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DOI
10.1049/iet-bmt.2018.5106

Publication date
2019

Document Version
Final published version

Published in
IET Biometrics

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Citation for published version (APA):
Semiparametric likelihood-ratio-based biometric score-level fusion via parametric copula

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Abstract: The authors present a mathematical framework for modelling dependence between biometric comparison scores in likelihood-based fusion by copula models. The pseudo-maximum likelihood estimator for the copula parameters and its asymptotic performance are studied. For a given objective performance measure in a realistic scenario, a resampling method for choosing the best copula pair is proposed. Finally, the proposed method is tested on some public biometric databases from fingerprint, face, speaker, and video-based gait recognitions under some common objective performance measures: maximising acceptance rate at fixed false acceptance rate, minimising half total error rate, and minimising discrimination loss.

1 Introduction

In a biometric verification system, biometric samples (images of faces, fingerprints, voices, gait etc.) of people are compared and classifiers indicate the level of similarity between any pair of samples by a comparison score. If two samples of the same person are compared, a genuine score is obtained. If a comparison concerns samples of different people, the resulting score is called an impostor score. Depending on the application, a biometric verification system may give either a hard decision or a soft decision. The hard decision means that the system decides whether two biometric samples (query and template) are from the same individual or not by comparing the score to a threshold. On the other hand, the soft decision can be used in a forensic scenario by only giving the likelihood ratio (LR) value as an evidential value and let the final decision to the judge [1]. A common performance measure for this scenario is the cost of log likelihood ratio that can be decomposed into discrimination and calibration performance [2].

When our biometric system has two or more classifiers, one has to fuse the resulting multiple scores into a new score, which is called score-level fusion. It is convenient if the fused score is again an LR because: (i) it is optimal for standard biometric verification [3] and (ii) it reflects evidential value in forensic individualisation [1]. By assuming independence between scores, the fused LR can be computed as the product of the individual LRs of the classifiers (henceforth called product of likelihood ratios (PLR) fusion). However, the score-level fusion problem becomes difficult if the scores are dependent. In this paper, we propose a score-level fusion method with the following advantages.

i. The fused score is an LR.
ii. It can deal with dependent scores.
iii. It is available as an open-source software framework, programmed in Matlab (http://scs.ewi.utwente.nl/downloads/show,Copula%20Fusion%20Framework/), that implements the method.

This paper uses the copula concept to handle dependence between the scores. Although the copula model has already been used in [4–7] for some different scenarios, none of them provides analytically how this model is built and why the estimation of parameters determining the model is reliable. After explaining some related works in Section 2, this paper will explain how a copula model splits the LR computation for two or more classifiers into a product of the individual LRs and a correction factor in Section 3. Section 4 introduces a semiparametric model of LR-based fusion and subsequently provides an estimator of the proposed model with its convergence analysis. Detailed procedures to apply for several different applications of our method is given in Section 5. Finally, our conclusions are presented in Section 6.

2 Score level fusion

There are three categories of score-level fusion: transformation-based [8], classifier based [9], and density based (henceforth called likelihood ratio-based) [10]. The transformation-based fusion is done by mapping all components of the vector of comparison scores to a common domain and applying some simple rules such as sum, mean, max, med, and so on. Apart from its simplicity, it is important that the training set is representative of the data. For instance, to normalise scores to the unit interval [0,1], one must have the minimum and maximum scores. However, if the training data has outlier(s) then this estimation will not be reliable and may destroy the fusion performance. The classifier-based fusion acts as a classifier of the vector of the comparison scores to distinguish between genuine and impostor scores. These two first categories cannot be used in a forensic scenario as the fused score is not always an LR value. The last category would be optimal for biometric verification if the underlying distributions were known according to the Neyman–Pearson lemma [3]. Moreover, the fused score, as an LR value, can be used for forensic evidence evaluation [1] in forensic individualisation as a multiplicative factor for the information before analysing the evidence (prior odds) to get the information after taking the evidence into account (posterior odds) via the Bayesian framework

\[ O_{\text{posterior}} = LR \times O_{\text{prior}}. \] (1)

The LR is defined as the ratio between the density functions of the genuine and impostor scores. There are two categories for computing the LR: (i) estimating the density functions of the genuine and impostor scores separately and (ii) estimating the LR directly. The common approaches of the first category are modelling the underlying densities parametrically (assuming normal, Weibull, Gaussian mixture etc.) and non-parametrically (kernel density estimation (KDE), histogram binning (HB) etc.). Parametric models are usually chosen because of their simplicity.
and non-parametric models because of their flexibility. However, the main problem in using parametric models is the difficulty in choosing the appropriate model whereas non-parametric estimators are sensitive to the choice of the bandwidth or other smoothing parameters, especially for our multivariate case. A common parametric model to compute the LR directly, is the logistic regression (Logit) method by assuming the LR having some parametric form such as linear, quadratic, and so on. Although this method can also be used for the multivariate case [11, 12], the same problem in choosing an appropriate model will appear. In [13] a number of fusion methods are compared among which several LR-based approaches. However, all of these LR-based methods use a parametric model. On the other hand, the non-parametric approach of pool adjacent violators (PAV), which seems promising because of its optimality in transforming scores into their LR values [2], is applicable only for one-dimensional (1D) scores, which means that it cannot be used to compute the LR for fusion.

Many studies of score-level fusion assume independence between classifiers [12, 14, 15]. However, the independence assumption is not realistic since the scores may rely on the same information. To incorporate the dependence between classifiers, we propose a semiparametric LR-based biometric fusion by modelling the marginal individual LRs non-parametrically and the dependence between them by parametric copulas, to trade-off between the limitations of parametric and the flexibility of non-parametric models.

3 LR computation via Copula

Suppose we have \( d \) classifiers and let \((s_1, \ldots, s_d)\) denote the concatenated vector of \( d \) similarity scores where \( s_k \) is the corresponding score from the \( k \)th classifier for \( k = 1, \ldots, d \). Let \( f_{\text{gen}} \) and \( f_{\text{imp}} \) be the densities of genuine and impostor scores, respectively. The LR at a point \((s_1, \ldots, s_d)\) is defined by

\[
\text{LR}(s_1, \ldots, s_d) = \frac{f_{\text{gen}}(s_1, \ldots, s_d)}{f_{\text{imp}}(s_1, \ldots, s_d)}. \tag{2}
\]

Using the copula concept, the densities \( f_{\text{gen}} \) and \( f_{\text{imp}} \) will be split into their marginal densities and a factor modelling their dependence.

A \( d \)-variate copula is a distribution function on the unit cube \([0, 1]^d\), of which the marginals are uniformly distributed. Sklar [16] shows the existence of a copula for any multivariate distribution function.

**Theorem 1:** (Sklar (1959)): Let \( d \geq 2 \), and suppose \( H \) is a distribution function on \( \mathbb{R}^d \) with 1D continuous marginal distribution functions \( F_1, \ldots, F_d \). Then there exists an unique copula \( C \) such that

\[
H(x_1, \ldots, x_d) = C(F_1(x_1), \ldots, F_d(x_d)) \tag{3}
\]

every \((x_1, \ldots, x_d) \in \mathbb{R}^d\).

By taking the derivative of (3) with respect to \( x_i \) we will get the joint density function

\[
h(x_1, \ldots, x_d) = c(F_1(x_1), \ldots, F_d(x_d)) \times \prod_{i=1}^d f_i(x_i) \tag{4}
\]

where \( c \) is the copula density and \( f_i \) is the \( i \)th marginal density for every \( i = 1, \ldots, d \). This implies that (2) can be written as

\[
\text{LR}(s_1, \ldots, s_d) = \frac{c_{\text{gen}}(F_{\text{gen}}(s_1), \ldots, F_{\text{gen}}(s_d))}{c_{\text{imp}}(F_{\text{imp}}(s_1), \ldots, F_{\text{imp}}(s_d))} \times \prod_{i=1}^d \frac{f_{\text{gen}}(s_i)}{f_{\text{imp}}(s_i)} \tag{5}
\]

where \( c_{\text{gen}} \) and \( c_{\text{imp}} \) are the copula densities of genuine copula \( C_{\text{gen}} \) and impostor copula \( C_{\text{imp}} \), respectively. The second factor of (5), which is the product of the individual LRs

\[
\text{PLR}(s) = \prod_{i=1}^d \frac{f_{\text{gen}}(s_i)}{f_{\text{imp}}(s_i)} \prod_{i=1}^d \text{LR}(s_i), \tag{6}
\]

will be called the Naïve Bayes part, while the first factor, which is the copula density ratio

\[
C_{\text{gen}}(s_{\text{gen}}) = \frac{c_{\text{gen}}(F_{\text{gen}}(s_1), \ldots, F_{\text{gen}}(s_d))}{c_{\text{imp}}(F_{\text{imp}}(s_1), \ldots, F_{\text{imp}}(s_d))} \tag{7}
\]

will be called the correction factor where the superscript \((C_{\text{gen}}, C_{\text{imp}})\) means that the CF is modelled by copulas \( C_{\text{gen}} \) and \( C_{\text{imp}} \) for genuine and impostor scores, respectively. We call (7) a correction factor because it corrects the Naïve Bayes part for dependence.

4 Semiparametric model for LR computation

The LR as defined in (5) could be computed exactly if the marginal and copula densities of genuine and impostor scores were known. However, they have to be estimated from training data, which will be done semiparametrically, modelling the Naïve Bayes part and distribution functions non-parametrically, and the dependence between them by parametric copulas. Note that we aim at incorporating dependence between classifiers. Therefore, the copula parameter is the main parameter that one is interested in, which is called the parameter of interest, while the marginal LRs and distribution functions are treated as nuisance parameters in the sense that they are less important than the copula parameter when modelling dependence between classifiers. However, in computing the LR itself, we need to estimate all parameters composing (5).

This section will present three main steps of computing the LR using our approach: (i) computing the Naïve Bayes part, (ii) computing the correction factor for a given copula pair of genuine and impostor scores, and (iii) choosing the best copula pair for a specific performance measure. Let

\[
W_1, \ldots, W_{n_{\text{gen}}} \tag{8}
\]

and

\[
B_1, \ldots, B_{n_{\text{imp}}} \tag{9}
\]

be i.i.d copies of \( d \)-dimensional random variable of genuine scores \( W = (W_1, \ldots, W_d) \) and impostor scores \( B = (B_1, \ldots, B_d) \), respectively. Here, we will assume that the random variables of genuine and impostor scores are continuous.

4.1 Naïve Bayes part

The Naïve Bayes part is typically easy to be computed because there are several methods of computing the LR for 1D scores. The most common ways are KDE, Logit, HB, and PAV methods; see [17] for a brief explanation of these methods. In this paper, we choose the PAV method because of its optimality [15].

For every \( k = 1, \ldots, d \), PAV sorts and assigns a posterior probability of 1 or 0 to the \( k \)th component of genuine and impostor scores in a training set, respectively. It then finds the non-monotonic adjacent group of probabilities and replaces it with the average of that group. This procedure is repeated until the whole sequence is monotonically increasing which estimates the posterior probability \( P(H_i(\cdot)) \) of the \( i \)th component of (8) and (9) where \( H_i \) corresponds to a genuine score. By assuming

\[
P(H_i) = \frac{\eta_{\text{gen}}}{\eta_{\text{gen}} + \eta_{\text{imp}}}.
\]
the corresponding LR of s of (8) and (9) can be computed according to the Bayesian formula by

\[ \hat{L}_{R_k}(\cdot) = \frac{P(H_1(\cdot))}{1 - P(H_1(\cdot))} \times \frac{n_{\text{imp}}}{n_{\text{gen}}} \]  

(10)

so that we have a numerical function that maps a score to its \( \hat{L}_{R_k} \). Finally, for every score for the \( k \)th classifier, its corresponding \( \hat{L}_{R_k} \) value can be computed by interpolation.

### 4.2 Semiparametric correction factor estimation

While the Naive Bayes part is modelled non-parametrically, the correction factor will be modelled semiparametrically by assuming \( C_{\text{gen}} \) and \( C_{\text{imp}} \) to be parametric copulas. Let \( \theta_{\text{gen}} \) and \( \theta_{\text{imp}} \) denote the dependence parameters determining \( C_{\text{gen}} \) and \( C_{\text{imp}} \), respectively. Since the marginal distributions are treated as nuisance parameters as noted before, we will focus on the estimation of parameter of interest \( \theta = \left[ \theta_{\text{gen}}, \theta_{\text{imp}} \right] \). Thus our correction factor model is defined by

\[ \mathcal{C} \mathcal{F} = \{ C_{\theta_{\text{gen}}F_{\text{gen}}}, C_{\theta_{\text{imp}}F_{\text{imp}}} : \theta \in \Theta, F \in \mathcal{F} \} \]  

(11)

where \( \Theta \subset \mathbb{R}^D \) is open and \( \mathcal{F} \) is a collection of continuous marginal distribution functions. Here, \( \theta \) is the dimensionality of \( \theta_{\text{gen}} \) and \( \theta_{\text{imp}} \) and

\[ F = (F_{\text{gen}}, \ldots, F_{\text{gen}}, a, F_{\text{imp}}, \ldots, F_{\text{imp}}, d) \]  

(12)

Note that if the marginal distributions \( F_{\text{gen}, k} \) and \( F_{\text{imp}, k} \) for \( k = 1, \ldots, d \) are known then the log-likelihood of the combined samples (8) and (9) can be written as

\[ L = \sum_{i=1}^{n_{\text{gen}}} \log c_{\theta_{\text{gen}}}(U_{\text{gen}, i}) + \sum_{j=1}^{n_{\text{imp}}} \log c_{\theta_{\text{imp}}}(U_{\text{imp}, j}) \]  

(13)

where

\[ U_{\text{gen}, i} = (F_{\text{gen}}(W_{1, i}), \ldots, F_{\text{gen}}(W_{d, i})) \]

and

\[ U_{\text{imp}, j} = (F_{\text{imp}}(B_{1, j}), \ldots, F_{\text{imp}}(B_{d, j})) \]

for \( i = 1, \ldots, n_{\text{gen}} \) and \( j = 1, \ldots, n_{\text{imp}} \) Differentiating (13) with respect to \( \theta \) gives

\[ \sum_{i=1}^{n_{\text{gen}}} \frac{\partial c_{\theta_{\text{gen}}}(U_{\text{gen}, i})}{c_{\theta_{\text{gen}}}(U_{\text{gen}, i})} \| \partial \theta_{\text{gen}} = 0 \]

and

\[ \sum_{j=1}^{n_{\text{imp}}} \frac{\partial c_{\theta_{\text{imp}}}(U_{\text{imp}, j})}{c_{\theta_{\text{imp}}}(U_{\text{imp}, j})} \| \partial \theta_{\text{imp}} = 0. \]

As a consequence, if \( F_{\text{gen}, k} \) and \( F_{\text{imp}, k} \) are replaced by their empirical distribution functions based on samples

\[ W_{k, 1}, \ldots, W_{k, n_{\text{gen}}} \]

and

\[ B_{k, 1}, \ldots, B_{k, n_{\text{imp}}} \]

respectively, we will get two-step estimators \( \hat{\theta}_{\text{gen}, n_{\text{gen}}} \) and \( \hat{\theta}_{\text{imp}, n_{\text{imp}}} \) called pseudo-maximum likelihood estimators of \( \theta_{\text{gen}} \) and \( \theta_{\text{imp}} \), respectively, as studied in [18] and extended in [19]. The terminology of the two-step estimator comes from the fact that one has to do two steps to obtain it: (i) transforming data to uniformly distributed and (ii) finding the maximum likelihood estimator of the transformed data. We use a modified version of the empirical distribution function to avoid singularity problems; it is defined as

\[ \hat{F}_n(x) = \frac{1}{n+1} \sum_{i=1}^{n} I(x_i \leq x), \forall x \in \mathbb{R} \]  

(14)

for a given sample \( X_1, \ldots, X_n \).

Under some regularity conditions, we can derive the convergence of

\[ \hat{\theta}_n = \begin{pmatrix} \hat{\theta}_{\text{gen}, n_{\text{gen}}} \\ \hat{\theta}_{\text{imp}, n_{\text{imp}}} \end{pmatrix} \]  

(15)

in the following theorem.

**Theorem 2:** Write \( n = n_{\text{gen}} + n_{\text{imp}} \) and assume \( 0 < \lim_{n \to \infty} n_{\text{gen}}/n < 1 \). If copula \( C_{\text{gen}} \) and \( C_{\text{imp}} \) satisfy some regularity conditions; see Section 3 of [19],

\[ \sqrt{n}(\hat{\theta}_n - \theta) \to A(0, \Sigma) \]  

(16)

holds as \( n \to \infty \) for some positive definite covariance matrix \( \Sigma \).

This theorem guarantees the convergence of \( \hat{\theta}_n \) with order \( 1/\sqrt{n} \). In a weaker statement, it tells that the estimated LR tends to the true LR if our parametric copulas correctly specify the true copula and the sample size is big enough.

### 4.3 Choosing the best copula pair

Note that the LR at score \( s = (s_1, \ldots, s_d) \) under correction factor model (11) can be computed by a rule of thumb

\[ \text{LR}(C_{\text{gen}}, C_{\text{imp}})(s) = \prod_{k=1}^{d} \hat{L}_{R_k}(s_k) \times C_{\theta_{\text{gen}}F_{\text{gen}}}(s) \]  

(17)

Here \( \hat{L}_{R_k}(\cdot) \) and \( \hat{\theta}_k \) are given by (10) and (15), respectively, while \( \hat{F}_n \) is the modified empirical version of (12), which is obtained by replacing all the components in \( F \) with their corresponding modified empirical distribution functions. However, the choice of the appropriate parametric copulas can be difficult in practice. Therefore, what we can do is assuming \( C_{\text{gen}} \) and \( C_{\text{imp}} \) belong to a family of parametric copulas and choosing the best copula pair. Interestingly, Theorem 2 is still valid if the copula pair \( (C_{\text{gen}}, C_{\text{imp}}) \) misspecified the true pair. It means that we will get a reasonable estimator of the dependence parameter whichever a copula pair is chosen.

As noted in Section 1, combining classifiers in biometric recognition may have different goals depending on the application or scenario. For a given classifier \( K \), let \( \varepsilon(K) \) be a performance measure of classifier \( K \). Assume that the smaller value of \( \varepsilon(K) \), the better performance of the classifier \( K \). For instance, \( \varepsilon(K) \) can be the equal error rate (EER), total error rate (TER), false rejection rate at a certain false accept rate, 1 minus area under receiver operating characteristic (ROC) curve, and so on.

Let

\[ \mathcal{C} = \{ C_1, \ldots, C_R \} \]

be a family of \( n_c \) candidate copulas. Since a goodness-of-fit test as provided in [20] will only give the copula pair that is closest to the pair \( (C_{\text{gen}}, C_{\text{imp}}) \), but whose ratio is not necessarily closest to the ratio \( C_{\text{gen}}/C_{\text{imp}} \), we propose to choose the best copula pair as follows. Let \( K(i, j) \) be the classifier using copula pair \( (C_i, C_j) \) in its correction factor model for \( 1 \leq i, j \leq n_c \), which is defined by
as given in (17). Given a performance measure \( e \), our model selection will choose \((C_g, C_i)\) as the best copula pair with respect \( e \) if \( e(K_{x_{ij}}) \) has the smallest value among other pairs, i.e.

\[
e(K_{x_{ij}}) \leq e(K_{x_{ij}}), \ \forall 1 \leq i, j, n.
\]

If there are two or more best pairs then we choose one of them at random. Note that it is always useful to include the independence copula (ind) in \( \mathcal{C} \) to guarantee that the chosen fused classifier performs at least as good as fusion under independence.

### 5 Applications

We will present how our correction factor works and improves the PLR fusion in some practical scenarios: in a standard biometric verification and in forensic scenarios. To approximate the performance measure \( e \), we employ the linear Logit as used in [12] for the Logit method while the gaussian mixture model (GMM) method, we employ the minimum message length criterion proposed in [21], which automatically estimates the number of mixture components using the minimum message length criterion with the minimum and maximum numbers of components being 1 and 20, respectively. Once all fusion strategies have been trained based on the training set, their performances with respect to the performance measure \( e \) are computed based on the testing set. The sample sizes of genuine and impostor scores for both training and testing sets of all databases are given in Table 1.

#### 5.1 Maximising true match rate (TMR) at fixed false match rate (FMR)

In standard biometric verification, one has to set a threshold \( \Delta \) such that a score greater than or equal to the threshold is recognised as genuine score while a score less than the threshold is recognised as impostor score. Therefore, a biometric recognition system can make two different errors: accept an impostor score as genuine score and reject a genuine score. The probability of accepting an impostor score is called the False Match Rate (FMR(\( \Delta \))) with a threshold \( \Delta \), while the probability of rejecting a genuine score is called the False Non-Match Rate (FNMR(\( \Delta \))). The complement of the FNMR(\( \Delta \)) is the True Match Rate (TMR(\( \Delta \))), which is defined as the probability of accepting a genuine score as genuine score. Since every genuine score will be either accepted or rejected by the system, we have TMR(\( \Delta \)) = 1 – FNMR(\( \Delta \)). The most common method to see the performance of a biometric person verification system is by plotting the relation between FMR(\( \Delta \)) and TMR(\( \Delta \)) for all \( \Delta \in (-\infty, \infty) \), which is known as ROC [22].

##### 5.1.1 Performance measure:

The threshold can also be determined by putting an FMR value in advance. For a given fixed FMR = \( a \), the corresponding TMR value can be estimated based on data. Let

\[
W_1, \ldots, W_{n_{\text{genuine}}} \quad (18) \\
B_1, \ldots, B_{n_{\text{impostor}}} \quad (19)
\]

be 1D genuine and impostor scores, respectively. In our case, these are the fused scores of the testing set. According to [23], the TMR(\( a \)) can be estimated by

\[
1 - \hat{F}_{\text{genuine}}(\hat{Q}_{\text{impostor}}^{-1}(1 - a))
\]

where \( \hat{F}_{\text{genuine}} \) and \( \hat{F}_{\text{impostor}} \) are left-continuous empirical distribution functions based on (18) and (19), respectively while \( \hat{Q}_{\text{impostor}}^{-1} \) is the empirical quantile function with respect to \( \hat{F}_{\text{impostor}} \). Slightly different from (14), the left-continuous empirical distribution function based on a sample \( X_1, \ldots, X_n \) is defined by

\[
\hat{F}_{\text{left}}(x) = \frac{1}{n} \sum_{i=1}^{n} I(x, x_i), \quad \forall x \in \mathbb{R}
\]

and its corresponding quantile function is defined by

\[
\hat{Q}_{\text{left}}(p) = \sup \{ y : \hat{F}_{\text{left}}(y) \leq p \}, \quad \forall p \in [0, 1].
\]

Since higher TMR leads to a better classifier, the performance measure in this standard verification scenario is \( e = 1 – \text{TMR}_a \).

##### 5.1.2 Databases:

We use NIST-finger [24] and Face-3D [25–28] data to simulate fingerprint and face authentication, respectively.

- NIST-finger: NIST-finger contains fingerprint similarity scores from one system run on images of 6000 subjects. Each subject has one left index and one right index fingerprint both in the gallery and probe sets. All comparison scores of all pairs of left index fingerprints and all pairs of right index fingerprints are then computed. Here, we can consider the comparison scores

<table>
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<tr>
<td>BSS1</td>
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<td>BSS2P1</td>
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<tr>
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based on the left and right index fingerprints to be the first and second classifiers that will be combined. We use the first 1000 subjects for training and the rest for testing.

- Face-3D: Face-3D is used in [27, 28] for 3D face recognition. The training and the testing set are already defined and contain very different images (taken with different cameras, backgrounds, poses, expressions, illuminations and time). In his paper, the author proposes 30 classifiers operating on 30 different facial regions. We only use 5 regions out of these 30: similarity of the full face, the left half, the right half, the bottom part, and the upper part of the face. This choice is made to have dependent classifiers.

5.1.3 Results: We train our method at FMR $10^{-3}$, $10^{-4}$, and $10^{-5}$. The best copula pairs and the TMRs for all scenarios are given by Tables 2 and 3, respectively. It is shown that on the NIST-finger database the improvement of our method compared to the PLR method is relatively small and that all fusion methods have almost the same performance as seen in Fig. 1, which shows that the ROC curves of all fusion methods almost coincide. On the other hand, the overlapping regions on the Face-3D database are not used to measure performance. To report fusion strategies used in [29] because it is already shown there that the pseudo-LR method is always among the first or second best results for all databases. Hence we are mostly interested in how our method can improve the PLR method. Interestingly, we can see that the PLR method performs worse than the best single classifier on the BSS2P1 database. This may be because ignoring the dependence on dependent classifiers will degrade the performance of the fusion. This is confirmed by the performance of our method, which does take the dependence into account. It is better than the best single classifier. We can also see that the GMM method is comparable to our method for all databases. Apparently, the GMM method fits the dependence structures quite well.

5.2 Minimising half TMR (HTMR)

5.2.1 Performance measure: Besides maximising the TMR at a given FMR, one may also be interested in minimising some types of error:

- Equal error rate (EER): Let $\Delta^*$ be the threshold value at which $\text{FMR}(\Delta^*) = \text{FNMR}(\Delta^*)$. Then EER is defined as the common value $\text{FMR}(\Delta^*) = \text{FNMR}(\Delta^*)$.
- TER(\Delta): The sum of the $\text{FMR}(\Delta)$ and the $\text{FNMR}(\Delta)$, i.e. $\text{TER}(\Delta) = \text{FMR}(\Delta) + \text{FNMR}(\Delta)$. One may also consider the HTER(\Delta) = TER(\Delta)/2, to keep the error value between 0 and 1.
- Weighted error rate WER(\beta,\Delta): A weighted sum of the $\text{FMR}(\Delta)$ and $\text{FNMR}(\Delta)$, i.e. $\text{WER}(\beta,\Delta) = \beta \cdot \text{FMR}(\Delta) + (1 - \beta) \cdot \text{FNMR}(\Delta)$, $\beta \in [0, 1]$. The weights are usually called cost of false acceptance and cost of false rejection.
- Area under ROC curve.

Here, $\Delta$ is the threshold to compute the FMR and FNMR. Note that for a given $\beta \in [0, 1]$, we can set the performance measure $e = \inf_{\Delta} \text{WER}(\beta,\Delta)$ and follow our procedure to get the best copula pair. To give an illustration, we will put $\beta = 0.5$, which leads to $\inf_{\Delta} \text{HTMR}(\Delta)$. Frequently, the minimum value of HTER is approximated by the EER. This EER is used to report a fusion performance in [14, 29]. However, as pointed out in [30], the corresponding $\Delta^*$ is only a decision threshold and hence EER should not be used to measure performance. To report fusion performance itself, they suggest to set the threshold $\Delta^*$ using the training set and to report the final performance by computing the HTER(\Delta*) on the testing set. Therefore, we train our method by following this procedure but adapted as follows. Once all 64 copula-based fusion strategies have been trained on the first subset of the training set, they are applied to the first subset of the training set to determine the threshold $\Delta^*$ and to the second subset of the training set to compute the HTER(\Delta*). For all our benchmark methods, the threshold $\Delta^*$ is determined using the fused scores of training data and the HTER(\Delta*) is computed using the fused scores of the testing data.

5.2.2 Databases: We use the same publicly available scores databases as used in [29] from video-based gait biometrics. There are four databases in which their training and testing sets are already clearly defined.

- BSS1: This database contains 3D scores based on the gait energy image (GEI), gait period, and height of the subject [31].
- BSS2P1: This database is composed of 3D scores based on the GEI and one and two times frequency elements in the frequency-domain feature [31].
- BSS2P2: This database is almost the same as the BSS2P1 database but the scores are computed based on the GEI, chronogait image, and gait low image [31].
- BSS3: This database is composed of 2D scores from a wearable accelerometer and a gyroscope sensor [32].

5.2.3 Results: The HTERs of different fusion strategies are reported in Table 4. We do not present the performance of other fusion strategies used in [29] because it is already shown there that the pseudo-LR method is always among the first or second best results for all databases. Hence we are mostly interested in how our method can improve the PLR method. Interestingly, we can see that the PLR method performs worse than the best single classifier on the BSS2P1 database. This may be because ignoring the dependence on dependent classifiers will degrade the performance of the fusion. This is confirmed by the performance of our method, which does take the dependence into account. It is better than the best single classifier. We can also see that the GMM method is comparable to our method for all databases. Apparently, the GMM method fits the dependence structures quite well.

5.3 Minimising discrimination loss

The last application of our method concerns forensic biometric scenarios. Unlike the standard biometric verification that gives a hard decision whether a score is genuine or impostor, the LR value in the forensic case only provides a soft decision, which can be used to support the judge in court to make an objective decision [1].

5.3.1 Performance measure: Fusion is hoped to integrate the complementary information from the individual classifiers. In a forensic scenario, one aims at increasing the discrimination power (the ability to distinguish between genuine and impostor scores). Brümmer and de Preez [2] introduce a measure called the cost of log likelihood ratio ($C_{\text{LLR}}$) in the field of speaker recognition, which may be interpreted as a summary statistic for an LR computation [33]. This measure is also used in forensic face scenarios [34]. Note that the scores are interpretable as LRs when computing this

Table 2 Best copula pair at several FMRs of our method on the NIST-finger and Face-3D databases

<table>
<thead>
<tr>
<th>Databases</th>
<th>Best pair at FMR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10^{-3}</td>
</tr>
<tr>
<td>NIST-finger</td>
<td>(Gu,Fr)</td>
</tr>
<tr>
<td>Face-3D</td>
<td>(ind,Fr)</td>
</tr>
</tbody>
</table>

Table 3 TMRs of different fusion strategies on the NIST-finger and Face-3D databases

<table>
<thead>
<tr>
<th>Methods</th>
<th>NIST-finger</th>
<th>Face-3D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TMR at FMR</td>
<td></td>
</tr>
<tr>
<td></td>
<td>10^{-3}</td>
<td>10^{-4}</td>
</tr>
<tr>
<td>BSM</td>
<td>0.793</td>
<td>0.835</td>
</tr>
<tr>
<td>PLR</td>
<td>0.882</td>
<td>0.912</td>
</tr>
<tr>
<td>Logit</td>
<td>0.883</td>
<td>0.911</td>
</tr>
<tr>
<td>GMM</td>
<td>0.878</td>
<td>0.910</td>
</tr>
<tr>
<td>Proposed</td>
<td>0.884</td>
<td>0.914</td>
</tr>
</tbody>
</table>

The bold number in every column is the best one.
obtained by plugging the scores after PA V transformation into (22),

To explain the name of this measure we note that our fused scores between the hypothesis of the prosecution, and impostor scores (19), which correspond to the hypothesis of the defense, the cost of log likelihood ratio $\text{LR}$. The minimum value of the calibration loss $C_{\text{llr}}^{\text{cal}}$ tends to zero if the scores are well-calibrated and grow without bound if the scores are miscalibrated. Since we are interested in having better discrimination power, we put the performance measure $e = C_{\text{llr}}^{\text{min}}$. Nevertheless, the $C_{\text{llr}}$ and discrimination loss will also be reported.

5.3.2 Databases: We use the following databases:

- XM2VTS: There are eight classifiers in this database: five face classifiers and three speech classifiers. In order to have an application in the field of speaker recognition, we only take the speech classifiers. Moreover, only the linear filter-bank cepstral coefficient (LFCC)-GMM and spectral subband centroid (SSC)-GMM classifiers are used in this experiment because they have the highest correlation value among all the pairs. The training and testing sets are already defined [30].
- Face-3D: The same database as used for the standard verification in Section 5.1.

5.3.3 Results: The $C_{\text{llr}}^{\text{min}}$ and $C_{\text{llr}}$ values of different fusion strategies and the best copula pair of our method on the XM2VTS and Face-3D databases are presented in Table 5. Our method outperforms other methods with respect to the performance measure $C_{\text{llr}}^{\text{min}}$. Moreover, the $C_{\text{llr}}$ of our method on the XM2VTS database is only slightly higher than the GMM method and on the Face-3D database, our method even has by far the smallest $C_{\text{llr}}$ among all methods. As before, the GMM method performs poorly even if it is compared to the best single classifier. Surprisingly, even though the Logit method performs better than the PLR method for the standard biometric verification scenario in Section 5.1, its performance is also worse than the best single classifier. This means that the Logit method can discriminate genuine and impostor scores quite well in the tails, but it fails in the middle. Another interesting thing is that if we use the best copula pair $\{\text{ind}, \text{Fr}\}$ chosen in Section 5.1 then the corresponding $C_{\text{llr}}^{\text{cal}}$ is 0.040 which is higher than the 0.038 for the copula pair $\{\text{ind}, t\}$, which is trained to minimise the $C_{\text{llr}}^{\text{cal}}$ here. It tells us that the copula pair $\{\text{ind}, t\}$ handles dependence on the whole scores better than the copula pair $\{\text{ind}, \text{Fr}\}$, which is trained to handle dependence in the tail. Finally, we also notify that the calibration loss of all fusion strategies (including our method) is pretty high on the Face-3D database as seen in Fig. 2. In order to reduce this calibration loss, we proposed in our previous work [7] a method called the two-step calibration method. Briefly, the first step of this method is computing both training and testing sets to their fused scores once the best copula pair has been found and the second step is calibrating the fused scores by the PAV algorithm trained based on the fused scores of the training set. Readers who are interested in the detailed explanation of the two-step calibration method may refer to [7].

6 Conclusion
We have presented the mathematical framework of a semiparametric LR-based score-level fusion method to improve via parametric copula families the PLR fusion strategy. Estimators of the dependence parameters have been provided and subsequently, their convergence has been analysed. It has also been shown in detail how our LR-based method is used and how the best copula pair is chosen that depends on a specific application. Finally, application to standard biometric verification and forensic scenarios has been demonstrated on real databases from fingerprint, face, speaker, and video-based gait recognition, and it has been confirmed that our LR-based method outperforms the GMM and Logit fusion methods, which are also designed to handle dependence.

Table 4 HTERs of different fusion strategies on the video-based gait databases

<table>
<thead>
<tr>
<th>Methods</th>
<th>BSS1</th>
<th>BSS2P1</th>
<th>BSS2P2</th>
<th>BSS3</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSM</td>
<td>0.042</td>
<td>0.050</td>
<td>0.048</td>
<td>0.150</td>
</tr>
<tr>
<td>PLR</td>
<td>0.035</td>
<td>0.056</td>
<td>0.042</td>
<td>0.132</td>
</tr>
<tr>
<td>Logit</td>
<td>0.034</td>
<td>0.052</td>
<td>0.041</td>
<td>0.136</td>
</tr>
<tr>
<td>GMM</td>
<td>0.034</td>
<td>0.047</td>
<td>0.034</td>
<td>0.134</td>
</tr>
<tr>
<td>proposed</td>
<td>0.034</td>
<td>0.047</td>
<td>0.035</td>
<td>0.131</td>
</tr>
<tr>
<td>best pair</td>
<td>(Gu,Cl)</td>
<td>(Gu,GC)</td>
<td>(t, t)</td>
<td>(Gu, Cl)</td>
</tr>
</tbody>
</table>

The bold number in every column is the best one.

Table 5 $C_{\text{llr}}^{\text{min}}$ and $C_{\text{llr}}$ values of different fusion strategies on the XM2VTS and Face-3D

<table>
<thead>
<tr>
<th>Methods</th>
<th>XM2VTS</th>
<th>Face-3D</th>
</tr>
</thead>
<tbody>
<tr>
<td>BSM</td>
<td>0.044</td>
<td>0.057</td>
</tr>
<tr>
<td>PLR</td>
<td>0.041</td>
<td>0.056</td>
</tr>
<tr>
<td>Logit</td>
<td>0.037</td>
<td>0.153</td>
</tr>
<tr>
<td>GMM</td>
<td>0.038</td>
<td>0.046</td>
</tr>
<tr>
<td>proposed</td>
<td>0.034</td>
<td>0.047</td>
</tr>
<tr>
<td>best pair</td>
<td>(Fr,Gu)</td>
<td>(ind, t)</td>
</tr>
</tbody>
</table>

The bold number in every column is the best one.

To explain the name of this measure we note that our fused scores are LR values and may be rewritten in terms of the logarithm of LR. The minimum value of the $C_{\text{llr}}$ (denoted by $C_{\text{llr}}^{\text{min}}$), which is obtained by plugging the scores after PAV transformation into (22), is called the discrimination loss. This measure can be seen as the opposite of discrimination power. The smaller the value of this quantity, the higher the discrimination power. The difference between the $C_{\text{llr}}$ and the $C_{\text{llr}}^{\text{min}}$ is called the calibration loss $C_{\text{llr}}^{\text{cal}} = C_{\text{llr}} - C_{\text{llr}}^{\text{min}}$. (23)
This research was supported by the Netherlands Organisation for Scientific Research (NWO) via the project Forensic Face Recognition, 727.011.008.

Acknowledgments
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References
[16] Sklar, M.: ‘Fonctions de répartition À N dimensions Et leurs Marges’ (Université Paris 8, France, 1959)

9 Appendix

Proof of Theorem 2: According to Proposition 2 of Chen and Fan [19], we have
\[ \sqrt{n} \left( \frac{\hat{\theta}_{\text{gen}, n_{\text{gen}}} - \theta_{\text{gen}}} {\Sigma_{\text{gen}}} \right) \rightarrow \mathcal{N}(0, \Sigma_{\text{gen}}) \]
and
\[ \sqrt{n} \left( \frac{\hat{\theta}_{\text{imp}, n_{\text{imp}}} - \theta_{\text{imp}}} {\Sigma_{\text{imp}}} \right) \rightarrow \mathcal{N}(0, \Sigma_{\text{imp}}) \]
for some positive definite matrices $\Sigma_{\text{gen}}$ and $\Sigma_{\text{imp}}$. Define
\[ \lambda_{\text{gen}} = n_{\text{gen}} \text{ and } \lambda_{\text{imp}} = \lambda. \]
Since $\hat{\theta}_{\text{gen}, n_{\text{gen}}}$ and $\hat{\theta}_{\text{imp}, n_{\text{imp}}}$ are independent then
\[ \sqrt{n} \left( \frac{\lambda_{\text{gen}} \left( \theta_{\text{gen}, n_{\text{gen}}} - \theta_{\text{gen}} \right) - \lambda_{\text{imp}} \left( \theta_{\text{imp}, n_{\text{imp}}} - \theta_{\text{imp}} \right)} {\lambda_{\text{gen}} \Sigma_{\text{gen}} + \lambda_{\text{imp}} \Sigma_{\text{imp}}} \right) \rightarrow \mathcal{N}(0, \Sigma) \]
where
\[ \Sigma = \begin{pmatrix} \lambda_{\text{gen}} & 0 \\ 0 & \lambda_{\text{imp}} (1 - \lambda) \end{pmatrix} \]