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### Complete subvarieties of moduli spaces of algebraic curves

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## CHAPTER 4

### A complete surface in $M_6$ in characteristic $> 2$

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**Abstract.** *In all characteristics  $p > 2$  we construct a complete surface in the moduli space of smooth genus 6 curves. The surface is contained in the locus of curves with automorphisms.*

We consider the following question: ‘What is the maximal number of essential parameters on which a complete family of smooth curves of genus  $g$  depends?’ or equivalently, ‘What is the maximal dimension of a complete subvariety of  $M_g$ , the moduli space of smooth curves of genus  $g$ ?’ In [14] Diaz provided an upper bound for the dimension of such a subvariety: for  $g \geq 2$  this dimension is at most  $g - 2$ . The moduli space  $M_g$  itself is irreducible, quasi-projective of dimension  $3g - 3$  ( $g \geq 2$ ). Diaz proved his result in characteristic 0, but his bound also holds in characteristic  $> 0$  (see [35]).

In order to see how good Diaz’s bound is one has to construct complete subvarieties of  $M_g$ . This turns out to be a difficult problem, in any characteristic. Only in genus  $\leq 3$  Diaz’s bound is known to be sharp, since it is known that  $M_g$  contains complete curves if  $g$  is at least 3 (see [16]). In higher genera almost nothing is known. The best result we know is a construction of complete subvarieties of arbitrary dimension  $d \geq 1$  in  $M_g$  with  $g \geq 2^{d+1}$ . This construction gives a complete surface in  $M_8$ . For  $g = 4, 5, 6$  and  $7$  we still do not know whether a complete surface in  $M_g$  exists.

#### 4.1 The construction

Starting from a complete curve in  $M_3$ , we construct a complete surface in  $M_6$ . However, this construction only works in characteristic  $\neq 0, 2$ . Our result is:

**Theorem 4.1** *In any characteristic  $p > 2$  the moduli space  $M_6$  of smooth genus 6 curves contains a complete surface.*

To construct in characteristic 0 a complete surface in  $M_6$  seems mor difficult. This is more or less similar to the fact that the moduli space  $A_g$  of principally polarized abelian varieties of dimension  $g$  contains in characteristic  $p > 0$  complete subvarieties of rather high dimension [45]. The corresponding situation in characteristic 0 is completely unknown.<sup>†</sup>

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<sup>†</sup> However, see Keel-Sadun [32].