Essays in financial economics

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Appendix B

Appendix to chapter 2

B.1 Proofs

B.1.1 Proof of Lemma 1

The expected trading profit of a speculator becoming informed about the idiosyncratic risk factor of firm i is

$$\left(1-\frac{\phi_i}{\theta_I}\right)\Bigg[(1-q_a)\frac{\phi_a}{\theta_a}\cdot 0+\left(1-\frac{\phi_a}{\theta_a}\right)\left(q_a q_I(R_{H,i}-R_{L,i})+R_{L,i}-R_{L,i}\right) \\ +q_a\frac{\phi_a}{\theta_a}\left[q_I(R_{H,i}-R_{L,i})+R_{L,i}-R_{L,i}\right]\Bigg]$$

if $A_i=0$ and

$$\left(1-\frac{\phi_i}{\theta_I}\right)\Bigg[(1-q_a)\frac{\phi_a}{\theta_a}\cdot 0+\left(1-\frac{\phi_a}{\theta_a}\right)\left(-q_a q_I(R_{H,i}-R_{L,i})-R_{L,i}+q_a(R_{H,i}-R_{L,i})+R_{L,i}\right) \\ +q_a\frac{\phi_a}{\theta_a}\left[-q_I(R_{H,i}-R_{L,i})-R_{L,i}+R_{H,i}\right]\Bigg]$$

if $A_i=1$. Simplifying these terms and taking the unconditional expectation yields the term in Lemma 1. The analogue holds for the incentives to become informed about the common risk factor A_a, except that the summation is over all securities.

B.1.2 Proof of Proposition 1

For the proof, note that welfare is a smooth function of ΔR. There are four cases to distinguish depending on whether the information acquisition equations (2.2) and (2.3) are binding:

($\phi_a>0, \phi_I>0$): solve the FOC (equation 2.5) to get the interior solution $\Delta R^{SP}_{\text{Int}}$. It goes to infinite as $\left(\frac{c_a}{q_a}+\frac{c_I}{q_I}\right)(K-x_L)\longrightarrow c_I\frac{\theta_I c_I}{q_a q_I(1-q_I)}+c_a\frac{\theta_a c_a}{q_I q_a(1-q_a)}$. Thus, if $\left(\frac{c_a}{q_a}+\frac{c_I}{q_I}\right)(K-x_L)>c_I\frac{\theta_I c_I}{q_a q_I(1-q_I)}+c_a\frac{\theta_a c_a}{q_I q_a(1-q_a)}$, the optimal solution is ΔR maximal. By assumption $\Delta R_{\max}>$

$\max[\overline{\Delta R_I}, \overline{\Delta R_a}]$, so this is possible. If $\left(\frac{c_a}{q_a} + \frac{c_I}{q_I}\right)(K - x_L) \leq c_I \frac{\theta_I c_I}{q_a q_I (1-q_I)} + c_a \frac{\theta_a c_a}{q_I q_a (1-q_a)}$, then it is $\Delta R^{SP}_{\text{Int}}$. For existence we need that $\Delta R^{SP}_{\text{Int}} > \max[\overline{\Delta R_I}, \overline{\Delta R_a}]$.

$(\phi_a = 0, \phi_I = 0)$: no information is acquired. Welfare is just $q_a q_I (x_H - x_L) + x_L - K$. Any $\Delta R < \min[\overline{\Delta R_I}, \overline{\Delta R_a}]$ is optimal.

$(\phi_a > 0, \phi_I = 0)$: Setting $\frac{\partial \phi_I}{\partial \Delta R} = \phi_I = 0$, the FOC reduces to

$$\left[\frac{\frac{\partial \phi_a}{\partial \Delta R}}{\theta_a}(1 - q_a)\right](K - x_L) - c_a \frac{\partial \phi_a}{\partial \Delta R}.$$

$\frac{\partial \phi_a}{\partial \Delta R}$ cancels out and we get $\frac{1}{\theta_a}(1 - q_a)(K - x_L) - c_a$. Therefore, if $K - x_L > \frac{\theta_a c_a}{1-q_a}$, then the FOC is positive for all ΔR. Setting ΔR maximally without violating $(\phi_a > 0, \phi_I = 0)$ gives $\Delta R = \overline{\Delta R_I}$. If on the other hand $K - x_L < \frac{\theta_a c_a}{1-q_a}$, then the FOC is negative for all ΔR. We would like to minimize ΔR. But this violates $\phi_a > 0$, so it is not possible that $K - x_L < \frac{\theta_a c_a}{1-q_a}$.

$(\phi_a = 0, \phi_I > 0)$: Setting $\frac{\partial \phi_a}{\partial \Delta R} = \phi_a = 0$, the FOC reduces to

$$\left[\frac{\frac{\partial \phi_I}{\partial \Delta R}}{\theta_I}(1 - q_I)\right](K - x_L) - c_I \frac{\partial \phi_I}{\partial \Delta R}.$$

$\frac{\partial \phi_I}{\partial \Delta R}$ cancels out and we get $\frac{1}{\theta_I}(1 - q_I)(K - x_L) - c_I$. If $K - x_L > \frac{\theta_I c_I}{1-q_I}$, then the FOC (2.5) is positive for all ΔR. Setting ΔR maximally without violating $(\phi_a = 0, \phi_I > 0)$ gives $\Delta R = \overline{\Delta R_a}$. If on the other hand $K - x_L < \frac{\theta_I c_I}{1-q_I}$, then the FOC is negative for ΔR. But minimizing ΔR violates $\phi_I > 0$, so it is not possible that $K - x_L < \frac{\theta_I c_I}{1-q_I}$.

Next, I show that the interior solution is in fact a local maximum. If $\Delta R^{SP}_{\text{Int}} > \max[\overline{\Delta R_I}, \overline{\Delta R_a}]$, then welfare is concave in ΔR around $\Delta R^{SP}_{\text{Int}}$ because the second derivative is negative as I will show:

$$\begin{aligned}\frac{\partial^2 W}{\partial \Delta R^2} = \frac{\partial^2 \phi_a}{\partial \Delta R^2}&\left[\frac{1 - q_a}{\theta_a}\left(1 - \frac{1 - q_I}{\theta_I}\phi_I\right)(K - x_L) - c_a\right] \\ &+ \frac{\partial^2 \phi_I}{\partial \Delta R^2}\left[\frac{1 - q_I}{\theta_I}\left(1 - \frac{1 - q_a}{\theta_a}\phi_a\right)(K - x_L) - c_I\right] \\ &- 2(K - x_L)\frac{1 - q_I}{\theta_I}\frac{1 - q_a}{\theta_a}\frac{\partial \phi_I}{\partial \Delta R}\frac{\partial \phi_a}{\partial \Delta R}\end{aligned}$$

Using $\frac{\partial^2 W}{\partial \Delta R^2} = -\frac{\partial W}{\partial \Delta R}\frac{2}{\Delta R}$, this yields

$$\begin{aligned}\frac{\partial^2 W}{\partial \Delta R^2} = -\frac{2}{\Delta R}\Bigg\{ &\frac{\partial^2 \phi_a}{\partial \Delta R^2}\left[\frac{1-q_a}{\theta_a}\left(1-\frac{1-q_I}{\theta_I}\phi_I\right)(K-x_L)-c_a\right] \\ &+\frac{\partial^2 \phi_I}{\partial \Delta R^2}\left[\frac{1-q_I}{\theta_I}\left(1-\frac{1-q_a}{\theta_a}\phi_a\right)(K-x_L)-c_I\right]\Bigg\} \\ &-2(K-x_L)\frac{1-q_I}{\theta_I}\frac{1-q_a}{\theta_a}\underbrace{\frac{\partial \phi_I}{\partial \Delta R}}_{>0}\underbrace{\frac{\partial \phi_a}{\partial \Delta R}}_{>0}.\end{aligned}$$

The term in curly brackets is equal to the FOC (2.5) and therefore 0, so welfare is concave around the interior solution.

Let us compare the interior solution to the other three possible corner solutions. The claim is that the interior solution is optimal if it exists. Note that in the cases where ($\phi_a > 0, \phi_I = 0$) or ($\phi_a = 0, \phi_I > 0$), welfare is increasing in ΔR. But then then the interior solution must be better because $\overline{\Delta R_I} < \Delta R_{\text{Interior}}$ resp. $\overline{\Delta R_a} < \Delta R_{\text{Interior}}$ and the fact that welfare is smooth in ΔR. Is it possible that choosing ΔR minimal such that ($\phi_a = 0, \phi_I = 0$) is better? No, as the difference between welfare under the interior solution and under no information only depends on linear terms containing ϕ_I^* and ϕ_a^*, without a constant (thus not affine).

If $0 < \Delta R_{\text{Int}}^{SP} < \max[\overline{\Delta R_I}, \overline{\Delta R_a}]$, then the interior solution does not exist. If $\overline{\Delta R_I} > \overline{\Delta R_a}$, then only cases ($\phi_a > 0, \phi_I = 0$) or ($\phi_a = 0, \phi_I = 0$) can exist, which means the optimal solution must either be $\overline{\Delta R_I}$ or $\Delta R_{\min}$. If $K - x_L > \frac{\theta_a c_a}{1-q_a}$, then the optimal solution is therefore $\overline{\Delta R_I}$, otherwise $\Delta R_{\min}$. The reasoning is the same when $\overline{\Delta R_I} > \overline{\Delta R_a}$.

If the optimal solution is not within the corner solutions, then naturally it must be the corner solution closest since the objective function is smooth.

B.1.3 Proof of Proposition 2

If $K - x_L < \frac{\theta_I c_I}{1-q_I}$, then the FOC (equation 2.6) is negative regardless of ϕ_a (the equilibrium ΔR). So in that case the unique equilibrium must be $\Delta R = \Delta R_{\min}$.

Simple Equilibrium If however $K - x_L \geq \frac{\theta_I c_I}{1-q_I}$, then potentially firms would like have some information. FOC 2.6 is a quadratic equation,

$$\begin{aligned}\Delta R_{\text{Int}}^{CE\,2}\theta_a c_a - \Delta R_{\text{Int}}^{CE}[\overline{\Delta R_I}((1-q_I)q_a(K-x_L)-\theta_I c_I)+\theta_a c_a\overline{\Delta R_a}] \\ -(1-q_I)\overline{\Delta R_I}(1-q_a)\overline{\Delta R_a}(K-x_L) = 0 \quad \text{(B.1)}\end{aligned}$$

Solving for the ΔR yields the term in the proposition. The determinant is always positive therefore a solution always exists. It is also easy to verify that the first term of $\Delta R_{\text{Int}}^{CE}$ is smaller than the second, thus the sign in front of the second term can not be negative.

For the interior solution to be an equilibrium, we need that $\Delta R_{\text{Int}}^{CE} > \max[\overline{\Delta R_I}, \overline{\Delta R_a}]$ such that $\phi_a^* > 0$ and $\phi_I^* > 0$ and that it is not profitable for a firm to deviate. At the interior solution, it is easy to show that firms are always at a local maximum so any (marginal) deviation $\Delta R'_j > \overline{\Delta R_I}$ from $\Delta R^{CE}_{\text{Int}}$ is not profitable. For a firm not to deviate to $\Delta R'_j < \overline{\Delta R_I}$, we need that

$$\Pi_{\Delta R^{CE}_{\text{Int}}} - \Pi_{\Delta R'_j} = \phi_I^{int} \left[\frac{1-q_I}{\theta_I} \left(1 - \frac{1-q_a}{\theta_a} \phi_a^{int}\right)(K - x_L) - c_I \right] - \phi_a^{int} c_a \left(1 - \frac{\Delta R'_j}{\Delta R^{CE}_{\text{Int}}}\right) \geq 0 \quad \text{(B.2)}$$

where ϕ_I^{int} and ϕ_a^{int} are the amount of speculators becoming informed in equilibrium under no deviation. Rearranging terms and simplifying yields

$$\phi_a^{int} c_a \left[\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}} - 1 - \left(1 - \frac{\Delta R'_j}{\Delta R^{CE}_{\text{Int}}}\right)\right] \geq 0.$$

The condition is least likely to hold when $\Delta R'_j$ is chosen minimally, $\Delta R_{\min}$. The term in square brackets then is exactly the condition in the proposition.

We also need that $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_a}$ and $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_I}$. Rewriting condition $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_a}$, it turns out that it is equivalent to $(1-q_I)(K-x_L) \geq \theta_I c_I$, so it always holds. Condition $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_I}$ always holds if the no-deviation condition $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}} - 1 \geq 1 - \frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$ holds. To see this, note that if $\Delta R_{\min} > \overline{\Delta R_I}$, then together with the no-deviation condition this directly implies that $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_I}$. If on the other hand $\Delta R_{\min} < \overline{\Delta R_I}$, then condition $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}} - 1 \geq 1 - \frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$ is equivalent to $\Delta R^{CE}_{\text{Int}} \geq \overline{\Delta R_I}\left(1 + \sqrt{1 - \frac{\Delta R_{\min}}{\overline{\Delta R_I}}}\right)$. From that condition it automatically follows that $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_I}$.

Mixed Equilibrium When $(1-q_I)(K-x_L) \geq \theta_I c_I$ and $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}} - 1 < 1 - \frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$ we will show that the mixed equilibrium described in the proposition is indeed an equilibrium and that it is unique. For it to be an equilibrium, we need that firms issuing $\overline{\Delta R_I}\left(1 + \sqrt{1 - \frac{\Delta R_{\min}}{\overline{\Delta R_I}}}\right)$ and firms issuing $\Delta R_{\min}$ do not deviate at the margin and that both types earn the same profits. Note that the equilibrium can only exist if $\Delta R_{\min} < \overline{\Delta R_I}$. Then condition $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}} - 1 < 1 - \frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$ becomes $\Delta R^{CE}_{\text{Mix}} < \widetilde{\Delta R} \equiv \overline{\Delta R_I}\left(1 + \sqrt{1 - \frac{\Delta R_{\min}}{\overline{\Delta R_I}}}\right)$.

Firms issuing $\Delta R_{\min}$ do not deviate at the margin because for them $\phi_I^* = 0$ (due to $\Delta R_{\min} < \overline{\Delta R_I}$), thus they only care about the adverse selection costs stemming from information about the common risk factor induced by the other firms.

The condition for all firms to earn the same profits is:

$$c_a \phi_a \big|_{\Delta R = \Delta R^{CE}_{\text{Mix}}} \left[\frac{\widetilde{\Delta R}}{\overline{\Delta R_I}} - 1 - \left(1 - \frac{\Delta R_{\min}}{\widetilde{\Delta R}}\right)\right] = 0.$$

Solving this quadratic equation for $\widetilde{\Delta R}$ yields $\widetilde{\Delta R} = \overline{\Delta R_I}\left(1 + \sqrt{1 - \frac{\Delta R_{\min}}{\overline{\Delta R_I}}}\right)$.

$\Delta R^{CE}_{\text{Mix}}$ is pinned down by the local optimality condition of the firms issuing $\widetilde{\Delta R}$. We can rewrite FOC (equation 2.6) with $\frac{\theta_I c_I}{q_a q_I (1-q_I) \widetilde{\Delta R}^2}$ in place of $\frac{\partial \Pi_i}{\partial \Delta R_i}$ and $\theta_a \Big(1 -$

$\frac{c_a}{q_I q_a(1-q_a)\Delta R^{CE}_{\text{Mix}}}\Big)$ in place of ϕ^*_a,

$$\Delta R^{CE\ 2}_{\text{Mix}}\overline{\Delta R_I}[(1-q_I)q_a(K-x_L)-\theta_I c_I] \\ -\Delta R^{CE}_{\text{Mix}}\left[\theta_a c_a\widetilde{\Delta R}^2-\overline{\Delta R_a}(1-q_a)\overline{\Delta R_I}(1-q_I)(K-x_L)\right]+\theta_a c_a\overline{\Delta R_a}\widetilde{\Delta R}^2=0.$$

Solving this equation yields

$$\Delta R^{CE}_{\text{Mix}}=\frac{1}{2\overline{\Delta R_I}[(1-q_I)q_a(K-x_L)-\theta_I c_I]}\Bigg\{\left[\theta_a c_a\widetilde{\Delta R}^2-\overline{\Delta R_a}(1-q_a)\overline{\Delta R_I}(1-q_I)(K-x_L)\right] \\ \pm\sqrt{\left[\theta_a c_a\widetilde{\Delta R}^2-\overline{\Delta R_a}(1-q_a)\overline{\Delta R_I}(1-q_I)(K-x_L)\right]^2-4\overline{\Delta R_I}[(1-q_I)q_a(K-x_L)-\theta_I c_I]\theta_a c_a\overline{\Delta R_a}\widetilde{\Delta R}^2}\Bigg\}.$$

I will prove that the discriminant is always positive. Thus we require that

$$\left[\theta_a c_a\widetilde{\Delta R}^2-\overline{\Delta R_a}(1-q_a)\overline{\Delta R_I}(1-q_I)(K-x_L)\right]^2>4\overline{\Delta R_I}[(1-q_I)q_a(K-x_L)-\theta_I c_I]\theta_a c_a\overline{\Delta R_a}\widetilde{\Delta R}^2$$

Using equation B.1 together with $\Delta R^{CE}_{\text{Mix}}<\widetilde{\Delta R}$, we also know that

$$\widetilde{\Delta R}^2\theta_a c_a-(1-q_I)\overline{\Delta R_I}(1-q_a)\overline{\Delta R_a}(K-x_L)> \\ \widetilde{\Delta R}[\overline{\Delta R_I}((1-q_I)q_a(K-x_L)-\theta_I c_I)+\theta_a c_a\overline{\Delta R_a}]. \quad \text{(B.3)}$$

Thus we can underestimate the LHS of the determinant condition,

$$\widetilde{\Delta R}^2[\overline{\Delta R_I}((1-q_I)q_a(K-x_L)-\theta_I c_I)+\theta_a c_a\overline{\Delta R_a}]^2> \\ 4\overline{\Delta R_I}[(1-q_I)q_a(K-x_L)-\theta_I c_I]\theta_a c_a\overline{\Delta R_a}\widetilde{\Delta R}^2.$$

Reformulating this condition yields

$$[\overline{\Delta R_I}((1-q_I)q_a(K-x_L)-\theta_I c_I)-\theta_a c_a\overline{\Delta R_a}]^2>0,$$

which always holds.

One can also show that the sign in front of the square root must be negative.

The fraction f^* of firms issuing $\widetilde{\Delta R}$ is then pinned down by

$$\Delta R^{CE}_{\text{Mix}}=f^*\widetilde{\Delta R}+(1-f^*)\Delta R_{\min}$$
$$\leftrightarrow f^*=\frac{\Delta R^{CE}_{\text{Mix}}-\Delta R_{\min}}{\widetilde{\Delta R}-\Delta R_{\min}}$$

Next I show that $f\in[0,1]$ if and only if $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}}-1<1-\frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$. This implies that a mixed equilibrium only exists in that region. For $f<1$ to hold, we require that $\Delta R^{CE}_{\text{Mix}}<\widetilde{\Delta R}$. Solving this inequality yields condition B.3, the condition that holds if and only if $\frac{\Delta R^{CE}_{\text{Int}}}{\overline{\Delta R_I}}-1<1-\frac{\Delta R_{\min}}{\Delta R^{CE}_{\text{Int}}}$ (the inequality sign flips *twice* when $(1-q_I)q_a(K-x_L)<\theta_I c_I$). Condition $f>0$ is equivalent to $\Delta R^{CE}_{\text{Mix}}>\Delta R_{\min}$. But firms can not issue a safer security than $\Delta R_{\min}$, so it is impossible that $f<0$. A mixed equilibrium with three or more different strategies can not exist because there are only two local maxima (one for $\Delta R<\overline{\Delta R_I}$ and one for $\Delta R>\overline{\Delta R_I}$). This implies that the mixed equilibrium in the proposition is unique.

B.1.4 Proof of Proposition 3

If $\frac{1}{\theta_I}(1-q_I)(K-x_L) < c_I$, then from proposition 2 it follows that firms issue that $\Delta R_{\min}$ in the private market equilibrium. Region 1 is defined exactly as the parameter values where $\Delta R_{\min}$ is also socially optimal (proposition 1), thus no intervention by the planner is necessary. Since region 2 is just the difference between the region defined by $\frac{1}{\theta_I}(1-q_I)(K-x_L) < c_I$ and region 1, the socially optimal security has $\Delta R > \Delta R_{\min}$. Thus the planner would like to increase information production.

Next take the case when $\frac{1}{\theta_I}(1-q_I)(K-x_L) \geq c_I$. We need to show that the socially optimal security is riskier than in equilibrium when c_a small and safer when c_a large. Since both the socially optimal and equilibrium security are smooth in c_a, then we can conclude that there must be a cutoff $\overline{c_a}$ which defines the regions 3 and 4 together with $\frac{1}{\theta_I}(1-q_I)(K-x_L) \geq c_I$. First I show that the socially optimal security is riskier than in equilibrium when c_a small. If $\frac{1}{\theta_I}(1-q_I)(K-x_L)q_a < c_I$, then the socially optimal security is $\overline{\Delta R_I}$ whereas in equilibrium firms issue $\Delta R_{\min}$ (follows from their FOC (2.6)). If on the other hand $\frac{1}{\theta_I}(1-q_I)(K-x_L)q_a > c_I$, then the socially optimal security is $\Delta R_{\max}$ for a $c_a >> 0$, whereas in equilibrium firms only issue $\Delta R_{\max}$ if $c_a \to 0$. To see this, note that From the first part of proposition 1, $\Delta R_{\max}$ is socially optimal whenever $\theta_I \overline{\Delta R_I} c_I + \theta_a \overline{\Delta R_a} c_a \leq \left(\overline{\Delta R_I}(1-q_I)q_a + \overline{\Delta R_a}(1-q_a)q_I\right)(K-x_L)$. The condition can be rewritten as

$$c_a\left(\theta_a \frac{c_a}{q_i q_a(1-q_a)} - \frac{K-x_L}{q_a}\right) \leq \overline{\Delta R_I}\left[(1-q_I)q_a(K-x_L) - \theta_I c_I\right].$$

The right hand side is positive. Thus the condition always holds for $\frac{1}{\theta_a q_I(1-q_a)(K-x_L)} > c_a > 0$.

Now I show that the socially optimal security is safer than in the private equilibrium when c_a is large. Note that when c_a is large, then $\overline{\Delta R_a} > \overline{\Delta R_I}$. We can also show that $\Delta R^{SP}_{\text{Int}} < \overline{\Delta R_a}$. Rewriting that inequality yields

$$\overline{\Delta R_I}[2(1-q_a)(1-q_I)(K-x_L) + (1-q_I)q_a(K-x_L) - \theta_I c_I] < \overline{\Delta R_a}[\theta_a c_a - (1-q_a)q_I(K-x_L)],$$

which holds if c_a large enough. Thus we have that $\Delta R^{SPP} = \overline{\Delta R_a}$. In the private market equilibrium the solution is interior if $\frac{1}{\theta_I}(1-q_I)(K-x_L) \geq c_I$. Rewriting the inequality yields exactly $\Delta R^{CE}_{\text{Int}} > \overline{\Delta R_a}$ yields $\frac{1}{\theta_I}(1-q_I)(K-x_L) > c_I$, thus it holds.