where, at resonance, the imaginary part of the $M_{1+}^{3/2}$ multipole is the dominant contribution to the neutral pion photoproduction amplitude. The transverse-transverse cross section should be quite sensitive to the resonant multipoles.

In summary, we have shown that the extracted $N-\Delta$ electromagnetic transition form factor is extremely dependent on the choice of background. Based on the inclusive electroproduction data, it is possible to construct models where both the resonance and background agree with the Bloom-Gilman duality, or to construct models in which the resonance falls like $1/Q^2$ compared to the background. In Stoler's approach for high Q^2 , the background quickly rises at threshold and dominates the cross section at the resonance energy. In our models, the PV Born terms dominate near threshold, but quickly level off to provide about 20% of the response at the resonance energy. The additional background is relatively small near threshold, but has a more rapid energy dependence than the Born terms. In model 1, the Δ is still large at the resonance energy, but the peak is not seen due to the background which rises quickly *after* the Δ resonance energy. Model 2 is in between model 1

and Stoler's approach. Which scenario is correct can only be decided by exclusive electroproduction experiments at facilities like CEBAF.

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