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DOI

[10.1515/rle-2016-0005](https://doi.org/10.1515/rle-2016-0005)

Publication date

2018

Document Version

Final published version

Published in

Review of Law and Economics

[Link to publication](#)

Citation for published version (APA):

Fabbri, M., & Britto, D. G. C. (2018). Distributive Justice, Public Policies and the Comparison of Legal Rules: Quantify the “Price of Equity”. *Review of Law and Economics*, 14(3), [20160005]. <https://doi.org/10.1515/rle-2016-0005>

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Distributive Justice, Public Policies and the Comparison of Legal Rules: Quantify the “Price of Equity”

<https://doi.org/10.1515/rle-2016-0005>

Abstract: This paper proposes a quantitative approach to study two methodological problems arising when a costly redistribution of resources is implemented through public policies or legal rules: (a) aggregating individual into social preferences and (b) choosing the object of maximization. We consider a redistribution intervention that reduces inequality but diminishes total wealth and we specify a set of social welfare functions combining different preferences aggregation methods and maximands. For each social welfare function, we calculate its “price of equity”, defined as the maximum fraction of total wealth that a society is willing to sacrifice in order to implement the redistribution. Comparing the prices for equity across different social welfare function specifications, we identify systematic relationships and we rank them according to the efficiency-equity orientation. Results show that social welfare functions characterized by aggregation methods conventionally considered equity-oriented may reject redistribution interventions that are evaluated as welfare-improving by social welfare functions using efficiency-oriented aggregation methods. Similarly, social welfare functions considered equity-oriented because using utility as object of maximization may reject distributive policies that are evaluated as welfare-improving by social welfare functions using wealth as maximand. We argue that the quantitative approach proposed, by expounding the trade-off between equity and efficiency connected to different social welfare functions, may prove useful in areas of public law where policy-makers have to engage in the choice of a normative criterion for the evaluation of social welfare. Additionally, our results may inform rule-makers interested in comparing the distributive effects of alternative legal rules in special circumstances where private remedies can efficiently achieve redistribution goals.

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Keywords: efficiency, law and economics, Nash, Rawls, social welfare function

JEL Classification: D04, D61, D63, K10

1 Introduction

A fundamental motivation to engage in economic analysis of the law consists in making comparative evaluations between different legal regimes and policy options. This task poses two methodological problems to law and economics practitioners. First, it demands the choice of a method for aggregating individual preferences into social preferences. Second, the analyst has to choose what the legal system should try to maximize.¹ To provide an example, imagine a policy intervention that reduces the payoff of agent A from \$10 to \$8 and that increases the payoff of agent B from \$50 to \$55. Assuming no one else is affected, should we consider this policy welfare increasing?² The policy analyst interested in maximizing the *sum* of agents' *wealth* would say yes, since the status quo total wealth increases from \$60 to \$63. However, were the policy analyst interested in increasing *proportional wealth*, that is, to maximize the product of agents' wealth, the policy would be evaluated as worsening social welfare, since $50 \times 10 = \$500 > \$55 \times \$8 = \440 . Similarly, were the analyst maximizing the sum of individuals' *utility* (assuming that utility is estimated to be logarithmic) the policy would not be implemented since $\ln 50 + \ln 10 = 6.21 > \ln 55 + \ln 8 = 6.08$. This example illustrates the importance for law and economics practitioners operating in important areas of public law, most notably constitutional law and social welfare law, of choosing a proper way to measure and a suitable method for aggregating individuals'

1 An overarching open issue, not treated in the present work, concerns the role of law and economics in designing legal institutions. With respect to this point, three main different schools of thought has developed: the so called "Positive school", known also as the "Chicago school", the "Normative school" known as the "Yale school", and the "Functional School" or "Virginia school" (Parisi, 2004; Parisi et al., 2016). To some extent, scholars within each of these schools agreed to common methodological solutions for dealing with the problems of evaluating social preferences and choosing the object of maximization of the legal system. For a detailed account of differences between schools of thought see Parisi and Klick (2004).

2 In recent years, some law and economics scholars started to move away from the concept of static efficiency as a criterion to evaluate legal rules. Those scholars contend that dynamic efficiency, measured as GDP growth per capita, may be a better criterion to evaluate different legal provisions, hence suggesting that the legal system should pay a special attention to promoting innovation and R&D investments (Cooter, 2014; Paccès, 2015). While the relationship between law and dynamic efficiency certainly deserve future investigations, in this article we do not directly address this emerging branch of the literature.

wellbeing. Moreover, while private law remedies are in general considered sub-optimal compared to taxation and other public remedies as means to achieve redistributive goals,³ nevertheless, in special circumstances carefully designed legal rules can result efficient in redistributing resources (Cooter, 2003). Examples are laws mandating the elimination of architectural barriers in public and private buildings; legal provisions directed toward the protection of minorities; and rules aimed at overcoming gender discrimination. In these examples, properly designed laws would target desired agents inducing predictable changes of behaviors at a low cost of enforcement. Moreover, distortion of incentives is modest. Hence, in special circumstances, the measure of individuals' well-being and the choice of an aggregation method acquires importance also for the comparison of different legal rules, each of them embodying a different trade-off between redistribution and efficiency. For decades, scholars have contributed to the debate concerning these two distinct normative issues (Adler, 2012; Brams and Taylor, 1996; Cooter and Helpman, 1974; Mueller, 1967, 2003). However, despite systematic attempts to deal with these problems, the discipline has not established a clear methodological benchmark yet (Nussbaum, 1997) and the solutions adopted by practitioners derive more from the need of analytical tractability than from a convincing normative analysis (Parisi, 2004).

This paper contributes to the methodology of normative analysis in law and economics by performing a quantitative analysis. Our starting point is that choosing the aggregation method and selecting the object of maximization are two specific aspects of a general problem, namely the function of the legal system in regulating the relationship between efficient resource allocation and distributive justice.⁴ We investigate the efficiency-equity relationship by adapting to the law and economics framework a methodological approach commonly used in operation research science. Scholars in this field, when having to identify a criterion for allocating scarce resources between multiple agents in a specific application (for examples, allocating airport slots among companies), estimate the monetary

3 Among the possible reasons commonly advanced against the inclusion of wealth redistribution in the general principles of private law there are: (1) difficulties of precisely target agents; (2) unintended consequences and side-effects (3) high transaction costs (4) large distortion of incentives. For a thorough discussion of these points see Cooter (1995).

4 The fact that in law and economics scholarship, the relationship between efficiency and justice is still an open issue is well summarized by Hatzis and Mercuro (2015, Preface, p. x): "But fundamental questions remain: is law and economics a utilitarian, market-friendly legal theory? Or is it a new legal paradigm which, despite its intellectual debts, has managed to realize the promise of sociological jurisprudence and legal realism? ...Or is it both?". For a similar argument, see also Parisi and Rowley (2005).

costs of moving from the allocative criterion that maximizes aggregated monetary outcomes to alternative less efficient solutions that mitigate allocative inequality.

The quantitative analysis we propose proceeds in the following way. First, we briefly present the methodological problem of aggregating the wellbeing of individuals into social welfare and the three methods most often used in the law and economics literature. Second, we divide between the cases where the selected object of maximization, the *maximand*, is either individuals' wealth or utility, which we further differentiate between the cases of exponential, polynomial and logarithmic utility functions. Then, we illustrate how the choice of the maximand also influences resources redistributions. Third, we consider a scenario where a transfer of resources happens between two agents endowed with given initial payoffs. The transfer reduces inequality but it is costly, since it reduces the total amount of available resources.

We then consider various combinations of aggregation methods and maximands which are characterized by a "social welfare function" (SWF).⁵ For each SWF, we derive the maximum "price of equity", that is, the maximum fraction of wealth a society is willing to give up in order to implement the redistribution.⁶ We then perform a comparison and rank the price of equity of the SWFs considered based on an efficiency-equity scale.

Our first main finding is that considering the choices of a proper aggregation method and of a suitable maximand as two independent normative problems may be often times misleading. This constitutes the first main contribution of the paper. For instance, we show that SWFs which aggregate individuals' welfare by multiplying them - an aggregation method conventionally considered in the literature as more equity-oriented - may reject distributive policies that are instead evaluated as welfare-improving by SWFs that sum individuals' welfare - an aggregation method labelled as the most efficiency oriented. Similarly, we

5 Following Sen (1970), we define a SWF as a real-valued function that ranks conceivable social states (alternative complete descriptions of the society) from lowest to highest. Inputs of the function include any variables considered to affect the economic welfare of a society. One use of SWF relevant for the present discussion is to represent prospective patterns of collective choice regarding alternative social states.

Formally, a SWF could be defined as follow:

$$SW(W): \mathbb{R}^n \longrightarrow \mathbb{R}$$

where $W = (V_1, V_2, V_3, \dots, V_n)$ is a vector containing the welfare of each single individual in the population. The only assumption we would like to impose is the function to be weakly increasing in the welfare of each single individual. Formally, if $W^1 \geq W^2$, then $SW(W^1) \geq SW(W^2)$.

6 In the Appendix B.3, we consider also the symmetric, less common situation in which a transfer of resources happens from the least well-off agent to the wealthier one and the transfer increases the total amount of wealth, hence generating a "price of efficiency".

show that SWFs whose objects of maximization are well-behaved utility functions may reject distributive policies that are considered welfare-improving by SWFs that employ as maximand individuals' wealth, an object of maximization often criticized because of its absence of equity considerations.

Our claim is that the choice of a normative criterion for the evaluation of social welfare should be guided by the effective efficiency-equity trade-off associated with each SWF specification. We argue that the two tasks of choosing the optimal aggregation method and selecting the optimal object of maximization cannot be discussed as independent problems.

Our second main contribution is quantifying the price of equity, or the taste for redistribution, implied by different combinations of aggregation methods and a maximands. More specifically, the contribution is showing that a number of SWF specifications, defined by an aggregation method and a maximand, can be well-ordered in terms of equity considerations. Thus, one important goal of this paper is to facilitate scholars and practitioners with the common task of choosing a specific normative criterion for evaluating policies and legal rules.

The remainder of this paper is structured as follows. In section 2, we provide a brief overview of the debate in law and economics concerning the aggregation of individuals' well-being into a representation of social preferences and the choice of the object of maximization. Readers familiar with this debate might skip Section 2 and jump directly to Section 3, where the quantitative analysis and our innovative contributions are presented. Section 4 concludes the paper, summarizing the results obtained and possible applications. Tables reporting quantitative results are provided in Appendix A and proofs in Appendix B.

2 Aggregating individual into social preferences and choosing the object of maximization

The most common criterion choosed by law and economics scholars for aggregating individual preferences is derived from the so called "classical utilitarianism" (Hicks, 1939; Kaldor, 1939). According to this criteria, policy interventions have the objective of maximizing the sum of agents' welfare, or, using the words of the British philosopher Jeremy Bentham who is considered the founding father of this school, "the greatest happiness of the greatest number that is the measure of right and wrong" (Bentham, 1776, Preface, p. ii). The Bentham approach had great influence on law and economics scholar and this aggregation method is the most commonly encountered in the literature. For instance, see Polinsky and Shavell (1998) in a study of punitive damages, Kaplow and Shavell (1996)

who analyzes property versus liability rules and Kaplow and Shavell (1994) who discuss the redistributive efficiency of the legal system, to cite a few.

Formally, we define a “Bentham” criterion for aggregation of individual into social preferences as:

$$SWB(W) = \sum_{i=1}^N V_i \quad (1)$$

The problem with the Bentham aggregation criterion is the absence of equity considerations. Hence, from an ethical standard, the acceptability of the utilitarian principle has been questioned from more perspectives (see for example Gauthier, 1963; Nagel, 1970; Rawls, 1971) and some scholars urge practitioners of law and economics to critically reconsider this methodological choice (Nussbaum, 1997).

An alternative approach to the utilitarian criterion is derived from Nash’s studies of bargaining solutions, the Nash standard of comparison (Nash, 1950).⁷ Under this framework, a transfer of resources between agents is justified if the percentage of welfare increase of the gainer is greater than the loser’s percentage loss. This criterion mitigates some important concerns regarding distributive justice that were problematic with the Bentham approach.⁸

While Nash is a less popular aggregation method in the law and economics literature compared to the Bentham criterion, it received substantial attention as an alternative normative criterion. For examples of contributions using the Nash criterion see Klick and Parisi (2005, 2015); Parisi and Klick (2004); Parisi (2004). Moreover, this criterion has also been used in numerous applications, see for instance Alvarez-Cuadrado and Van Long (2009); Brandt et al. (2012); Parisi (1997, 2003); Ramezani and Endriss (2010).

⁷ Kalai and Smorodinsky (1975) proposes and axiomatizes an alternative solution to the Nash standard of comparison. The two solutions differ for the set of axioms that they are able to satisfy. In particular, the Nash solution satisfies Pareto optimality, symmetry, affine invariance and independence of irrelevant alternatives. The Kalai-Smorodinsky solution, beside the first three axioms of the Nash solution, satisfies also monotonicity, however it is not able to satisfy the independence of irrelevant alternatives. Kaneko and Nakamura (1979) attributes the multiplicative form of social welfare function to Nash.

⁸ Indeed, according to the Bentham criterion, transferring one dollar from a homeless to a billionaire does not affect social welfare (assuming the transfer is costless), since the aggregate welfare remains unchanged. However, this argument clashes with intuitive reasoning. In particular, the Bentham criterion fails to consider that utility is marginally diminishing. Hence, an equal transfer of resources would benefit more disadvantaged members of society. In contrast, the Nash approach captures this characteristic of utility by attaching a heavier weight to less well-off members of the society.

The “Nash” criterion for aggregating individual into social preferences can be defined as:

$$SWN(W) = \prod_{i=1}^N V_i \quad (2)$$

Finally, we consider an aggregation criterion inspired by the work of the American philosopher Rawls (1971, 1974a,b). According to this theoretical position, the welfare of a society is constituted by the welfare of its least well-off individual.⁹ According to the Rawls criterion, any welfare-enhancing policy should involve maximizing the welfare of the worst-off individual. The Rawls criterion served as normative benchmark in various fields of application, for instance environmental policy (Haddad, 2005), intergenerational equity (Asheim, 2010) or administrative law and economics (Graham, 2008). Further applications of the Rawls’ principle are discussed by Adler and Sanchirico (2006).

Formally, a “Rawls” or “maxmin” criterion could be expressed as:

$$SWR(W) = \min_{i=1}^N \{V_i\} \quad (3)$$

Given a scenario involving a costly transfer of resources between parties that reduces total resources available, the three aggregation methods described could fairly represent the full range of preferences concerning the trade-off between efficiency and distributive justice. The Bentham criterion implies a full concern for efficiency and no distributive justice considerations. Conversely, the Rawls SWF implies that only the least well-off party represents the measure of societal well-being.¹⁰ Therefore, given a “leaky bucket” redistribution that implies a decrease of total wealth, the Rawls SWF is concerned about equality of parties’ wellbeing and does not consider efficiency, defined as the measure of total wealth available. Finally, the Nash criterion assumes an intermediate position between Bentham

9 Rawls defends his position elaborating a complex theory of social justice. He argues that every man would ex-ante choose an equal allocation of resources among different alternatives if, while expressing the choice, his knowledge of the ex-post effective distribution would be prevented by a “veil of ignorance”.

10 As correctly noted by one Referee, according to the “difference principle” proper of Rawl’s philosophy, a policy triggering an increase in inequality is considered welfare improving as long as also the welfare of the least well-off individual is increasing. For example, a situation where both individuals’ wealth increase, but the rich gains more than the poor, could increase inequality but be desirable according to the Rawl’s criterion. However, as stated in the introduction, in this work we follow the customary assumption of a costly redistribution between two parties that diminishes total wealth. Thus in this scenario the difference principle does not apply.

and Rawls. On the one hand, it is concerned about efficiency and in some situations might consider a welfare improvement an activity that further increases the welfare of the better-off agent while it reduces the welfare of the worse-off agent. On the other hand, it implicitly weights more the welfare of the poorest agent, since to consider welfare-improving a policy it requires that the percentage of welfare increase of the gainer is greater than the loser's percentage loss.

The second methodological problem for the law and economics practitioner is determining what is the object which the law has to maximize. Scholars involved in the debate often ask if the law should be concerned with wealth maximization or with the maximization of some measure of utility as a description of individual well-being.

The choices of wealth or utility as object of maximization differ in two fundamental aspects that are closely related. First, since marginal utility of wealth is commonly assumed to be strictly decreasing, choosing utility as a maximand is equivalent to stating that individuals are risk averse to some degree. However, the same does not apply to wealth, which implies risk neutrality concerning individuals' welfare. The second point is that, due to decreasing marginal utility, the use of the utility function as a maximand embodies some distributional concerns, because one extra unit of wealth is more valuable to a worse-off individual in comparison to someone better-off. The degree to which the utility function embodies risk aversion and distributional concerns hinges on the particular choice of the utility function and its degree of concavity.

Traditionally, welfare economics theory considers utility as a maximand. Indeed, it seems obvious that the ultimate goal of any policy intervention would be to maximize human happiness, therefore the all-encompassing definition of utility captures better the human dimension of the policy intervention. Conversely, scholars in the law and economics tradition tend to prefer using wealth as object of maximization. The reason lies in the difficulty to identify reliable measures for utility and in the practical and theoretical problems that interpersonal comparison of utility creates. Nonetheless, the law and economics literature still resorts to utility measures when analyzing problems of uncertainty. In such cases, risk aversion is clearly a relevant issue which is brought to the analysis through the use of marginally decreasing utility, which in turn implicitly embodies some redistributive concerns. In the next section, we show how specific utility form choices inherently embody redistributive tastes for social welfare criteria.

In the present work we consider three forms of utility function commonly used in law and economics: polynomial (constant relative risk aversion), logarithmic (constant relative risk aversion) and exponential (constant absolute risk aversion). Therefore, we consider the following maximands:

- Wealth: $V_i = w_i$, where w_i is the wealth of the individual i
- Polynomial Utility: $V_i = u(w_i) = w_i^\alpha$, where $\alpha \in (0, 1)$
- Logarithmic Utility: $V_i = u(w_i) = \ln(w_i)$
- Exponential Utility: $V_i = u(w_i) = 1 - e^{-\alpha w_i}$, where $\alpha \in (0, 1)$

3 The price of equity

In this section, we apply a quantitative approach to the comparison of different criteria for the evaluation of social welfare. This approach is often used by scholars studying the problem of resources allocation in fields such as applied engineering and operational research (Bertsimas et al., 2011; Khodadadi et al., 2006; Terrab and Odoni, 1993). The basic idea consists in taking as a reference point the allocation that minimizes the sum of monetary costs (i.e. the optimal allocation) and calculating the loss of efficiency implied by switching from the optimal allocation to other schemes more equity-oriented (i.e. the price of equity).

For a set of SWF specifications, we calculate and then rank the price of equity in a two-agents scenario, showing that there are systematic relationships between some of the criteria considered. We analyze the effects of a rule which yields distributive consequences, transferring to agent A an amount of wealth ϵ taken from agent B. The aggregation methods and maximands considered in the analysis are: three aggregation criteria, Bentham (B), Nash (N) and Rawls (R), combined with four possible individual utility function specifications, utility equal to wealth (W), polynomial utility (P), logarithmic utility (L) and exponential utility (E). The transfer of resources is happening from the richest to the poorer individual and we name this kind of transfers “Robin-hood” (RH).¹¹ The transfer is costly, in the sense that agent A receives only a fraction $k \in [0;1]$ of the amount of wealth taken from B. Hence, while the redistribution reduces inequality, it also decreases total wealth. Table 2 in Appendix A reports, for each SWF, the fraction k of the total endowment that has to be preserved in order to rank the new allocation generated by the redistribution weakly preferred to the status quo.

¹¹ In Appendix B.3, we also consider the symmetric situation in which a transfer occurs from the less wealthy to the wealthier individual and at the same time it increases total wealth. We label this kind of transfer “Efficiency-increasing” (EI). The intuition is that a policy implementing EI transfers would further increase inequality, but could nonetheless be desirable from a social perspective if the wealthiest group’s gains more than compensate the losses borne by the poorest group. For instance, recent empirical evidence confirms that transferring resources from poor to rich is less costly than a redistribution from rich to poor and endowing rich members of the population with additional resources may in some circumstances generate Kaldor-Hicks improvements (Hendren, 2014).

For each combination of aggregation method and maximand, we calculate the associated price of equity, that is equal to $(1-k)$. Hence, the higher the price associated to a specific SWF, the more resources a society is willing to give up in order to implement a distributive policy, the more equity-oriented the welfare evaluation criterion is. We compare the price of equity across different SWFs. We identify two subsets of SWFs where it is possible to rank the different specifications from the more efficiency-oriented one to the more equity-oriented one.

Result 1. The price of equity can be ranked in increasing order for two subsets of SWF specifications:

- Subset 1) Bentham-Wealth < Bentham-Polynomial < Bentham-Logarithmic = Nash-Wealth = Nash-Polynomial < Nash-Logarithmic < any Rawls specification;
- Subset 2) Bentham-Wealth < Bentham-Exponential < Nash-Exponential < any Rawls specification.

Proof. We proceed by showing that the size of k (reported in Table 2 in the Appendix) are well-ordered for each SWF specification in Result 1. For instance, in order to show that any redistribution implying an improvement under Bentham-Wealth is also welfare-improving under Bentham-Polynomial, we show that the minimum k required by the former is greater than the minimum k required by the latter.

Subset 1:

- B-W implies B-P

We need to show that:

$$(1/\epsilon)\{[w_1^\alpha + w_2^\alpha - (w_2 - \epsilon)^\alpha]^{1/\alpha} - w_1\} \leq 1$$

Rearrange terms and note that $\epsilon = \lambda(w_2 - w_1)$, $\lambda \in (0, 1)$, since transfers do not increase inequality by assumption:

$$w_1^\alpha + w_2^\alpha \leq [(1 - \lambda)w_1 + \lambda w_2]^\alpha + [(1 - \lambda)w_2 + \lambda w_1]^\alpha$$

Since w^α is a concave function, we can write:

$$\begin{aligned}\lambda_1 w_1^\alpha + (1 - \lambda_1)w_2^\alpha &\leq [\lambda_1 w_1 + (1 - \lambda_1)w_2]^\alpha \\ \lambda_2 w_1^\alpha + (1 - \lambda_2)w_2^\alpha &\leq [\lambda_2 w_1 + (1 - \lambda_2)w_2]^\alpha\end{aligned}$$

where $\lambda_1, \lambda_2 \in (0, 1)$ Setting $\lambda_1 = 1 - \lambda$ and $\lambda_2 = \lambda$, and summing up the inequalities:

$$w_1^\alpha + w_2^\alpha \leq [(1-\lambda)w_1 + \lambda w_2]^\alpha + [(1-\lambda)w_2 + \lambda w_1]^\alpha$$

- B-P implies B-L

We show that:

$$\frac{w_1}{w_2 - \epsilon} \leq (1/\epsilon)\{[w_1^\alpha + w_2^\alpha - (w_2 - \epsilon)^\alpha]^{1/\alpha} - w_1\}$$

By rearranging terms and, once again, taking $\epsilon = \lambda(w_2 - w_1)$, we find:

$$\{(w_1)^\alpha + (w_2)^\alpha - [(1-\lambda)w_2 + \lambda w_1]^\alpha\}[(1-\lambda)w_2 + \lambda w_1]^\alpha \geq w_1^\alpha w_2^\alpha$$

Define the left term from the inequality as $f(\lambda)$. Note that $f(0) = w_1^\alpha w_2^\alpha$ and $f(1) = w_1^\alpha w_2^\alpha$. We show that this is a concave function, thus by continuity it directly follows that the inequality holds for any $\lambda \in (0, 1)$. Calculate the second derivative:

$$f''(\lambda) = (\alpha^2 - \alpha)(w_1^\alpha + w_2^\alpha)(w_1 - w_2)^2[(1-\lambda)w_2 + \lambda w_1]^{\alpha-2} \leq 0$$

The second derivative is always smaller than zero, since $\lambda \in (0, 1)$, $w_2 > w_1 > 0$ and $\alpha \in (0, 1)$.

- Equivalence of B-L, N-W, N-P

Consider a Nash-wealth SWF:

$$SWN^w(W) = \prod_{i=1}^N w_i$$

by raising by α we obtain:

$$= \prod_{i=1}^N w_i^{\alpha} = SWN^{pol}(W)$$

this proves the equivalence with SWF Nash Polynomial.

Then take the logarithm and divide it by α :

$$= \sum_{i=1}^N \ln w_i = SWB^{log}(W)$$

this proves the equivalence with SWF Bentham Logarithmic.

Since all of the transformations above are monotonic with a positive derivative, this implies that all three specifications rank alternative states of the world in the same order.

- B-L, N-W and N-P imply N-L

We show that:

$$(1/\epsilon)\{e^{\left[\frac{(\ln w_A)^j (\ln w_B)^{N-j}}{(\ln w_B - \epsilon)^{N-j}}\right]^{1/j}} - w_A\} \leq \frac{w_1}{w_2 - \epsilon}$$

By rearranging terms and taking $\epsilon = \lambda(w_2 - w_1)$, we find:

$$\ln\left(\frac{w_1 w_2}{(1-\lambda)w_2 + \lambda w_1}\right) \ln[(1-\lambda)w_2 + \lambda w_1] \geq \ln w_1 \ln w_2$$

Define the left term from the inequality as $f(\lambda)$ and note that $f(0) = \ln w_1 \ln w_2$ and $f(1) = \ln w_1 \ln w_2$. If we can show that this is a concave function, by continuity it follows that the inequality holds for any $\lambda \in (0, 1)$ and we are done. The second derivative is as follows:

$$f''(\lambda) = -(w_1 - w_2)^2 [(1-\lambda)w_2 + \lambda w_1]^{-2} \ln\left(\frac{w_1 w_2}{(1-\lambda)w_2 + \lambda w_1}\right) \leq 0$$

Indeed, the second derivative is always smaller than zero, since $\lambda \in (0, 1)$, $w_2 > w_1 > 1$ in the Nash-Logarithm form (otherwise, we risk aggregating negative utilities by multiplying them among each other).

– N-L implies Rawls

In the two groups case, any RH transfer will be desirable under Rawls since it benefits the poorest group. In a more general setup, any RH transfer is weakly preferred, given that it strictly improves welfare when it enriches the least well-off group. Otherwise, in the case in which the individual granted with the transfer is not the least well-off, the new allocation simply bears the same level of welfare under the Rawlsian principle.

Subset 2:

– B-W implies B-E

We must show that:

$$(1/\epsilon)\{- (1/\alpha) \ln[e^{-\alpha w_1} + e^{-\alpha w_2} - e^{-\alpha(w_2-\epsilon)}] - w_1\} \leq 1$$

Rearranging terms and taking $\epsilon = \lambda(w_2 - w_1)$ results in:

$$e^{\alpha[(1-\lambda)w_1 + \lambda w_2]} [e^{-\alpha w_1} + e^{-\alpha w_2} - e^{-\alpha[\lambda w_1 + (1-\lambda)w_2]}] \geq 1$$

Define the left term from the inequality as $f(\lambda)$ and note that $f(0) = 1$ and $f(1) = 1$, satisfying the inequality. We study the sign of the derivative:

$$f'(\lambda) = \alpha(w_2 - w_1)e^{\alpha[(1-\lambda)w_1 + \lambda w_2]} [e^{-\alpha w_1} + e^{-\alpha w_2} - 2e^{-\alpha[\lambda w_1 + (1-\lambda)w_2]}]$$

We note that $f'(0) = \alpha(w_2 - w_1)e^{\alpha w_1} [e^{-\alpha w_1} - e^{-\alpha w_2}] > 0$, $f'(1) = \alpha(w_2 - w_1)e^{\alpha w_2} [e^{-\alpha w_2} - e^{-\alpha w_1}] < 0$ and that $f'(\lambda)$ sign depends exclusively on the term inside the brackets. This term is strictly decreasing in λ and, therefore, the derivative sign only changes from positive to negative at one point. This implies that the inequality holds since $f(\lambda)$ is continuous and increasing at

$\lambda = 0$, has only one maximum point for $\lambda \in (0, 1)$ and satisfies the inequality at $\lambda = 0$ and $\lambda = 1$.

- B-E implies N-E

We show that:

$$\begin{aligned} & \frac{1}{\epsilon} \left\{ \left(\frac{-1}{\alpha} \right) \ln \left[1 - \left[\frac{(1 - e^{-\alpha w_1})(1 - e^{-\alpha w_2})}{1 - e^{-\alpha(w_2 - \epsilon)}} \right] \right] - w_1 \right\} \leq \\ & \leq \frac{1}{\epsilon} \left\{ \left(\frac{-1}{\alpha} \right) \ln [e^{-\alpha w_1} + e^{-\alpha w_2} - e^{-\alpha(w_2 - \epsilon)}] - w_1 \right\} \end{aligned}$$

Rearranging terms and taking $\epsilon = \lambda(w_2 - w_1)$, we find:

$$[e^{-\alpha w_1} + e^{-\alpha w_2}]e^{-\alpha[\lambda w_1 + (1-\lambda)w_2]} - e^{-2\alpha[\lambda w_1 + (1-\lambda)w_2]} - e^{-\alpha w_1}e^{-\alpha w_2} \geq 0$$

As before, we define the left term from the inequality as $f(\lambda)$ and note that $f(0) = 0$ and $f(1) = 0$, satisfying the inequality. Subsequently, we study the sign of the derivative to prove our proposition:

$$f'(\lambda) = \alpha(w_2 - w_1)e^{\alpha[\lambda w_1 + (1-\lambda)w_2]}[e^{-\alpha w_1} + e^{-\alpha w_2} - 2e^{-2\alpha[\lambda w_1 + (1-\lambda)w_2]}]$$

Note that $f'(0) = \alpha(w_2 - w_1)e^{\alpha w_2}[e^{-\alpha w_1} + e^{-\alpha w_2} - 2e^{-2\alpha w_2}] > 0$ and that $f'(\lambda)$ sign depends exclusively on the term inside the brackets. This term is strictly decreasing in λ and, therefore, the derivative only changes sign at one point, if ever. This implies that the inequality holds since $f(\lambda)$ is continuous and increasing at $\lambda = 0$, has at most one maximum point for $\lambda \in (0, 1)$ and satisfies the inequality at $\lambda = 0$ and $\lambda = 1$. This proves our proposition.

- N-E implies Rawls

The same reasoning used to show that N-P implies Rawls also holds here. ■

Table 1 provides a summary of our results. For each SWF specification, it reports which other specifications yield a greater or equal price of equity. This means that if a R-H policy is desirable under a given specification, it will also be desirable to any other specification yielding a larger price of equity.

It is worth noting and discussing three points related to this result. First, for a set of SWFs, using wealth or utility as the maximand does not influence the policy evaluation result. This result is surprising, since the choice of wealth or utility as the object of maximization has been the argument of a harsh debate in law and economics for more than forty years.¹² (Calabresi, 1979; Dworkin, 1979,

12 The most famous example of this debate is probably the organization of the “twin symposia” on efficiency and wealth-maximization in legal theory (acts of the symposia are published in the

Table 1: Generality of results under RH transfers.

Maximand/SWF Form	Bentham	Nash	Rawls
Wealth	$\{\forall B; \forall N; \forall R\}$	$\{B-L; N-P; N-L; \forall R\}$	$\{\forall R\}$
Polynomial	$\{B-L; N-W; N-P; N-L; \forall R\}$	$\{B-L; N-W; N-L; \forall R\}$	$\{\forall R\}$
Logarithmic	$\{N-W; N-P; N-L; \forall R\}$	$\{\forall R\}$	$\{\forall R\}$
Exponential	$\{N-E; \forall R\}$	$\{\forall R\}$	$\{\forall R\}$

1980; Ehrlich and Posner, 1974; Van Praag, 1993; Posner, 2015; Rawls, 1985). Our result suggests that the discussion on the comparison between wealth and utility as maximands may be in some circumstances an empty one.

Second, the Bentham and the Nash aggregation methods, when paired with specific maximands, yield precisely the same price of equity. This result might sound counter-intuitive, since it is commonly argued that Bentham and Nash embody two very distinct normative perspectives regarding the trade-off between equity and efficiency (Adler, 2012; Dolan and Tsuchiya, 2009; Ellerman, 2014; Gauthier, 1963; Mueller, 2003; Nagel, 1970; NG, 1981; Parisi, 2004). We clarify that the choice of a specific aggregation method alone does not automatically imply in a greater or smaller taste for redistribution. Therefore, a quantitative analysis that estimates the cost for equity connected to distributive policies may be a more appropriate tool to guide the normative choice of the welfare evaluation criterion.

Third, the findings from above could also be useful for scholars engaging in policy or legal evaluation who are interested in verifying to what extent results based on a specific SWF generalize. A common concern is that results derived may be just an artifact of the specific SWF assumed in the study. For instance, Brennan and Pincus in a contribution on fiscal equity in federal systems devote an entire subsection to report results of “experiments with alternative utility functions” in order to verify if employing different SWF specifications “alter the basic results” (Brennan and Pincus, 2010:359). Our results provide a robustness check for the possibility to generalize results obtained by employing a certain SWF specification.

3.1 Justice and extra-welfarist considerations

The above results are derived following the “anonymity principle” typical of neoclassical welfare economics, stating that each individual’s well-being affects social welfare in a symmetric manner. However, some scholars argue that violating the anonymity principle is sometimes necessary when evaluating policy interventions that concern minorities and disadvantaged groups (Brouwer et al.,

Journal of Legal Studies, 9:2 March 1980 and in the Hofstra Law Review, 9:3-4, Spring Summer 1980)

2008; Coleman, 1984). In this section, we analyze a distributive policy where the analyst explicitly assigns different weights to individuals or groups of interest and we see how the price of equity associated to different SWFs modifies when unequal weights are introduced.¹³

We provide a graphical example with the purpose of conveying the message in an intuitive way. Figure 1 compares an example of policy analysis results in a situation where the anonymity principle holds (left panel) compared with a situation where the wealthier agent has a weight equal to $\frac{3}{4}$ of poorer agent's wealth (right panel). In the example, Agent 1 has a wealth of 6.5 while Agent 2 has a wealth of 12. The horizontal axis reports the transfer of resources that the policy proposes to reallocate from Agent 2 to Agent 1. The vertical axis reports the price of equity (i.e. the minimum level of efficiency of the distributive system) which is required for the policy to be deemed welfare improving. For each combination of aggregation method and maximand, we plot how the price of equity change when the amount of resources transferred is increasing.

We see that, for each SWF considered,¹⁴ the price of equity required in the situation where the poorer agent's well-being is weighted more (right panel) is lower than that when the anonymity principle is preserved (left panel). This result is expected, since the well-being of the poorest agent is now weighted more than the well-being of the wealthier one. Furthermore, we notice that this reduction is larger for some SWF specifications than for others. This happens because attaching weights to agents introduces an element of non-linearity in some of the

Figure 1: An evaluation of a RH transfer where the anonymity principle is violated. The graph on the left has homogeneous agents, while in the graph on the right the well-being of the wealthier agent is weighted $\frac{3}{4}$ of the well-being of the poorest one.

¹³ A possible alternative interpretation, leading to the same results, is that we are considering groups of different sizes, assuming that wealth is homogeneous among groups. Therefore, in the groups of interest interpretation we assume that $w_1 = w_2 = \dots = w_j$ and $w_{j+1} = w_{j+2} = \dots = w_N$.

¹⁴ The parameter α in the polynomial SWF is assumed to be 0.3.

equations determining k , while it only results in linear transformations for other specifications.

We report in Table 3 in the Appendix the price of equity for different SWF specifications calculated including the possibility to assign unequal weights to agents. Table 4 summarizes the results for these set of transfers.

4 Conclusion

This paper contributes to the literature in law and economics regarding two fundamental methodological problems arising when alternative policy interventions and legal rules are compared. One problem concerns the aggregation of individuals' well-being into social preferences. The second problem is to determine the object that the legal system should maximize. We summarized the debate and the most significant contributions, pointing out different positions and schools of thought. Throughout the paper, we illustrate how these methodological problems are part of the broader discussion regarding the role of efficiency versus justice in the economic analysis of the law. The paper contributes to this debate by engaging in a quantitative analysis of different criteria for evaluating alternative public interventions and legal rules. We compare different SWF specifications by calculating for each of them the price of equity, that is the maximum fraction of total wealth that a society is willing to sacrifice in order to implement an equity-increasing redistribution.

Results show that considering the combined effects of aggregation methods and objects of maximization may lead to policy evaluation outcomes opposite to what moral philosophers and law and economics scholars predict. Indeed, aggregation methods conventionally considered equity-oriented may reject distributive policies that are instead implemented by efficiency-oriented aggregation methods. Similarly, choosing as the object of maximization utility instead of wealth does not automatically imply that the distributive policy or rule evaluated grants a more equitable allocation of the resources. Conversely, our results show that, by comparing the prices of equity associated to each SWF, it is possible to identify systematic relationships and to rank the criteria for the evaluation of social welfare with respect to their equity-efficiency orientation.

These findings support policy analysts and rule-makers having to choose between alternative rules and policy options. Indeed, they could identify the equity-efficiency trade-off implicit in the SWF chosen as welfare evaluation criterion. Moreover, the results have academic relevance for scholars that employ formal social welfare analysis. In fact, these scholars could easily verify to what degree the predictions derived in their studies are sensitive to the specific criterion used to guide the welfare analysis.

Acknowledgment: The idea of this paper was inspired by a conversation of the authors with Francesco Parisi. We are grateful to professor Parisi for his suggestions and comments on older versions of the paper. We also thank Emanuela Carbonara, Robert Cooter, Giuseppe Dari-Mattiacci, Michael Faure, Jonathan Klick, Louis Visscher, conference and workshop participants at the 2013 European Association of Law and Economics Annual Meeting, the 2013 German Association of Law and Economics Annual Meeting, the Institute of Law and Economics at Erasmus University Rotterdam and two anonymous referees for helpful comments. The usual disclaimer applies.

Appendix A

Table 2: Minimal level of K required to guarantee welfare improvement - homogeneous groups.

Utility Func. Form	Bentham	Nash	Rawls
Wealth	1	$\frac{w_A}{w_B - \epsilon}$	$\emptyset$
Polynomial	$(1/\epsilon)\{[w_A^\alpha + w_B^\alpha - (w_B - \epsilon)^\alpha]^{1/\alpha} - w_1\}$	$\frac{w_A}{w_B - \epsilon}$	$\emptyset$
Logarithmic	$\frac{w_A}{w_B - \epsilon}$	$(1/\epsilon)[e^{\frac{\ln w_A \ln w_B}{\ln w_B - \epsilon}} - w_A]$	$\emptyset$
Exponential	$(1/\epsilon)\{- (1/\alpha) \ln[e^{-\alpha w_A} + e^{-\alpha w_B} - e^{-\alpha(w_B - \epsilon)}] - w_A\}$	$\frac{1}{\epsilon} \left\{ \left(\frac{-1}{\alpha} \right) \ln \left[1 - \frac{(1 - e^{-\alpha w_A})(1 - e^{-\alpha w_B})}{1 - e^{-\alpha(w_B - \epsilon)}} \right] \right\} - w_A \}$	$\emptyset$

Table 3: Minimal level of K required to guarantee welfare improvement - non-homogeneous groups.

Utility	Bentham	Nash	Rawls
Wealth	$\frac{N-j}{j}$	$(1/\epsilon)[(\frac{w_A^j w_B^{N-j}}{(w_B - \epsilon)^{N-j}})^{(1/j)} - w_A]$	$\emptyset$
Poly.	$(1/\epsilon)\{[\frac{jw_A^\alpha + (N-j)w_B^\alpha - (N-j)(w_B - \epsilon)^\alpha}{j}]^{1/\alpha} - w_A\}$	$(1/\epsilon)[(\frac{w_A^j w_B^{N-j}}{(w_B - \epsilon)^{N-j}})^{(1/j)} - w_A]$	$\emptyset$
Log.	$(1/\epsilon)[(\frac{w_A^j w_B^{N-j}}{(w_B - \epsilon)^{N-j}})^{(1/j)} - w_A]$	$(1/\epsilon)\{e^{[\frac{(\ln w_A)^j (\ln w_B)^{N-j}}{(\ln w_B - \epsilon)^{N-j}}]^{1/j}} - w_A\}$	$\emptyset$
Exp.	$(1/\epsilon)\{- (1/\alpha) \ln [\frac{j e^{-\alpha w_A} + (N-j) e^{-\alpha w_B} - (N-j) e^{-\alpha(w_B - \epsilon)}}{j}] - w_1\}$	$(1/\epsilon)\{(\frac{-1}{\alpha}) \ln [1 - \frac{(1 - e^{-\alpha w_A})^j (1 - e^{-\alpha w_B})^{N-j}}{(1 - e^{-\alpha(w_B - \epsilon)})^{N-j}}]^{1/j} - w_A\}$	$\emptyset$

Appendix B

Appendix B.1 Prices of equity for homogeneous groups: Proofs of level of k reported in Table 2

For each case, we derive the minimum k such that $SW(W') \geq SW(W)$, where $W' = (w_1 + k\epsilon, w_2 - \epsilon)$ $W = (w_1, w_2)$.

Bentham Case:

Wealth: $u(w_i) = w_i$

$$(w_1 + k\epsilon) + (w_2 - \epsilon) \geq w_1 + w_2$$

$$\iff k \geq 1$$

Polynomial: $u(w_i) = w_i^\alpha$

$$(w_1 + k\epsilon)^\alpha + (w_2 - \epsilon)^\alpha \geq w_1^\alpha + w_2^\alpha$$

$$\iff k \geq \frac{1}{\epsilon} \{ [w_1^\alpha + w_2^\alpha - (w_2 - \epsilon)^\alpha]^{1/\alpha} - w_1 \}$$

Logarithmic: $u(w_i) = \ln(w_i)$

$$\ln(w_1 + k\epsilon) + \ln(w_2 - \epsilon) \geq \ln(w_1) + \ln(w_2)$$

$$\iff k \geq \frac{w_1}{w_2 - \epsilon}$$

Exponential: $u(w_i) = 1 - e^{-\alpha w_i}$

$$(1 - e^{-\alpha(w_1 + k\epsilon)}) + (1 - e^{-\alpha(w_2 - \epsilon)}) \geq (1 - e^{-\alpha w_1}) + (1 - e^{-\alpha w_2})$$

$$\iff k \geq \left(\frac{-1}{\alpha\epsilon} \right) \ln[1 + e^{-\alpha(w_2 - w_1)}(1 - e^{-\alpha\epsilon})]$$

Nash Case:

Wealth: $u(w_i) = w_i$

$$(w_1 + k\epsilon)(w_2 - \epsilon) \geq w_1 w_2$$

$$\iff k \geq \frac{w_1}{w_2 - \epsilon}$$

Polynomial: $u(w_i) = w_i^\alpha$

$$(w_1 + k\epsilon)^\alpha (w_2 - \epsilon)^\alpha \geq w_1^\alpha w_2^\alpha$$

$$\iff k \geq \frac{w_1}{w_2 - \epsilon}$$

Logarithmic: $u(w_i) = \ln(w_i)$

$$\begin{aligned} \ln(w_1 + k\epsilon) \ln(w_2 - \epsilon) &\geq \ln(w_1) \ln(w_2) \\ \iff k &\geq \frac{1}{\epsilon} \left\{ \exp\left[\frac{\ln(w_1) \ln(w_2)}{\ln(w_2 - \epsilon)}\right] - w_1 \right\} \end{aligned}$$

Exponential: $u(w_i) = 1 - e^{-\alpha w_i}$

$$\begin{aligned} (1 - e^{-\alpha(w_1 + k\epsilon)})(1 - e^{-\alpha(w_2 - \epsilon)}) &\geq (1 - e^{-\alpha w_1})(1 - e^{-\alpha w_2}) \\ \iff k &\geq \left(\frac{-1}{\alpha\epsilon}\right) \ln\left[\frac{1 - e^{-\alpha w_2} + e^{-\alpha(w_2 - w_1)}}{1 + e^{-\alpha(w_2 - \epsilon)}}\right] \end{aligned}$$

Appendix B.2 Prices of equity for non-homogeneous groups: Proofs of level of k reported in Table 3

Let $W = [jw_1, (N - j)w_2]$ be the status quo and $W' = [jw_1 + k\epsilon, (N - j)w_2 - \epsilon]$, $j \in \{0, N\}$, be the new allocation in which agent A increases her wealth by $k\epsilon$, and agent B has her wealth reduced by ϵ .¹⁵ Table 3 in Appendix A shows the price of equity for each SWF combination.

Bentham Case:

Wealth: $u(w_i) = w_i$

$$\begin{aligned} \sum_{i=1}^j (w_i + k\epsilon) + \sum_{i=j+1}^N (w_i - \epsilon) &\geq \sum_{i=1}^N w_i \\ \iff k &\geq \frac{(N - j)}{j} \end{aligned}$$

Polynomial: $u(w_i) = w_i^\alpha$

$$\begin{aligned} \sum_{i=1}^j (w_i + k\epsilon)^\alpha + \sum_{i=j+1}^N (w_i - \epsilon)^\alpha &\geq \sum_{i=1}^N (w_i)^\alpha \\ \iff k &\geq (1/\epsilon) \left\{ \left[\frac{jw_A^\alpha + (N - j)w_B^\alpha - (N - j)(w_B - \epsilon)^\alpha}{j} \right]^{1/\alpha} - w_A \right\} \end{aligned}$$

¹⁵ In the interpretation where we preserve the anonymity principle but we consider groups of interest of different sizes, let $W = (w_1, w_2, \dots, w_j, w_{j+1}, \dots, w_N)$ be the status quo and $W' = (w_1 + k\epsilon, w_2 + k\epsilon, \dots, w_j + k\epsilon, w_{j+1} - \epsilon, \dots, w_N - \epsilon)$ be the new allocation in which individuals belonging to group A increase their wealth by $k\epsilon$ each, and group B individuals have their wealth reduced by ϵ .

Logarithmic: $u(w_i) = \ln(w_i)$

$$\sum_{i=1}^j \ln(w_i + k\epsilon) + \sum_{i=j+1}^N \ln(w_i - \epsilon) \geq \sum_{i=1}^N \ln(w_i)$$

$$\iff k \geq \frac{1}{\epsilon} \left[\left(\frac{w_A^j w_B^{(N-j)}}{(w_B - \epsilon)^{(N-j)}} \right)^{1/j} - w_A \right]$$

Exponential: $u(w_i) = 1 - e^{-\alpha w_i}$

$$\sum_{i=1}^j (1 - e^{-\alpha(w_i + k\epsilon)}) + \sum_{i=j+1}^N (1 - e^{-\alpha(w_i - \epsilon)}) \geq \sum_{i=1}^N (1 - e^{-\alpha(w_i)})$$

$$\iff k \geq \left(\frac{-1}{\alpha\epsilon} \right) \ln \left[1 + \frac{(N-j)}{j} e^{-\alpha(w_B - w_A)} (1 - e^{\alpha\epsilon}) \right]$$

Nash Case:

Wealth: $u(w_i) = w_i$

$$\prod_{i=1}^j (w_i + k\epsilon) \prod_{i=j+1}^N (w_i - \epsilon) \geq \prod_{i=1}^N w_i$$

$$\iff k \geq \frac{1}{\epsilon} \left[\left(\frac{w_A^j w_B^{N-j}}{(w_B - \epsilon)^{N-j}} \right)^{(1/j)} - w_A \right]$$

Polynomial: $u(w_i) = w_i^\alpha$

$$\prod_{i=1}^j (w_i + k\epsilon)^\alpha \prod_{i=j+1}^N (w_i - \epsilon)^\alpha \geq \prod_{i=1}^N w_i^\alpha$$

$$\iff k \geq \frac{1}{\epsilon} \left[\left(\frac{w_A^j w_B^{N-j}}{(w_B - \epsilon)^{N-j}} \right)^{(1/j)} - w_A \right]$$

Logarithmic: $u(w_i) = \ln(w_i)$

$$\prod_{i=1}^j \ln(w_i + k\epsilon) \prod_{i=j+1}^N \ln(w_i - \epsilon) \geq \prod_{i=1}^N \ln(w_i)$$

$$\iff k \geq \frac{1}{\epsilon} \left\{ \exp \left[\frac{(\ln w_A)^j (\ln w_B)^{N-j}}{(\ln(w_B - \epsilon))^{N-j}} \right]^{1/j} - w_A \right\}$$

Exponential: $u(w_i) = 1 - e^{-\alpha w_i}$

$$\prod_{i=1}^j (1 - e^{-\alpha(w_i + k\epsilon)}) \prod_{i=j+1}^N (1 - e^{-\alpha(w_i - \epsilon)}) \geq \prod_{i=1}^N (1 - e^{-\alpha w_i})$$

$$\iff k \geq \frac{1}{\epsilon} \left\{ \left(\frac{-1}{\alpha} \right) \ln \left[1 - \frac{(1 - e^{-\alpha w_A})^j (1 - e^{-\alpha w_B})^{N-j}}{(1 - e^{-\alpha(w_B - \epsilon)})^{N-j}} \right]^{1/j} - w_A \right\}$$

(4)

Appendix B.3 Efficiency-improving transfers

The proofs of results for E-I transfers summarized in the table above follow analogously to those of the R-H case. There are only two differences, respectively that $w_2 - w_1 < 0$ and that ϵ is now bounded by w_2 (an individual cannot be left with negative wealth) instead of $w_2 - w_1$.

Table 4: Generality of results under EI transfers.

Maximand/SWF Form	Bentham	Nash	Rawls
Wealth	$\emptyset$	$\{B-W; B-P; B-L; N-P\}$	EI-R
Polynomial	$\{B-W\}$	$\{B-W; B-P; B-L; N-W\}$	EI-R
Logarithmic	$\{B-W; B-P\}$	$\{B-W; B-P; B-L; N-W; N-P\}$	EI-R
Exponential	$\{B-W\}$	$\{B-W; B-E\}$	EI-R

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