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Two-Dimensional Bose-Einstein Condensate under Extreme Rotation

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We show that a Bose-condensed gas, under extreme rotation in a 2D anisotropic trap, forms a novel elongated quantum fluid which has a roton-maxon excitation spectrum. For a sufficiently large interaction strength, the roton energy reaches zero and the system undergoes a second order quantum transition to the state with a periodic structure—rows of vortices. The number of rows increases with the interaction, and the vortices eventually form a triangular Abrikosov lattice.

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Rotating Bose-Einstein condensates (BECs) of trapped atoms constitute a novel many-body system where nucleated quantized vortices form a triangular lattice [1], and a fast rotation is expected to change dramatically the properties of the gas. In a harmonically trapped BEC rotating at a frequency Ω close to the trap frequency in the rotation plane $\omega_\perp$, the vortex lattice can melt when the number of vortices approaches the number of particles [2,3]. In this respect, there is an analogy with type-II superconductors, which undergo a transition to the normal phase above a critical value of the magnetic field [4]. Possible signatures of the melting of the vortex lattice in rapidly rotating BECs were observed in experiments at JILA and at ENS [5]. The proposals that are put forward to describe the state of the rapidly rotating Bose gas include yrast states [6], correlated quantum Hall states of bosons [3,7], and a giant vortex state [8].

In this Letter we show that an anisotropy in the trapping potential can drastically change the picture. As found at ENS [9], a Bose gas under critical rotation ($\Omega \sim \omega_\perp$) can become very elongated in one direction in the rotating frame. We focus on this case and assume that Ω is close to the smallest of the trap frequencies in the rotation plane. Then the condensate becomes free along the direction of the weaker confinement and forms a novel quantum fluid in a narrow channel. The excitation spectrum of this fluid has a "roton-maxon" character and becomes unstable at a critical interaction strength. This instability leads to the formation of a periodic structure which represents vortex rows. An increase in the interaction (decrease in the anisotropy) increases the number of rows and reduces the correlation length, and the gas ultimately enters the strongly correlated regime.

We consider a two-dimensional (2D) Bose-condensed gas at zero temperature, rotating with frequency Ω and harmonically trapped with frequencies $\omega_{x,y} = \omega_\perp\sqrt{1 \mp \epsilon}$ along the x, y axes in the rotating frame. The Hamiltonian of the system in this frame reads (see [10])

$$H = \int d^2r \tilde{\Psi}^\dagger \left[\frac{(-i\hbar\vec{\nabla} + m\vec{r}\times\vec{\Omega})^2}{2m} + V_{\rm eff}(\vec{r}) + \frac{g}{2}\tilde{\Psi}^\dagger\tilde{\Psi}\right]\tilde{\Psi}, \quad (1)$$

where $\tilde{\Psi}^\dagger$ and $\tilde{\Psi}$ are bosonic field operators, m is the particle mass, g is the coupling constant for the mean-field interaction, and the effective trapping potential is $V_{\rm eff}(\vec{r}) = (m/2)[(\omega_x^2 - \Omega^2)x^2 + (\omega_y^2 - \Omega^2)y^2]$. The Hamiltonian (1) is analogous to that of charged particles in the magnetic field, and in this respect the quantity $m\vec{r}\times\vec{\Omega}$ is the gauge field. We consider the limit of extreme rotation, where $\Omega = \omega_x \equiv \omega_\perp\sqrt{1-\epsilon}$ and the gas becomes free along the x direction. Then, assuming that in this direction the atoms are confined in a large rectangular box of size L, the gas becomes a long cigar. After the gauge transformation $\tilde{\Psi} = \Psi e^{im\Omega xy/\hbar}$, the Hamiltonian in the Landau gauge can be written as

$$H = \int d^2r \Psi^\dagger \left[\frac{(p_x + 2m\Omega y)^2 + p_y^2}{2m} + \frac{m\omega_-^2 y^2}{2} + \frac{g}{2}\Psi^\dagger\Psi\right]\Psi, \quad (2)$$

where $\omega_- = \omega_\perp\sqrt{2\epsilon} \ll \Omega$, assuming a small ellipticity ϵ.

Omitting the interaction term, the single particle eigenstates are the Landau levels [11] separated from each other by an energy gap $\sim 2\hbar\Omega$. In the dilute limit, where the mean-field interaction $gn_{2D} \ll \hbar\Omega$ (n_{2D} is the 2D density), we may restrict our discussion within the lowest Landau level. Then the field operator can be written in the form $\Psi = \sum_k \phi_k a_k$, where a_k is the creation operator of a particle with momentum k along the x direction and ϕ_k is the corresponding eigenfunction:

$$\phi_k(x, y) = \frac{\exp(ikx)}{(\pi l_0^2 L^2)^{1/4}} \exp\left\{-\frac{1}{2}\left(\frac{y}{l_0} + \frac{\Omega k l_0}{\tilde{\Omega}}\right)^2\right\}, \quad (3)$$

where $\tilde{\Omega} = \sqrt{\Omega^2 + \omega_-^2/4}$, and $l_0 = (\hbar/2m\tilde{\Omega})^{1/2}$. Then, after the spatial integration of Eq. (2), we obtain an effective one-dimensional (1D) Hamiltonian:

$$H = \sum_k (\hbar^2 k^2/2m^*) a_k^\dagger a_k + (g^*/2L) \sum_{k,k',q} a_{k+q}^\dagger a_{k'-q}^\dagger a_k a_{k'} \exp\{-l_0^2[(k-k'+q)^2 + q^2]/2\}, \quad (4)$$

where $g^* = g/\sqrt{2\pi} l_0$ is an effective 1D coupling constant. The Hamiltonian (4) describes particles with a large effective mass $m^* = m(2\tilde{\Omega}/\omega_-)^2 \gg m$. The fact that m^* is not infinite and the kinetic energy term is still present originates from the asymmetry of the trapping potential. This asymmetry leads to a small difference between the frequencies Ω and $\tilde{\Omega}$. However, once the finite kinetic energy term is extracted one may put $\tilde{\Omega} = \Omega$, and we have done this in the second term on the right-hand side of Eq. (4).

The momentum dependence of the interaction term in the Hamiltonian (4) originates from the presence of the gauge field. The wave functions of particles which have opposite momenta in the x direction are shifted in opposite directions along the y axis, which decreases their overlap and reduces the interaction amplitude.

The behavior of the system is governed by the particles with momenta $k \lesssim l_0^{-1}$, for which the extension of the wave function in the y direction is l_0. If the 1D density $n = N/L$ (N is the total number of particles) satisfies the condition $nl_0 \ll 1$, then we are dealing with a 1D Bose gas. In this case, characteristic momenta that are important are of the order of n or smaller. They satisfy the condition $kl_0 \ll 1$, and the exponential term in Eq. (4) is equal to unity. Then the Hamiltonian (4) corresponds to the well-described Lieb-Liniger model for the 1D Bose gas [12].

We, therefore, focus on the other extreme, where

$$nl_0 \gg 1. \quad (5)$$

Then the system can be viewed as a 2D gas in a narrow channel. The 2D density is $\sim n/l_0$ and the interaction energy per particle is $I \sim ng/l_0$. The characteristic kinetic energy of a particle at the mean distance from other particles is $K \sim \hbar^2 n/m^* l_0$, and for $K \gg I$ the wave function of the particle at such interparticle distances is not influenced by the interactions. The gas is then in the weakly interacting mean-field regime. The criterion of weak interactions takes the form (see, for example, [13])

$$m^* g/\hbar^2 \ll 1. \quad (6)$$

Thus, under the conditions (5) and (6) we have a weakly interacting 2D Bose gas in a narrow channel.

As well as in the 1D Bose gas, the ground state can be a quasicondensate in which the density fluctuations are suppressed, but the phase fluctuates in the x direction on a distance scale smaller than the size L [13]. However, locally the quasicondensate is indistinguishable from a true BEC. The analysis in the density-phase representation [13] gives the same excitation spectrum as the one obtained in the Bogoliubov approach assuming that most particles are in the condensate. Employing this approach, we first reduce the Hamiltonian (4) to a bilinear form:

$$H_b = \sum_k [\hbar^2 k^2/2m^* + 2ng^* \exp\{-k^2 l_0^2/2\}] a_k^\dagger a_k + (ng^*/2) \sum_k \exp\{-k^2 l_0^2\}(a_k^\dagger a_{-k}^\dagger + a_k a_{-k}). \quad (7)$$

Diagonalizing H_b we obtain the excitation spectrum:

$$\epsilon^2(k) = [\hbar^2 k^2/2m^* + g^* n\{2\exp(-k^2 l_0^2/2) - 1\}]^2 - g^{*2} n^2 \exp\{-2k^2 l_0^2\}. \quad (8)$$

The key feature of the excitation energy (8) is the momentum dependence of the interaction terms proportional to g^*. The structure of the spectrum depends on the ratio of the mean-field interaction to the kinetic energy at momentum $k \approx 1/l_0$. This ratio can take both small and large values and is given by

$$\beta = \frac{ng^*}{\hbar^2/2m^* l_0^2} = \sqrt{\frac{2}{\pi}} \left(\frac{m^* g}{\hbar^2}\right) nl_0. \quad (9)$$

In units of $m\hbar\tilde{\Omega}/m^*$, the excitation energy is a universal function of β and kl_0. For small β, the interaction terms proportional to g^* in Eq. (8) are important only at $k \ll 1/l_0$, where they become momentum independent. Then Eq. (8) gives the ordinary Bogoliubov spectrum, with a small sound velocity $c_s = \sqrt{ng^*/m^*}$.

For $\beta \gtrsim 1$ the situation drastically changes. The interaction is already important for momenta $k \gtrsim 1/l_0$, where the interaction dependent terms in Eq. (8) decrease with increasing k. For this reason the spectrum develops the roton-maxon structure for $\beta > 2.6$ (see Fig. 1). This structure is well known in the physics of superfluid ^{4}He and is predicted for trapped dipolar condensates [14] and studied for lattice bosons with long-range interactions [15]. The roton minimum is located at $k \sim 1/l_0$, and the corresponding excitation energy decreases with increasing β. For a critical value $\beta = 4.9$, the roton minimum reaches zero at $k = k_c = 1.6/l_0$, and a further increase in β makes the

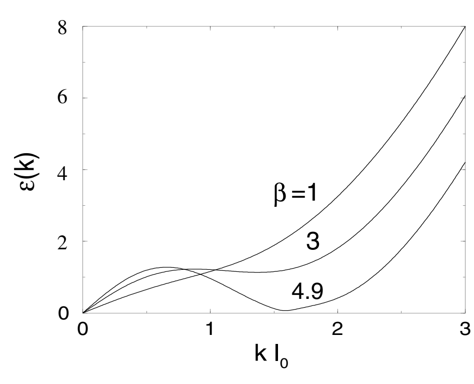

FIG. 1. Excitation energy (in units of $m\hbar\tilde{\Omega}/m^*$) versus kl_0.

system unstable. Thus our weakly interacting 2D Bose-condensed gas in a narrow channel is stable for $\beta \leq 4.9$, exhibiting a roton-maxon spectrum in the range $2.6 < \beta < 4.9$. For $\beta > 4.9$, where the Bose-condensed state is unstable, one has to find a new ground state.

At the instability point the excitations with momenta $\pm k_c$ can be excited without any cost of energy. This indicates that the ground state macroscopic wave function can contain several momentum components. A general form of this type of wave function reads

$$\psi = \sqrt{N}\left[C_0\phi_0 + \sum_{i=1}^{j}(C_{k_i}\phi_{k_i} + C_{-k_i}\phi_{-k_i})\right]. \qquad (10)$$

The absence of current in the x direction requires $|C_{k_i}| = |C_{-k_i}|$, and the normalization condition reads $|C_0|^2 + 2\sum_i^j |C_{k_i}|^2 = 1$. Note that Eq. (10) gives two possibilities: a $(2j+1)$ component wave function with $C_0 \neq 0$ and a $2j$ component wave function for which $C_0 = 0$.

To understand the instability of the single component state, we consider a wave function with three components:

$$\psi = \sqrt{N}[C_k\phi_k + C_0 e^{i\theta}\phi_0 + C_{-k}\phi_{-k}]. \qquad (11)$$

Here all the C coefficients are real, and we may put $C_k = C_{-k}$. We then find the critical value of β, above which this 3-component state has lower energy than the single component one. The wave function (11) leads to $a_0 = C_0$, $a_{\pm k} = C_k$ in Eq. (4), and we obtain the energy

$$E[C_k, k]/N = A(k)|C_k|^4 + B(k)|C_k|^2 + g^*n/2, \qquad (12)$$

where the coefficients $A(k)$ and $B(k)$ are given by

$$A(k) = g^*n[3 - 8t + 2t^4 - 4t^2\cos(2\theta)], \qquad (13)$$

$$B(k) = \hbar^2k^2/m^* - g^*n[2 - 4t - 2t^2\cos(2\theta)], \qquad (14)$$

with $t = \exp\{-k^2l_0^2/2\}$. For $|C_k|^2 < 1/2$, the energy is minimized for $\cos(2\theta) = -1$, and we have $A(k) > 0$. Minimizing E with respect to $|C_k|^2$ yields $|C_k|^2 = -B(k)/2A(k)$. The energy of the 3-component state is

$$E/N = g^*n/2 - B^2/4A,$$

and it is lower than the energy of the single component state. The physically acceptable solution requires $B(k) < 0$. Therefore, the transition from the single to 3-component state occurs when the minimum value of $B(k)$ reaches zero. Using Eq. (14) we find that this happens at $k = k_c$ and $\beta = \beta_c = 4.9$. So, the 3-component solution becomes the ground state for $\beta > 4.9$. This breaks the translational symmetry and leads to a modulation of the density along the x axis with a period of $2\pi/k_c$, which is similar to a scenario proposed for superfluid ^{4}He flowing with a velocity exceeding the critical Landau velocity [16].

A 3-component wave function can be viewed as two vortex rows along the x axis. The nodes in the x, y plane are obtained straightforwardly, and near the transition point they are very far from the line $y = 0$. The transition from the single to 3-component wave function can be treated as a second order quantum transition. The energy and chemical potential μ change continuously, whereas the quantity $\frac{\partial\mu}{\partial n}$ undergoes a jump at the transition point. For the 3-component state it is smaller by an amount $0.5g^*$.

Near the transition point quantum fluctuations increase due to the vanishing excitation energy at the roton minimum. This energy can be expressed as

$$\epsilon(k) = \left(\frac{\hbar^2}{2m^*l_0^2}\right)\left[\frac{2}{\beta_c}(\beta_c - \beta) + \gamma l_0^4(k^2 - k_c^2)^2\right]^{1/2},$$

where $\gamma \approx 0.12$. For the mean square fluctuations of the density we then obtain

$$\left(\frac{\delta n}{n}\right)^2 = \int\frac{dk}{2n\pi}\left[\frac{\hbar^2k^2}{2m^*\epsilon(k)} - 1\right] \approx \frac{1}{nl_0}\ln\left(\frac{1}{\beta_c - \beta}\right).$$

These fluctuations become large for $\delta\beta = (\beta_c - \beta) \lesssim \exp(-nl_0)$. Thus, the transition from a single to 3-component wave function occurs in the interval $\delta\beta$, which is exponentially narrow due to the inequality (5). The behavior of the system in this region requires a special investigation and is beyond the scope of this Letter.

It is important that already for $\beta = 5.4$ the ground state wave function changes from 3- to 2-component, and it again becomes 3-component at $\beta = 20$. With increasing β (either increasing g^*n or decreasing ω_-), more momentum states are macroscopically occupied. This is because an increase of β is equivalent to increasing the effective mass m^*, which makes momentum states in the lowest Landau level (LLL) more degenerate.

Our results for the number of momentum components in the ground state wave function are displayed in Fig. 2. These states describe one or several vortex rows along the x axis. For example, a two component state represents one vortex row [see Fig. 3(I)]. Eventually, a triangular Abrikosov vortex lattice [17] is formed when increasing β to very large values. It should be mentioned here that the structure of vortex rows has been obtained in the studies of type-II superconductors [18] and in the studies of condensates in rotating anisotropic traps [19].

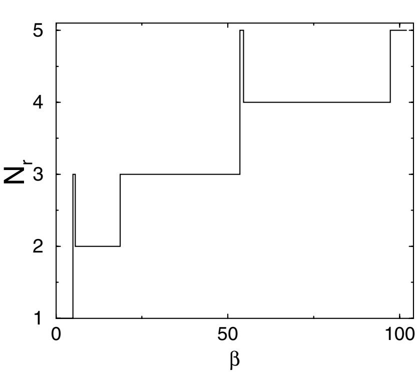

FIG. 2. The number of components (N_r) in the ground state macroscopic wave function for various values of β.

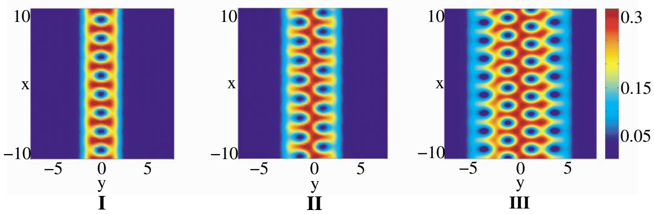

FIG. 3 (color online). Density profile of 2-component (I), 3-component (II), and 5-component (III) states. (Lengths are in units of l_0.)

In the presence of many rows of vortices, we may rely on an average vortex description in the LLL. Using the relation $\langle \phi_k | y^2/l_0^2 | \phi_k \rangle = k^2 l_0^2 + 1/2$, we write the energy functional of the system in the form $E[\rho] = \int d^2 r [m\omega_-^2 y^2 \rho + g\rho^2]/2$. Minimizing the energy, we obtain the coarse grained density $\rho(y) = \frac{1}{g}[\mu - \frac{1}{2} m\omega_-^2 y^2]$, which smooths out density modulations introduced by vortex rows. The size of the vortex core is always $\sim l_0$, and the number of the vortex rows is $\sim \beta^{1/3}$. The energy per particle is given by

$$E/N \approx 3\hbar\tilde{\Omega}(\omega_-/2\Omega)^2 (3\sqrt{2\pi}\beta/4)^{2/3}/5. \qquad (15)$$

The result of Eq. (15) deviates by less than 10% from the energy obtained by the full numerical minimization.

The vortex lattice is expected to melt when the number of vortices N_v approaches the number of particles N [3,20]. In our case the condition $N_v \sim N$ transforms into $m^* g/\hbar^2 \sim n^2 l_0^2$. However, our approach of the weakly interacting mean-field regime in a narrow channel breaks down before the melting transition. This approach requires inequalities (5) and (6) and, hence, is surely expected to be invalid for $m^* g/\hbar^2 \gtrsim n^2 l_0^2$. Therefore, the description of the melting transition requires a more elaborate treatment of quantum fluctuations.

In conclusion, we have shown that a 2D BEC at extreme rotation frequency in an elliptic trap forms a novel quantum fluid in a narrow channel. The behavior of this fluid is determined by the parameter β Eq. (9) which increases with the interaction strength and effective mass m^*. For $\beta < \beta_c = 4.9$, the excitation spectrum of the fluid has a small sound velocity and exhibits a roton-maxon character. At a critical interaction strength β_c, the roton energy reaches zero and the uniform (in the long direction) ground state becomes unstable. The system then undergoes a second order quantum transition to the state with a periodic structure which can be viewed as rows of vortices. For $\beta > \beta_c$, with increasing the interaction parameter β, more vortex rows are nucleated. Finally, the vortices form the Abrikosov lattice which can melt due to quantum fluctuations.

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